



EL-MOASSER

By a group of supervisors

THE MAIN BOOK

2nd
PREP.
FIRST TERM

Maths



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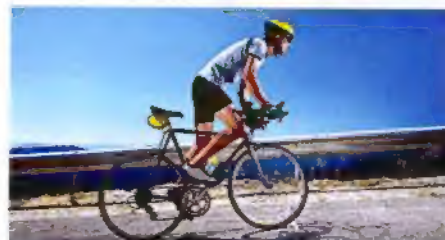
First Algebra and Statistics

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Unit **2** Relation between Two Variables.



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First Algebra and Statistics

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Revision

The sets of numbers

You had studied before the following sets of numbers :

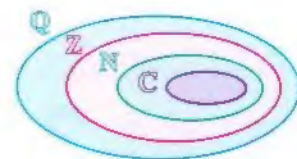
- The set of counting numbers: $\mathbb{C} = \{1, 2, 3, 4, \dots\}$
- The set of natural numbers : $\mathbb{N} = \{0, 1, 2, 3, \dots\} = \mathbb{C} \cup \{0\}$
- The set of integers : $\mathbb{Z} = \{\dots, 3, 2, 1, 0, -1, -2, -3, \dots\}$
- The set of positive integers : $\mathbb{Z}_+ = \{1, 2, 3, \dots\} = \mathbb{C}$
- The set of negative integers : $\mathbb{Z}_- = \{-1, -2, -3, \dots\}$
- The set of rational numbers : $\mathbb{Q} = \left\{ \frac{a}{b} : a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0 \right\}$

Examples of rational numbers : $\frac{2}{3}$, $-\frac{1}{2}$, zero, 3, -5, 0.2, 25%, ...

Notice that :

$$\mathbb{C} \subset \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$$

The opposite figure shows that.



Writing a rational number in its simplest form

To put the rational number $\frac{a}{b}$ in its simplest form, divide each of its terms by the highest common factor (H.C.F.) between them if it exists.

For example:

To put the rational number $\frac{8}{12}$ in its simplest form.

Divide each of its terms 8 and 12 by the highest common factor (H.C.F.) between them which is 4 as follows:

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

, then $\frac{2}{3}$ is the simplest form of the rational number $\frac{8}{12}$

The absolute value of a rational number

We denote the absolute value of the number a by $|a|$ where $|a| \geq 0$

For example:

$$|4| = 4$$

$$|-4| = 4$$

$$|0| = 0$$

! Remark

If $|X| = a$, then $X = a$ or $X = -a$

For example:

If $|X| = 5$, then $X = 5$ or $X = -5$

Indices

If a and b are two rational numbers, m and n are two integers then :

1	$a^{-n} = \frac{1}{a^n}$	$5^{-1} = \frac{1}{5}$	For example :
2	$a^m \times a^n = a^{m+n}$	$2^3 \times 2^2 = 2^{3+2} = 2^5 = 32$	
3	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{3^2}{3^{-1}} = 3^{2-(-1)} = 3^{2+1} = 3^3 = 27$	
4	$(ab)^n = a^n b^n$	$(5 \times 10)^2 = 5^2 \times (10)^2 = 25 \times 100 = 2500$	
5	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$	
6	$(a^m)^n = a^{mn}$	$(2^2)^3 = 2^{2 \times 3} = 2^6 = 64$	

The standard form of the rational number

The number is written in its standard form (scientific notation) if it is in the form :

$$a \times 10^n \text{ where } n \in \mathbb{Z}, 1 \leq |a| < 10$$

For example:

- The standard form of the number 32.4×10^5 is 3.24×10^6
- The standard form of the number 0.000423 is 4.23×10^{-4}

The perfect square rational number

It is the rational number that can be written in the form of a square of a rational number
i.e. In the form $(\text{rational number})^2$

For example:

The number 9 is a perfect square rational number because it can be written in the form

$$(3)^2 \text{ or } (-3)^2$$

Examples of perfect square rational numbers : zero, 1, 4, $\frac{9}{25}$, $\frac{16}{49}$, 2.25, ...

REVISION

The square root of the perfect square rational number

The square root of the perfect square rational number (a) is the rational number whose square equals (a)

For example:

- 25 has two square roots which are 5 and -5

Because : $(5)^2 = 25$, $(-5)^2 = 25$

- $\frac{16}{49}$ has two square roots which are $\frac{4}{7}$ and $-\frac{4}{7}$

Notice that :

The two square roots of the rational number , each of them is the additive inverse of the other and their sum = zero.

! Remark

The symbol $\sqrt{\quad}$ means the positive square root of a number , then we find that :

- $\sqrt{16} = 4$, $-\sqrt{16} = -4$, $\pm\sqrt{16} = \pm 4$ • $\sqrt{0} = 0$
- $\sqrt{\text{negative number}}$ is meaningless
- $\sqrt{a^2} = |a|$

For example:

$$\sqrt{3^2} = |3| = 3 \quad , \quad \sqrt{(-6)^2} = |-6| = 6 \quad , \quad \sqrt{\left(-\frac{2}{5}\right)^2} = \left|-\frac{2}{5}\right| = \frac{2}{5}$$

- Sometimes , you need to factorize a number to its prime factors to facilitate finding its square root , then you take a factor from each two equal factors , then the product of these taken factors is the square root of this number.

For example:

$$\begin{aligned} \therefore 441 &= \underbrace{3 \times 3} \times \underbrace{7 \times 7} \\ \therefore \sqrt{441} &= 3 \times 7 \\ &= 21 \end{aligned}$$

$$\begin{array}{r|l} 441 & 3 \\ 147 & 3 \\ 49 & 7 \\ 7 & 7 \\ 1 & \end{array} \quad \begin{array}{l} \textcircled{3} \\ \textcircled{7} \end{array}$$

You can use your calculator to check your answer.

Solving equations

Example Find the solution set of each of the following equations :

1 $x + 2 = |-2|$, $x \in \mathbb{N}$

3 $x^2 - 4 = 5$, $x \in \mathbb{Q}$

2 $2x - 5 = 13$, $x \in \mathbb{Q}$

4 $x^2 + 25 = 0$, $x \in \mathbb{Q}$

Solution

1 $\therefore x + 2 = 2$

$\therefore x = 0$

2 $\therefore 2x - 5 = 13$

$\therefore 2x = 18$

$\therefore x = 9$

3 $\therefore x^2 - 4 = 5$

$\therefore x^2 = 5 + 4$

$\therefore x^2 = 9$

$\therefore x = \pm\sqrt{9}$

$\therefore x = \pm 3$

$\therefore \text{The S.S.} = \{3, -3\}$

4 $\therefore x^2 + 25 = 0$

$\therefore x = \pm\sqrt{-25} \notin \mathbb{Q}$

(There is no square root for a negative rational number in \mathbb{Q})

$\therefore \text{The S.S.} = \emptyset$

$\therefore x = 2 - 2$

$\therefore \text{The S.S.} = \{0\}$

$\therefore 2x = 13 + 5$

$\therefore x = \frac{18}{2}$

$\therefore \text{The S.S.} = \{9\}$

Notice that :

We used the concept of the square root to find the value of x according to the following remark :

If $x^2 = a$, then $x = \pm\sqrt{a}$

$\therefore x^2 = -25$



UNIT

1

Real Numbers

Lessons of the unit :

1. The cube root of a rational number.
2. The set of irrational numbers \mathbb{Q}
3. The set of real numbers \mathbb{R} – Ordering numbers in \mathbb{R}
4. Intervals.
5. Operations on the real numbers.
6. Operations on the square roots.
7. The two conjugate numbers.
8. Operations on the cube roots.
9. Applications on the real numbers.
10. Solving equations and inequalities of the first degree in one variable in \mathbb{R}

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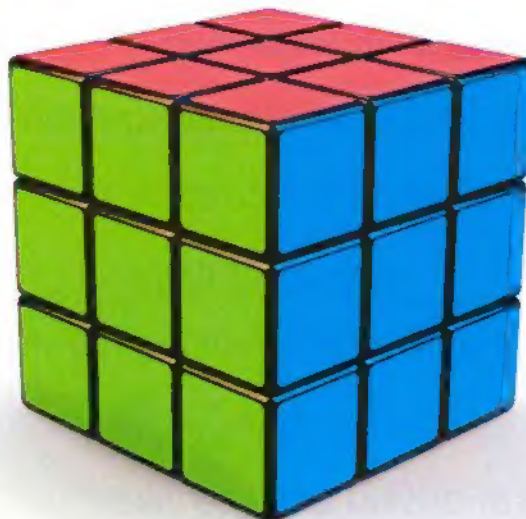


Unit Objectives :

By the end of this unit, student should be able to :

- recognize the cube root of a rational number.
 - find the cube root of a rational number.
 - recognize the set of irrational numbers.
 - represent the irrational number on the number line.
 - recognize the set of real numbers.
 - perform the operations on the intervals.
 - perform the arithmetic operations on the real numbers.
 - solve equations and inequalities of the first degree in one variable in \mathbb{R}
 - perform the operations on the square roots and the cube roots.
 - recognize two conjugate numbers.
 - apply what he studied in the real numbers to find the volumes and the areas of some of the solids.
-

The cube root of a rational number



- The product of a number by itself three times is the cube of that number.

For example: **64** is the **cube** of **4** because $4 \times 4 \times 4 = 64$



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- The reverse of finding the cube is finding the cube root.
- Finding the cube root of a number is finding another number if multiplied by itself three times, we get the first number.

For example: **4** is the **cube root** of **64** because $64 = 4 \times 4 \times 4$

Definition

The cube root of the number “a” is the number whose cube equals a

- The symbol $\sqrt[3]{}$ (read as “the cube root of”) is used to designate the cube root.

For example: $\sqrt[3]{64}$ designates the cube root of 64

- The cube root of a positive number is positive and the cube root of a negative number is negative.

For example: $\sqrt[3]{64} = 4$ and $\sqrt[3]{-64} = -4$

i.e. The cube root of any number has the same sign of this number.

Finding the cube root of a rational number (representing a perfect cube)

- The perfect cube rational number is the number which can be written as a cube of a rational number i.e. (rational number)³ as the numbers : $8 = 2^3$, $27 = (3)^3$
- The cube root of a perfect cube rational number is also a rational number.

For example: $\sqrt[3]{8} = 2$, $\sqrt[3]{-27} = -3$

- If a number is not a perfect cube , then you indicate its cube root by using the cube root symbol.

For example: The cube root of 4 is $\sqrt[3]{4}$ because 4 is not a perfect cube.

• $\sqrt[3]{a^3} = a$ **For example:** $\sqrt[3]{5^3} = 5$, $\sqrt[3]{(-5)^3} = -5$

• $\sqrt[3]{a^n} = a^{\frac{n}{3}}$ where $n \in \mathbb{Z}$ **For example:** $\sqrt[3]{a^6} = a^{\frac{6}{3}} = a^2$

- You can use factorization to find the cube root of a perfect cube number , as in the following example.

Example 1 Find each of the following :

1 $\sqrt[3]{216}$

2 $\sqrt[3]{-\frac{8}{125}}$

3 $\sqrt[3]{0.064}$



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Solution

1 $\sqrt[3]{216} = 2 \times 3 = 6$

2 $\sqrt[3]{-\frac{8}{125}} = -\frac{2}{5}$

3 $\sqrt[3]{0.064} = \sqrt[3]{\frac{64}{1000}} = \frac{2 \times 2}{2 \times 5} = \frac{4}{10} = 0.4$

$$\begin{array}{r|l} 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array} \quad \textcircled{2}$$

$$\begin{array}{r|l} 125 & 5 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array} \quad \textcircled{5}$$

$$\begin{array}{r|l} 64 & 2 \\ 16 & 2 \\ 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array} \quad \textcircled{2}$$

$$\begin{array}{r|l} 1000 & 2 \\ 250 & 2 \\ 125 & 5 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array} \quad \textcircled{2}$$

Example 2 Choose the correct answer from those given :

1 $\sqrt[3]{\dots\dots\dots} = -5$

- (a) 125 (b) 25 (c) -25 (d) -125

2 $\sqrt{4} - \sqrt[3]{-8} = \dots\dots\dots$

- (a) -4 (b) -2 (c) 4 (d) 8

3 $\sqrt{(-7)^2} - \sqrt[3]{(-7)^3} = \dots\dots\dots$

- (a) -14 (b) zero (c) 7 (d) 14

4 If $\sqrt[3]{x} = \sqrt{4}$, then $x = \dots\dots\dots$

- (a) 2 (b) 4 (c) 8 (d) ± 8

5 $\sqrt{x^4} = \sqrt[3]{\dots\dots\dots}$

- (a) x (b) x^2 (c) x^4 (d) x^6

Solution

1 (d) The reason : $(-5)^3 = -125$

2 (c) The reason : $\sqrt{4} - \sqrt[3]{-8} = 2 - (-2) = 2 + 2 = 4$

3 (d) The reason : $\sqrt{(-7)^2} - \sqrt[3]{(-7)^3} = 7 - (-7) = 7 + 7 = 14$

4 (c) The reason : $\because \sqrt[3]{x} = \sqrt{4} \quad \therefore \sqrt[3]{x} = 2 \quad \therefore x = 2^3 = 8$

5 (d) The reason : $\because \sqrt{x^4} = x^2 \quad \therefore (x^2)^3 = x^6 \quad \therefore \sqrt{x^4} = \sqrt[3]{x^6}$

TRY 1

Complete the following :

1 $\sqrt[3]{512} = \dots\dots\dots$

2 $\sqrt{64} - \sqrt[3]{64} = \dots\dots\dots$

3 $\sqrt[3]{27} = \sqrt{\dots\dots\dots}$

4 If $\sqrt[3]{x} = 6$, then $x = \dots\dots\dots$

Final answers
of try by yourself
questions
are at the end of each
lesson to check
your answer.

Solving equations in \mathbb{Q}

- If “a” is a perfect cube number ,

then the equation : $x^3 = a$ has a unique solution in \mathbb{Q} , which is $\sqrt[3]{a}$

For example :

- The equation : $x^3 = 8$ has a unique solution in \mathbb{Q} which is $\sqrt[3]{8} = 2$
- The equation : $x^3 = 9$ has no solution in \mathbb{Q} because 9 is not a perfect cube.

Example 3 Solve each of the following equations in \mathbb{Q} :

1 $40x^3 - 1 = -136$

2 $(y - 2)^3 = -343$

Solution 1 $\therefore 40x^3 - 1 = -136$

$\therefore 40x^3 = -136 + 1$

$\therefore 40x^3 = -135$

$\therefore x^3 = -\frac{135}{40}$

$\therefore x^3 = -\frac{27}{8}$

$\therefore x = \sqrt[3]{-\frac{27}{8}}$

$\therefore x = -\frac{3}{2}$

2 $\therefore (y - 2)^3 = -343$

Taking the cube root of each side :

$\therefore \sqrt[3]{(y - 2)^3} = \sqrt[3]{-343}$

$\therefore y - 2 = -7$

$\therefore y = -7 + 2$

$\therefore y = -5$

2

Find in \mathbb{Q} the S.S. of each of the following equations :

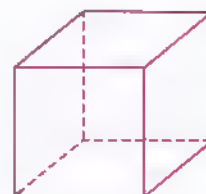
1 $27x^3 - 2 = 62$

2 $(5x - 3)^3 - 2 = 6$

Applications

**Remember that**

- The volume of a cube = the edge length \times itself \times itself
- The area of one face of a cube = the edge length \times itself
- The lateral area of a cube = the area of one face $\times 4$
- The total area of a cube = the area of one face $\times 6$



For example: If the volume of a cube is 8 cm^3 , then :

- The edge length $= \sqrt[3]{8} = 2 \text{ cm}$.
- The area of one face $= 2 \times 2 = 4 \text{ cm}^2$
- The lateral area $= 4 \times 4 = 16 \text{ cm}^2$
- The total area $= 4 \times 6 = 24 \text{ cm}^2$

Example 4

Find each of the following :

- 1 The length of the inner edge of a vessel in the shape of a cube if its capacity = 8 litres.
- 2 The radius length of a sphere of volume $\frac{36}{125} \pi \text{ cm}^3$
Knowing that : The volume of the sphere $= \frac{4}{3} \pi r^3$
where r is the radius length of the sphere, π is the ratio between the circumference of the circle and its diameter length.
- 3 The diameter length of a sphere of volume equals 38808 cm^3 ($\pi \approx \frac{22}{7}$)

Solution

- 1 \therefore The capacity of the vessel = 8 litres = $8 \times 1000 = 8000 \text{ cm}^3$

$$\therefore \text{The inner edge length} = \sqrt[3]{8000} \\ = 20 \text{ cm.}$$

- 2 \therefore The volume of the sphere $= \frac{4}{3} \pi r^3$

$$\therefore \frac{4}{3} \pi r^3 = \frac{36}{125} \pi \quad \therefore \frac{4}{3} r^3 = \frac{36}{125}$$

$$\therefore r^3 = \frac{36}{125} \times \frac{3}{4} \quad \therefore r^3 = \frac{27}{125}$$

$$\therefore r = \sqrt[3]{\frac{27}{125}} = \frac{3}{5} \text{ cm.} \quad \therefore \text{The radius length of the sphere} = \frac{3}{5} \text{ cm.}$$

**Remember that**

$$1 \text{ litre} = 1000 \text{ cm}^3$$

3 \therefore The volume of the sphere = $\frac{4}{3} \pi r^3$

$\therefore \frac{4}{3} \pi r^3 = 38808$

$\therefore \frac{4}{3} \times \frac{22}{7} r^3 = 38808$

$\therefore \frac{88}{21} r^3 = 38808$

$\therefore r^3 = 38808 \times \frac{21}{88}$

$\therefore r^3 = 9261$

$\therefore r = \sqrt[3]{9261}$

$\therefore r = 3 \times 7 = 21 \text{ cm.}$

\therefore The diameter length = $21 \times 2 = 42 \text{ cm.}$

Notice that : You can use the calculator to find $\sqrt[3]{9261}$ directly.

$$\begin{array}{r} 9261 \\ 3087 \\ 1029 \\ 343 \\ 49 \\ 7 \\ 1 \end{array} \left. \begin{array}{l} \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \end{array} \right\} \begin{array}{l} 3 \\ 7 \\ 7 \end{array}$$

TRY YOURSELF 3

- Find the length of the inner edge of a vessel in the shape of a cube with capacity equals 27 litres.
- Find the length of the diameter of a sphere of volume $36 \pi \text{ cm}^3$
(Knowing that : the volume of the sphere = $\frac{4}{3} \pi r^3$)

At the end of each lesson, you will find the final answers of try by yourself questions in the same form.

4 216

3 9

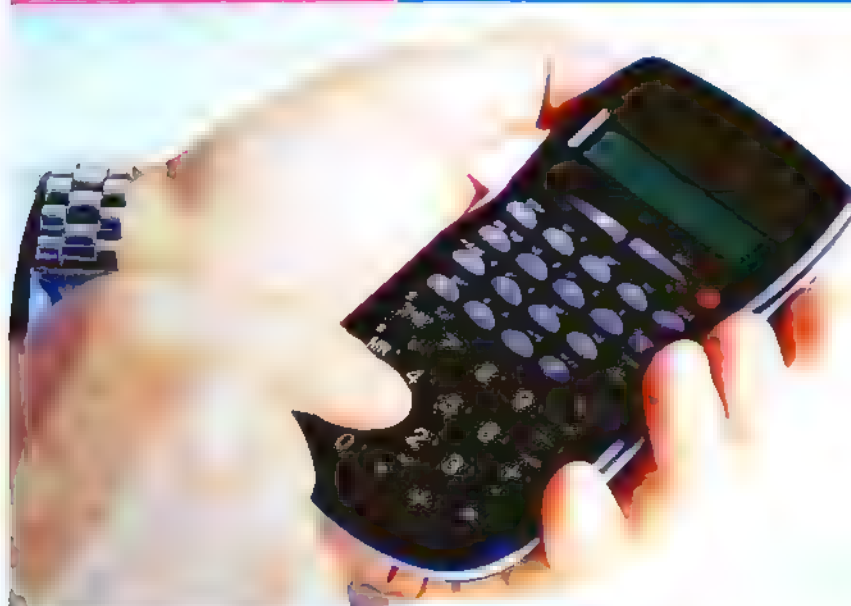
2 6 cm.
2 {1}
2 4

3 1 30 cm.
2 1 {3/4}
1 1 8

Answers of try by yourself

2

The set of irrational numbers



Prelude

* You studied before that a rational number is the number that can be written as $\frac{a}{b}$ where a and b are integers and $b \neq 0$, and the set of rational numbers is denoted by \mathbb{Q}

* Based on the previous, you know that :

<p>All integers are rational numbers</p> <p>For example: 3 is a rational number because it can be expressed as $\frac{3}{1}$ or $\frac{6}{2}$ or ...</p>	<p>All decimals are rational numbers</p> <p>For example: 2.5 is a rational number because it can be expressed as $\frac{25}{10}$ or $\frac{5}{2}$ or ...</p>	<p>All percentages are rational numbers</p> <p>For example: 15 % is a rational number because it can be expressed as $\frac{15}{100}$ or $\frac{150}{1000}$ or ...</p>
<p>The square root of a perfect square rational number is a rational number</p> <p>For example: $\sqrt{36}$, $\sqrt{\frac{4}{25}}$, $\sqrt{0.09}$ are all rational numbers where $\sqrt{36} = 6$, $\sqrt{\frac{4}{25}} = \frac{2}{5}$ $\sqrt{0.09} = \sqrt{\frac{9}{100}} = \frac{3}{10}$</p>	<p>The cube root of a perfect cube rational number is a rational number</p> <p>For example: $\sqrt[3]{8}$, $\sqrt[3]{-64}$, $\sqrt[3]{\frac{27}{1000}}$ are all rational numbers where $\sqrt[3]{8} = 2$, $\sqrt[3]{-64} = -4$ $\sqrt[3]{\frac{27}{1000}} = \frac{3}{10}$</p>	

Irrational numbers

<p>The square root of a rational number which is not a perfect square is not a rational number</p> <p>For example:</p> <p>$\sqrt{2} \notin \mathbb{Q}$ because there is no rational number whose square is 2 , so $\sqrt{2}$ cannot be written as $\frac{a}{b}$ where a and b are integers , $b \neq 0$</p>	<p>The cube root of a rational number which is not a perfect cube is not a rational number</p> <p>For example:</p> <p>$\sqrt[3]{4} \notin \mathbb{Q}$ because there is no rational number whose cube is 4 , so $\sqrt[3]{4}$ cannot be written as $\frac{a}{b}$ where a and b are integers , $b \neq 0$</p>
<p>π is not a rational number</p> <p>(However $\frac{22}{7}$, 3.14 and 3.142 are rational numbers , each of them represents an approximating value of π)</p>	<p>Other examples of numbers not rational</p> <p>$\sqrt{5} + 1$, $1 - \sqrt[3]{7}$, $2\sqrt{7}$, $-\frac{\sqrt[3]{9}}{5}$</p>
<p>The set of irrational numbers is denoted by \mathbb{Q}</p> <p>Notice that : \mathbb{Q} and \mathbb{Q} are disjoint sets. i.e. $\mathbb{Q} \cap \mathbb{Q} = \emptyset$</p>	

! Remarks

$$\bullet (\sqrt{a})^2 = \sqrt{a} \times \sqrt{a} = a, \text{ where } a \geq 0$$

For example: $(\sqrt{2})^2 = \sqrt{2} \times \sqrt{2} = 2$

$$\bullet (\sqrt[3]{a})^3 = \sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a, \text{ where } a \in \mathbb{Q}$$

For example: $(\sqrt[3]{-7})^3 = \sqrt[3]{-7} \times \sqrt[3]{-7} \times \sqrt[3]{-7} = -7$

Example 1 Show which of the following numbers belongs to \mathbb{Q} and which of them belongs to \mathbb{Q} :

1 $\sqrt{0.49}$

2 $\sqrt[3]{-0.064}$

3 $\sqrt{\frac{25}{49}}$

4 $\sqrt[3]{\frac{25}{49}}$

5 $\sqrt{25 + \sqrt[3]{16}}$

Solution 1 $\because \sqrt{0.49} = 0.7 = \frac{7}{10}$

$\therefore \sqrt{0.49} \in \mathbb{Q}$

2 $\because \sqrt[3]{-0.064} = -0.4 = -\frac{4}{10}$

$\therefore \sqrt[3]{-0.064} \in \mathbb{Q}$



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$$3 \quad \therefore \sqrt{\frac{25}{49}} = \sqrt{\left(\frac{5}{7}\right)^2} = \frac{5}{7}$$

$$\therefore \sqrt{\frac{25}{49}} \in \mathbb{Q}$$

$$4 \quad \therefore \sqrt[3]{\frac{25}{49}} \notin \mathbb{Q} \text{ because there is no rational number whose cube is } \frac{25}{49}$$

$$\therefore \sqrt[3]{\frac{25}{49}} \notin \mathbb{Q}$$

$$5 \quad \therefore \sqrt{25} + \sqrt[3]{16} = 5 + \sqrt[3]{16} \quad \therefore \text{There is no rational number whose cube is 16}$$

$$\therefore \sqrt[3]{16} \notin \mathbb{Q} \quad \therefore (5 + \sqrt[3]{16}) \notin \mathbb{Q} \quad \therefore (\sqrt{25} + \sqrt[3]{16}) \notin \mathbb{Q}$$

TRY
by yourself

Complete using one of the symbols \mathbb{Q} or \mathbb{Q}^c :

$$1 \quad 3 \in \dots$$

$$2 \quad \sqrt{3} \in \dots$$

$$3 \quad 9 \in \dots$$

$$4 \quad \sqrt{9} \in \dots$$

$$5 \quad -8 \in \dots$$

$$6 \quad \sqrt[3]{-8} \in \dots$$

$$7 \quad 5 \in \dots$$

$$8 \quad \sqrt[3]{5} \in \dots$$

$$9 \quad \sqrt[3]{-9} \in \dots$$

Solving equations in \mathbb{Q}

Example 2 If $x \in \mathbb{Q}$, find the S.S. of each of the following equations :

$$1 \quad x^2 = 5$$

$$2 \quad x^3 = 7$$

$$3 \quad \frac{2}{5} x^2 = \frac{4}{25}$$

$$4 \quad 64x^3 - 2 = -29$$

$$5 \quad (x^2 - 10)(x^3 - 4) = 0$$

Solution 1 $\therefore x^2 = 5$

$$\therefore x = \pm\sqrt{5}$$

$$\therefore \text{The S.S.} = \{\sqrt{5}, -\sqrt{5}\}$$

$$2 \quad \therefore x^3 = 7$$

$$\therefore x = \sqrt[3]{7}$$

$$\therefore \text{The S.S.} = \{\sqrt[3]{7}\}$$

$$3 \quad \therefore \frac{2}{5} x^2 = \frac{4}{25}$$

$$\therefore x^2 = \frac{4}{25} \times \frac{5}{2}$$

$$\therefore x^2 = \frac{2}{5}$$

$$\therefore x = \pm \sqrt{\frac{2}{5}}$$

$$\therefore \text{The S.S.} = \left\{ \sqrt{\frac{2}{5}}, -\sqrt{\frac{2}{5}} \right\}$$

$$4 \quad \therefore 64x^3 - 2 = -29$$

$$\therefore 64x^3 = -29 + 2$$

$$\therefore 64x^3 = -27$$

$$\therefore x^3 = -\frac{27}{64}$$

$$\therefore x = \sqrt[3]{-\frac{27}{64}}$$

$$\therefore x = -\frac{3}{4}$$

$$\therefore -\frac{3}{4} \in \mathbb{Q}$$

$$\therefore -\frac{3}{4} \notin \mathbb{Q}^c$$

$$\therefore \text{The S.S.} = \emptyset$$

$$\begin{aligned}
 5 \quad & \because (x^2 - 10)(x^3 - 4) = 0 \\
 & \therefore x^2 - 10 = 0 \quad \text{or} \quad x^3 - 4 = 0 \\
 & \therefore x^2 = 10 \qquad \qquad \qquad \therefore x^3 = 4 \\
 & \therefore x = \pm\sqrt{10} \qquad \qquad \qquad \therefore x = \sqrt[3]{4} \\
 & \therefore \text{The S.S.} = \{ \sqrt{10}, -\sqrt{10}, \sqrt[3]{4} \}
 \end{aligned}$$

Remember that

For any two numbers x, y :
 If $xy = \text{zero}$, then
 $x = \text{zero}$ or $y = \text{zero}$

TRY YOURSELF 2

Find the S.S. in \mathbb{Q} for each of the following :

① $2x^3 - 7 = 3$

② $\frac{1}{2}x^2 - 5 = 3$

Finding an approximated value of an irrational number

If you use the calculator to find the values of some irrational numbers , you will find that :

$$\sqrt{2} \approx 1.4142 \dots, \quad \sqrt{3} \approx 1.73205 \dots, \quad \sqrt{5} \approx 2.236 \dots$$

i.e. The irrational number is represented by an infinite decimal and not recurring.

And you can deduce an approximated value of the irrational number without using the calculator.

For example:

You can deduce an approximated value of the irrational number $\sqrt{5}$ as follows :

$\because 4 < 5 < 9$ (notice that we chose 4 and 9 because each of them is a perfect square , and the number 5 includes between them) and by taking the square root for all the terms.

$$\therefore \sqrt{4} < \sqrt{5} < \sqrt{9} \qquad \therefore 2 < \sqrt{5} < 3$$

i.e. $\sqrt{5} = 2 + \text{decimal less than } 1$

To find an approximated value of the number $\sqrt{5}$, you search for the values of the following numbers : $(2.1)^2$, $(2.2)^2$ and $(2.3)^2$

, then you find that $(2.1)^2 = 4.41$, $(2.2)^2 = 4.84$, $(2.3)^2 = 5.29$

$$\therefore 4.84 < 5 < 5.29 \qquad \therefore \sqrt{4.84} < \sqrt{5} < \sqrt{5.29} \qquad \therefore 2.2 < \sqrt{5} < 2.3$$

We can say that 2.2 and 2.3 are approximated values of $\sqrt{5}$ and thus we can get more accurate values for the irrational number $\sqrt{5}$ and we can use the calculator to check the approximated value of the number $\sqrt{5}$

! Remark

Each irrational number lies between two rational numbers.

Example 3 Prove that :

- 1 $\sqrt{3}$ lies between 1.7 and 1.8 2 $\sqrt[3]{12}$ lies between 2.2 and 2.3

Solution 1 $\therefore (\sqrt{3})^2 = \sqrt{3} \times \sqrt{3} = 3$, $(1.7)^2 = 2.89$, $(1.8)^2 = 3.24$
 $\therefore 2.89 < 3 < 3.24$ $\therefore \sqrt{2.89} < \sqrt{3} < \sqrt{3.24}$ $\therefore 1.7 < \sqrt{3} < 1.8$
i.e. $\sqrt{3}$ lies between 1.7 and 1.8

You can solve the problem using the calculator as follows

$$\therefore \sqrt{3} \approx 1.73, \therefore 1.7 < 1.73 < 1.8$$

$$\therefore 1.7 < \sqrt{3} < 1.8 \quad \therefore \sqrt{3} \text{ lies between 1.7 and 1.8}$$

$$2 \therefore (\sqrt[3]{12})^3 = \sqrt[3]{12} \times \sqrt[3]{12} \times \sqrt[3]{12} = 12, (2.2)^3 = 10.648, (2.3)^3 = 12.167$$

$$\therefore 10.648 < 12 < 12.167 \quad \therefore \sqrt[3]{10.648} < \sqrt[3]{12} < \sqrt[3]{12.167}$$

$$\therefore 2.2 < \sqrt[3]{12} < 2.3$$

i.e. $\sqrt[3]{12}$ lies between 2.2 and 2.3

You can solve the problem using the calculator as follows

$$\sqrt[3]{12} \approx 2.289 \quad \therefore 2.2 < 2.289 < 2.3$$

$$\therefore 2.2 < \sqrt[3]{12} < 2.3 \quad \therefore \sqrt[3]{12} \text{ lies between 2.2 and 2.3}$$

TRY yourself 3

- Find two consecutive integers such that $\sqrt{13}$ lies between them.
- Prove that : $\sqrt{7}$ lies between 2.6 and 2.7



• If you draw the right-angled triangle ABC at B such that :

AB = 1 length unit, BC = 2 length units, then according to Pythagoras' theorem you find :

$$(AC)^2 = (AB)^2 + (BC)^2 = (1)^2 + (2)^2 = 1 + 4 = 5$$

$$\therefore AC = \sqrt{5} \text{ length unit.}$$

i.e. The length of \overline{AC} represents the irrational number $\sqrt{5}$



- If you draw the number line and you open the compasses with a distance equal to the length of \overline{AC} and using O which represents zero as a centre and draw an arc cutting the number line at the point X on the right of the point O , then the point X represents the number $\sqrt{5}$ on the number line.



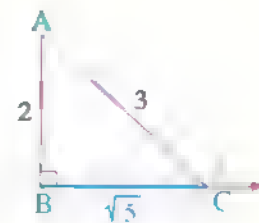
- And with the same length of \overline{AC} , if you use O as a centre and draw an arc cutting the number line at the point Y on the left side of O , then the point Y represents the number $-\sqrt{5}$ on the number line.

Generally

Each irrational number can be represented by a point on the number line.

! Remark

If you draw the right-angled triangle ABC at B such that $AB = 2$ length units , $AC = 3$ length units , then $(BC)^2 = (AC)^2 - (AB)^2 = (3)^2 - (2)^2 = 9 - 4 = 5$ i.e. $BC = \sqrt{5}$ length unit, then you can use the length of \overline{BC} to determine the point which represents $\sqrt{5}$ or $-\sqrt{5}$

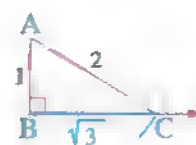


From the previous , we deduce that :

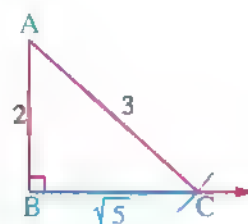
To get a line segment with length that equals the irrational number \sqrt{a} , we search for two numbers , the sum of their squares or the difference between their squares = a , then we use them to draw a right-angled triangle.

The following figures can help you to get two numbers such that the difference between their squares equals the square of the irrational number.

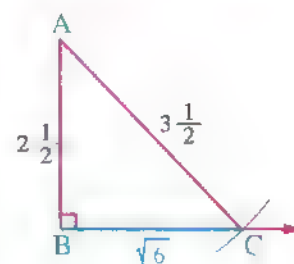
- To draw a line segment with length $\sqrt{3}$ length unit , then the length of one of the two sides of the right-angle = $\frac{3-1}{2} = 1$ length unit. and the length of the hypotenuse = $\frac{3+1}{2} = 2$ length units.



- To draw a line segment with length $\sqrt{5}$ length unit ,
then the length of one of the two sides of
the right-angle $= \frac{5-1}{2} = 2$ length units.
and the length of the hypotenuse $= \frac{5+1}{2} = 3$ length units.

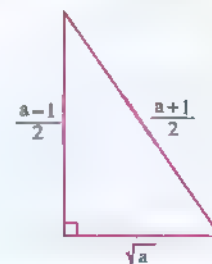


- To draw a line segment with length $\sqrt{6}$ length unit ,
then the length of one side of the right-angle
 $= \frac{6-1}{2} = 2\frac{1}{2}$ length units.
and the length of the hypotenuse $= \frac{6+1}{2} = 3\frac{1}{2}$ length units.



Generally

To draw a line segment with length \sqrt{a} length unit where $a > 1$
 , draw a right-angled triangle in which
 the length of one side of the right-angle $= \frac{a-1}{2}$ length unit.
 and the length of the hypotenuse $= \frac{a+1}{2}$ length unit.



Example 4 Draw a line segment with length $\sqrt{7}$ length unit , then use it to determine the points which represent the following numbers on the number line :

- | | | |
|------------------|---------------|------------------|
| 1 $\sqrt{7}$ | 2 $-\sqrt{7}$ | 3 $1 + \sqrt{7}$ |
| 4 $2 - \sqrt{7}$ | 5 $2\sqrt{7}$ | |

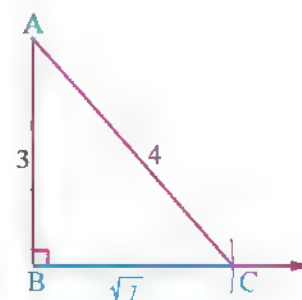
Solution Draw the right-angled triangle ABC at B such that :

$$AB = \frac{7-1}{2} = 3 \text{ length units.}$$

$$\therefore AC = \frac{7+1}{2} = 4 \text{ length units.}$$

$$\therefore (BC)^2 = (AC)^2 - (AB)^2 = 16 - 9 = 7$$

$$\therefore BC = \sqrt{7} \text{ length unit.}$$



- Using the compasses with a distance equal to the length of \overline{BC} taking O as a centre , draw an arc to cut the number line on the right side of O at the point X , then X is the point which represents $\sqrt{7}$



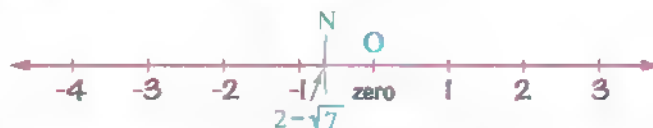
- Using the same previous distance and taking O as a centre , draw an arc to cut the number line on the left side of O at the point Y , then Y is the point which represents the number $-\sqrt{7}$



- Using the same previous distance and taking the point which represents the number 1 on the number line as a centre, draw an arc to cut the number line on the right side of the previous point at Z , then Z represents the number $(1 + \sqrt{7})$



- Using the same previous distance and taking the point which represents the number 2 on the number line as a centre , draw an arc to cut the number line on the left side of this point at the point N , then N is the point which represents the number $(2 - \sqrt{7})$

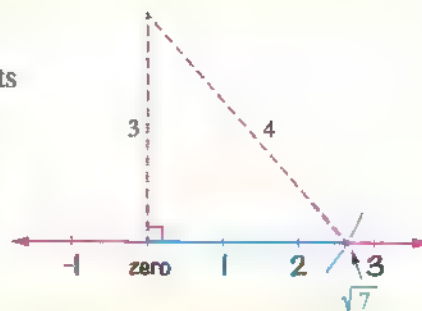


- Using the same previous distance and taking the point O on the number line as a centre , draw an arc to cut the number line on the right side of O at the point D , then taking D as a centre and with the same previous distance in the same direction, draw an arc to cut the number line at E , then E is the point which represents the number $2\sqrt{7}$



! Remark

In the previous example we can determine the point which represents the number $\sqrt{7}$ on the number line by drawing the right-angled triangle directly on the number line as in the opposite figure.

**Example 5** Find the length of the diagonal of a square whose area = 5 cm^2 **Solution**

Let the side length of the square be $L \text{ cm}$.

$$\therefore \text{The area of the square} = L \times L = L^2 \text{ cm}^2$$

$$\therefore L^2 = 5 \qquad \therefore L = \sqrt{5} \text{ cm.}$$

(Notice that : The square side length must be positive, so L equals $\sqrt{5}$ not $-\sqrt{5}$)

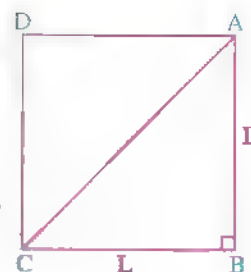
$\therefore \triangle ABC$ is a right-angled triangle at B ,

(from the properties of the square $ABCD$)

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (\sqrt{5})^2 + (\sqrt{5})^2 = 5 + 5 = 10$$

$$\therefore AC = \sqrt{10} \text{ cm.}$$

$$\therefore \text{The length of the diagonal of the square} = \sqrt{10} \text{ cm.}$$

**TRY 4**

Determine the point which represents the number $\sqrt{11}$ on the number line.

4 determine by yourself.

3 1 3, 4

2 1 3, 5

7

4

1

2 prove by yourself.

2

8

5

2

9

9

3

of try by yourself

Answer

The set of real numbers \mathbb{R} and ordering numbers in \mathbb{R}



The set of real numbers

It is the set obtained from the union of the set of rational numbers and the set of irrational numbers. It is denoted by \mathbb{R}

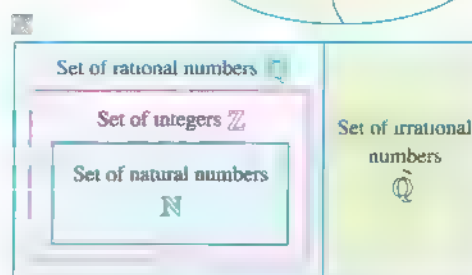
i.e. $\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}^c$ (as shown in the opposite figure)

Noticing that : $\mathbb{Q} \cap \mathbb{Q}^c = \emptyset$

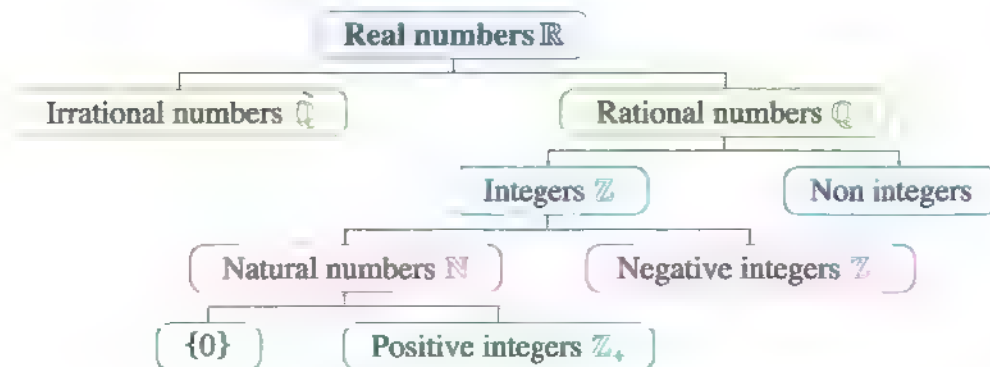
• The opposite Venn diagram shows that :

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

$$\text{and } \mathbb{Q}^c \subset \mathbb{R}$$



The following diagram shows the relation among the sets of numbers that we studied till now :



Ordering numbers in \mathbb{R}

- Each real number is represented by a unique point on the number line.
- The set of real numbers is an ordered set.
- If the point representing the number x on the number line lies on the left of the point representing the number y as shown in the figure, then $x < y$ or $y > x$



- Each real number represented by a point lying on the right side of the origin O is greater than zero, and all these numbers form a set called “the set of the positive real numbers” denoted by \mathbb{R}_+

$$\mathbb{R}_+ = \{x : x \in \mathbb{R}, x > \text{zero}\}$$

- Each real number represented by a point lying on the left side of the origin O is less than zero and all these numbers form a set called “the set of the negative real numbers” denoted by \mathbb{R}_-

$$\mathbb{R}_- = \{x : x \in \mathbb{R}, x < \text{zero}\}$$



! Remarks

- $\mathbb{R}_+ \cap \mathbb{R}_- = \emptyset$
- $\mathbb{R} = \mathbb{R}_+ \cup \{0\} \cup \mathbb{R}_-$
- The number zero is neither positive nor negative.
- $\mathbb{R}_+ \cup \{0\} = \{x : x \in \mathbb{R}, x \geq 0\}$
and it is called the set of the non-negative real numbers.
- $\mathbb{R}_- \cup \{0\} = \{x : x \in \mathbb{R}, x \leq 0\}$
and it is called the set of the non-positive real numbers.
- The set of real numbers without zero (The non-zero real numbers) is denoted by \mathbb{R}^*
i.e. $\mathbb{R}^* = \mathbb{R} - \{0\} = \mathbb{R}_+ \cup \mathbb{R}_-$

Example 1 Arrange the following numbers ascendingly :

$$\sqrt{75}, \sqrt{68}, -\sqrt{45}, -8, 7 \text{ and } -\sqrt{32}$$

Solution • Arrange the positive numbers which are $\sqrt{75}, \sqrt{68}$ and 7

$$\because 7 = \sqrt{49}$$

$$\therefore 49 < 68 < 75 \quad \therefore \sqrt{49} < \sqrt{68} < \sqrt{75}$$

$$\text{i.e. } 7 < \sqrt{68} < \sqrt{75}$$

• Arrange the negative numbers which are $-\sqrt{45}, -8$ and $-\sqrt{32}$

$$\because 8 = \sqrt{64}$$

$$\therefore 64 > 45 > 32 \quad \therefore \sqrt{64} > \sqrt{45} > \sqrt{32}$$

$$\therefore -\sqrt{64} < -\sqrt{45} < -\sqrt{32}$$

$$\text{i.e. } -8 < -\sqrt{45} < -\sqrt{32}$$

$$\therefore \text{The ascending order is : } -8, -\sqrt{45}, -\sqrt{32}, 7, \sqrt{68} \text{ and } \sqrt{75}$$

! Remark

You can use the calculator to get the solution by finding approximated values of the roots.

Example 2 Write three irrational numbers included between 11 and 12

Solution $\because (11)^2 = 121, (12)^2 = 144$

$$\therefore 125, 126 \text{ and } 130 \text{ are three integers included between } 121 \text{ and } 144$$

$$\therefore 121 < 125 < 126 < 130 < 144$$

$$\therefore \sqrt{121} < \sqrt{125} < \sqrt{126} < \sqrt{130} < \sqrt{144}$$

$$\therefore \text{The required irrational numbers are : } \sqrt{125}, \sqrt{126} \text{ and } \sqrt{130}$$

(Notice that : There are other irrational numbers included between 11 and 12)

Example 3 Find the S.S. in \mathbb{R} for each of the following equations :

1 $3x^2 + 125 = 221$

2 $\frac{1}{6}x^3 - 8 = 28$

3 $2x^2 + 6 = 4$

Solution

1 $\therefore 3x^2 + 125 = 221$

$\therefore 3x^2 = 221 - 125$

$\therefore 3x^2 = 96$

$\therefore x^2 = \frac{96}{3}$

$\therefore x^2 = 32$

$\therefore x = \pm\sqrt{32}$

$\therefore \text{The S.S.} = \{\sqrt{32}, -\sqrt{32}\}$

2 $\therefore \frac{1}{6}x^3 - 8 = 28$

$\therefore \frac{1}{6}x^3 = 36$

$\therefore x^3 = 6 \times 36$

$\therefore x^3 = 216$

$\therefore x = \sqrt[3]{216}$

$\therefore x = 6$

$\therefore \text{The S.S.} = \{6\}$

3 $\therefore 2x^2 + 6 = 4$

$\therefore 2x^2 = 4 - 6$

$\therefore 2x^2 = -2$

$\therefore x^2 = -\frac{2}{2}$

$\therefore x^2 = -1$

$\therefore x = \pm\sqrt{-1}$

$\therefore \sqrt{-1} \notin \mathbb{R}, -\sqrt{-1} \notin \mathbb{R} \therefore \text{The S.S.} = \emptyset$



Complete each of the following using one of the symbols $>$ or $<$:

1 $\sqrt{2} \dots\dots 1$

2 $-\sqrt{3} \dots\dots -1$

3 $\sqrt[3]{9} \dots\dots 3$

4 $-\sqrt[3]{7} \dots\dots -2$

5 $\sqrt{7} \dots\dots 2.6$

6 $-\sqrt[3]{16} \dots\dots -2.52$

6 $<$

3 $<$

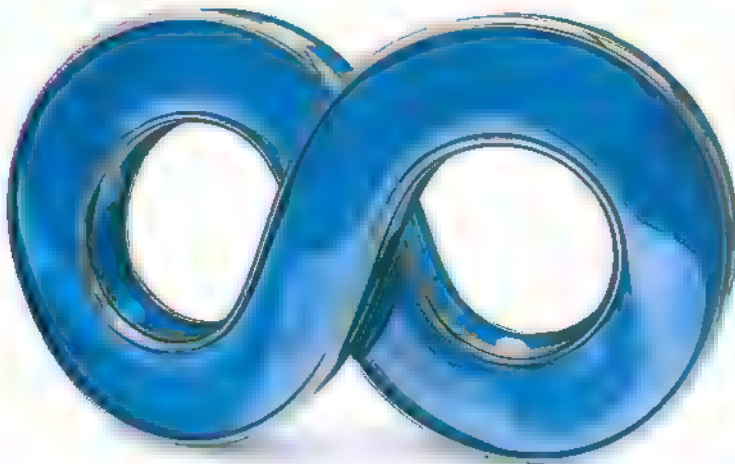
5 $<$

2 $<$

4 $<$

1 $<$

Answers of try by yourself



Prelude Through your previous study , you knew different methods to express a subset of the set of natural numbers and a subset of the set of integers and you learnt how to represent them on the number line.

For example:

If X = the set of integers which are greater than or equal to -3 and less than 2

* Then you can express the set X by

the description method as follows : $\longrightarrow X = \{a : a \in \mathbb{Z} , -3 \leq a < 2\}$

* You can also express it by listing

method as follows : $\longrightarrow X = \{-3, -2, -1, 0, 1\}$

* The set X is represented on the number

line as shown in the figure : \longrightarrow



- And now the question is : Is it possible to use the same previous methods to express a subset of the set of real numbers and represent it on the number line ?

Assuming that : K = the set of real numbers that are greater than or equal to -3 and less than 2

* You can express the set K by the description

method as follows : $\longrightarrow K = \{a : a \in \mathbb{R} , -3 \leq a < 2\}$

* But it is impossible to express the set K by listing method because there are an infinity of real numbers existing between -3 and 2

* For the same reason , it is impossible to represent this set K by separate points on the number line as shown in the previous figure therefore we use another method to express a subset of the set of real numbers , which is **the intervals**.

- In the following , we will show the types of intervals :

First Limited intervals

A Closed interval

- The set $\{x : x \in \mathbb{R}, -3 \leq x \leq 2\}$ expresses the set of real numbers which consists of the two numbers -3 and 2 and all the real numbers included between them.

We denote it by $[-3, 2]$ and it is called a «closed interval».

- It is represented on the number line as shown in the figure :



Notice that : $-3 \in [-3, 2]$, $2 \in [-3, 2]$

We express this by drawing two shaded circles at the two points representing the two numbers -3 and 2

B Opened interval

- The set $\{x : x \in \mathbb{R}, -3 < x < 2\}$ expresses the set of real numbers included between the two numbers -3 and 2 such that the two numbers -3 and 2 are not contained in this set.

We denote this set by $] -3, 2[$ and it is called an «opened interval».

- It is represented on the number line as in the figure :



Notice that : $-3 \notin] -3, 2[$ and $2 \notin] -3, 2[$

We express this by drawing two unshaded circles at the two points representing the two numbers -3 and 2

C Half opened interval (Half closed interval)

- The set $\{x : x \in \mathbb{R}, -3 \leq x < 2\}$ expresses the number -3 and all the real numbers included between -3 and 2 without the number 2 , we denote it by $[-3, 2[$ and it is called a «half opened interval» or «half closed interval».

- It is represented on the number line as in the figure :



Notice that : $-3 \in [-3, 2[$, $2 \notin [-3, 2[$

- The set $\{x : x \in \mathbb{R}, -3 < x \leq 2\}$ expresses the number 2 and all the real numbers included between -3 and 2 without the number -3 , we denote it by $] -3, 2]$ and it is called a «half opened interval» or «half closed interval».

- It is represented on the number line as in the figure :



Notice that : $-3 \notin] -3, 2]$, $2 \in] -3, 2]$

Second Unlimited Intervals

- 1 • The set $\{x : x \in \mathbb{R}, x \geq 2\}$ expresses the set of real numbers which consists of the number 2 and all the real numbers which are greater than 2 with no end.

It is denoted by $[2, \infty[$ where the symbol « ∞ » is read as **positive infinity** and it doesn't represent a real number

- It is represented on the number line as shown in the figure :



Notice that : $2 \in [2, \infty[$

- 2 • The set $\{x : x \in \mathbb{R}, x > 2\}$ expresses the set of all real numbers which are greater than the number 2 with no end. It is denoted by $]2, \infty[$

- It is represented on the number line as shown in the figure :



Notice that : $2 \notin]2, \infty[$

- 3 • The set $\{x : x \in \mathbb{R}, x \leq 2\}$ expresses the set of real numbers which consists of the number 2 and all the real numbers which are smaller than the number 2 with no end.

It is denoted by $]-\infty, 2]$ where the symbol « $-\infty$ » is read as **negative infinity** and it doesn't represent a real number.

- It is represented on the number line as shown in the figure :



Notice that : $2 \in]-\infty, 2]$

- 4 • The set $\{x : x \in \mathbb{R}, x < 2\}$ expresses the set of all real numbers which are smaller than the number 2 with no end. It is denoted by $]-\infty, 2[$

- It is represented on the number line as shown in the figure :



Notice that : $2 \notin]-\infty, 2[$

- We can express the previous symbolically in the following table assuming that :
 $a \in \mathbb{R}, b \in \mathbb{R}$ and $a < b$



Types of intervals		The interval	Expression by distinguished property	Representation on the number line	Notice that
The limited intervals	Closed	$[a, b]$	$\{x : x \in \mathbb{R}, a \leq x \leq b\}$		<ul style="list-style-type: none"> $a \in [a, b]$ $b \in [a, b]$
	Opened	$]a, b[$	$\{x : x \in \mathbb{R}, a < x < b\}$		<ul style="list-style-type: none"> $a \notin]a, b[$ $b \notin]a, b[$
	half opened (half closed)	$[a, b[$	$\{x : x \in \mathbb{R}, a \leq x < b\}$		<ul style="list-style-type: none"> $a \in [a, b[$ $b \notin [a, b[$
		$]a, b]$	$\{x : x \in \mathbb{R}, a < x \leq b\}$		<ul style="list-style-type: none"> $a \notin]a, b]$ $b \in]a, b]$
The unlimited intervals		$[a, \infty[$	$\{x : x \in \mathbb{R}, x \geq a\}$		$a \in [a, \infty[$
		$]a, \infty[$	$\{x : x \in \mathbb{R}, x > a\}$		$a \notin]a, \infty[$
		<math]-\infty, a]<="" math=""></math]-\infty,>	$\{x : x \in \mathbb{R}, x \leq a\}$		$a \in]-\infty, a]$
		<math]-\infty, a[<="" math=""></math]-\infty,>	$\{x : x \in \mathbb{R}, x < a\}$		$a \notin]-\infty, a[$

! Remarks

- $\mathbb{R} =]-\infty, \infty[$
- $\mathbb{R}_+ =]0, \infty[$
- $\mathbb{R} =]-\infty, 0[$
- The set of non-negative real numbers $= \mathbb{R}_+ \cup \{0\} = [0, \infty[$
- The set of non-positive real numbers $= \mathbb{R} \cup \{0\} =]-\infty, 0]$

Example 1

Write each of the following sets in the form of an interval, then represent it on the number line :

- $\{x : x \in \mathbb{R}, -3 < x \leq 0\}$
- $\{a : a \in \mathbb{R}, 1 \geq a \geq -2\}$
- $\{x : x \in \mathbb{R}, x > 0\}$
- $\{y : y \in \mathbb{R}, -1 \geq y\}$

Solution

1 $]-3, 0]$



2 $[-2, 1]$



3 $]0, \infty[$



4 $]-\infty, -1]$


TRY 1

1 Write each of the following in an interval form, then represent it on the number line :

1 $\{x : x \in \mathbb{R}, -4 < x \leq 2\}$

2 $\{y : y \in \mathbb{R}, y \geq -5\}$

2 Represent each of the following on the number line and express it by the description method :

1 $]-3, 0[$

2 $]-\infty, 2]$

Example 2

Choose the correct answer from those given :

1 $4 \in \dots\dots\dots$

(a) $]4, 7[$

(b) $[-4, 4[$

(c) $]2, 5[$

(d) $[-11, -4]$

2 $\sqrt[3]{-8} \dots\dots\dots [-8, -2[$

(a) \in

(b) \notin

(c) \subset

(d) $\not\subset$

3 $\{1, 6\} \dots\dots\dots]1, 6]$

(a) \in

(b) \notin

(c) \subset

(d) $\not\subset$

4 If $x \in [-5, \infty[$, then $\dots\dots\dots$

(a) $x > -5$

(b) $x \geq -5$

(c) $x < -5$

(d) $x \leq -5$

5 The sum of the real numbers in the interval $[-3, 3[$ is $\dots\dots\dots$

(a) -6

(b) -3

(c) zero

(d) 6

Solution

1 (c)

2 (b) The reason : $\because \sqrt[3]{-8} = -2$, $[-8, -2[$ is open at -2

$\therefore -2 \notin [-8, -2[$

3 (d) The reason : $1 \notin]1, 6]$ because the interval is open at 1

4 (b)

5 (b) The reason : Each number belongs to the interval has its additive inverse except -3 because $3 \notin [-3, 3[$

TRY 2 by yourself

Complete using one of the symbols \in, \notin, \subset or $\not\subset$:

1 $-1 \dots\dots [-4, -1[$

2 $\frac{1}{2} \dots\dots]0, 1[$

3 $\{2, 4\} \dots\dots [2, 5[$

4 $\sqrt{7} \dots\dots]2, 3]$

5 $\{-1, 0, 1\} \dots\dots [0, 1]$

6 $|-5| \dots\dots]-\infty, 0]$

Operations on intervals

You studied before the sets and how to carry out the operations of intersection , union , difference and complement on them.

For example:

If $X = \{1, 2, 3, 4\}$, $Y = \{3, 4, 5, 6\}$, then :

- $X \cap Y$ = the set of elements which are common in X and $Y = \{3, 4\}$
- $X \cup Y$ = the set of all elements in X or Y without repeating = $\{1, 2, 3, 4, 5, 6\}$
- $X - Y$ = the set of elements which are in X and not in $Y = \{1, 2\}$
- $Y - X$ = the set of elements which are in Y and not in $X = \{5, 6\}$
- If the universal set $U = \{1, 2, 3, 4, 5, 6, 7\}$,

then the complement of X which is denoted by $\bar{X} = U - X$

i.e. \bar{X} = the set of elements which are in U and not in $X = \{5, 6, 7\}$

The following examples show how to carry out the operations of intersection , union and difference on intervals :

Example 3 If $X = [-3, 3]$ and $Y = [-1, 5[$, find using the number line :

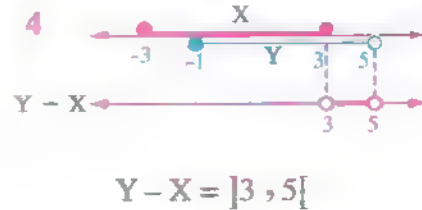
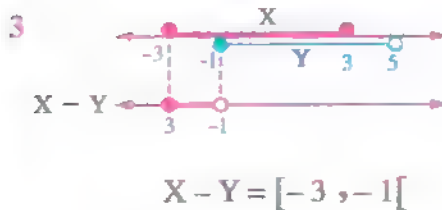
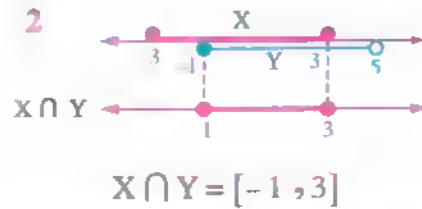
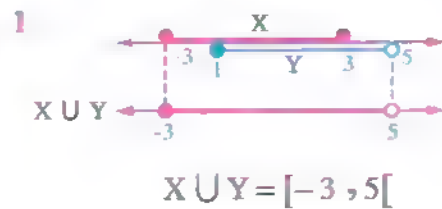
1 $X \cup Y$

3 $X - Y$

2 $X \cap Y$

4 $Y - X$

Solution



Example 4 Find each of the following :

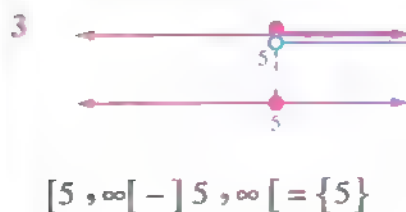
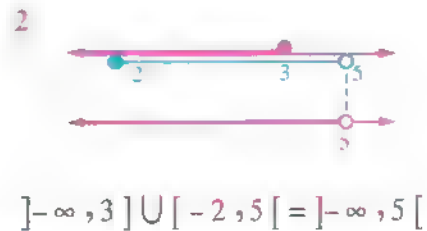
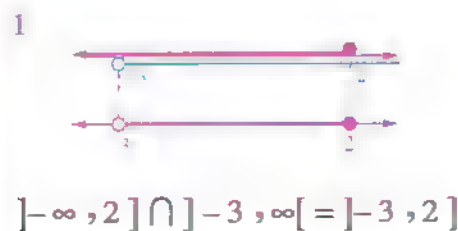
1 $]-\infty, 2] \cap]-3, \infty[$

3 $[5, \infty[-]5, \infty[$

2 $]-\infty, 3] \cup [-2, 5[$

4 $[2, \infty[\cap]-\infty, 2[$

Solution



Example 5 If $X =] -\infty, 2[$ and $Y = [-1, 5]$, find using the number line :

1 $X \cup Y$

3 $X - Y$

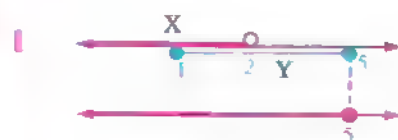
5 \bar{X}

2 $X \cap Y$

4 $Y - X$

6 \bar{Y}

Solution



$$X \cup Y =] -\infty, 5]$$



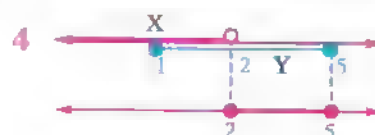
$$X - Y =] -\infty, -1[$$



$$\bar{X} = [2, \infty[$$



$$X \cap Y = [-1, 2[$$



$$Y - X = [2, 5]$$



$$\bar{Y} =] -\infty, -1[\cup] 5, \infty[\\ = \mathbb{R} - [-1, 5]$$

Example 6 If $X = [1, 4[$ and $Y = \{1, 4\}$, find :

1 $X \cap Y$

3 $X - Y$

2 $X \cup Y$

4 $Y - X$

Solution

1 $X \cap Y = \{1\}$

2 $X \cup Y = [1, 4]$

3 $X - Y =]1, 4[$

4 $Y - X = \{4\}$



TRY 3

If $X = [-1, 3[$ and $Y =]0, 4]$, find using the number line :

- | | | |
|---------------------|--------------|-----------------------------------|
| 1 $X \cap Y$ | 2 $X \cup Y$ | 3 $X - [0, \infty[$ |
| 4 $]0, \infty[- Y$ | 5 X^c | 6 $X \cup \{-2, -1, 0, 1, 2, 3\}$ |

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- | | | |
|-----------------|---|---------------------|
| 4 $]4, \infty[$ | 5 $] - \infty, -1[\cup]3, \infty[= \mathbb{R} - [-1, 3]$ | 6 $\{-1, 0, 1, 2\}$ |
| 3 $]0, 3]$ | 2 $[-1, 4]$ | 3 $[-1, 0]$ |
| 4 \in | 5 \notin | 6 \notin |
| 2 \notin | 2 \in | 3 \subset |

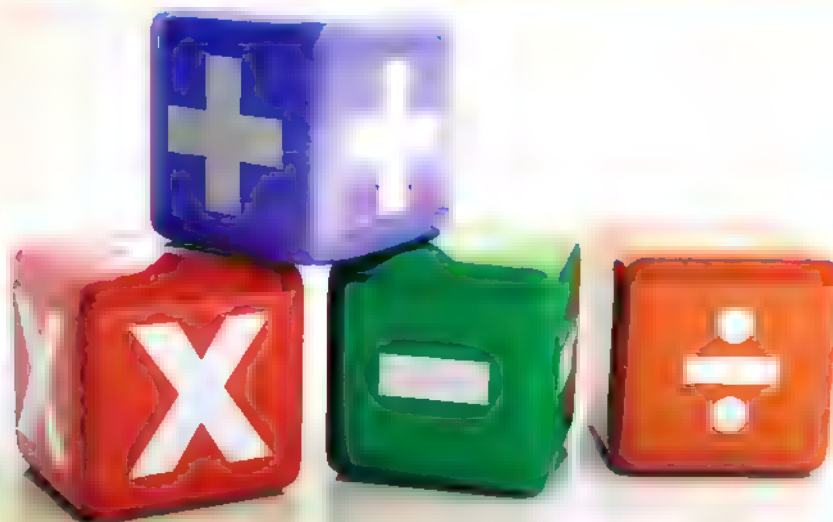
(2) $\{x: x \in \mathbb{R}, x \leq 2\}$

(1) $\{x: x \in \mathbb{R}, -3 < x < 0\}$

(2) $[-5, \infty[$

1 (1) $[-4, 2]$

Answers



First Addition

- We know that $2x$ and $3x$ are two like algebraic terms and their sum is an algebraic term like them.

$$\text{Where } 2x + 3x = (2 + 3)x = 5x$$

$$\text{Then we deduce that : } 2\sqrt{5} + 3\sqrt{5} = (2 + 3)\sqrt{5} \\ = 5\sqrt{5}$$

Remember that

The real number $2\sqrt{5}$ is produced by multiplying the rational number 2 by the irrational number $\sqrt{5}$

- We know that $2x$ and $3y$ are two unlike algebraic terms and we express their sum by an algebraic expression whose simplest form is $2x + 3y$

Therefore we deduce that :

The two real numbers $2\sqrt{3}$ and $3\sqrt{2}$, their sum is expressed by a real number whose simplest form is $2\sqrt{3} + 3\sqrt{2}$

Properties of addition of real numbers

Closure

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ we find that $(a + b) \in \mathbb{R}$

i.e. The sum of any two real numbers is a real number, therefore we say \mathbb{R} is closed under addition.

For example: $\sqrt{5} \in \mathbb{R}$ and $2\sqrt{5} \in \mathbb{R}$, we find that : $\sqrt{5} + 2\sqrt{5} = 3\sqrt{5} \in \mathbb{R}$

Commutative property

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a + b = b + a$

For example: $5\sqrt[3]{2} + 4\sqrt[3]{2} = 9\sqrt[3]{2}$, $4\sqrt[3]{2} + 5\sqrt[3]{2} = 9\sqrt[3]{2}$

$$\text{i.e. } 5\sqrt[3]{2} + 4\sqrt[3]{2} = 4\sqrt[3]{2} + 5\sqrt[3]{2}$$

Associative property

For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a + b) + c = a + (b + c) = a + b + c$

For example: $(\sqrt{3} + 2\sqrt{3}) + 5\sqrt{3} = 3\sqrt{3} + 5\sqrt{3} = 8\sqrt{3}$,

$$\sqrt{3} + (2\sqrt{3} + 5\sqrt{3}) = \sqrt{3} + 7\sqrt{3} = 8\sqrt{3}$$

$$\text{i.e. } (\sqrt{3} + 2\sqrt{3}) + 5\sqrt{3} = \sqrt{3} + (2\sqrt{3} + 5\sqrt{3})$$

The additive neutral

For every $a \in \mathbb{R}$ it will be $a + 0 = 0 + a = a$

i.e. Zero is the additive neutral.

For example: $\sqrt{2} + 0 = 0 + \sqrt{2} = \sqrt{2}$, $-\sqrt[3]{5} + 0 = 0 + (-\sqrt[3]{5}) = -\sqrt[3]{5}$

The additive inverse of every real number

For every $a \in \mathbb{R}$ there is $(-a) \in \mathbb{R}$ where $a + (-a) = \text{zero (the additive neutral)}$

For example: • The additive inverse of $\sqrt{3}$ is $-\sqrt{3}$ and vice versa because $\sqrt{3} + (-\sqrt{3}) = 0$

• The additive inverse of $2 + \sqrt{5}$ is $-(2 + \sqrt{5})$ and equals $-2 - \sqrt{5}$

• The additive inverse of $3 - \sqrt{2}$ is $-(3 - \sqrt{2})$ and equals $\sqrt{2} - 3$

• The additive inverse of zero is itself.

! Remark

Since every real number has an additive inverse , then the subtraction operation is possible entirely in \mathbb{R} , and it is defined as follows :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a - b = a + (-b)$

i.e. The subtraction operation $(a - b)$ means adding the number a to the additive inverse of the number b

And we can deduce that :

Subtraction operation in \mathbb{R} is not commutative and it is not associative.

Example 1

Choose the correct answer from those given :

- 1 $\sqrt{7} + \sqrt{7} = \dots\dots\dots$
 (a) $\sqrt{14}$ (b) $2\sqrt{7}$ (c) 7 (d) 14
- 2 $2\sqrt{2} - 3\sqrt{2} = \dots\dots\dots$
 (a) -1 (b) $-\sqrt{2}$ (c) $\sqrt{2}$ (d) $5\sqrt{2}$
- 3 $4 + \sqrt{3} - 7 - \sqrt{3} = \dots\dots\dots$
 (a) $-3 - 2\sqrt{3}$ (b) $-3 + 2\sqrt{3}$ (c) -3 (d) 3
- 4 If $X = 9\sqrt{5}$, $y = 5\sqrt{3}$, then $X - y = \dots\dots\dots$
 (a) $4\sqrt{2}$ (b) $4\sqrt{5}$ (c) $4\sqrt{3}$ (d) $9\sqrt{5} - 5\sqrt{3}$
- 5 The additive inverse of $\sqrt{7} - \sqrt{5}$ is $\dots\dots\dots$
 (a) $-\sqrt{7} - \sqrt{5}$ (b) $\sqrt{7} + \sqrt{5}$ (c) $\sqrt{5} - \sqrt{7}$ (d) $\sqrt{7} - \sqrt{5}$
- 6 If $\sqrt{2} + X = 0$, then $X - \sqrt{2} = \dots\dots\dots$
 (a) zero (b) $-\sqrt{2}$ (c) $-2\sqrt{2}$ (d) $2\sqrt{2}$

Solution

- 1 (b)
- 2 (b) The reason : $2\sqrt{2} - 3\sqrt{2} = (2 - 3)\sqrt{2} = -\sqrt{2}$
- 3 (c) The reason : $4 + \sqrt{3} - 7 - \sqrt{3} = (4 - 7) + (\sqrt{3} - \sqrt{3}) = -3 + 0 = -3$
- 4 (d) The reason : $X - y = 9\sqrt{5} - 5\sqrt{3}$ and this is the simplest form of the difference.
- 5 (c) The reason : The additive inverse of $\sqrt{7} - \sqrt{5}$ is $-(\sqrt{7} - \sqrt{5})$
 which is $-\sqrt{7} + \sqrt{5}$ or $\sqrt{5} - \sqrt{7}$
- 6 (c) The reason : X is the additive inverse of $\sqrt{2}$ which is $-\sqrt{2}$
 $\therefore X - \sqrt{2} = -\sqrt{2} - \sqrt{2} = -2\sqrt{2}$

**TRY
yourself 1**

- 1 Write the additive inverse for each of the following numbers :

$$\sqrt{2}, -\sqrt[3]{5}, \sqrt{2} + \sqrt{7}, \sqrt[3]{5} - 3, -\sqrt{6} - \sqrt[3]{7}$$

- 2 Simplify to the simplest form :

$$1 \quad 2 + 2\sqrt{7} - 1 - 5\sqrt{7}$$

$$2 \quad 3\sqrt{5} + \sqrt{3} - 3\sqrt{5} + 5\sqrt{3}$$

Second Multiplication

- We know that : $3 \times 2x = (3 \times 2)x = 6x$

Therefore we find that : $3 \times 2\sqrt{3} = (3 \times 2)\sqrt{3} = 6\sqrt{3}$

- We know also $2x \times 5x = (2 \times 5)(x \times x) = 10x^2$

Therefore we find that : $2\sqrt{3} \times 5\sqrt{3} = (2 \times 5) \times (\sqrt{3} \times \sqrt{3}) = 10(\sqrt{3})^2 = 10 \times 3 = 30$

Example 2 Find the result of each of the following :

1 $-2 \times 3\sqrt{5}$

2 $4\sqrt{2} \times \sqrt{2}$

3 $-2\sqrt{7} \times 4\sqrt{7}$

Solution 1 $-2 \times 3\sqrt{5} = (-2 \times 3)\sqrt{5} = -6\sqrt{5}$

2 $4\sqrt{2} \times \sqrt{2} = 4(\sqrt{2})^2 = 4 \times 2 = 8$

3 $-2\sqrt{7} \times 4\sqrt{7} = (-2 \times 4) \times (\sqrt{7})^2 = -8 \times 7 = -56$

Properties of multiplication of real numbers**Closure**

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b \in \mathbb{R}$

i.e. The product of any two real numbers is a real number therefore we say :

\mathbb{R} is closed under multiplication.

For example: $\sqrt{3} \in \mathbb{R}$ and $2\sqrt{3} \in \mathbb{R}$

We find that : $\sqrt{3} \times 2\sqrt{3} = 2 \times 3 = 6 \in \mathbb{R}$

Commutative property

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b = b \times a$

For example: $2\sqrt{5} \times 3\sqrt{5} = 6 \times 5 = 30$, $3\sqrt{5} \times 2\sqrt{5} = 6 \times 5 = 30$

i.e. $2\sqrt{5} \times 3\sqrt{5} = 3\sqrt{5} \times 2\sqrt{5}$

Associative property

For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a \times b) \times c = a \times (b \times c) = a \times b \times c$

For example: $(2\sqrt{7} \times 4\sqrt{7}) \times \sqrt{7} = 56 \times \sqrt{7} = 56\sqrt{7}$,

$2\sqrt{7} \times (4\sqrt{7} \times \sqrt{7}) = 2\sqrt{7} \times 28 = 56\sqrt{7}$

i.e. $(2\sqrt{7} \times 4\sqrt{7}) \times \sqrt{7} = 2\sqrt{7} \times (4\sqrt{7} \times \sqrt{7})$

The multiplicative neutral

For every $a \in \mathbb{R}$ it will be $a \times 1 = 1 \times a = a$

i.e. One is the multiplicative neutral in \mathbb{R}

For example: $\sqrt[3]{5} \times 1 = 1 \times \sqrt[3]{5} = \sqrt[3]{5}$

The multiplicative inverse of any non-zero real number

For every real number $a \neq 0$, there is a real number $\frac{1}{a}$ where $a \times \frac{1}{a} = 1$ which is the multiplicative neutral.

For example:

- The multiplicative inverse of $\sqrt{3}$ is $\frac{1}{\sqrt{3}}$
because $\sqrt{3} \times \frac{1}{\sqrt{3}} = 1$
- The multiplicative inverse of $-\frac{\sqrt{2}}{5}$ is $-\frac{5}{\sqrt{2}}$
- The multiplicative inverse of 1 is itself
and also the multiplicative inverse of -1 is itself.

Notice that :

- Both the number and its multiplicative inverse have the same sign.
- There is no multiplicative inverse for zero because $\frac{1}{\text{zero}}$ is meaningless (**i.e.** Undefined)

! Remark

Since each non-zero real number has a multiplicative inverse, then the division operation by any real number does not equal zero is possible in \mathbb{R} and it is defined as follows :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}^*$ it will be $a \div b = a \times \frac{1}{b}$

i.e. The division operation ($a \div b$) means multiplying the number a by the multiplicative inverse of the number b such that $b \neq 0$

And we can deduce that :

Division operation in \mathbb{R} is not commutative and it is not associative.

Example 3 Find the result of : $\frac{\sqrt{5}}{5} \times \frac{4\sqrt{5}}{12\sqrt{2}} \div \frac{1}{3\sqrt{2}}$

Solution $\left(\frac{\sqrt{5}}{5} \times \frac{4\sqrt{5}}{12\sqrt{2}} \right) \div \frac{1}{3\sqrt{2}} = \frac{5}{15\sqrt{2}} \div \frac{1}{3\sqrt{2}} = \frac{1}{3\sqrt{2}} \times 3\sqrt{2} = 1$

Example 4 Write each of the following such that the denominator is an integer :

1 $\frac{9}{\sqrt{3}}$

2 $-\frac{3}{\sqrt{2}}$

3 $\frac{5}{3\sqrt{5}}$

Solution 1 Multiplying the two terms of $\frac{9}{\sqrt{3}}$ by $\sqrt{3}$

$$\therefore \text{we get } \frac{9}{\sqrt{3}} = \frac{9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{3} = 3\sqrt{3}$$

Notice that : $\frac{\sqrt{3}}{\sqrt{3}} = 1$ "The multiplicative neutral"

2 $-\frac{3}{\sqrt{2}} = -\frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-3\sqrt{2}}{2}$

3 $\frac{5}{3\sqrt{5}} = \frac{5}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{3 \times 5} = \frac{\sqrt{5}}{3}$

Another solution :

$$\therefore \sqrt{5} \times \sqrt{5} = 5 \quad \therefore \frac{5}{3\sqrt{5}} = \frac{\sqrt{5} \times \sqrt{5}}{3\sqrt{5}} = \frac{\sqrt{5}}{3}$$

Example 5 Choose the correct answer from those given :

1 The multiplicative inverse of $\frac{\sqrt{5}}{10}$ is

(a) $\sqrt{10}$

(b) $\sqrt{5}$

(c) $2\sqrt{5}$

(d) $-2\sqrt{5}$

2 The additive inverse of $\frac{7}{\sqrt{7}}$ is

(a) $\frac{\sqrt{7}}{7}$

(b) 7

(c) $-\sqrt{7}$

(d) -7

3 The multiplicative inverse of $\frac{3\sqrt{2}}{4}$ equals $\frac{\dots}{3}$

(a) $4\sqrt{2}$

(b) $2\sqrt{2}$

(c) $\sqrt{2}$

(d) $\frac{4}{3\sqrt{2}}$

Solution

1 (c) The reason : The multiplicative inverse of $\frac{\sqrt{5}}{10}$

$$\text{is } \frac{10}{\sqrt{5}} = \frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$$

2 (c) The reason : The additive inverse of $\frac{7}{\sqrt{7}}$

$$\begin{aligned} \text{is } \frac{7}{\sqrt{7}} &= -\frac{7}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = -\frac{7\sqrt{7}}{7} \\ &= -\sqrt{7} \end{aligned}$$

3 (b) The reason : The multiplicative inverse of $\frac{3\sqrt{2}}{4}$

$$\begin{aligned} \text{is } \frac{4}{3\sqrt{2}} &= \frac{4}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{6} \\ &= \frac{2\sqrt{2}}{3} \end{aligned}$$

TRY**2**

1 Find each of the following :

1 $\sqrt{5} \times \frac{1}{\sqrt{5}} \times \sqrt{5}$

2 $\frac{\sqrt{3}}{3} \times \frac{4\sqrt{5}}{20} \times \frac{5\sqrt{3}}{\sqrt{5}}$

2 Make the denominator an integer :

1 $\frac{3}{\sqrt{7}}$

2 $\frac{9}{2\sqrt{6}}$

Distributing multiplication on addition and subtraction

For any three real numbers a , b and c it will be :

• $a(b \pm c) = ab \pm ac$

• $(b \pm c)a = ba \pm ca$



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Example 6

Find each of the following :

1 $2\sqrt{3}(5\sqrt{3}-4)$

2 $(2+\sqrt{3})(\sqrt{3}+7)$

3 $(7\sqrt{2}-5)(7\sqrt{2}+5)$

4 $(5\sqrt{3}-2)^2$

Solution

$$1 \quad 2\sqrt{3} (5\sqrt{3} - 4) = 2\sqrt{3} \times 5\sqrt{3} + 2\sqrt{3} \times (-4)$$

$$= 10 \times 3 - 8 \times \sqrt{3} = 30 - 8\sqrt{3}$$

$$2 \quad (2 + \sqrt{3})(\sqrt{3} + 7) = 2(\sqrt{3} + 7) + \sqrt{3}(\sqrt{3} + 7)$$

$$= 2 \times \sqrt{3} + 2 \times 7 + \sqrt{3} \times \sqrt{3} + \sqrt{3} \times 7$$

$$= 2\sqrt{3} + 14 + 3 + 7\sqrt{3}$$

$$= (2\sqrt{3} + 7\sqrt{3}) + (14 + 3) = 9\sqrt{3} + 17$$

$$3 \quad (7\sqrt{2} - 5)(7\sqrt{2} + 5) = 98 + 35\sqrt{2} - 35\sqrt{2} - 25 = 73$$

Another solution by multiplying by inspection :

$$(7\sqrt{2} - 5)(7\sqrt{2} + 5) = (7\sqrt{2})^2 - (5)^2$$

$$= 7^2 \times (\sqrt{2})^2 - 5^2$$

$$= 49 \times 2 - 25 = 98 - 25 = 73$$

Notice that :

$$(a + b)(a - b) = a^2 - b^2$$

4 Multiplying by inspection

$$\therefore (5\sqrt{3} - 2)^2 = (5\sqrt{3})^2 - 2 \times 5\sqrt{3} \times 2 + (-2)^2$$

$$= 5^2 \times (\sqrt{3})^2 - 20\sqrt{3} + 4$$

$$= 25 \times 3 - 20\sqrt{3} + 4$$

$$= 75 - 20\sqrt{3} + 4 = 79 - 20\sqrt{3}$$

Notice that :

$$\bullet (a + b)^2 = a^2 + 2ab + b^2$$

$$\bullet (a - b)^2 = a^2 - 2ab + b^2$$

Example 7If $x = 5\sqrt{3} - 2$, $y = 5\sqrt{3} + 2$, find the value of the expression : $x^2 + 2xy + y^2$ **Solution** From multiplying by inspection , we find that :

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$\therefore x^2 + 2xy + y^2 = (5\sqrt{3} - 2 + 5\sqrt{3} + 2)^2$$

$$= (10\sqrt{3})^2 = (10)^2 \times (\sqrt{3})^2 = 100 \times 3 = 300$$

Example 8 Give an estimation for the result of :

$(5 + \sqrt{10})(3 - \sqrt[3]{7})$, then check your answer using the calculator.

Solution **First** : The estimation of $\sqrt{10}$ is 3 (because $\sqrt{9} = 3$)

\therefore The estimation of $(5 + \sqrt{10})$ is $5 + 3 = 8$

, the estimation of $\sqrt[3]{7}$ is 2 (because $\sqrt[3]{8} = 2$)

\therefore The estimation of $(3 - \sqrt[3]{7})$ is $3 - 2 = 1$

\therefore The estimation of $(5 + \sqrt{10})(3 - \sqrt[3]{7})$ is $8 \times 1 = 8$

Second : By using the calculator, we find that the result approximated to the nearest thousandths is 8.873

i.e. The estimation is accepted.

TRY YOURSELF 3

1 Find the result of each of the following in the simplest form :

1 $5\sqrt{2}(3\sqrt{2} - 2)$

2 $(2\sqrt{3} - 3)(2\sqrt{3} + 3)$

2 If $x = 2\sqrt{3} - 1$ and $y = 2\sqrt{3} + 1$

, find the value of the expression : $x^2 - 2xy + y^2$

3 Give an estimation for the result of : $(1 + \sqrt{15})(4 - \sqrt{8})$

, then check your answer by using the calculator.

3 5 (Check by yourself)

2 4

3 1 (1) 30 - 10 $\sqrt{2}$

2 1 (1) $3\sqrt{7}$

(2) $3\sqrt{6}$

2 1 (1) $\sqrt{5}$

(2) 1

2 (1) $1 - 3\sqrt{7}$

(2) $6\sqrt{3}$

1 1 $-\sqrt{2}$, $\sqrt[3]{5}$, $-\sqrt{2} - \sqrt{7}$, $-\sqrt[3]{5} + 3$, $\sqrt{6} + \sqrt[3]{7}$

Answers / of try by yourself

Operations on the square roots



If a and b are two non negative real numbers , then :

$$1 \quad \sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

For example: • $\sqrt{3} \times \sqrt{12} = \sqrt{36} = 6$

• $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

$$2 \quad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \quad (\text{where } b \neq 0)$$

For example: • $\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$

• $\sqrt{\frac{16}{49}} = \frac{\sqrt{16}}{\sqrt{49}} = \frac{4}{7}$

$$3 \quad \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b} \quad (\text{where } b \neq 0)$$

This operation is carried out to make the denominator an integer.

For example: • $\frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$

• $\sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

! Remarks

① $\sqrt{a^2 + b^2} \neq a + b$, $\sqrt{a^2 - b^2} \neq a - b$

For example:

• $\sqrt{6^2 + 8^2} \neq 6 + 8$ because $\sqrt{6^2 + 8^2} = \sqrt{100} = 10$

• $\sqrt{25 - 9} \neq 5 - 3$ because $\sqrt{25 - 9} = \sqrt{16} = 4$

② $a\sqrt{b} = \sqrt{a^2 b}$

For example:

• $2\sqrt{\frac{1}{2}} = \sqrt{4 \times \frac{1}{2}} = \sqrt{2}$

• $15\sqrt{\frac{1}{3}} = 5 \times 3\sqrt{\frac{1}{3}} = 5\sqrt{9 \times \frac{1}{3}} = 5\sqrt{3}$

Example 1 Write each of the following in the form $a\sqrt{b}$ where a and b are two integers , b is the least possible value :

1 $\sqrt{27}$

2 $5\sqrt{54}$

3 $3\sqrt{\frac{2}{3}}$

4 $\frac{\sqrt{84}}{\sqrt{7}}$

Solution

1 $\sqrt{27} = \sqrt{9 \times 3}$

$= \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$

2 $5\sqrt{54} = 5\sqrt{9 \times 6} = 5 \times \sqrt{9} \times \sqrt{6}$

$= 5 \times 3 \times \sqrt{6} = 15\sqrt{6}$

3 $3\sqrt{\frac{2}{3}} = 3 \times \frac{\sqrt{2}}{\sqrt{3}} = 3 \times \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 3 \times \frac{\sqrt{6}}{3} = \sqrt{6}$

Another solution :

3 $\sqrt{\frac{2}{3}} = \sqrt{3^2 \times \frac{2}{3}} = \sqrt{3 \times 2} = \sqrt{6}$

4 $\frac{\sqrt{84}}{\sqrt{7}} = \sqrt{\frac{84}{7}} = \sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

Example 2 Simplify to the simplest form :

1 $\sqrt{45} - 2\sqrt{20} + 2\sqrt{5}$

2 $2\sqrt{18} + \sqrt{50} - 42\sqrt{\frac{1}{2}}$

3 $2\sqrt{27} - 3\sqrt{\frac{1}{3}} - \frac{6}{\sqrt{3}}$

Solution 1 $\sqrt{45} - 2\sqrt{20} + 2\sqrt{5} = \sqrt{9 \times 5} - 2\sqrt{4 \times 5} + 2\sqrt{5}$

$$= \sqrt{9} \times \sqrt{5} - 2 \times \sqrt{4} \times \sqrt{5} + 2\sqrt{5}$$

$$= 3\sqrt{5} - 2 \times 2\sqrt{5} + 2\sqrt{5}$$

$$= 3\sqrt{5} - 4\sqrt{5} + 2\sqrt{5} = \sqrt{5}$$

2 $2\sqrt{18} + \sqrt{50} - 42\sqrt{\frac{1}{2}} = 2\sqrt{9 \times 2} + \sqrt{25 \times 2} - 42 \times \frac{\sqrt{1}}{\sqrt{2}}$

$$= 2 \times \sqrt{9} \times \sqrt{2} + \sqrt{25} \times \sqrt{2} - 42 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= 2 \times 3\sqrt{2} + 5\sqrt{2} - 21\sqrt{2} = -10\sqrt{2}$$

3 $2\sqrt{27} - 3\sqrt{\frac{1}{3}} - \frac{6}{\sqrt{3}} = 2\sqrt{9 \times 3} - 3 \times \frac{\sqrt{1}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} - \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$$= 6\sqrt{3} - \sqrt{3} - \frac{6\sqrt{3}}{3}$$

$$= 6\sqrt{3} - \sqrt{3} - 2\sqrt{3} = 3\sqrt{3}$$

Example 3 Find the result of each of the following :

1 $2\sqrt{3}(\sqrt{6} + 5)$

2 $(3\sqrt{2} - 5)(3\sqrt{2} + 5)$

3 $(\sqrt{2} + \sqrt{6})^2$

Solution 1 $2\sqrt{3}(\sqrt{6} + 5) = 2\sqrt{3} \times \sqrt{6} + 2\sqrt{3} \times 5$

$$= 2\sqrt{18} + 10\sqrt{3}$$

$$= 2\sqrt{9 \times 2} + 10\sqrt{3}$$

$$= 6\sqrt{2} + 10\sqrt{3}$$

2 $(3\sqrt{2} - 5)(3\sqrt{2} + 5) = (3\sqrt{2})^2 - (5)^2$

$$= 3^2 \times (\sqrt{2})^2 - (5)^2$$

$$= 9 \times 2 - 25$$

$$= 18 - 25 = -7$$

 **Remember that**

$$(a - b)(a + b) = a^2 - b^2$$

$$\begin{aligned}
 3 \quad (\sqrt{2} + \sqrt{6})^2 &= (\sqrt{2})^2 + 2 \times \sqrt{2} \times \sqrt{6} + (\sqrt{6})^2 \\
 &= 2 + 2\sqrt{12} + 6 \\
 &= 8 + 2\sqrt{4 \times 3} = 8 + 4\sqrt{3}
 \end{aligned}$$



Remember that

$$\bullet (a + b)^2 = a^2 + 2ab + b^2$$

$$\bullet (a - b)^2 = a^2 - 2ab + b^2$$

Example 4

If $a = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{2}}$, find the value of $a^2 + 2\sqrt{3}$

Solution

To facilitate the solution, we will make the denominator an integer by multiplying both the numerator and the denominator by $\sqrt{2}$

$$\begin{aligned}
 \therefore a &= \frac{\sqrt{6} - \sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6} \times \sqrt{2} - \sqrt{2} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{12} - 2}{2} \\
 &= \frac{\sqrt{4 \times 3} - 2}{2} = \frac{2\sqrt{3} - 2}{2} = \frac{2(\sqrt{3} - 1)}{2} = \sqrt{3} - 1
 \end{aligned}$$

$$\therefore a^2 = (\sqrt{3} - 1)^2 = (\sqrt{3})^2 - 2 \times \sqrt{3} \times 1 + 1 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$$

$$\therefore a^2 + 2\sqrt{3} = 4 - 2\sqrt{3} + 2\sqrt{3} = 4$$

Another method to simplify a :

$$\therefore a = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{2}} \quad \therefore a = \frac{\sqrt{6}}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{\frac{6}{2}} - 1 = \sqrt{3} - 1$$

TRY YOURSELF 1

1 Simplify to the simplest form :

1 $\sqrt{75} - 2\sqrt{27} + \sqrt{3}$

2 $2\sqrt{50} - 3\sqrt{2} - 4\sqrt{\frac{1}{8}}$

2 Write each of the following such that the denominator is an integer :

1 $\frac{5\sqrt{3}}{2\sqrt{5}}$

2 $\frac{1 + \sqrt{3}}{3\sqrt{3}}$

(1) $\frac{9}{3 + \sqrt{3}}$
(2) $6\sqrt{2}$

(1) $\frac{2}{\sqrt{15}}$
(2) 0

of try by yourself

Answer



If a and b are two positive rational numbers

Then each of the two numbers $(\sqrt{a} + \sqrt{b})$ and $(\sqrt{a} - \sqrt{b})$ is conjugate to the other one and we find that :

• Their sum = $(\sqrt{a} + \sqrt{b}) + (\sqrt{a} - \sqrt{b}) = 2\sqrt{a} = \text{twice the first term.}$

• Their product = $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$

the square
of
1st term
the square
of
2nd term

For example: $(\sqrt{3} - \sqrt{2})$ its conjugate is $(\sqrt{3} + \sqrt{2})$, then we find that

• Their sum = $2\sqrt{3}$

• Their product = $3 - 2 = 1$

! Remark

The product of the two conjugate numbers is always a rational number.

! Remark

If we have a real number whose denominator is written in the form $(\sqrt{a} + \sqrt{b})$ or $(\sqrt{a} - \sqrt{b})$, we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator.

Example 1

Choose the correct answer from those given :

- 1 The number $\frac{4}{\sqrt{7}-\sqrt{3}}$ in the simplest form is
 (a) $\sqrt{7}-\sqrt{3}$ (b) $\sqrt{7}+\sqrt{3}$
 (c) $4\sqrt{7}-4\sqrt{3}$ (d) $4\sqrt{7}+4\sqrt{3}$
- 2 The conjugate of $\frac{1}{\sqrt{3}-\sqrt{2}}$ is
 (a) $\sqrt{3}-\sqrt{2}$ (b) $\sqrt{3}-2$ (c) $\sqrt{3}+\sqrt{2}$ (d) $\sqrt{3}+2$
- 3 The multiplicative inverse of $1-\sqrt{2}$ is
 (a) $\sqrt{2}-1$ (b) $1-\sqrt{2}$ (c) $-1-\sqrt{2}$ (d) $1+\sqrt{2}$
- 4 If $\frac{1}{x} = \sqrt{10}-3$, then $x =$
 (a) $\sqrt{10}+3$ (b) $\sqrt{10}-3$ (c) $3-\sqrt{10}$ (d) $-3-\sqrt{10}$

Solution

- 1 (b) **The reason :** Multiplying the two terms of the number by the conjugate of the denominator which is $(\sqrt{7}+\sqrt{3})$

$$\begin{aligned}\therefore \frac{4}{\sqrt{7}-\sqrt{3}} &= \frac{4}{\sqrt{7}-\sqrt{3}} \times \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} \\ &= \frac{4(\sqrt{7}+\sqrt{3})}{(\sqrt{7})^2 - (\sqrt{3})^2} = \frac{4(\sqrt{7}+\sqrt{3})}{7-3} \\ &= \sqrt{7}+\sqrt{3}\end{aligned}$$

- 2 (a) **The reason :** Multiplying the two terms of the number by the conjugate of the denominator which is $(\sqrt{3}+\sqrt{2})$

$$\begin{aligned}\therefore \frac{1}{\sqrt{3}-\sqrt{2}} &= \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} \\ &= \frac{\sqrt{3}+\sqrt{2}}{3-2} = \sqrt{3}+\sqrt{2}\end{aligned}$$

$$\therefore \text{The conjugate of } \frac{1}{\sqrt{3}-\sqrt{2}} \text{ is } \sqrt{3}-\sqrt{2}$$

3 (c) The reason : The multiplicative inverse of $1 - \sqrt{2}$ is $\frac{1}{1 - \sqrt{2}}$,

by multiplying the two terms of the number by the conjugate of the denominator which is $(1 + \sqrt{2})$

$$\begin{aligned}\therefore \frac{1}{1 - \sqrt{2}} &= \frac{1}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}} = \frac{1 + \sqrt{2}}{1 - 2} \\ &= \frac{1 + \sqrt{2}}{-1} = 1 - \sqrt{2}\end{aligned}$$

4 (a) The reason : $\because \frac{1}{x} = \sqrt{10} - 3 \quad \therefore x = \frac{1}{\sqrt{10} - 3}$

$$\therefore x = \frac{1}{\sqrt{10} - 3} \times \frac{\sqrt{10} + 3}{\sqrt{10} + 3} = \frac{\sqrt{10} + 3}{10 - 9} = \sqrt{10} + 3$$

Example 2 If $x = \frac{4}{2 - \sqrt{2}}$ and $y = \frac{3 - 2\sqrt{2}}{3 + 2\sqrt{2}}$, write each of x and y such that its denominator is a rational number, then find $x + y$

Solution $\therefore x = \frac{4}{2 - \sqrt{2}} \times \frac{2 + \sqrt{2}}{2 + \sqrt{2}} = \frac{4(2 + \sqrt{2})}{4 - 2} = \frac{4(2 + \sqrt{2})}{2}$

$$= 2(2 + \sqrt{2}) = 4 + 2\sqrt{2}$$

$$\begin{aligned}y &= \frac{3 - 2\sqrt{2}}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}} \\ &= \frac{(3 - 2\sqrt{2})^2}{9 - 8} = \frac{9 - 12\sqrt{2} + 8}{1} = 17 - 12\sqrt{2}\end{aligned}$$

$$\therefore x + y = 4 + 2\sqrt{2} + 17 - 12\sqrt{2} = 21 - 10\sqrt{2}$$



Write each of the following such that the denominator is a rational number :

1 $\frac{12}{\sqrt{6} - \sqrt{2}}$

2 $\frac{\sqrt{8}}{3 + 2\sqrt{2}}$

! Important remarks from direct product (multiplying by inspection)

• We know that : $(X - y)(X + y) = X^2 - y^2$

• And we know also :

$$(X + y)^2 = X^2 + 2Xy + y^2$$

Then

• $X^2 + Xy + y^2 = (X + y)^2 - Xy$

• $X^2 + y^2 = (X + y)^2 - 2Xy$

$$(X - y)^2 = X^2 - 2Xy + y^2$$

Then

• $X^2 - Xy + y^2 = (X - y)^2 + Xy$

• $X^2 + y^2 = (X - y)^2 + 2Xy$

Example 3

If $X = \frac{2}{\sqrt{5} - \sqrt{3}}$ and $y = \sqrt{5} - \sqrt{3}$, prove that X and y are conjugate

numbers, then find the value of each of :

1 $X^2 + 2Xy + y^2$

2 $X^2 + Xy + y^2$

Solution

$$\begin{aligned} \therefore X &= \frac{2}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{2(\sqrt{5} + \sqrt{3})}{5 - 3} \\ &= \frac{2(\sqrt{5} + \sqrt{3})}{2} = \sqrt{5} + \sqrt{3} \end{aligned}$$

• $\therefore y = \sqrt{5} - \sqrt{3}$

$\therefore X$ and y are conjugate numbers.

$$\begin{aligned} 1 \quad X^2 + 2Xy + y^2 &= (\sqrt{5} + \sqrt{3})^2 + 2(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) + (\sqrt{5} - \sqrt{3})^2 \\ &= (5 + 2\sqrt{15} + 3) + 2(5 - 3) + (5 - 2\sqrt{15} + 3) \\ &= 8 + 2\sqrt{15} + 4 + 8 - 2\sqrt{15} = 20 \end{aligned}$$

Another solution using the previous remarks :

Since $X^2 + 2Xy + y^2 = (X + y)^2$

$$\begin{aligned} \therefore X^2 + 2Xy + y^2 &= [(\sqrt{5} + \sqrt{3}) + (\sqrt{5} - \sqrt{3})]^2 \\ &= (2\sqrt{5})^2 = 4 \times 5 = 20 \end{aligned}$$

$$\begin{aligned}
 2 \quad x^2 + xy + y^2 &= (\sqrt{5} + \sqrt{3})^2 + (\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) + (\sqrt{5} - \sqrt{3})^2 \\
 &= (5 + 3 + 2\sqrt{15}) + (2) + (5 + 3 - 2\sqrt{15}) = 18
 \end{aligned}$$

Another solution using the previous remarks :

$$\begin{aligned}
 x^2 + xy + y^2 &= (x + y)^2 - xy = (\sqrt{5} + \sqrt{3} + \sqrt{5} - \sqrt{3})^2 - (\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) \\
 &= (2\sqrt{5})^2 - 2 = 20 - 2 = 18
 \end{aligned}$$

TRY 1

If $x = \frac{3}{2\sqrt{2} - \sqrt{5}}$ and $y = 2\sqrt{2} - \sqrt{5}$, find the value of the expression :
 $x^2 - y^2$

$$\boxed{2} \quad 8\sqrt{10}$$

$$\boxed{1} \quad 3\sqrt{6} + 3\sqrt{2}$$

$$\boxed{2} \quad -8 + 6\sqrt{2}$$

Answers / of try by yourself



If a and b are two real numbers , then :

1 $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{ab}$

For example:

- $\sqrt[3]{3} \times \sqrt[3]{9} = \sqrt[3]{3 \times 9} = \sqrt[3]{27} = 3$
- $\sqrt[3]{2} \times \sqrt[3]{-4} = \sqrt[3]{2 \times -4} = \sqrt[3]{-8} = -2$
- $\sqrt[3]{16} = \sqrt[3]{8 \times 2} = \sqrt[3]{8} \times \sqrt[3]{2} = 2\sqrt[3]{2}$
- $\sqrt[3]{-54} = \sqrt[3]{-27 \times 2} = \sqrt[3]{-27} \times \sqrt[3]{2} = -3\sqrt[3]{2}$

2 $\frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\frac{a}{b}}$ (where $b \neq 0$)

For example:

- | | |
|---|--|
| • $\frac{\sqrt[3]{32}}{\sqrt[3]{4}} = \sqrt[3]{\frac{32}{4}} = \sqrt[3]{8} = 2$ | • $\frac{\sqrt[3]{54}}{\sqrt[3]{-2}} = \sqrt[3]{-\frac{54}{2}} = \sqrt[3]{-27} = -3$ |
| • $\sqrt[3]{\frac{8}{125}} = \frac{\sqrt[3]{8}}{\sqrt[3]{125}} = \frac{2}{5}$ | • $\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$ |

Example 1 Find the result of each of the following in its simplest form :

1 $\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{\frac{4}{9}}$

2 $\sqrt[3]{\frac{5}{4}} \div \sqrt[3]{\frac{2}{25}}$

Solution

1 $\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{\frac{4}{9}} = \sqrt[3]{\frac{2}{3} \times \frac{4}{9}} = \sqrt[3]{\frac{8}{27}} = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

2 $\sqrt[3]{\frac{5}{4}} \div \sqrt[3]{\frac{2}{25}} = \sqrt[3]{\frac{5}{4} \div \frac{2}{25}} = \sqrt[3]{\frac{5}{4} \times \frac{25}{2}} = \sqrt[3]{\frac{125}{8}} = \sqrt[3]{\frac{125}{8}} = \frac{5}{2}$

! Remarks

If a and b are two real numbers , then :

① $\sqrt[3]{a^3 + b^3} \neq a + b$, $\sqrt[3]{a^3 - b^3} \neq a - b$

② $\sqrt[3]{-a} = -\sqrt[3]{a}$

③ $a\sqrt[3]{b} = \sqrt[3]{a^3b}$

For example: • $3\sqrt[3]{\frac{1}{9}} = \sqrt[3]{27 \times \frac{1}{9}} = \sqrt[3]{3}$

• $8\sqrt[3]{\frac{1}{4}} = 4 \times 2\sqrt[3]{\frac{1}{4}} = 4\sqrt[3]{8 \times \frac{1}{4}} = 4\sqrt[3]{2}$

④ $\sqrt[3]{\frac{a}{b}} = \sqrt[3]{\frac{a}{b} \times \frac{b^2}{b^2}} = \sqrt[3]{\frac{ab^2}{b^3}} = \frac{1}{b}\sqrt[3]{ab^2}$ (Where $b \neq 0$)

For example: $\sqrt[3]{\frac{1}{3}} = \sqrt[3]{\frac{1}{3} \times \frac{9}{9}} = \sqrt[3]{\frac{9}{27}} = \frac{1}{3}\sqrt[3]{9}$

Example 2 Put each of the following in its simplest form :

1 $\sqrt[3]{24} + \sqrt[3]{3} - \sqrt[3]{81}$

2 $\sqrt[3]{54} + 6\sqrt[3]{16} - 6\sqrt[3]{\frac{1}{4}}$

3 $\sqrt[3]{81} + \sqrt[3]{12} - 2\sqrt[3]{3} - 2\sqrt[3]{3}$

Solution

$$\begin{aligned}
 1 \quad \sqrt[3]{24} + \sqrt[3]{3} - \sqrt[3]{81} &= \sqrt[3]{8 \times 3} + \sqrt[3]{3} - \sqrt[3]{27 \times 3} \\
 &= \sqrt[3]{8} \times \sqrt[3]{3} + \sqrt[3]{3} - \sqrt[3]{27} \times \sqrt[3]{3} \\
 &= 2\sqrt[3]{3} + \sqrt[3]{3} - 3\sqrt[3]{3} = \text{zero}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad \sqrt[3]{54} + 6\sqrt[3]{16} - 6\sqrt[3]{\frac{1}{4}} &= \sqrt[3]{27 \times 2} + 6\sqrt[3]{8 \times 2} - 3 \times 2\sqrt[3]{\frac{1}{4}} \\
 &= \sqrt[3]{27} \times \sqrt[3]{2} + 6 \times \sqrt[3]{8} \times \sqrt[3]{2} - 3 \times \sqrt[3]{8 \times \frac{1}{4}} \\
 &= 3 \times \sqrt[3]{2} + 6 \times 2 \times \sqrt[3]{2} - 3 \times \sqrt[3]{2} \\
 &= 3\sqrt[3]{2} + 12\sqrt[3]{2} - 3\sqrt[3]{2} = 12\sqrt[3]{2}
 \end{aligned}$$

Another solution :

$$\begin{aligned}
 \therefore \sqrt[3]{\frac{1}{4}} &= \sqrt[3]{\frac{1}{4} \times \frac{16}{16}} = \sqrt[3]{\frac{16}{64}} = \frac{\sqrt[3]{16}}{\sqrt[3]{64}} = \frac{1}{4}\sqrt[3]{16} \\
 &= \frac{1}{4}\sqrt[3]{8 \times 2} = \frac{1}{4} \times 2\sqrt[3]{2} = \frac{1}{2}\sqrt[3]{2}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \sqrt[3]{54} + 6\sqrt[3]{16} - 6\sqrt[3]{\frac{1}{4}} &= 3\sqrt[3]{2} + 6 \times 2\sqrt[3]{2} - 6 \times \frac{1}{2}\sqrt[3]{2} \\
 &= 3\sqrt[3]{2} + 12\sqrt[3]{2} - 3\sqrt[3]{2} = 12\sqrt[3]{2}
 \end{aligned}$$

One more solution :

$$\therefore \sqrt[3]{\frac{1}{4}} = \sqrt[3]{\frac{1}{4} \times \frac{2}{2}} = \sqrt[3]{\frac{2}{8}} = \frac{\sqrt[3]{2}}{\sqrt[3]{8}} = \frac{\sqrt[3]{2}}{2}$$

$$\begin{aligned}
 \therefore \sqrt[3]{54} + 6\sqrt[3]{16} - 6\sqrt[3]{\frac{1}{4}} &= 3\sqrt[3]{2} + 12\sqrt[3]{2} - 6 \times \frac{\sqrt[3]{2}}{2} \\
 &= 3\sqrt[3]{2} + 12\sqrt[3]{2} - 3\sqrt[3]{2} = 12\sqrt[3]{2}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad \sqrt[3]{81} + \sqrt[3]{12} - 2\sqrt[3]{3} - 2\sqrt{3} &= \sqrt[3]{27 \times 3} + \sqrt[3]{4 \times 3} - 2\sqrt[3]{3} - 2\sqrt{3} \\
 &= \sqrt[3]{27} \times \sqrt[3]{3} + \sqrt[3]{4} \times \sqrt[3]{3} - 2\sqrt[3]{3} - 2\sqrt{3} \\
 &= 3\sqrt[3]{3} + 2\sqrt[3]{3} - 2\sqrt[3]{3} - 2\sqrt{3} = \sqrt[3]{3}
 \end{aligned}$$

Example 3 Find in the simplest form : $2\sqrt[3]{4} \left(5\sqrt[3]{\frac{1}{2}} - \sqrt[3]{32} \right)$

Solution

$$\begin{aligned} 2\sqrt[3]{4} \left(5\sqrt[3]{\frac{1}{2}} - \sqrt[3]{32} \right) &= 2 \times 5 \sqrt[3]{4 \times \frac{1}{2}} - 2 \times \sqrt[3]{4 \times 32} \\ &= 10\sqrt[3]{2} - 2 \times \sqrt[3]{128} = 10\sqrt[3]{2} - 2 \times \sqrt[3]{64 \times 2} \\ &= 10\sqrt[3]{2} - 2 \times 4\sqrt[3]{2} = 10\sqrt[3]{2} - 8\sqrt[3]{2} = 2\sqrt[3]{2} \end{aligned}$$

Example 4 If $x = \sqrt[3]{5} + 2$ and $y = \sqrt[3]{5} - 2$, find the value of $(x + y)^3 - (x - y)^3$

Solution

$$\begin{aligned} \therefore x + y &= \sqrt[3]{5} + 2 + \sqrt[3]{5} - 2 = 2\sqrt[3]{5} \\ x - y &= \sqrt[3]{5} + 2 - (\sqrt[3]{5} - 2) = \sqrt[3]{5} + 2 - \sqrt[3]{5} + 2 = 4 \\ \therefore (x + y)^3 - (x - y)^3 &= (2\sqrt[3]{5})^3 - (4)^3 = 2^3 \times (\sqrt[3]{5})^3 - 4^3 \\ &= 8 \times 5 - 64 = 40 - 64 = -24 \end{aligned}$$

TRY
yourself

Simplify each of the following to the simplest form :

1 $5\sqrt[3]{2} - \sqrt[3]{16} + \sqrt[3]{-54}$

2 $\sqrt[3]{72} + \sqrt[3]{\frac{1}{3}} + \sqrt[3]{-9}$

2 $6\sqrt[3]{\frac{3}{4}}$

1 0

of try by yourself

Answers

Applications on the real numbers

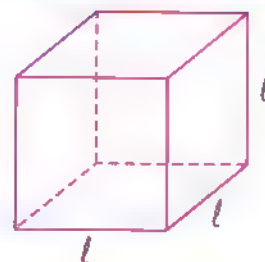


The cube

It is a solid whose six faces are congruent squares.

i.e. All its edges are equal in length.

Assuming that the edge length of the cube = l length unit, then :



- 1 The area of each face = l^2 square unit.
- 2 Its lateral area = $4l^2$ square unit.
- 3 Its total area (the area of its 6 faces) = $6l^2$ square unit.
- 4 Its volume = l^3 cube unit.

Example 1

Choose the correct answer from those given :

- 1 A cube of volume 64 cm^3 , then the sum of its edge lengths is
 (a) 16 cm. (b) 32 cm. (c) 48 cm. (d) 64 cm.
- 2 A cube of volume 125 cm^3 , then its total area =
 (a) 200 cm^2 (b) 150 cm^2 (c) 125 cm^2 (d) 25 cm^2
- 3 A cube of volume 216 cm^3 , then its lateral area =
 (a) 36 cm^2 (b) 72 cm^2 (c) 144 cm^2 (d) 216 cm^2

- 4 The lateral area of a cube is 4 cm^2 , then its volume = ..
 (a) 1 cm^3 (b) 2 cm^3 (c) 4 cm^3 (d) 16 cm^3

- 5 The total area of a cube is 294 cm^2 , then its lateral area =
 (a) 28 cm^2 (b) 49 cm^2 (c) 196 cm^2 (d) 343 cm^2

Solution

- 1 (c) The reason : \because The volume of the cube $= l^3$ where l is its edge length
 $\therefore l^3 = 64$ $\therefore l = \sqrt[3]{64} = 4 \text{ cm}$.
 \therefore The sum of the edge lengths $= 12l = 12 \times 4 = 48 \text{ cm}$.
- 2 (b) The reason : \because The volume of the cube $= l^3$ where l is its edge length
 $\therefore l^3 = 125$ $\therefore l = \sqrt[3]{125} = 5 \text{ cm}$.
 \therefore The total area of the cube $= 6l^2 = 6 \times 5^2 = 150 \text{ cm}^2$.
- 3 (c) The reason : \because The volume of the cube $= l^3$ where l is its edge length
 $\therefore l^3 = 216$ $\therefore l = \sqrt[3]{216} = 6 \text{ cm}$.
 \therefore The lateral area of the cube $= 4l^2 = 4 \times 6^2 = 144 \text{ cm}^2$.
- 4 (a) The reason : \because The lateral area of the cube $= 4l^2$ where l is its edge length
 $\therefore 4l^2 = 4$ $\therefore l^2 = 1$ $\therefore l = \sqrt{1} = 1 \text{ cm}$.
 \therefore The volume of the cube $= l^3 = 1^3 = 1 \text{ cm}^3$.
- 5 (c) The reason : \because The total area of the cube $= 6l^2$ where l is its edge length
 $\therefore 6l^2 = 294$ $\therefore l^2 = \frac{294}{6} = 49$
 \therefore The lateral area $= 4l^2 = 4 \times 49 = 196 \text{ cm}^2$.

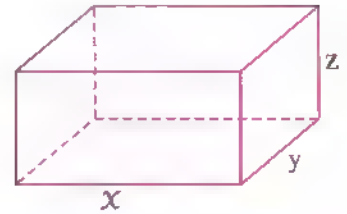
TRY 1

Complete the following table :

	Edge length of the cube	Area of one face	Lateral area	Total area	Volume
1	3 cm.
2	49 cm^2
3	144 cm^2
4	150 cm^2
5	64 cm^3

The cuboid

It is a solid that contains 6 faces, each of them is a rectangle and each two opposite faces are congruent. Assuming that the lengths of the edges of the cuboid are x , y and z length unit, then :



- 1 Its lateral area = the perimeter of the base \times height = $2(x + y) \times z$ square unit.
- 2 Its total area (the area of its six faces) = the lateral area + twice the area of the base

$$= 2(x + y) \times z + 2xy$$

$$= 2(xy + yz + zx) \text{ square unit.}$$
- 3 Its volume = the area of the base \times the height

$$= x \times y \times z \text{ cube unit.}$$



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Remarks

- The cuboid may contain two opposite faces, each of them is a square.
- The cube is a special case of the cuboid.
i.e. The cube is a cuboid with edges having the same length.

Example 2

The height of a cuboid is 4 cm. and its base is a square of side length 5 cm. Find :

- 1 Its volume.
- 2 Its lateral area.
- 3 Its total area.

Solution

- 1 The volume of the cuboid = the area of the base \times the height

$$= 5 \times 5 \times 4 = 100 \text{ cm}^3$$
- 2 The lateral area of the cuboid = the perimeter of the base \times the height

$$= 4 \times 5 \times 4 = 80 \text{ cm}^2$$
- 3 The total area of the cuboid

$$= \text{the lateral area} + \text{twice the area of the base} = 80 + 2 \times 25 = 130 \text{ cm}^2$$

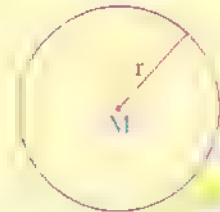
TRY 2

The dimensions of a cuboid are 3 cm, 4 cm, and 5 cm. Calculate its volume and its total area.

The circle

If M is a circle with radius length r , then :

- 1 The circumference of the circle $= 2 \pi r$ length unit.
- 2 The area of the circle $= \pi r^2$ square unit.



Example 3 The area of a circle is $25 \pi \text{ cm}^2$. Calculate its circumference in terms of π

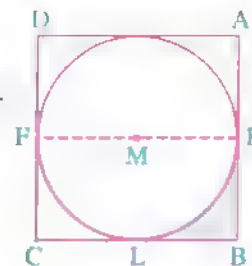
Solution

$$\begin{aligned} \therefore \text{The area of the circle} &= \pi r^2 & \therefore \pi r^2 &= 25 \pi \\ \therefore r^2 &= 25 & \therefore r &= \sqrt{25} = 5 \text{ cm.} \\ \therefore \text{The circumference of the circle} &= 2 \pi r \\ &= 2 \times 5 \times \pi = 10 \pi \text{ cm.} \end{aligned}$$

Example 4 In the opposite figure :

A circle M is drawn inside a square (touching its sides).
If the area of the square $= 196 \text{ cm}^2$, find :

- 1 The area of the shaded part.
- 2 The perimeter of the shaded part.



Solution

$$\begin{aligned} \therefore \text{The area of the square} &= 196 \text{ cm}^2 \\ \therefore \text{The side length of the square} &= \sqrt{196} = 14 \text{ cm.} \\ \therefore \text{the side length of the square} &= 2r \\ \therefore 14 &= 2r & \therefore r &= 7 \text{ cm.} \end{aligned}$$

- 1 The area of the shaded part

$$\begin{aligned} &= (\text{the area of the square} - \text{the area of the circle}) \div 4 \\ &= \left(196 - \frac{22}{7} \times 7 \times 7 \right) \div 4 = 42 \div 4 = 10.5 \text{ cm}^2 \end{aligned}$$

- 2 The perimeter of the shaded part

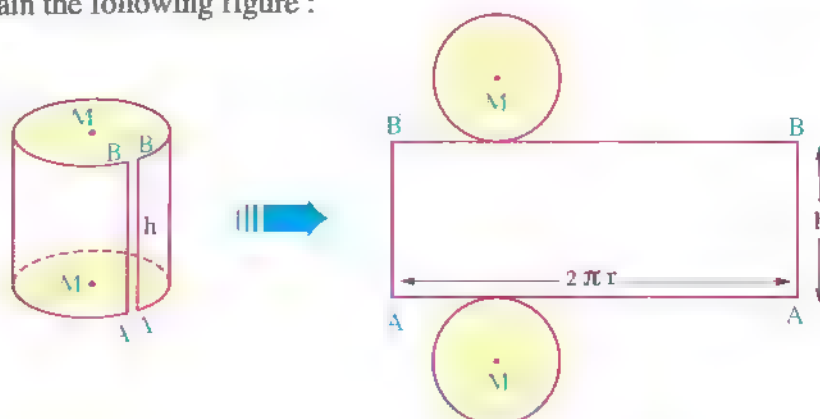
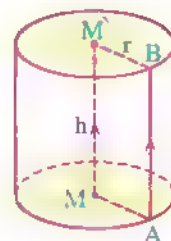
$$\begin{aligned} &= BE + BL + \frac{1}{4} \text{ circumference of the circle} = 7 + 7 + \left(\frac{1}{4} \times 2 \times \frac{22}{7} \times 7 \right) \\ &= 14 + 11 = 25 \text{ cm.} \end{aligned}$$

TRY YOURSELF 3

The circumference of a circle is 88 cm. Find its area. $(\pi = \frac{22}{7})$

The right circular cylinder

- It is a solid having two parallel congruent bases, each of them is a circular-shaped surface while its lateral surface is a curved surface which is called cylindrical surface.
- The line segment $\overline{MM'}$ drawn between the two centres of the two bases is perpendicular to each plane of the two bases and it is called the height of the cylinder.
- If we draw \overline{AB} on the cylindrical surface such that $A \in \text{the circle } M$, $B \in \text{the circle } M'$, $\overline{AB} \parallel \overline{MM'}$ and if we cut the lateral surface of the cylinder at \overline{AB} and flattened it out, then we will obtain the following figure :



This figure consists of the surface of the rectangle $ABB'A'$ and it is the same cylindrical surface of the cylinder in addition to the two surfaces of two circles which represent the two bases of the cylinder, then we find :

$AB = \text{the height of the cylinder.}$

$A'A = \text{the circumference of the base of the cylinder.}$

$\therefore \text{The lateral area of the cylinder} = \text{the area of the rectangle } ABB'A' = A'A \times AB$

$= \text{the circumference of the base of the cylinder} \times \text{its height}$

and if we assume that the length of the radius of the base = r and its height = h , then :

- 1 The lateral area of the cylinder = $2 \pi r h$ square unit.
- 2 The total area of the cylinder = the lateral area of the cylinder + twice the area of the base = $2 \pi r h + 2 \pi r^2$ square unit.
- 3 The volume of the cylinder = the area of the base \times height = $\pi r^2 h$ cube unit.



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Example 5 A right circular cylinder is of height 10 cm. and its volume is 1540 cm^3 . Find its total area ($\pi = \frac{22}{7}$)

Solution \therefore The volume of the cylinder = $\pi r^2 h$

$$\therefore 1540 = \frac{22}{7} \times r^2 \times 10$$

$$\therefore 1540 = \frac{220}{7} r^2$$

$$\therefore r^2 = 1540 \times \frac{7}{220} = 49$$

$$\therefore r = \sqrt{49} = 7 \text{ cm.}$$

$$\therefore \text{The total area of the cylinder} = 2 \pi r h + 2 \pi r^2$$

$$= 2 \times \frac{22}{7} \times 7 \times 10 + 2 \times \frac{22}{7} \times 7^2$$

$$= 440 + 308 = 748 \text{ cm}^2$$

TRY YOURSELF 4

A right circular cylinder is of volume $90 \pi \text{ cm}^3$ and its height is 10 cm. Find the diameter length of its base.

The sphere

- It is a solid with a curved surface whose all points are equidistant from a fixed point inside the sphere.
 - The equal distances are called the radius length of the sphere.
 - The fixed point is called the centre of the sphere.
 - If we cut the sphere by a plane passing through its centre, then the resulted section is a circle having the same centre of the sphere and its radius length is the same of the sphere.
- Assuming that the radius length of the sphere = r , then :





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1 The area of the sphere = $4 \pi r^2$ square unit.

2 The volume of the sphere = $\frac{4}{3} \pi r^3$ cube unit.

Example 6 The volume of a sphere = $\frac{500}{3} \pi \text{ cm}^3$. Find the length of its diameter.

Solution

$$\therefore \text{The volume of the sphere} = \frac{4}{3} \pi r^3 \qquad \therefore \frac{500}{3} \pi = \frac{4}{3} \pi r^3$$

$$\therefore r^3 = \frac{500}{3} \times \frac{3}{4} = 125 \qquad \therefore r = \sqrt[3]{125} = 5 \text{ cm.}$$

$$\therefore \text{The diameter length of the sphere} = 2 \times 5 = 10 \text{ cm.}$$

Example 7 A right circular cylinder is of height 6 cm. and its volume = $\frac{2}{3}$ the volume of a sphere whose radius length is 3 cm.

Find the radius length of the base of the cylinder.

Solution

Let the radius length of the sphere be r_1 cm. and the radius length of the base of the cylinder be r_2 cm.

$$\therefore \text{The volume of the sphere} = \frac{4}{3} \pi r_1^3 = \frac{4}{3} \pi (3)^3 = 36 \pi \text{ cm}^3$$

$$\therefore \text{The volume of the cylinder} = \frac{2}{3} \text{ the volume of the sphere.}$$

$$\therefore \pi r_2^2 h = \frac{2}{3} \times 36 \pi \qquad \therefore r_2^2 \times 6 = 24$$


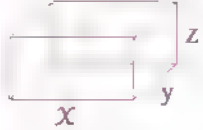


$$\therefore r_2^2 = 4 \qquad \therefore r_2 = \sqrt{4} = 2 \text{ cm.}$$

$$\therefore \text{The radius length of the base of the cylinder} = 2 \text{ cm.}$$

TRY YOURSELF 5

The area of a sphere is $36 \pi \text{ cm}^2$. Find its volume in terms of π

In the following , we will summarize the previous rules of areas and volumes of some solids :

The solid		The lateral area	The total area	The volume
The cube		$4l^2$	$6l^2$	l^3
The cuboid		$2(x+y) \times z$	$2(xy + yz + zx)$	xyz
The cylinder		$2\pi rh$	$2\pi rh + 2\pi r^2$ $= 2\pi r(h+r)$	$\pi r^2 h$
The sphere		—	$4\pi r^2$	$\frac{4}{3}\pi r^3$

Answers of try by yourself

1 9 cm², 36 cm², 54 cm², 27 cm³

2 7 cm², 196 cm², 294 cm², 343 cm³

3 6 cm², 36 cm², 216 cm³

4 5 cm², 25 cm², 100 cm², 125 cm³

5 4 cm², 16 cm², 64 cm², 96 cm²

2 The volume = 60 cm³, the total area = 94 cm²

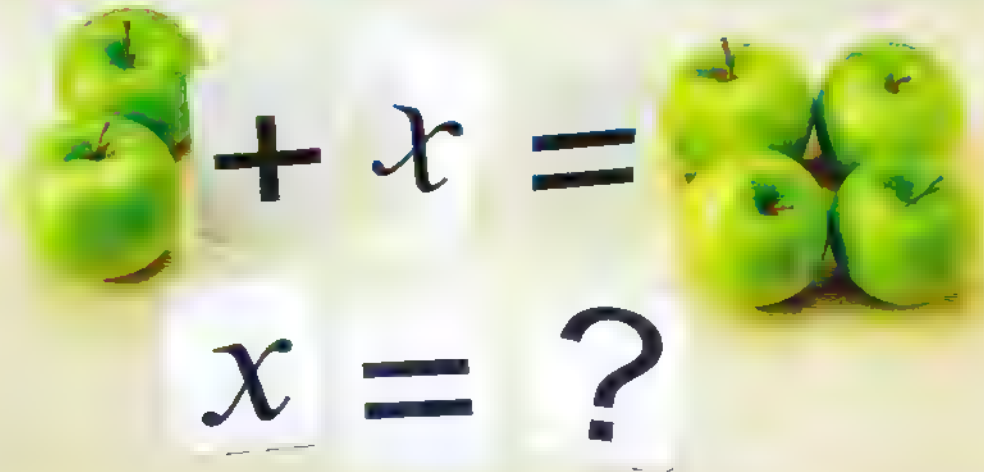
3 616 cm²

4 6 cm²

5 36 π cm³

10

Solving equations and inequalities of the first degree in one variable in \mathbb{R}



First Solving equations of the first degree in one unknown in \mathbb{R}

* Each of the equations :

$$2x - 5 = 3$$

$$\sqrt{3}x - 1 = 8$$

$$\frac{1}{2}x - \sqrt{5} = 0$$

is called an **equation** of the first degree in one variable (**one unknown**) which is x because the exponent of the variable x equals one.

* Solving the equation of the first degree in one variable means finding the real number which satisfies this equation.

* The following examples will show how to solve an equation of the first degree in one variable :

Example 1 Find in \mathbb{R} the S.S. of each of the following equations , then represent the solution on the number line :

1 $3x + 2 = 1$

2 $\sqrt{3}x - 1 = 2$

3 $7x - \sqrt{7} = 6\sqrt{7}$

4 $x - \sqrt{5} = 1$

Solution 1 $\therefore 3x + 2 = 1$ (adding -2 to both sides)

$$\therefore 3x + 2 - 2 = 1 - 2 \quad \therefore 3x = -1$$

(multiplying both sides by $\frac{1}{3}$ the multiplicative inverse of the coefficient of x)

$$\therefore 3x \times \frac{1}{3} = -1 \times \frac{1}{3} \quad \therefore x = -\frac{1}{3} \quad \therefore \text{The S.S.} = \left\{-\frac{1}{3}\right\}$$

• We can represent the number $-\frac{1}{3}$ on the number line as follows :



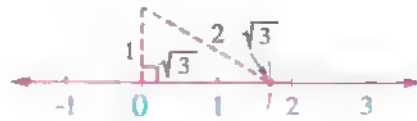
$$2 \quad \therefore \sqrt{3}x - 1 = 2 \quad \therefore \sqrt{3}x = 2 + 1$$

$$\therefore \sqrt{3}x = 3 \quad \therefore x = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\therefore x = \frac{3\sqrt{3}}{3} \quad \therefore x = \sqrt{3}$$

$$\therefore \text{The S.S.} = \{\sqrt{3}\}$$

• We can represent the number $\sqrt{3}$ on the number line as follows :



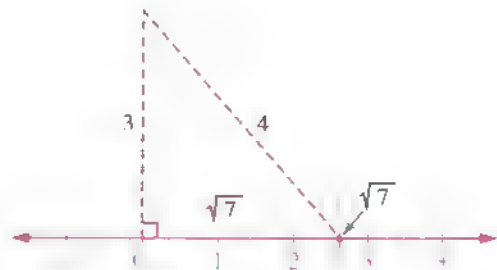
$$3 \quad \therefore 7x - \sqrt{7} = 6\sqrt{7} \quad \therefore 7x = 6\sqrt{7} + \sqrt{7}$$

$$\therefore 7x = 7\sqrt{7} \quad \therefore x = \frac{7\sqrt{7}}{7}$$

$$\therefore x = \sqrt{7}$$

$$\therefore \text{The S.S.} = \{\sqrt{7}\}$$

• We can represent the number $\sqrt{7}$ on the number line as follows :

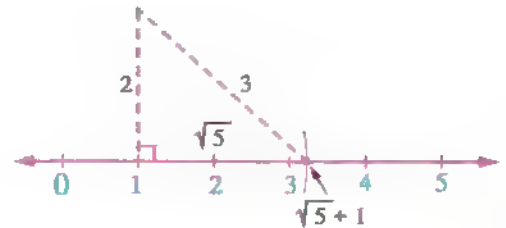


$$4 \because x - \sqrt{5} = 1$$

$$\therefore x = 1 + \sqrt{5}$$

$$\therefore \text{The S.S.} = \{1 + \sqrt{5}\}$$

- We can represent the number $(1 + \sqrt{5})$ on the number line as follows :



TRY YOURSELF 1

Find in \mathbb{R} the S.S. of each of the following equations, then represent the solution on the number line :

$$\textcircled{1} 2x + 5 = 4$$

$$\textcircled{2} \sqrt{5}x - 1 = 4$$

$$\textcircled{3} x - \sqrt{3} = 2$$

Solving inequalities of the first degree in one unknown in \mathbb{R}

- Each of the inequalities :

$$2x < 5$$

$$3x + 2 \leq 1$$

$$5 + x > 2x - 1 \geq 3 + x$$

is called an inequality of the first degree in one unknown denoted by x

- Solving the inequality means finding all values of the unknown (x) which satisfy this inequality.
- The S.S. of the inequality in \mathbb{R} will be written as an interval as will be shown later.



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The methods of solving these inequalities in \mathbb{R} depend on the properties of the inequality relation which will be summarized in the following :

Let a , b and c be three real numbers and assuming that $a < b$, then :

$a + c < b + c$ $\xrightarrow{\text{whether}}$ c is **positive** or **negative** (the addition property)

$ac < bc$ $\xrightarrow{\text{if}}$ c is **positive** (the property of multiplying by a positive real number)

$ac > bc$ $\xrightarrow{\text{if}}$ c is **negative** (the property of multiplying by a negative real number)

i.e. when we multiply (or divide) the two sides of an inequality by a negative number, we should change the symbol of the inequality.



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Example 2 Find in \mathbb{R} the S.S. of each of the following inequalities , then represent the solution on the number line :

1 $2x + 6 < 2$

2 $5 - 4x \leq -3$



Solution

1 $\therefore 2x + 6 < 2$ (adding the additive inverse of the number 6 (it is -6) to both sides)

$$\therefore 2x + 6 - 6 < 2 - 6$$

$$\therefore 2x < -4$$

(multiplying both sides by the multiplicative inverse of the number 2 (it is $\frac{1}{2}$))

$$\therefore 2x \times \frac{1}{2} < -4 \times \frac{1}{2}$$

$$\therefore x < -2$$

\therefore The S.S. is all the real numbers which are less than -2

i.e. The S.S. = $] -\infty, -2[$



2 $\therefore 5 - 4x \leq -3$ (adding -5 to both sides)

$$\therefore -4x \leq -8$$
 (dividing both sides by -4)

$$\therefore x \geq 2$$

(Notice the change in the symbol of the inequality because we divided by a negative number)

\therefore The S.S. = $[2, \infty[$



Example 3 Find in \mathbb{R} the S.S. of each of the following inequalities , then represent the solution on the number line :

1 $-3 < 2x - 1 \leq 5$

2 $3 < 3 - 5x < 13$

Solution 1 $\therefore -3 < 2x - 1 \leq 5$ (adding 1 to all sides)

$$\therefore -2 < 2x \leq 6$$
 (dividing all sides by 2)

$$\therefore -1 < x \leq 3$$

\therefore The S.S. = $] -1, 3]$



2 $\therefore 3 < 3 - 5x < 13$ (subtracting 3 from all sides)

$$\therefore 0 < -5x < 10$$
 (dividing all sides by -5)

$$\therefore 0 > x > -2$$

(Notice the change in the symbols of the inequality because we divided by a negative number)

\therefore The S.S. = $] -2, 0[$



Example 4 Find in \mathbb{R} the S.S. of each of the following inequalities :

1 $x - 2 \geq 3x - 5$

2 $x - 1 < 3x - 3 \leq x + 5$

Solution

1 $\therefore x - 2 \geq 3x - 5$ (adding 2 to both sides)

$\therefore x \geq 3x - 3$ (adding $-3x$ to both sides)

$\therefore -2x \geq -3$ (multiplying both sides by $-\frac{1}{2}$)

$\therefore x \leq \frac{3}{2}$ (Notice the change in the symbol of the inequality)

\therefore The S.S. = $]-\infty, \frac{3}{2}]$

2 $\therefore x - 1 < 3x - 3 \leq x + 5$ (adding 3 to all sides)

$\therefore x + 2 < 3x \leq x + 8$ (adding $-x$ to all sides)

$\therefore 2 < 2x \leq 8$ (multiplying by $\frac{1}{2}$)

$\therefore 1 < x \leq 4 \quad \therefore$ The S.S. = $]1, 4]$

Another solution for number (2) :

We can divide this inequality into two inequalities as follows :

$x - 1 < 3x - 3 \longrightarrow (1)$ and $3x - 3 \leq x + 5 \longrightarrow (2)$

Then the solution set of the origin inequality is the intersection set of the two sets of solutions of the two inequalities (1) and (2)

• **Finding the S.S. of the inequality (1) :**

$\therefore x - 1 < 3x - 3$ (adding 1 to both sides)

$\therefore x < 3x - 2$ (adding $-3x$ to both sides)

$\therefore -2x < -2$ (multiplying both sides by $-\frac{1}{2}$)

$\therefore x > 1 \quad \therefore$ The S.S. = $]1, \infty[$

• **Finding the S.S. of the inequality (2) :**

$$\therefore 3x - 3 \leq x + 5 \quad (\text{adding } 3 \text{ to both sides})$$

$$\therefore 3x \leq x + 8 \quad (\text{adding } -x \text{ to both sides})$$

$$\therefore 2x \leq 8 \quad \left(\text{multiplying both sides by } \frac{1}{2} \right)$$

$$\therefore x \leq 4 \quad \therefore \text{The S.S.} =]-\infty, 4]$$

• The S.S. of the origin inequality = $]1, \infty[\cap]-\infty, 4] =]1, 4]$

TRY
by Yourself

2

Find in \mathbb{R} the S.S. of each of the following inequalities :

1 $3x - 1 > 8$

2 $2 - 2x \geq -6$

3 $-16 < 5x + 4 \leq 9$

4 $2x + 1 > 4x - 3 > 2x - 11$

4 $] -4, 2[$

3 $] -4, 1[$

3 $\{2 + \sqrt{3}\}$

2 $] -\infty, 4]$

2 $\{\sqrt{5}\}$

2 $] 3, \infty[$

1 $\{-\frac{1}{2}\}$

Answers / of try by yourself



UNIT

2

Relation between Two Variables

Lessons of the unit :

1. Relation between two variables.
2. Slope of straight line.
3. Real life applications on the slope.

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Unit Objectives :

By the end of this unit, student should be able to :

- recognize the relation between two variables of first degree.
- represent the relation between two variables of first degree graphically.
- recognize the slope of the straight line.
- find the slope of the straight line passing through two given points.
- recognize the slope of the straight line parallel to x-axis and the slope of the straight line parallel to y-axis.
- verify using the slope of the straight line that the three points are collinear or not.
- find the uniform velocity of a car by using the slope of the straight line.
- solve applications on the slope of the straight line.

• •

Relation between two variables



The concept of the relation between two variables

- Islam has 50 pounds. If Islam went to the amusement park, he would find two kinds of favourite games :

The first kind

costs 5 pounds for playing one game.



The second kind

costs 10 pounds for playing one game.

- What are the different possibilities for playing the two kinds such that he spends all his money ?
- To find all the possibilities :

- Assume that he will play x games of the first kind and y games of the second kind.
- Then the cost of playing the first kind is $5x$ pounds and the cost of playing the second kind is $10y$ pounds.
- In order to spend all his money, it should be : $5x + 10y = 50$
- This is an algebraic relation between the two variables x and y and it is called an equation of the first degree in two variables.

- We can simplify the previous relation by dividing all terms by 5 to get an equivalent equation which is : $x + 2y = 10$

It can be written also in the form : $2y = 10 - x$

i.e. $y = \frac{10 - x}{2}$

$$5x + 10y = 50 \quad \div 5$$

$$x + 2y = 10$$

$$2y = 10 - x \quad \div 2$$

$$y = \frac{10 - x}{2}$$

For example:

- If Islam decided that he will not play the first kind.

i.e. $x = 0$, then $y = \frac{10 - 0}{2} = 5$

- i.e. He can spend all his money by playing 5 games of the second kind.

We express that by the ordered pair (0 , 5)

- If he decided to play one game of the first kind.

i.e. $x = 1$, then $y = \frac{10 - 1}{2} = 4\frac{1}{2}$

but in this case , he cannot play $4\frac{1}{2}$ games of the second kind because the number of games must be a natural number.

- If he decided to play two games of the first kind

i.e. $x = 2$, then $y = \frac{10 - 2}{2} = 4$

- i.e. He can spend all his money by playing 2 games of the first kind and 4 games of the second kind . We express that by the ordered pair (2 , 4)

Thus we can know the different possibilities and put them in a table such as the following :

Number of games of the 1 st kind (x)	0	2	4	6	8	10
Number of games of the 2 nd kind (y)	5	4	3	2	1	0

! Remarks

- There is an infinite number of ordered pairs which satisfy the previous relation but some of them can't represent the possible numbers of each games because the number of games must be a natural number.
 - As we mentioned before $(1, 4\frac{1}{2})$ satisfies the relation but it is not possible to represent the number of games because $4\frac{1}{2} \notin \mathbb{N}$
 - Similarly $(-2, 6)$ satisfies the relation but it is not to be used because $-2 \notin \mathbb{N}$
- To find all the possibilities , we write the equation : $x + 2y = 10$ putting y in one hand side as : $y = \frac{10 - x}{2}$
 We can also put x in one hand side as : $x = 10 - 2y$
 And the following example shows that.

Example 1

What are the different possibilities for a person to pay L.E. 45 using two kinds of bills (banknotes) of L.E. 5 and L.E. 10 ?

Solution

Let the number of bills of L.E. 5 be x , then its value = $5x$ pounds
and the number of bills of L.E. 10 be y , then its value = $10y$ pounds.

$$\therefore 5x + 10y = 45 \text{ , dividing the two sides by 5}$$

$$\therefore x + 2y = 9$$

Putting x in one hand side , then the equation will be in the form :

$$x = 9 - 2y$$

The following table shows all possibilities to pay the sum of money :

y	x	(x, y)	Number of bills of each kind
0	$9 - 2 \times 0 = 9$	$(9, 0)$	9 bills of 5 pounds
1	$9 - 2 \times 1 = 7$	$(7, 1)$	7 bills of 5 pounds and 1 bill of 10 pounds
2	$9 - 2 \times 2 = 5$	$(5, 2)$	5 bills of 5 pounds and 2 bills of 10 pounds
3	$9 - 2 \times 3 = 3$	$(3, 3)$	3 bills of 5 pounds and 3 bills of 10 pounds
4	$9 - 2 \times 4 = 1$	$(1, 4)$	1 bill of 5 pounds and 4 bills of 10 pounds

Notice that :

If $y = 5$, then $x = -1 \notin \mathbb{N}$, then $y = 5$ is impossible.

TRY 1
by yourself

Find the different possibilities for a person to pay L.E. 65 of bills (banknotes) of L.E. 5 and L.E. 20

The linear relation

- The linear relation is a relation of the first degree between two variables x and y , it is in the form

$ax + by = c$, where a , b and c are real numbers , a and b are not both equal to zero

- There is an infinite number of ordered pairs which satisfy this relation.

- If we represent it graphically, the graph will be a straight line therefore it is called a linear relation, this will be shown later when we study the graphic representation of the linear relation.

Example 2 Find three ordered pairs satisfying each of the following relations :

1 $3x + y = 5$

2 $3x - 2y = 6$

3 $2x = 3$

4 $y = -2$

Solution We can find these ordered pairs by setting a value for x and substituting in the relation to get its corresponding value of y or we do the converse :

- 1 • Set $x = 0$

$$\therefore 3 \times 0 + y = 5$$

$$\therefore y = 5$$

$\therefore (0, 5)$ satisfies the relation.

- Set $x = 1$

$$\therefore 3 \times 1 + y = 5$$

$$\therefore y = 5 - 3 = 2$$

$\therefore (1, 2)$ satisfies the relation.

- Set $x = -2$

$$\therefore 3 \times (-2) + y = 5$$

$$\therefore y = 5 + 6 = 11$$

$\therefore (-2, 11)$ satisfies the relation.

- 2 By substituting directly as we did in 1 we can get the ordered pairs but we will present another method of solution by putting one of the two variables in one hand side alone.

$$\therefore 3x - 2y = 6$$

$$\therefore -2y = 6 - 3x \text{ (multiply by } (-1) \text{)}$$

$$\therefore 2y = 3x - 6$$

$$\therefore y = \frac{3x - 6}{2}$$

- Set $x = 0$

$$\therefore y = \frac{3 \times 0 - 6}{2} = \frac{-6}{2} = -3$$

$\therefore (0, -3)$ satisfies the relation.

- Set $x = 1$

$$\therefore y = \frac{3 \times 1 - 6}{2} = \frac{-3}{2} = -1\frac{1}{2}$$

$\therefore (1, -1\frac{1}{2})$ satisfies the relation.

- Set $x = 2$

$$\therefore y = \frac{3 \times 2 - 6}{2} = 0$$

$\therefore (2, 0)$ satisfies the relation.

$$3 \quad \because 2x = 3 \qquad \therefore x = \frac{3}{2} \qquad \therefore x = 1\frac{1}{2}$$

This relation will be satisfied for all ordered pairs (x, y) where $x = 1\frac{1}{2}$ whatever the value of y such as $(1\frac{1}{2}, 0)$, $(1\frac{1}{2}, 1)$ and $(1\frac{1}{2}, 2)$

$$4 \quad y = -2$$

This relation will be satisfied for all ordered pairs (x, y) , where $y = -2$, whatever the value of x such as $(0, -2)$, $(1, -2)$ and $(2, -2)$

Example 3

Choose the correct answer from those given :

1 Which of the following ordered pairs satisfies the relation $2x - y = 1$?

- (a) $(0, 1)$ (b) $(5, 3)$ (c) $(3, 5)$ (d) $(-2, 5)$

2 If $(2, -3)$ satisfies the relation $2x - y = c$, then $c = \dots\dots\dots$

- (a) -7 (b) -1 (c) 1 (d) 7

3 If $(-2, 1)$ satisfies the relation $3x + by = 1$, then $b = \dots\dots\dots$

- (a) -7 (b) -5 (c) c (d) 7

4 If $(k, 2k)$ satisfies the relation $5x - y = 6$, then $k = \dots\dots\dots$

- (a) -18 (b) -2 (c) 2 (d) 18

5 If $(k, -2)$ satisfies the relation $5x + 4y = 7$, then $k = \dots\dots\dots$

- (a) -3 (b) $-\frac{1}{5}$ (c) $\frac{1}{5}$ (d) 3

Solution

1 (c) The reason : By substituting each ordered pair in the given relation, we find that $(3, 5)$ satisfies the relation as follows : putting : $x = 3, y = 5$
 $\therefore 2x - y = 2(3) - 5 = 6 - 5 = 1$
 $\therefore (3, 5)$ satisfies the relation.

2 (d) The reason : $\because (2, -3)$ satisfies the relation $2x - y = c$
 $\therefore 2(2) - (-3) = c$
 $\therefore 4 + 3 = c$
 $\therefore c = 7$

3 (d) The reason : $\because (-2, 1)$ satisfies the relation : $3x + by = 1$

$$\therefore 3(-2) + b \times 1 = 1 \qquad \therefore -6 + b = 1$$

$$\therefore b = 1 + 6 \qquad \therefore b = 7$$

4 (c) The reason : $\because (k, 2k)$ satisfies the relation : $5x - y = 6$

$$\therefore 5k - 2k = 6 \qquad \therefore 3k = 6$$

$$\therefore k = 2$$

5 (d) The reason : $\because (k, -2)$ satisfies the relation : $5x + 4y = 7$

$$\therefore 5k + 4(-2) = 7 \qquad \therefore 5k - 8 = 7$$

$$\therefore 5k = 15 \qquad \therefore k = 3$$

TRY 2

1 Find four ordered pairs satisfying the relation : $3x + y = 2$

2 If $(3k, 2k)$ satisfies the relation : $x - 3y = 9$, find the value of k

The graphic representation of the linear relation

- We mentioned that linear relation between two variables x and y is usually written in the form : $ax + by = c$, where a , b and c are real numbers, a and b are not both equal to zero.

This linear relation is represented graphically by a straight line (that is why it is called linear).

- To graph a linear relation, you need to graph at least two ordered pairs satisfying this relation. You can add a third ordered pair to check that the three points lie on the same straight line which is the graphic representation of the relation.



WATCH VIDEO

Example 4 Represent the relation : $2x - y = 3$ graphically

Solution To represent this relation graphically, we should determine three ordered pairs satisfying the relation : $2x - y = 3$, as follows :

$$\bullet \text{ Set } x = 0 \qquad \therefore 2 \times 0 - y = 3 \qquad \therefore -y = 3 \qquad \therefore y = -3$$

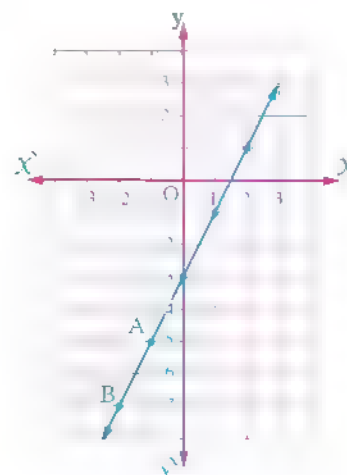
$$\bullet \text{ Set } x = 1 \qquad \therefore 2 \times 1 - y = 3 \qquad \therefore -y = 1 \qquad \therefore y = -1$$

$$\bullet \text{ Set } x = 2 \qquad \therefore 2 \times 2 - y = 3 \qquad \therefore -y = -1 \qquad \therefore y = 1$$

It is preferable to put the values of x and y in a table as the following :

x	0	1	2
y	-3	-1	1

Then we determine the points which represent these ordered pairs : $(0, -3)$, $(1, -1)$ and $(2, 1)$ on orthogonal coordinates system, then we draw the straight line passing through these points, it will be the graphic representation of the relation : $2x - y = 3$



! Remark

All the points of the straight line which represents the relation determine ordered pairs which satisfy the relation.

For example:

The point A determines the ordered pair $(-1, -5)$ which satisfies the relation when we put $x = -1$ we find that $2 \times (-1) - y = 3$ i.e. $y = -5$ and also the point B $(-2, -7)$

TRY your self 3

Represent the relation : $y - 2x = -1$ graphically.

Special cases

We studied before the relation : $ax + by = c$, where a, b are not both equal to zero and it is called a linear relation and it is represented graphically by a straight line and now we study the following cases :

1 If $a = 0, b \neq 0$

Then the relation becomes in the form :

$$by = c$$

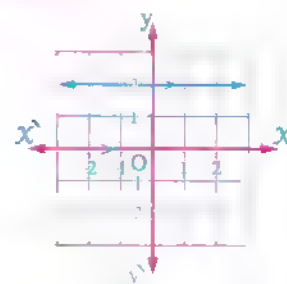
and it is represented graphically by a straight line parallel to x -axis and intersects y -axis at the point $(0, \frac{c}{b})$

Examples

For example :

The relation : $2y = 4$

i.e. $y = 2$ is represented by a straight line parallel to x -axis and intersects y -axis at the point $(0, 2)$



Notice that :

The relation : $y = 0$ is represented by x -axis

2 If $b = 0, a \neq 0$

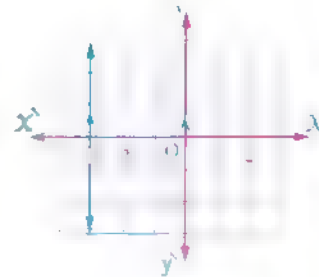
Then the relation becomes in the form :

$$aX = c$$

and it is represented graphically by a straight line parallel to y-axis and intersects X-axis at the point $(\frac{c}{a}, 0)$

For example :

The relation : $X = -3$ is represented by a straight line parallel to y-axis and intersects X-axis at the point $(-3, 0)$



Notice that :

The relation : $X = 0$ is represented by y-axis

3 If $c = 0$

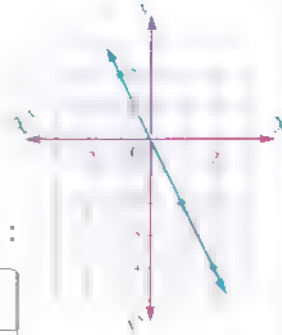
Then the relation becomes :

$$aX + bY = 0$$

and it is represented by a straight line passing through the origin point $(0, 0)$

For example :

The relation : $2X + Y = 0$ is represented graphically by a straight line passing through the origin point as shown in the opposite graph :



X	1	-1	2
y	-2	2	-4

Example 5 Graph the straight line which represents the relation : $2X + 5Y = 10$ and if this straight line intersects X-axis at the point A and y-axis at the point B , find the area of ΔOAB where O is the origin point.

Solution $\therefore 2X + 5Y = 10$

$$\therefore X = \frac{10 - 5Y}{2}$$

• Set $y = 0$

$\therefore (5, 0)$ satisfies the relation.

• Set $y = 2$

$\therefore (0, 2)$ satisfies the relation.

• Set $y = 4$

$\therefore (-5, 4)$ satisfies the relation

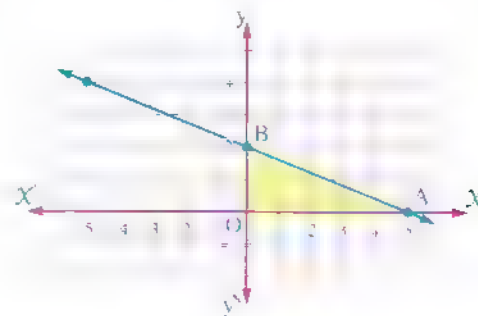
$$\therefore 2X = 10 - 5Y$$

$$\therefore X = \frac{10 - 5(0)}{2} = 5$$

$$\therefore X = \frac{10 - 5(2)}{2} = 0$$

$$\therefore X = \frac{10 - 5(4)}{2} = -5$$

x	5	0	-5
y	0	2	4



\therefore The straight line intersects x -axis at the point (5 , 0)

\therefore OA = 5 length units.

\therefore the straight line intersects y -axis at the point (0 , 2)

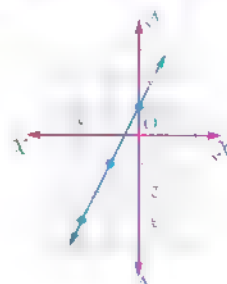
\therefore OB = 2 length units.

\therefore The area of Δ OAB = $\frac{1}{2}$ OA \times OB = $\frac{1}{2} \times 5 \times 2 = 5$ square units.

! Remark

In the previous example , we can get the points of intersection of the straight line representing the relation : $2x + 5y = 10$ and the coordinate axes without using the graph as the following :

- Set $y = 0$ $\therefore 2x + 5 \times 0 = 10$
 $\therefore 2x = 10$ $\therefore x = 5$
 \therefore The point of intersection with x -axis is (5 , 0)
- Set $x = 0$ $\therefore 2(0) + 5y = 10$
 $\therefore 5y = 10$ $\therefore y = 2$
 \therefore The point of intersection with y -axis is (0 , 2)



3

2

13 bills of 5 pounds - 9 bills of 5 pounds and 1 bill of 20 pounds - 5 bills of 5 pounds and 2 bills of 20 pounds - 1 bill of 5 pounds and 3 bills of 20 pounds.

2

1

13 bills of 5 pounds - 9 bills of 5 pounds and 1 bill of 20 pounds - 5 bills of 5 pounds and 2 bills of 20 pounds - 1 bill of 5 pounds and 3 bills of 20 pounds.

1

Answers of try by yourself

Slope of straight line



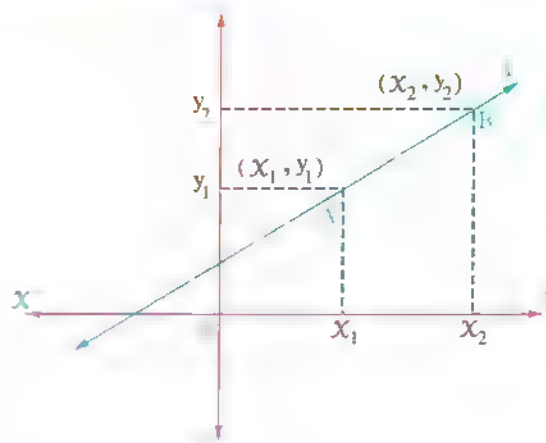
If a point moves on a straight line L from the location $A (x_1, y_1)$ to the location

$B (x_2, y_2)$, then :

– The change in the x -coordinates $= x_2 - x_1$
It is called (the horizontal change).

– The change in the y -coordinates $= y_2 - y_1$
It is called (the vertical change).

The ratio of the change in the y -coordinates to the change in the x -coordinates is called the slope of the straight line (S).



Definition

The slope of the straight line $= \frac{\text{the change in } y\text{-coordinates}}{\text{the change in } x\text{-coordinates}} = \frac{\text{the vertical change}}{\text{the horizontal change}}$

i.e. $S = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$

• S is undefined if $x_1 = x_2$



WATCH VIDEO

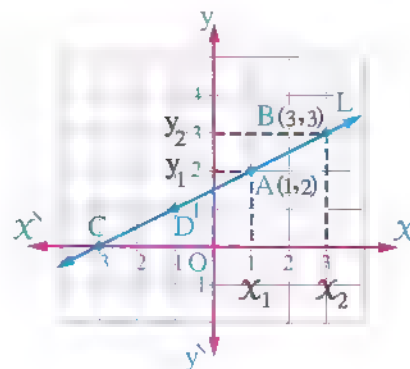
Example 1 In the opposite figure :

Find the slope of the straight line L.

SolutionWe determine two points on the straight line such as $A = (1, 2)$ and $B = (3, 3)$

$$\therefore S = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore S = \frac{3 - 2}{3 - 1} = \frac{1}{2}$$

**! Remark**

In the previous example, notice that if we used another two points of the straight line to find its slope as the points $C(-3, 0)$ and $D(-1, 1)$ we find that :

$$S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{-1 - (-3)} = \frac{1}{2} \text{ (the same result)}$$

i.e. The slope of the straight line is constant for any two selected points on it.

Example 2

Find the slope of the straight line passing through each pair of points in the following :

1 $(2, 4), (4, 5)$

2 $(1, 3), (4, 2)$

3 $(-2, -3), (-4, 1)$

4 $(3, 1), (-1, 0)$

Solution

1 $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 4}{4 - 2} = \frac{1}{2}$

2 $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 3}{4 - 1} = -\frac{1}{3}$

3 $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-3)}{-4 - (-2)} = \frac{4}{-2} = -2$

4 $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 1}{-1 - 3} = \frac{-1}{-4} = \frac{1}{4}$

TRY by yourself 1

Find the slope of the straight line passing through each pair of points in the following :

1 $(2, 1), (3, 4)$

2 $(3, -5), (-4, 2)$

3 $(-3, -1), (1, 0)$

4 $(-6, 3), (-4, 2)$

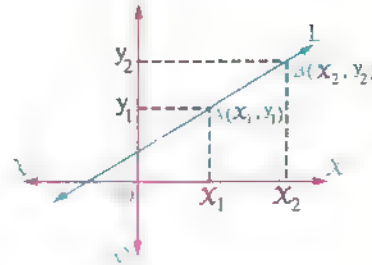
! Remarks

- If a point moves on a straight line from the location A (X_1, y_1) to the location B (X_2, y_2), where $X_2 > X_1$, then

1 If $y_2 > y_1$

i.e. y increases as X increases, then the slope of the straight line is a **positive** number.

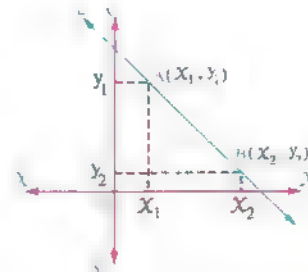
i.e. $S > 0$



2 If $y_2 < y_1$

i.e. y decreases as X increases, then the slope of the straight line is a **negative** number.

i.e. $S < 0$

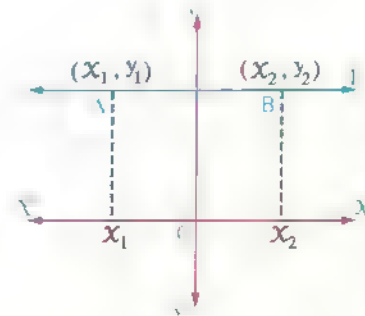


3 If $y_2 = y_1$

i.e. y is constant as X changes, then the slope of the straight line = **zero**

i.e. $S = 0$

i.e. The slope of the straight line parallel to X -axis = zero

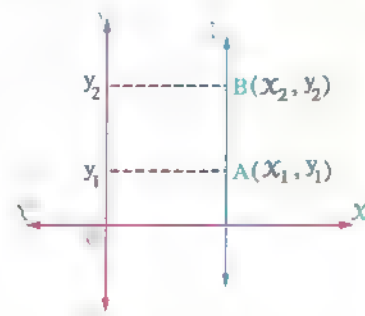


4 If $X_2 = X_1$

, then the slope of the straight line is **undefined** because there is no change in the X -axis.

i.e. $X_2 - X_1 = 0$

i.e. The slope of the straight line parallel to y -axis is undefined.



Example 3

In the opposite figure :

ABC is a triangle in which
 $\overrightarrow{BC} \parallel \overrightarrow{xx}$, $\overrightarrow{AD} \perp \overrightarrow{BC}$

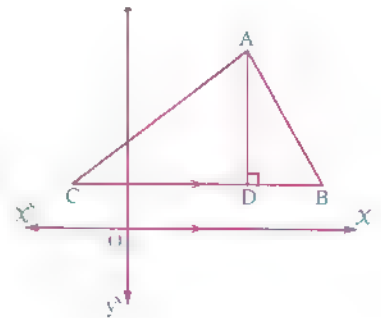
Complete the following using one of the words (positive , negative , zero , undefined) in the spaces :

1 The slope of \overrightarrow{AB} is

2 The slope of \overrightarrow{BC} is

3 The slope of \overrightarrow{AC} is

4 The slope of \overrightarrow{AD} is



Solution

1 Negative

2 Zero

3 Positive

4 Undefined

Example 4

If the slope of the straight line passing through the two points $(-3, 4)$ and $(1, y)$ is 2 , find the value of y

Solution

$$\therefore S = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore 2 = \frac{y - 4}{1 - (-3)}$$

$$\therefore 2 = \frac{y - 4}{4}$$

$$\therefore y - 4 = 2 \times 4$$

$$\therefore y - 4 = 8$$

$$\therefore y = 12$$

! An important remark

In the previous , we found that the slope of the straight line is constant and it does not change whatever the two selected points on the line , therefore to prove that the three points A , B and C are collinear , then we find the slope of \overrightarrow{AB} and the slope of \overrightarrow{BC}

If the slope of \overrightarrow{AB} = the slope of \overrightarrow{BC} , then A , B and C are collinear.

Example 5

Prove that the points A (2 , 3) , B (4 , 2) and C (8 , 0) are collinear.

Solution

$$\therefore S = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore \text{The slope of } \overrightarrow{AB} = \frac{2-3}{4-2} = -\frac{1}{2} , \text{ the slope of } \overrightarrow{BC} = \frac{0-2}{8-4} = \frac{-2}{4} = -\frac{1}{2}$$

, \therefore the slope of \overrightarrow{AB} = the slope of \overrightarrow{BC} and the point B is common.

\therefore The points A , B and C are collinear.

Example 6

If the points A, B and C are collinear where A (3, 2), B (5, -1) and C (1, k), find the value of k

Solution

$$\therefore S = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore \text{The slope of } \overrightarrow{AB} = \frac{-1 - 2}{5 - 3} = \frac{-3}{2}$$

$$\therefore \text{the slope of } \overrightarrow{BC} = \frac{k - (-1)}{1 - 5} = \frac{k + 1}{-4}$$

\therefore A, B and C are collinear, the slope of the straight line is constant for any two points on it.

$$\therefore \text{The slope of } \overrightarrow{AB} = \text{the slope of } \overrightarrow{BC}$$

$$\therefore \frac{-3}{2} = \frac{k + 1}{-4}$$

$$\therefore 2(k + 1) = -3 \times (-4)$$

$$\therefore 2k + 2 = 12$$

$$\therefore 2k = 10$$

$$\therefore k = 5$$

TRY YOURSELF 2

1 If the slope of the straight line passing through the two points (3, -1), (7, a) is $\frac{3}{4}$, find the value of a

2 Prove that: $C(-1, 2) \in \overrightarrow{AB}$, where A (1, 3) and B (3, 4)

2 Prove by yourself [Hint: Prove that the slope of \overrightarrow{AC} = the slope of \overrightarrow{AB}].

of try by yourself

2 1 2
3 1 1

2 - 1

3 $\frac{1}{4}$

4 $-\frac{2}{1}$



- We studied before that if there is a linear relation between two variables X and y , then :

The slope of the straight line which represents this relation = $\frac{\text{the change in } y\text{-coordinates}}{\text{the change in } x\text{-coordinates}}$

i.e. The slope of the straight line (S) expresses the rate of change of y with respect to x

- In our life, there are many applications which we need to know the rate of change in dealing with them.

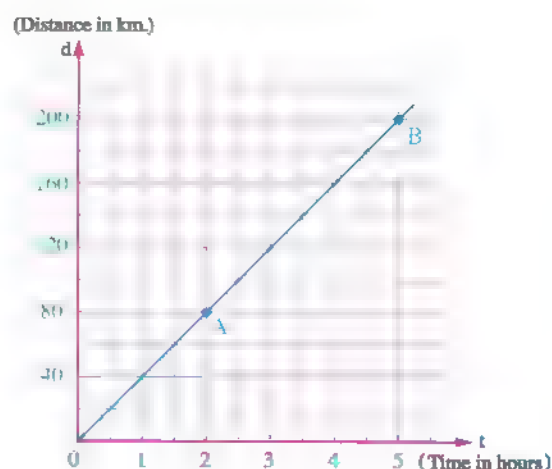
For example:

- 1 If the opposite graph represents the motion of a car, then :

The uniform velocity of the car (v)
= the rate of change of the distance (d) with respect to the time (t)

i.e. The uniform velocity of the car (v)
= the slope of the straight line (S)
and by selecting two points on the straight line as A (2, 80) and B (5, 200)

$$\therefore v = \frac{d_2 - d_1}{t_2 - t_1} = \frac{200 - 80}{5 - 2} = \frac{120}{3} = 40 \text{ km./hr}$$



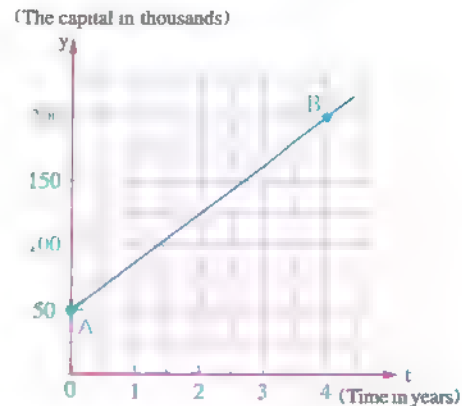
2 If the opposite graph represents the change in the capital of a company (y) within the time (t) , then :

The rate of change in the capital of the company
= the slope of the straight line \overrightarrow{AB}

\therefore The rate of change of the capital of the company

$$= \frac{y_2 - y_1}{t_2 - t_1} = \frac{200 - 50}{4 - 0}$$

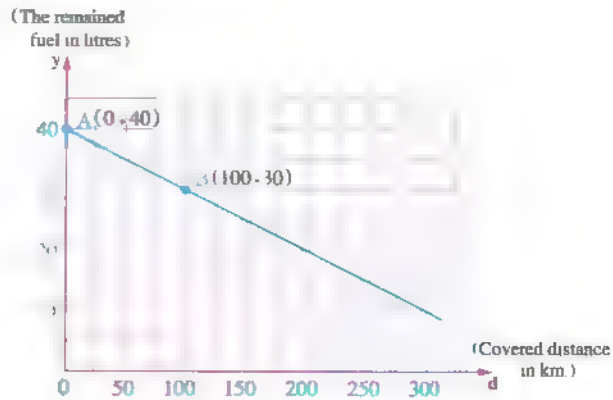
$$= \frac{150}{4} = 37.5 \text{ thousand pounds / year.}$$



i.e. The capital of the company increases in the rate = $37.5 \times 1000 = 37500$ pounds/year.

3 A person filled the tank of his car whose capacity is 40 litres with fuel. After he covered a distance 100 km. , he found that the remained fuel in the tank = 30 litres.

The opposite figure shows the relation between the covered distance in km. (d) and the amount of the remained fuel in the tank in litres (y) , then :



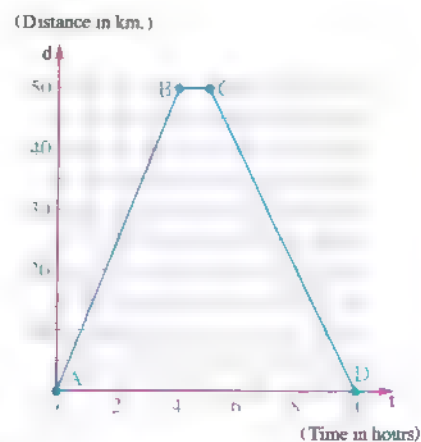
The rate of consumption of fuel = the slope of \overrightarrow{AB}

i.e. The rate of consumption of fuel = $\frac{y_2 - y_1}{d_2 - d_1} = \frac{30 - 40}{100 - 0} = \frac{-10}{100} = -\frac{1}{10}$ litre/km.

(The negative sign denotes the amount of fuel decreases in the rate of one litre for each 10 km.)

Example 1 Waleed rode his bicycle from Cairo to Benha , then he returned back to Cairo.
The opposite graph represents the bicycle motion during going and returning back :

- 1 Find his velocity in going trip.
- 2 Find his velocity in returning back trip.
- 3 Find the average velocity during all trips.
- 4 What do you say about the horizontal line segment in the graph ?



Solution

- 1 Taking the two points A (0 , 0) and B (4 , 50)

$$\therefore v \text{ (during going trip)} = \frac{50 - 0}{4 - 0} = 12.5 \text{ km./hr.}$$

- 2 Taking the two points C (5 , 50) and D (10 , 0)

$$\therefore v \text{ (during returning back trip)} = \frac{0 - 50}{10 - 5} = -\frac{50}{5} = -10 \text{ km./hr.}$$

(The negative sign means that Waleed moved in the opposite direction of his first motion returning back to Cairo with velocity 10 km./hr.)

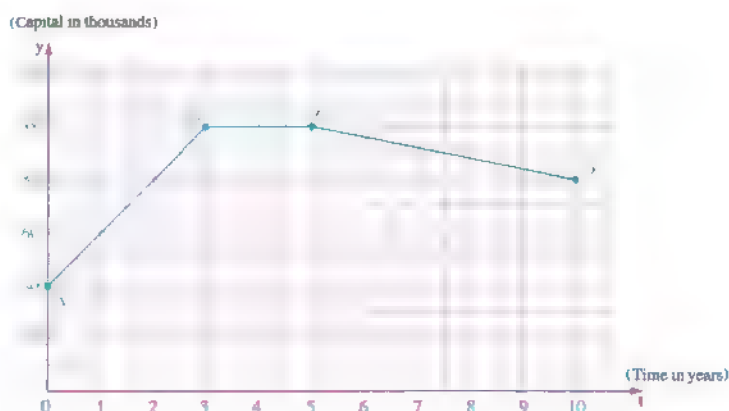
- 3 The average velocity = $\frac{\text{the total distance}}{\text{the total time}} = \frac{100}{10} = 10 \text{ km./hr.}$

- 4 The horizontal line segment in the graph shows that Waleed stopped for an hour after he covered a distance equal to 50 km. , then he returned back to the start point.

Example 2

The following graph shows the change of the capital of a company within 10 years :

- Find the slope of each of \overrightarrow{AB} , \overrightarrow{BC} and \overrightarrow{CD} What is the meaning of each of them ?
- Calculate the capital of the company at the beginning.

**Solution**

$$\therefore A (0 , 40) , B (3 , 100) , C (5 , 100) \text{ and } D (10 , 80)$$

- 1 • The slope of $\overrightarrow{AB} = \frac{100 - 40}{3 - 0} = \frac{60}{3} = 20$

It expresses the increase in the capital of the company within the first three years from the beginning in the rate of 20000 pounds/year.

• The slope of $\overrightarrow{BC} = \frac{100 - 100}{5 - 3} = \frac{0}{2} = 0$

It expresses that the capital of the company is still constant without increasing or decreasing within the fourth and the fifth years from the beginning.

• The slope of $\overrightarrow{CD} = \frac{80 - 100}{10 - 5} = \frac{-20}{5} = -4$

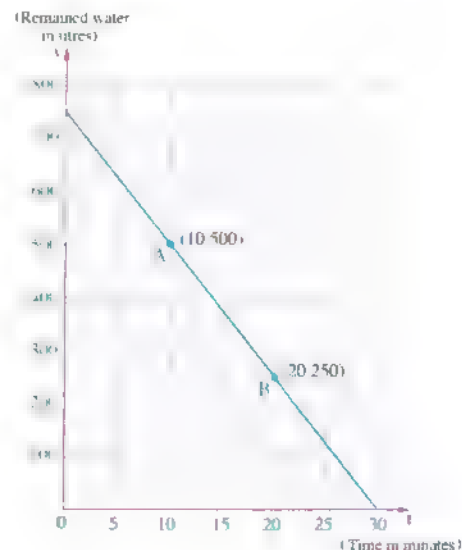
It expresses the decrease in the capital of the company within the last five years in the rate of 4000 pounds/year.

2 $\therefore A(0, 40)$

\therefore The capital of the company in the beginning = 40000 pounds.

Example 3

A tank of water is filled with water completely.
A tap is opened below the tank to empty it. The opposite graph represents the relation between the time (t) in minutes and the amount of water remained in the tank (v) in litres :



- 1 What is the greatest capacity of the tank ?
- 2 What is the time needed to empty the tank ?
- 3 What is the amount remained in the tank after 20 minutes ?
- 4 What is the rate of emptying the tank ?

Solution

- 1 From the graph , we find that \overrightarrow{AB} intersects the axis which represents the amount of remained water (v) at the point (0 , 750)
 \therefore The greatest capacity of the tank = 750 litres.
- 2 From the graph , we find that \overrightarrow{AB} intersects the axis which represents the time (t) at the point (30 , 0)
 \therefore The needed time for emptying the tank is 30 minutes.

3 \therefore The point $(20, 250) \in \overleftrightarrow{AB}$

\therefore After 20 minutes, the remained amount of water in the tank is 250 litres.

4 The rate of emptying the tank = the slope of \overleftrightarrow{AB}

$$= \frac{v_2 - v_1}{t_2 - t_1} = \frac{250 - 500}{20 - 10} = \frac{-250}{10} = -25$$

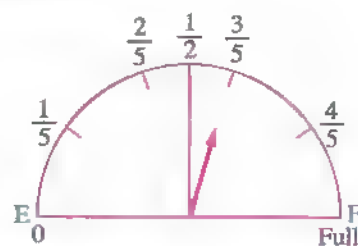
\therefore The tank is emptied by the rate 25 litres/minute.

Example 4

Hossam filled the tank of his car with fuel given that its capacity is 50 litres.

After Hossam covered a distance 200 km.

, he noticed that fuel meter shows that the tank has fuel = $\frac{3}{5}$ its capacity.



Graph the relation between the distance covered by the car and the amount of fuel in the tank and calculate the distance covered by the car till the tank becomes empty.

Solution

Let the covered distance = d (km.)

and the remained amount of fuel = y (litres)

\therefore In the beginning, the distance = 0 km.

i.e. $d = 0$ and the amount of fuel in the tank = 50 litres.

i.e. $y = 50$

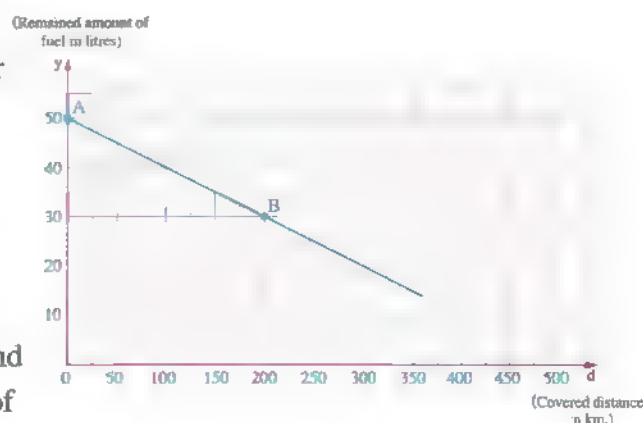
\therefore The point A $(0, 50)$ expresses the amount of fuel in the tank in the beginning of motion.

$\therefore \frac{3}{5}$ the capacity of the tank = $\frac{3}{5} \times 50 = 30$ litres.

\therefore The point B $(200, 30)$

expresses the amount of fuel in the tank after a covered distance 200 km. from the beginning.

$\therefore \overleftrightarrow{AB}$ represents the relation between the covered distance (d) and the remained amount of fuel in the tank (y)



∴ The rate of decrease of fuel = the slope of \overrightarrow{AB}

$$= \frac{y_2 - y_1}{d_2 - d_1} = \frac{30 - 50}{200 - 0} = \frac{-20}{200} = -\frac{1}{10} \text{ litre/km.}$$

i.e. The amount of fuel in the tank decreases with rate of one litre per 10 km.

∴ The covered distance from beginning the motion till the tank becomes empty

$$= \frac{\text{the amount of fuel in the beginning}}{\text{rate of decrease of fuel}} = \frac{50}{\frac{1}{10}} = 50 \times 10 = 500 \text{ km.}$$

! Remark

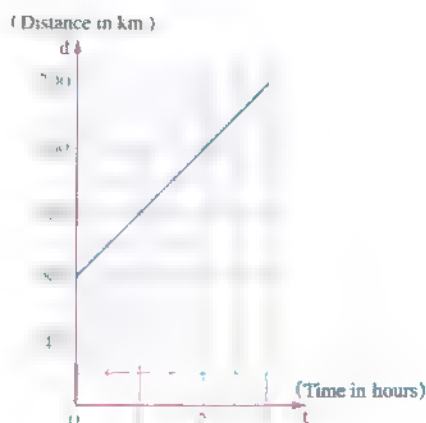
We can find the covered distance from the beginning till the tank becomes empty from the graph by finding the point of intersection of \overrightarrow{AB} with the axis which represents the distance d which is $(500, 0)$

i.e. The covered distance by the car when the tank becomes empty = 500 km.

TRY

The opposite graph represents the motion of a car measured from a fixed point A :

- 1 Determine the uniform velocity of the car.
- 2 Calculate the covered distance after two hours from the beginning of the motion.



Answer of try by yourself

1 40 km./hour
2 80 km.



■ Lessons of the unit :

1. Collecting and organizing data.
2. The ascending and descending cumulative frequency tables and their graphical representation.
3. Mean.
4. Median.
5. Mode.

► Use your smart phone or tablet to scan the QR Code and enjoy watching videos.

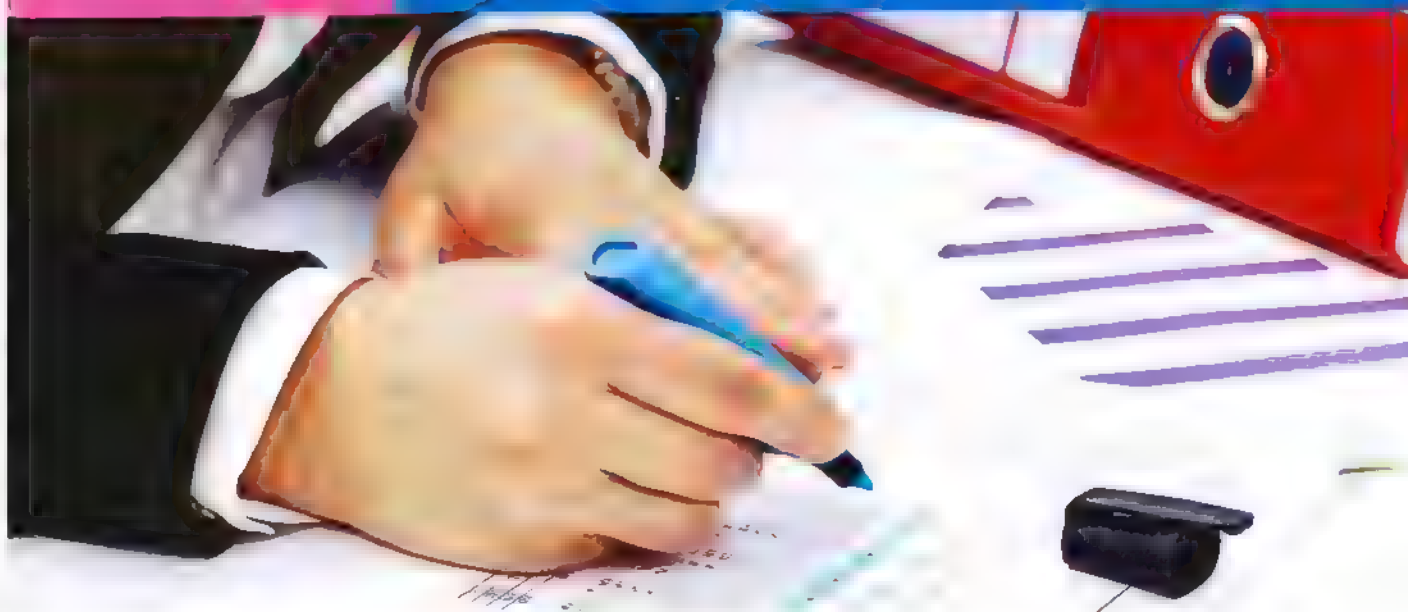


I Unit Objectives :

By the end of this unit, student should be able to :

- organize data in frequency tables with sets.
- form each of the ascending and descending cumulative frequency tables.
- graph each of the ascending and descending cumulative frequency tables.
- find the mean of a set of data organized in a frequency table with sets.
- find the median of a frequency distribution with sets.
- calculate the mode from a frequency table with sets.

Collecting and organizing data



In the last year, you knew how to organize data and put them in a simple frequency table, but when summarizing large masses of data, it is useful to distribute them into sets, and determine the number of individuals belonging to each set.

The table consisting of sets and their corresponding frequencies is called “frequency table with sets”. The following example shows how to organize data into a frequency table with sets.

Example

In the following table, these are the marks of 54 students in one of the classes in grade two preparatory in a school, which they took in an exam in mathematics where the full mark is 60



WATCH VIDEO

42	54	36	46	34	45	51	40	48
48	40	47	25	48	45	36	56	44
38	47	30	37.5	40	20	42	28	50
47	55	27	45	30	42	51	43	46
29	43	59	35	44.5	32	24	39	54
41	36	45	39	42	58	35	50	45

The required is forming the frequency table with sets.

Solution 1 Determine the range

(it is the difference between the greatest mark and the smallest mark)

∴ The smallest mark is 20 and the greatest mark is 59

∴ The range = $59 - 20 = 39$

- 2 Divide these data into a suitable number of sets of marks , say 10 disjoint sets , the length of each of them is 4 , then you obtain the following sets :

• **The first set :**

The students who obtain 20 marks till less than 24 marks , which is written as (20 –)

• **The second set :**

The students who obtain 24 marks till less than 28 marks , it is written as (24 –)

• **The third set :**

The students who obtain 28 marks till less than 32 marks , it is written as (28 –) and so on till you reach the tenth set.

• **The tenth set :**

The students who obtain 56 marks till less than 60 , it is written as (56 –)

- 3 Form the tally table as follows :

Sets	Tallies	Frequency
20 –	/	1
24 –	///	3
28 –	////	4
32 –	////	4
36 –	//// //	7
40 –	//// ////	10
44 –	//// //// //	12
48 –	//// //	7
52 –	///	3
56 –	///	3
Total		54

(The tally table)

- 4 Omit the tallies column from the table to get the final form of the frequency table with sets. It can be written vertically or horizontally.

The following is the horizontal form of the frequency table :

Sets	20-	24-	28-	32-	36-	40-	44-	48-	52-	56-	Total
Frequency	1	3	4	4	7	10	12	7	3	3	54

From the previous table , we deduce that :

- The set that has the greatest frequency is 44 -
- The set that has the least frequency is 20 -



The following is the weights of 50 persons :

52	35	40	57	43	40	36	49	43	58
47	48	51	30	59	36	45	41	44	37
42	54	38	55	42	47	46	34	53	44
47	32	41	62	50	39	58	46	43	49
40	41	64	44	54	45	38	40	48	41

Form the frequency table with sets.

Sets	30 -	35 -	40 -	45	50 -	55 -	60 -
Frequency	3	7	16	11	6	5	2

Answers of try by yourself



Prelude

- In the previous lesson , you learnt how to form a frequency table with sets and how to get some information from it as the following table which represents the distribution of weekly wages of 50 workers in one factory :

Sets of wages	54 -	58 -	62 -	66 -	70 -	Total
No. of workers (Frequency)	5	12	22	7	4	50

From this table , you can know the number of workers (the frequency) in each set.

For example:

- The number of workers whose wages lie between 58 and less than 62 pounds is 12 workers.
- The number of workers whose wages lie between 66 and less than 70 pounds is 7 workers.
- But some other information cannot be obtained directly from this table such as :
 - The number of workers who obtain wages less than 62 pounds.
 - The number of workers who obtain wages equal to 58 pounds or more.
- In order to be able to know such information , you need to study how to form another type of tables called **cumulative frequency tables (ascending and descending)** and this what will be shown in the following examples :

Example 1

The following frequency table shows the weekly wages in pounds of 50 workers in one factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

Form the ascending cumulative frequency table and represent it graphically , then find :

- 1 The number of workers whose weekly wages are less than 60 pounds.
- 2 The percentage of the number of workers whose weekly wages are less than 60 pounds.

Solution

- Form the ascending cumulative frequency table as follows :

The upper boundaries of sets	Frequency	Sets of wages	54 –	58 –	62 –	66 –	70 –
		Number of workers (Frequency)	5	12	22	7	4
Less than 54	zero	Less than 54 = 0					
Less than 58	5	Less than 58 = 5 + 0 = 5					
Less than 62	17	Less than 62 = 5 + 12 = 17					
Less than 66	39	Less than 66 = 5 + 12 + 22 = 39					
Less than 70	46	Less than 70 = 5 + 12 + 22 + 7 = 46					
Less than 74	50	Less than 74 = 5 + 12 + 22 + 7 + 4 = 50					

The ascending cumulative frequency table.

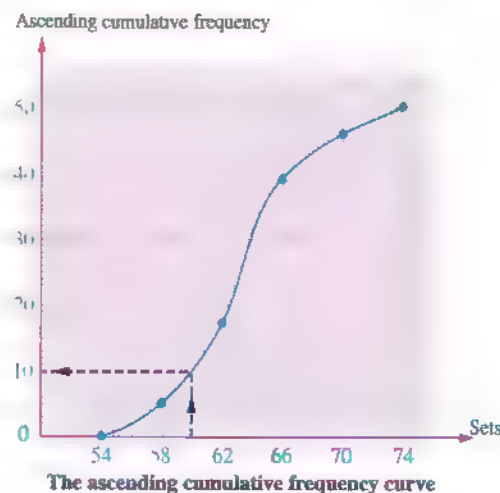
Notice that : The ascending cumulative frequency begins with zero and ends at the total frequency.

To represent the ascending cumulative frequency table graphically , do as follows :

- 1 Specialize the horizontal axis for sets and the vertical axis for the ascending cumulative frequency.
- 2 Choose a suitable scale to represent data on the vertical axis so that it contains the ascending cumulative frequency easily.
- 3 Represent the ascending cumulative frequency of each set , then draw the graph (the curve) such that it passes through the points which we located as shown in the opposite figure.

• From the graph , we find that :

- 1 The number of workers whose weekly wages are less than 60 pounds = 10 workers.
- 2 The percentage of the number of workers whose weekly wages are less than 60 pounds = $\frac{10}{50} \times 100\%$
= 20%



Example 2

The following frequency table shows the weekly wages of 50 workers in one factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

Form the descending cumulative frequency table and represent it graphically , then find :

- 1 The number of workers whose weekly wages are 60 pounds or more.
- 2 The percentage of the number of workers whose weekly wages are 60 pounds or more.

Solution

• Form the descending cumulative frequency table as follows :

Sets of wages	54 –	58 –	62 –	66 –	70 –	The lower boundaries of sets	Frequency
Number of workers (Frequency)	5	12	22	7	4		
54 and more =	$5 + 12 + 22 + 7 + 4 = 50$					54 and more	50
58 and more =	$12 + 22 + 7 + 4 = 45$					58 and more	45
62 and more =	$22 + 7 + 4 = 33$					62 and more	33
66 and more =	$7 + 4 = 11$					66 and more	11
70 and more =	4					70 and more	4
74 and more =	0					74 and more	zero

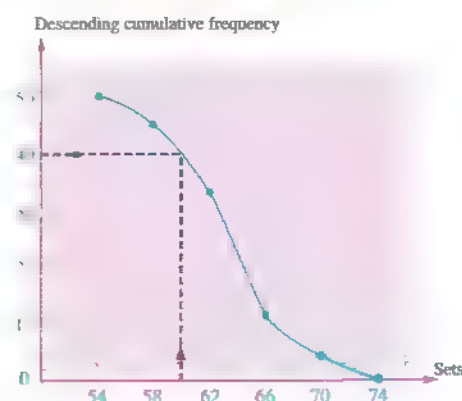
The descending cumulative frequency table

Notice that The descending cumulative frequency begins with the total frequency and ends with zero.

- To represent this table graphically , follow the same previous steps in the ascending cumulative frequency table to get the opposite graph.

• From the graph , we find that :

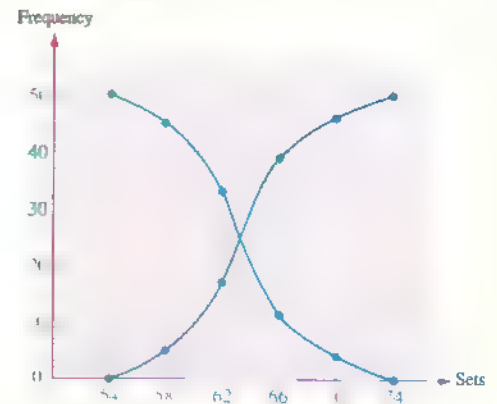
- 1 The number of workers whose weekly wages are 60 pounds or more = 40 workers.
- 2 The percentage of those workers = $\frac{40}{50} \times 100\% = 80\%$



The descending cumulative frequency curve

! Remark

You can graph the two curves of the ascending and descending cumulative frequency of a frequency distribution in one sketch as shown in the opposite graph.



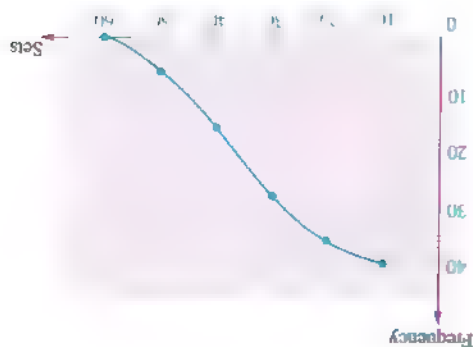
TRY

The following table shows the frequency distribution of marks of 40 students in math exam :

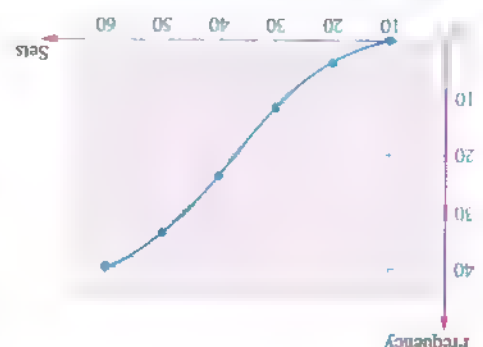
Sets	10 –	20 –	30	40 –	50 –	Total
Frequency	4	8	12	10	6	40

Graph each of :

- 1 The ascending cumulative frequency curve.
- 2 The descending cumulative frequency curve.

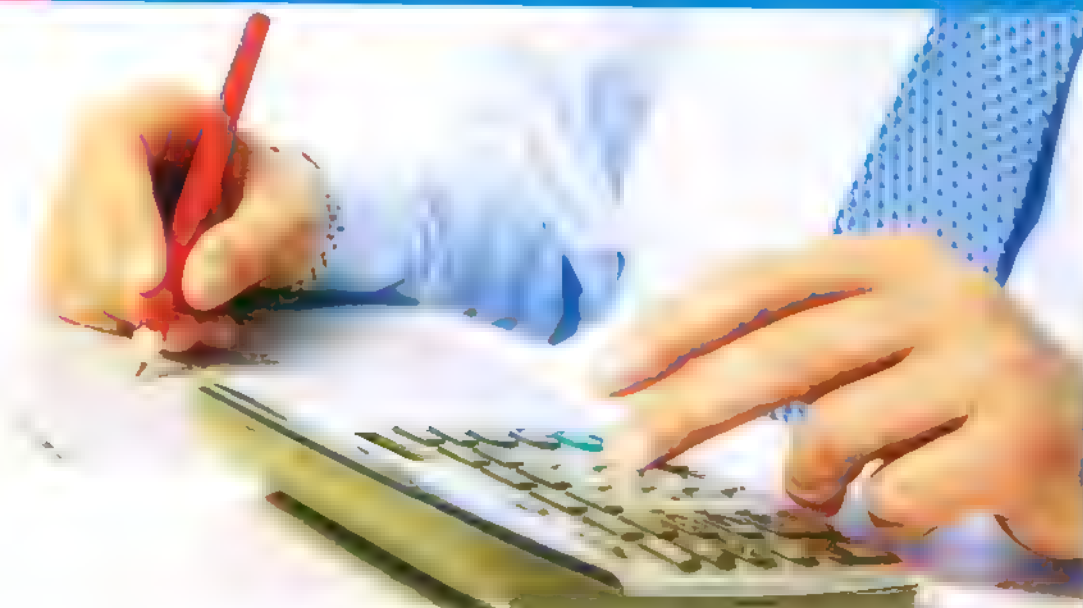


2



1

Answers of try by yourself



You studied last year some of the measures of central tendency of a set of values which are the mean , the median and the mode.

Now you will study how you can find these three measures of a set of data organized in a frequency table with sets.



Remember that

To calculate the mean of a set of values , do as follows :

- 1 Find the sum of these values.
- 2 Divide this sum by the number of these values

$$\text{i.e. The mean of a set of values} = \frac{\text{The sum of values}}{\text{Number of values}}$$

For example:

If the marks of 5 students are 25 , 23 , 21 , 22 , 24

$$\text{, then the mean of marks} = \frac{25 + 23 + 21 + 22 + 24}{5} = 23 \text{ marks.}$$

Notice that : $23 \times 5 = 115$

$$\text{, the sum of marks of the 5 students} = 25 + 23 + 21 + 22 + 24 = 115$$

i.e. The mean is the value which is given to each item of a set , then the sum of these new values is the same sum of the original values.

Finding the mean of data from the frequency table with sets

Example The following table shows the distribution of the marks of 50 students in mathematics :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	8	12	14	9	7	50

Find the mean of these marks.

Solution 1 Determine the centres of sets according to the rule :

$$\text{The centre of a set} = \frac{\text{the lower limit} + \text{the upper limit}}{2}$$

$$\text{, then the centre of the first set} = \frac{10 + 20}{2} = 15$$

$$\text{, the centre of the second set} = \frac{20 + 30}{2} = 25 \dots \text{and so on.}$$

Since the lengths of the subsets are equal and each of them = 10 therefore we consider the upper limit of the last set = 60

$$\text{, then its centre} = \frac{50 + 60}{2} = 55$$

2 Form the vertical table :

Set	Centre of the set « X »	Frequency « f »	X × f
10 –	15	8	120
20 –	25	12	300
30 –	35	14	490
40 –	45	9	405
50 –	55	7	385
Total		50	1700

$$3 \text{ The mean} = \frac{\text{The sum of } (X \times f)}{\text{The sum of } f} = \frac{1700}{50} = 34 \text{ marks.}$$

TRY
yourself

The following table shows the daily wages in pounds of 50 workers in a factory :

Sets	5 –	15 –	25	35 –	45 –	Total
Frequency	7	10	12	13	8	50

Find the mean of the wage of the worker in pounds.

31 Pounds.

of try by yourself

Answers



Remember that

The median is the middle value in a set of values after arranging it ascendingly or descendingly such that the number of values which are less than it is equal to the number of values which are greater than it.

- To find the median of a set of values , do as follows :

Arrange the values ascendingly or descendingly

then

If the values number is **odd**

Then :

The median is the value lying in the middle exactly.

For example:

- If the values are :

42, 23, 17, 30, 20

- We arrange them ascendingly as follows

17, 20, 23, 30, 42



The median = 23

If the values number is **even**

Then :

The sum of the two values lying in the middle

The median = $\frac{\quad}{2}$

For example:

- If the values are :

27, 13, 23, 24, 13, 21

- We arrange them ascendingly as follows

13, 13, 21, 23, 24, 27



The median = $\frac{21 + 23}{2} = 22$

Finding the median of a frequency distribution with sets graphically

To find the median of a frequency distribution with sets graphically, do the following steps :

- 1 Form the ascending or the descending cumulative frequency table, then draw the cumulative frequency curve of it.
- 2 Find the order of the median = $\frac{\text{The total of frequency}}{2}$
- 3 Determine the point which represents the order of the median on the vertical axis, from this point, draw a horizontal straight line to intersect the curve at a point, then from this point, draw a perpendicular to the horizontal axis to intersect it at a point which represents the median.

The following example shows how to find the median using the two curves (the ascending or the descending cumulative frequency curve).

Example

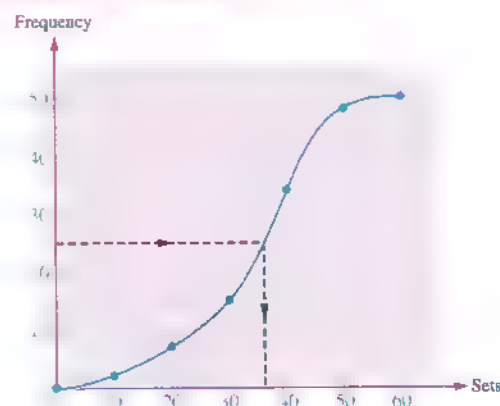
The following table shows the frequency distribution of marks of 50 students in math exam :

Sets of marks	0 –	10 –	20	30 –	40 –	50 –	Total
Number of students	2	5	8	19	14	2	50

Find the median mark of the students.

Solution * First : Using the ascending cumulative frequency curve :

The upper boundaries of sets	Frequency
Less than 0	0
Less than 10	2
Less than 20	7
Less than 30	15
Less than 40	34
Less than 50	48
Less than 60	50

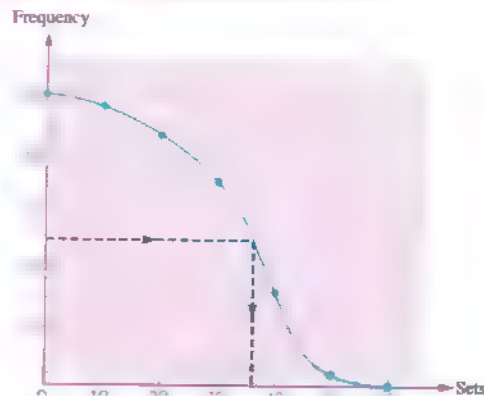


$$\therefore \text{The order of the median} = \frac{50}{2} = 25$$

\therefore From the graph, the median = 36 approximately

Second : Using the descending cumulative frequency curve :

The lower boundaries of sets	Frequency
0 and more	50
10 and more	48
20 and more	43
30 and more	35
40 and more	16
50 and more	2
60 and more	0



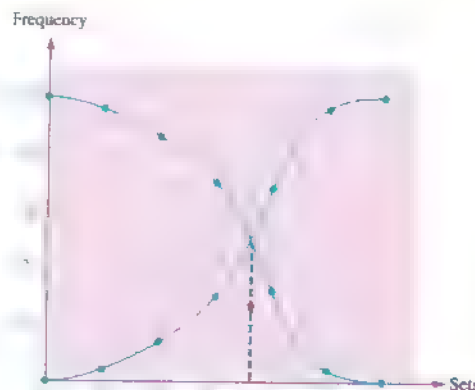
$$\therefore \text{The order of the median} = \frac{50}{2} = 25$$

\therefore From the graph , the median = 36 approximately

! Remark

You can find the median by more accurate method , this by drawing the two curves (the ascending and descending cumulative frequency curves) together in one graph to intersect at one point.

From this point , draw a vertical straight line to meet the horizontal axis at a point which represents the median as shown in the opposite graph to get the median = 36 approximately.



Using the ascending or descending cumulative frequency curve , find the median of the following frequency distribution :

Sets	4 –	8 –	12 –	16 –	20 –	Total
Frequency	2	4	8	6	4	24

15 approximately.

of try by yourself



Remember that

The mode of a set of values is the most common value in the set , or in other words , it is the value which is repeated more than any other values.

For example: The mode of the set of the values

7 , 3 , 4 , 1 , 7 , 9 , 7 , 4 is **7**

Finding the mode for a frequency distribution with sets

The following example shows how to find the mode of a frequency distribution with sets :

Example

The following is the frequency distribution of marks of 100 students in an exam :

Sets of marks	10 –	20 –	30	40 –	50 –	Total
Number of students	16	24	30	20	10	100

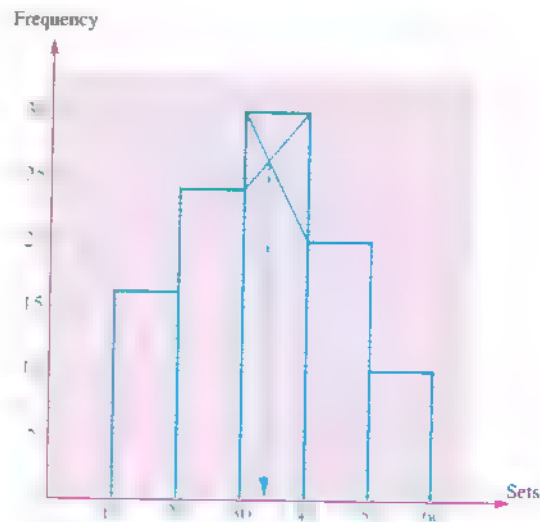
Find the mode mark for these students.

Solution

You can find the mode of that distribution graphically using the histogram as follows :

- 1 Draw two orthogonal axes , one of them is horizontal and the other is vertical to represent the frequency of each set.

- 2 Divide the horizontal axis into a number of equal parts with a suitable drawing scale to represent the sets.
 - 3 Divide the vertical axis into a number of equal parts with a suitable drawing scale to represent the greatest frequency in the sets.
 - 4 Draw a rectangle whose base is the set (10 –) and its height equals the frequency (16)
 - 5 Draw a second rectangle adjacent to the first one whose base is the set (20 –) and its height equals the frequency (24)
 - 6 Repeat drawing the remained adjacent rectangles till the last set (50 –)
 - 7 Determine the set which has the greatest frequency then draw two lines as shown in the histogram to intersect at a point.
- From this point, draw a vertical line to intersect the horizontal axis at a point which represents the value of the mode.



i.e. The mode mark is 34 approximately.

TRY

Find the mode for the following frequency distribution :

Sets	2 –	4 –	6 –	8 –	10 –	Total
Frequency	3	10	12	10	5	40

7 approximately.

of try by yourself

Second

Geometry

Revision 117

Unit

4

**Medians of Triangle –
Isosceles Triangle.**

122

Unit

5

Inequality.

152



Revision

1 Some relations between angles

Complementary angles :

Two angles are said to be complementary, if the sum of their measures is 90°

In the opposite figure :

$\angle A$ and $\angle B$ are complementary angles.



Supplementary angles :

Two angles are said to be supplementary, if the sum of their measures is 180°

In the opposite figure :

$\angle A$ and $\angle C$ are supplementary angles.

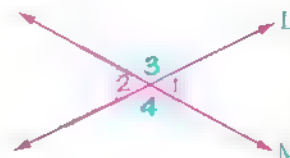


Vertically opposite angles (V.O.A.):

If two straight lines intersect, then each two vertically opposite angles are equal in measure.

In the opposite figure :

$$m(\angle 1) = m(\angle 2) \quad , \quad m(\angle 3) = m(\angle 4)$$

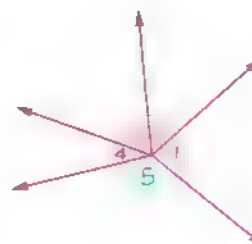


Accumulative angles at a point :

The sum of measures of the accumulative angles at a point is 360°

In the opposite figure :

$$m(\angle 1) + m(\angle 2) + m(\angle 3) + m(\angle 4) + m(\angle 5) = 360^\circ$$



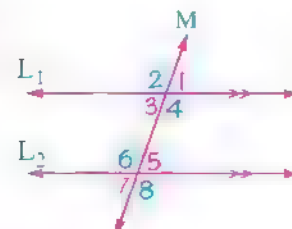
2 Parallelism

If a straight line intersects two parallel straight lines , then :

Each two alternate angles are equal in measure.

• $m(\angle 3) = m(\angle 5)$ "Alternate angles"

• $m(\angle 4) = m(\angle 6)$ "Alternate angles"



Each two corresponding angles are equal in measure.

• $m(\angle 1) = m(\angle 5)$ "Corresponding angles"

• $m(\angle 2) = m(\angle 6)$ "Corresponding angles"

• $m(\angle 3) = m(\angle 7)$ "Corresponding angles"

• $m(\angle 4) = m(\angle 8)$ "Corresponding angles"

Each two interior angles in the same side of the transversal are supplementary.

• $m(\angle 3) + m(\angle 6) = 180^\circ$ "Interior angles in the same side of the transversal"

• $m(\angle 4) + m(\angle 5) = 180^\circ$ "Interior angles in the same side of the transversal"

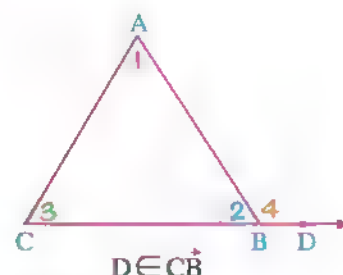
3 The triangle

• The sum of measures of the interior angles of a triangle = 180°

$$m(\angle 1) + m(\angle 2) + m(\angle 3) = 180^\circ$$

• The measure of the exterior angle of a triangle equals the sum of measures of its non-adjacent interior angles.

$$m(\angle 4) = m(\angle 1) + m(\angle 3)$$



Pythagoras' Theorem :

In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

In $\triangle ABC$ which is right-angled at A :

- $(BC)^2 = (AB)^2 + (AC)^2$
- $(AB)^2 = (BC)^2 - (AC)^2$
- $(AC)^2 = (BC)^2 - (AB)^2$



Cases of congruence of two triangles :

Two triangles are congruent if one of the following cases is satisfied :

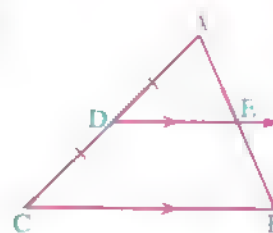
- 1 Congruence of two sides and the included angle of one triangle to the corresponding parts of the other triangle.
- 2 Congruence of two angles and the side drawn between their vertices of one triangle to the corresponding parts of the other triangle.
- 3 Congruence of each side of one triangle to the corresponding side of the other triangle.
- 4 Two right-angled triangles are congruent, if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.



- The ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side.

In the opposite figure :

If D is the midpoint of \overline{AC} , $\overline{DE} \parallel \overline{BC}$ such that $E \in \overline{AB}$,
then E is the midpoint of \overline{AB} (i.e. $AE = EB$)



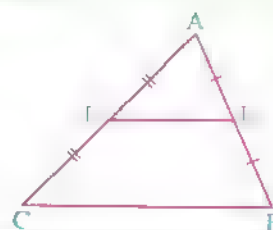
- The line segment joining the midpoints of two sides in a triangle is parallel to the third side and its length equals half the length of this side.

In the opposite figure :

If D is the midpoint of \overline{AB} and E is the midpoint of \overline{AC} , then :

1 $\overline{DE} \parallel \overline{BC}$

2 $DE = \frac{1}{2} BC$



REVISION

4 The polygon

- The sum of measures of the interior angles of a polygon with n sides equals $(n - 2) \times 180^\circ$

For example:

- The sum of measures of the interior angles of the quadrilateral $= (4 - 2) \times 180^\circ = 360^\circ$
- The sum of measures of the interior angles of the pentagon $= (5 - 2) \times 180^\circ = 540^\circ$
- The measure of each interior angle in a regular polygon with n sides $= \frac{(n - 2) \times 180^\circ}{n}$

For example:

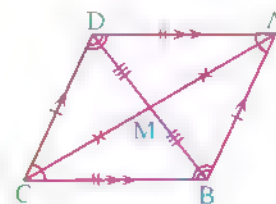
- The measure of the interior angle of the equilateral triangle $= \frac{(3 - 2) \times 180^\circ}{3} = 60^\circ$
- The measure of the interior angle of the regular hexagon $= \frac{(6 - 2) \times 180^\circ}{6} = 120^\circ$

5 The parallelogram and its special cases

Properties of a parallelogram :

In the opposite figure :

ABCD is a parallelogram whose diagonals
 \overline{AC} and \overline{BD} intersect at M



We can deduce the following properties :

- 1** The sum of measures of each two consecutive angles in a parallelogram is 180°

- i.e. • $m(\angle A) + m(\angle B) = 180^\circ$ • $m(\angle B) + m(\angle C) = 180^\circ$
• $m(\angle C) + m(\angle D) = 180^\circ$ • $m(\angle D) + m(\angle A) = 180^\circ$

- 2** In a parallelogram , each two opposite angles are equal in measure.

- i.e. • $m(\angle A) = m(\angle C)$ • $m(\angle B) = m(\angle D)$

- 3** In a parallelogram , each two opposite sides are equal in length and parallel.

- i.e. • $AB = CD$, $\overline{AB} \parallel \overline{CD}$ • $AD = BC$, $\overline{AD} \parallel \overline{BC}$

- 4** The two diagonals in a parallelogram bisect each other.

- i.e. • $AM = CM$ • $BM = DM$

- The perimeter of the parallelogram = The sum of two consecutive side lengths $\times 2$

A quadrilateral is a parallelogram if one of the following cases is satisfied

Each two opposite sides are parallel



Each two opposite sides are equal in length



Two opposite sides are parallel and equal in length



The two diagonals bisect each other



Each two opposite angles are equal in measure



A parallelogram is a

Rectangle

If one of its angles is right.

or

The diagonals are equal in length.

Rhombus

If two adjacent sides are equal in length.

or

Its diagonals are perpendicular.

Square

If one of its angles is right and two adjacent sides are equal in length.

or

One of its angles is right and its diagonals are perpendicular.

or

The diagonals are equal in length and perpendicular.

or

Two adjacent sides are equal in length and its diagonals are equal in length.

— Notice that :

- A square is a rectangle with two adjacent sides equal in length.
- A square is a rhombus with a right angle , or a rhombus with two diagonals equal in length.
- To prove that a quadrilateral is a rectangle , a rhombus or a square , you must first prove that it is a parallelogram.



UNIT

4

Medians of Triangle – Isosceles Triangle

Lessons of the unit :

1. Medians of triangle.
2. Medians of triangle “follow”.
3. The isosceles triangle.
4. The converse of the isosceles triangle theorem.
5. Corollaries of the isosceles triangle theorems.

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Unit Objectives :

By the end of this unit, student should be able to :

- recognize the median of a triangle.
- recognize the intersection point of medians of a triangle and the ratio that the point divides each median.
- deduce the relation between the length of the median from the vertex of the right angle in the right-angled triangle and the length of the hypotenuse.
- recognize thirty and sixty triangle.
- recognize the properties of isosceles triangle.
- recognize the properties of equilateral triangle.
- recognize the axis of symmetry of the line segment.
- recognize the axis of symmetry of the isosceles triangle.
- solve miscellaneous problems on the equilateral triangle and the isosceles triangle.
- appreciate the role of geometry in solving of real life problems.

Medians of triangle



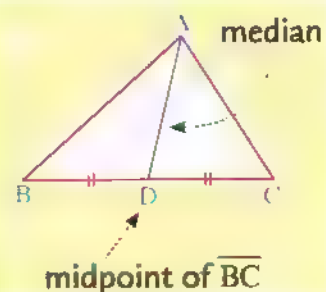
Definition

The median of a triangle is the line segment drawn from any vertex of this triangle to the midpoint of the opposite side of this vertex.

For example:

In the opposite figure :

If D is the midpoint of \overline{BC}
 , then \overline{AD} is a median of $\triangle ABC$



Notice that :

Any triangle has three medians.

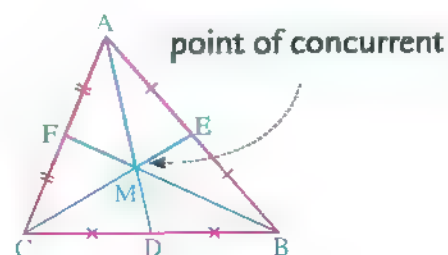
Theorem 1

The medians of a triangle are concurrent.

For example:

In the opposite figure :

\overline{AD} , \overline{BF} and \overline{CE} are the three medians of $\triangle ABC$,
 and they are concurrent at M
 (i.e. $\overline{AD} \cap \overline{BF} \cap \overline{CE} = \{M\}$)



Example 1 In the opposite figure :

$\triangle ABC$ is a right-angled triangle at B in which :

$AC = 10$ cm. , $BC = 8$ cm. ,

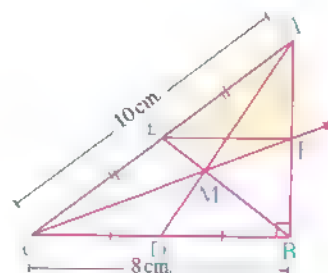
D and E are the midpoints of \overline{BC} and \overline{AC}

respectively

where $\overline{AD} \cap \overline{BE} = \{M\}$

Draw \overline{CM} to cut \overline{AB} at F

Find the perimeter of $\triangle AFE$

**Solution**

Given $m(\angle ABC) = 90^\circ$, $AC = 10$ cm. , $BC = 8$ cm. , D is the midpoint of \overline{BC} ,
E is the midpoint of \overline{AC}

R.T.F. The perimeter of $\triangle AFE$

Proof In $\triangle ABC$:

$$\therefore m(\angle ABC) = 90^\circ$$

$$\therefore (AB)^2 = (AC)^2 - (BC)^2 = 100 - 64 = 36 \quad \therefore AB = 6 \text{ cm.}$$

, \therefore D is the midpoint of \overline{BC}

$\therefore \overline{AD}$ is a median in $\triangle ABC$

, \therefore E is the midpoint of \overline{AC}

$\therefore \overline{BE}$ is a median in $\triangle ABC$

$$\therefore \overline{AD} \cap \overline{BE} = \{M\}$$

\therefore M is the intersection point of the medians of $\triangle ABC$

$$\therefore M \in \overline{CF}$$

$\therefore \overline{CF}$ is a median in $\triangle ABC$

\therefore F is the midpoint of \overline{AB}

$$\therefore AF = \frac{1}{2} AB = 3 \text{ cm.}$$

, \therefore E is the midpoint of \overline{AC}

$$\therefore AE = \frac{1}{2} AC = 5 \text{ cm.}$$

, in $\triangle ABC$:

\therefore F and E are the midpoints of \overline{AB} and \overline{AC} respectively.

$$\therefore FE = \frac{1}{2} BC = 4 \text{ cm.}$$

\therefore The perimeter of $\triangle AFE = AF + FE + AE$

$$= 3 + 4 + 5 = 12 \text{ cm.}$$

(The req.)

Theorem 2

The point of concurrence of the medians of the triangle divides each median in the ratio of 1 : 2 from its base.



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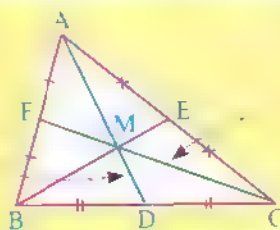
For example:

In the following figure :

M is the point of concurrence of the medians of $\triangle ABC$, then :

1 $MD = \frac{1}{2} AM$

If $AM = 6$ cm. ,
then $MD = 3$ cm.



2 $CM = 2 FM$

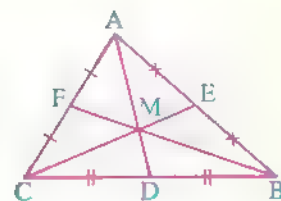
If $FM = 4$ cm. ,
then $CM = 8$ cm.

Remarks

- The point of concurrence of the medians of the triangle divides each of them in the ratio of 2 : 1 from the vertex.
- In the opposite figure :

If ABC is a triangle , M is the point of concurrence of its medians \overline{AD} , \overline{BF} and \overline{CE} , then:

$$MD = \frac{1}{3} AD \text{ and } AM = \frac{2}{3} AD$$



For example:

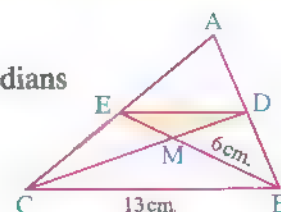
If $AD = 9$ cm. , then $MD = \frac{1}{3} AD = 3$ cm. , $AM = \frac{2}{3} AD = 6$ cm.

Similarly : $MF = \frac{1}{3} BF$, $BM = \frac{2}{3} BF$, $ME = \frac{1}{3} CE$ and $CM = \frac{2}{3} CE$

Example 2 In the opposite figure :

ABC is a triangle in which : \overline{CD} and \overline{BE} are two medians intersecting at M , $BM = 6$ cm. , $BC = 13$ cm. and $DC = 12$ cm.

Find the perimeter of $\triangle DME$

**Solution**

Given ABC is a triangle in which : \overline{CD} and \overline{BE} are two medians , M is the point of their intersection , $BM = 6$ cm. , $BC = 13$ cm. and $DC = 12$ cm.

R.T.F. The perimeter of $\triangle DME$

Proof $\therefore \overline{CD}$ and \overline{BE} are medians intersecting at the point M
 $\therefore M$ is the point of intersection of the medians of $\triangle ABC$

$$\therefore ME = \frac{1}{2} BM = \frac{1}{2} \times 6 = 3 \text{ cm.}$$

$$\therefore DM = \frac{1}{3} DC = \frac{1}{3} \times 12 = 4 \text{ cm.}$$

$\therefore \overline{CD}$ and \overline{BE} are two medians in $\triangle ABC$

$\therefore D$ is the midpoint of \overline{AB} and E is the midpoint of \overline{AC}

$$\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 13 = 6.5 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle DME = ME + DM + DE = 3 + 4 + 6.5 = 13.5 \text{ cm.}$$

(The req.)



The point which divides the median in a triangle by the ratio of 1 : 2 from the base is the point of intersection of the medians of this triangle.

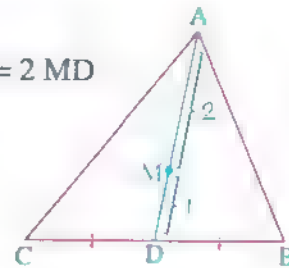
In the opposite figure :

If \overline{AD} is a median in $\triangle ABC$ and $M \in \overline{AD}$

such that $AM = 2 MD$,

then M is the point of intersection of the medians of $\triangle ABC$

$$AM = 2 MD$$



Example 3 In the opposite figure :

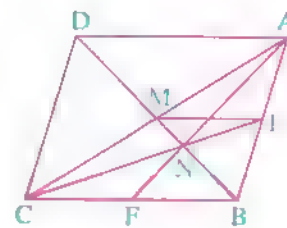
$ABCD$ is a parallelogram ,

M is the point of intersection of its diagonals ,

$N \in \overline{BM}$ where $BN = 2 NM$

and $\overline{CN} \cap \overline{AB} = \{E\}$

Prove that : $EM = \frac{1}{2} BC$



Solution

Given $ABCD$ is a parallelogram , M is the point of intersection of its diagonals ,
 $BN = 2 NM$, $N \in \overline{BM}$ and $\overline{CN} \cap \overline{AB} = \{E\}$

R.T.P. $EM = \frac{1}{2} BC$

Proof $\therefore ABCD$ is a parallelogram.

\therefore The two diagonals bisect each other.

$\therefore M$ is the midpoint of \overline{AC}

$\therefore \overline{BM}$ is a median in $\triangle ABC$

, $\therefore N \in \overline{BM}$ where $BN = 2 NM$

$\therefore N$ is the point of intersection of the medians of $\triangle ABC$

, $\therefore \overline{CE}$ passes through the point N $\therefore \overline{CE}$ is a median in $\triangle ABC$

$\therefore E$ is the midpoint of \overline{AB}

In $\triangle ABC$

$\therefore E$ is the midpoint of \overline{AB} and M is the midpoint of \overline{AC}

$\therefore EM = \frac{1}{2} BC$ (Q.E.D.)

TRY by yourself

In the opposite figure :

ABC is a triangle and M is the point of intersection of its medians.

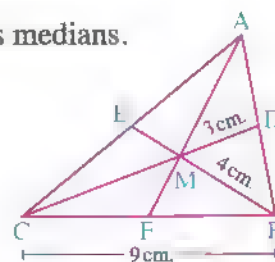
If $MD = 3$ cm. , $BM = 4$ cm. and $BC = 9$ cm. ,

complete the following :

① $BF = \dots$ cm.

② $MC = \dots$ cm.

③ $ME = \dots$ cm.



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3 2

2 6

1 4 5

Answers / of try by yourself

2

Medians of triangle "Follow"



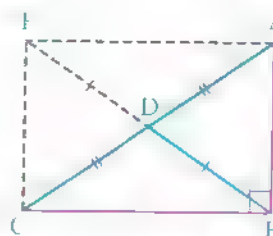
Theorem

In the right-angled triangle, the length of the median from the vertex of the right angle equals half the length of the hypotenuse.

Given ABC is a triangle in which $m(\angle ABC) = 90^\circ$,
BD is a median in the triangle ABC

R.T.P. $BD = \frac{1}{2} AC$

Construction Draw BD and take the point E \in BD such that $BD = DE$



Proof In the figure ABCE :

$\therefore \overline{AC}$ and \overline{BE} bisect each other.

\therefore The figure ABCE is a parallelogram.

$\therefore m(\angle ABC) = 90^\circ \therefore$ The figure ABCE is a rectangle.

$\therefore BE = AC$

$\therefore BD = \frac{1}{2} BE \therefore BD = \frac{1}{2} AC$

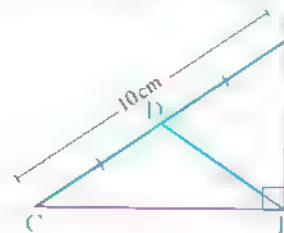
(Q.E.D.)

For example:

In the opposite figure :

ΔABC is a right-angled triangle at B ,

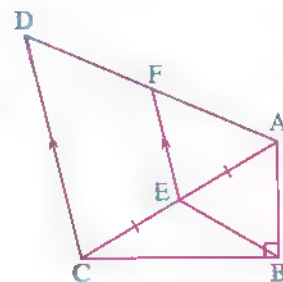
D is the midpoint of \overline{AC} and $AC = 10$ cm. , then $DB = 5$ cm.



Example 1 In the opposite figure :

ABCD is a quadrilateral in which $m(\angle ABC) = 90^\circ$
 $AC = CD$, E is the midpoint of \overline{AC}
 and $F \in \overline{AD}$ such that $\overline{EF} \parallel \overline{CD}$

Prove that : $BE = EF$

**Solution**

Given $m(\angle ABC) = 90^\circ$, $AC = CD$, E is the midpoint of \overline{AC} and $\overline{EF} \parallel \overline{CD}$

R.T.P. $BE = EF$

Proof In $\triangle ABC$:

$$\begin{aligned} \because m(\angle ABC) = 90^\circ \text{ and } \overline{BE} \text{ is a median} & \therefore BE = \frac{1}{2} AC \\ \because AC = CD & \therefore BE = \frac{1}{2} CD \end{aligned} \quad (1)$$

In $\triangle ACD$:

$$\begin{aligned} \because E \text{ is the midpoint of } \overline{AC} \text{ and } \overline{EF} \parallel \overline{CD} & \therefore F \text{ is the midpoint of } \overline{AD} \\ \therefore EF = \frac{1}{2} CD & \end{aligned} \quad (2)$$

$$\text{From (1) and (2) :} \quad \therefore BE = EF \quad (\text{Q.E.D.})$$

TRY 1

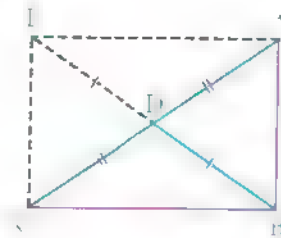
Choose the correct answer from those given :

- 1 In the right-angled triangle, the ratio between the length of the hypotenuse and the length of the median drawn from the vertex of the right angle is
 (a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 2 : 3
- 2 In $\triangle ABC$ which is right at B, if $AC = 12$ cm., D is the midpoint of \overline{AC} , then $BD = \dots\dots\dots$ cm.
 (a) 24 (b) 12 (c) 6 (d) 3
- 3 $\triangle ABC$ is right at A, the length of the median drawn from A is 4 cm., then $BC = \dots\dots\dots$ cm.
 (a) 12 (b) 8 (c) 4 (d) 2
- 4 $\triangle XYZ$ is right at Y, if $XY = 6$ cm., $YZ = 8$ cm., E is the midpoint of \overline{XZ} , then $YE = \dots\dots\dots$ cm.
 (a) 4 (b) 5 (c) 10 (d) 20

The converse of theorem 3

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right.

Given	ABC is a triangle, \overline{BD} is a median and $DA = DB = DC$
R.T.P.	$m(\angle ABC) = 90^\circ$
Construction	Draw \overrightarrow{BD} , then take the point $E \in \overrightarrow{BD}$ such that $BD = DE$
Proof	$\therefore BD = \frac{1}{2} BE = \frac{1}{2} AC$ $\therefore BE = AC$ \therefore In the figure $ABCE$: \overline{AC} and \overline{BE} are equal in length and bisect each other. \therefore The figure $ABCE$ is a rectangle. $\therefore m(\angle ABC) = 90^\circ$



(Q.E.D.)

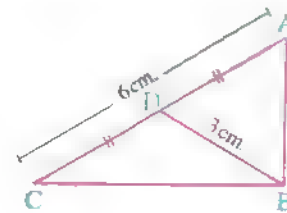
For example:

In the opposite figure :

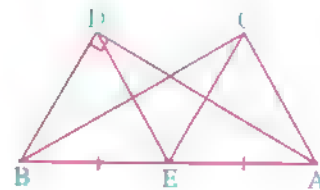
If \overline{BD} is a median in $\triangle ABC$,

$BD = 3$ cm, and $AC = 6$ cm,

then $m(\angle ABC) = 90^\circ$ "because $BD = \frac{1}{2} AC$ "



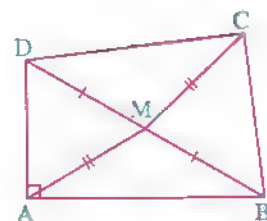
Example 2 In the opposite figure :
 ABD is a right-angled triangle at D ,
 E is the midpoint of \overline{AB} and $CE = DE$
Prove that : $m(\angle ACB) = 90^\circ$



Solution

Given	E is the midpoint of \overline{AB} , $m(\angle ADB) = 90^\circ$, $CE = DE$
R.T.P.	$m(\angle ACB) = 90^\circ$
Proof	<p>In $\triangle ADB$:</p> $\therefore m(\angle ADB) = 90^\circ$, \overline{DE} is a median $\therefore DE = \frac{1}{2} AB$ But $CE = DE$ $\therefore CE = \frac{1}{2} AB$ \therefore In $\triangle ACB$: \overline{CE} is a median with length equals half the length of \overline{AB} $\therefore m(\angle ACB) = 90^\circ$

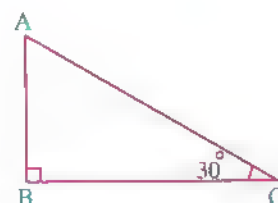
(Q.E.D.)

TRY YOURSELF 2**In the opposite figure :**ABCD is a quadrilateral in which $m(\angle BAD) = 90^\circ$,M is the midpoint of \overline{BD} and $CM = AM$ **Prove that :** $m(\angle BCD) = 90^\circ$ **Corollary**

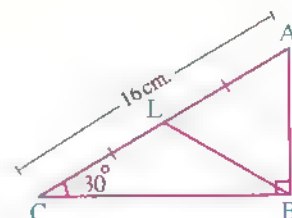
The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.

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i.e.

In the opposite figure :If $\triangle ABC$ is right-angled at B and $m(\angle C) = 30^\circ$, then $AB = \frac{1}{2} AC$ **For example:**If $AC = 20$ cm. , then $AB = 10$ cm.**! Remark**

The right-angled triangle whose measure of one of its angles is 30° , then the measure of the third angle is 60° is called thirty and sixty triangle.

Example 3 In the opposite figure :ABC is a triangle in which $m(\angle ABC) = 90^\circ$, $m(\angle C) = 30^\circ$, $AC = 16$ cm. andL is the midpoint of \overline{AC} **Find :** 1 The length of each of \overline{AB} and \overline{BL} 2 The perimeter of $\triangle ABL$ 

Solution

Given | $m(\angle ABC) = 90^\circ$, $m(\angle C) = 30^\circ$,
 $AC = 16$ cm. and L is the midpoint of \overline{AC}

R.T.F. | **1** AB , BL **2** The perimeter of $\triangle ABL$

Proof | $\therefore \triangle ABC$ is right-angled at B , $m(\angle C) = 30^\circ$

$$\therefore AB = \frac{1}{2} AC = 8 \text{ cm.}$$

$\therefore \overline{BL}$ is a median in $\triangle ABC$

$$\therefore BL = \frac{1}{2} AC = 8 \text{ cm.}$$

(First req.)

$$\therefore AL = \frac{1}{2} AC = 8 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle ABL = 8 + 8 + 8 = 24 \text{ cm.}$$

(Second req.)

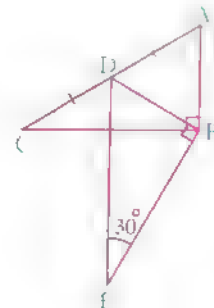
TRY 3

In the opposite figure :

$$m(\angle ABC) = m(\angle DBE) = 90^\circ,$$

D is the midpoint of \overline{AC} and $m(\angle E) = 30^\circ$

Prove that : $AC = DE$



Answer / of try by yourself

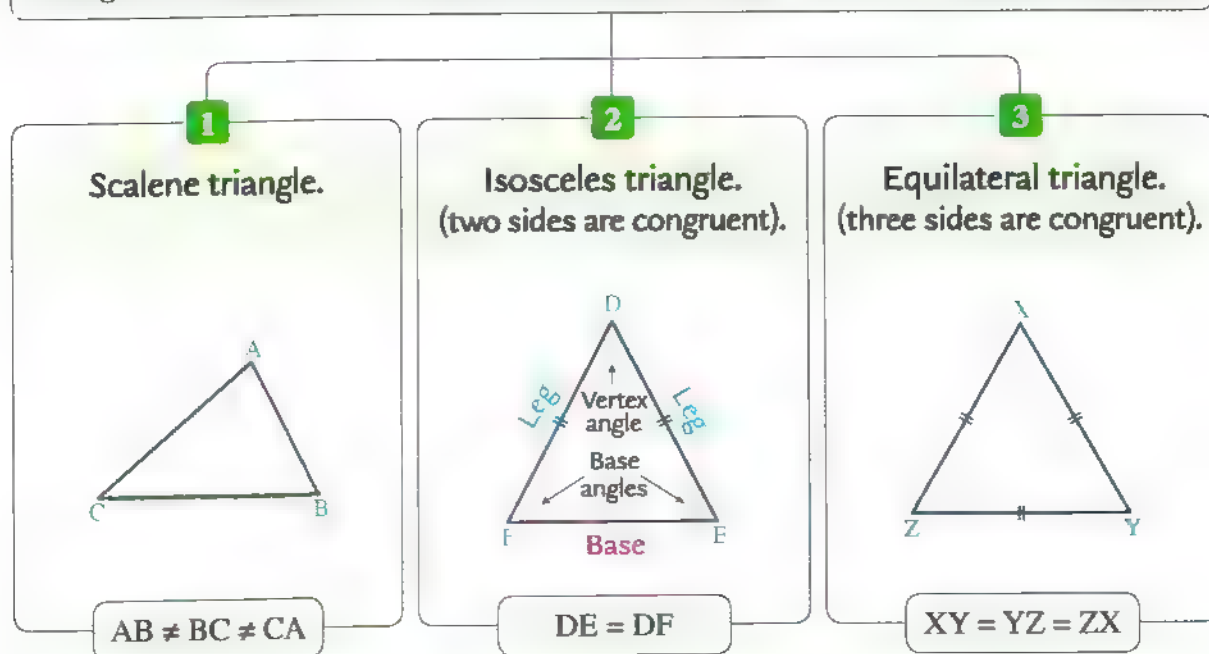
- 3** Prove by yourself. (Hint : Prove that $BD = \frac{1}{2} AC$ and $BD = \frac{1}{2} DE$)
- 2** Prove by yourself. (Hint : $AM = \frac{1}{2} BD$, $CM = AM$)
- 1** c
- 2** c
- 3** b
- 4** b

The isosceles triangle



Prelude

Triangles are classified according to the lengths of their sides into three types which are :



- In the following we will study the relations between the angles in the isosceles triangle and in the equilateral triangle.

The Isosceles Triangle Theorem

Theorem

The base angles of the isosceles triangle are congruent.



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Given ABC is a triangle in which $\overline{AB} \cong \overline{AC}$

R.T.P. $\angle B \cong \angle C$

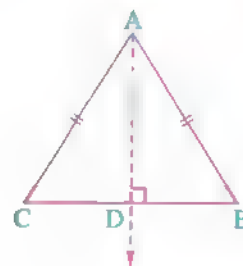
Construction Draw $\overline{AD} \perp \overline{BC}$ where $\overline{AD} \cap \overline{BC} = \{D\}$

Proof $\therefore \triangle ADB, ADC$ in which :

$$\begin{cases} m(\angle ADB) = m(\angle ADC) = 90^\circ & (\text{const.}) \\ \overline{AB} \cong \overline{AC} & (\text{given}) \\ \overline{AD} \text{ is a common side} \end{cases}$$

$\therefore \triangle ADB \cong \triangle ADC$,

then we deduce that $\angle B \cong \angle C$



(Q.E.D)

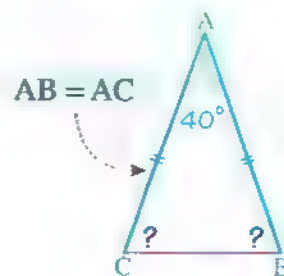
For example:

In the opposite figure :

If ABC is a triangle in which :

$AB = AC$, $m(\angle A) = 40^\circ$,

then $m(\angle B) = m(\angle C) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$



Remarks

- Both of the base angles in the isosceles triangle are acute.
- The vertex angle in the isosceles triangle may be acute, right or obtuse angle.

Example 1

Choose the correct answer from those given :

- ABC is a triangle in which $AB = AC$, $m(\angle B) = 70^\circ$, then $m(\angle A) = \dots\dots\dots$
 (a) 40° (b) 50° (c) 55° (d) 70°
- In $\triangle XYZ$, $XY = XZ$, $m(\angle X) = 100^\circ$, $m(\angle Z) = \dots\dots\dots$
 (a) 20° (b) 40° (c) 80° (d) 100°
- $\triangle XYZ$ is right at Y , if $XY = YZ$, then $m(\angle Z) = \dots\dots\dots$
 (a) 30° (b) 45° (c) 60° (d) 90°

4 LMN is a triangle in which $LM = MN$, then $\angle N$ is ...

- (a) acute. (b) right. (c) obtuse. (d) reflex.

5 XYZ is an isosceles triangle, $m(\angle X) = 110^\circ$

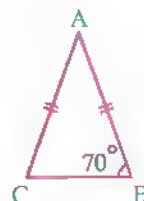
, then $m(\angle Y) = \dots\dots\dots$

- (a) 30° (b) 35° (c) 40° (d) 45°

Solution 1 (a) The reason : $\because AB = AC$

$$\therefore m(\angle B) = m(\angle C) = 70^\circ$$

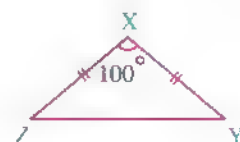
$$\therefore m(\angle A) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$$



2 (b) The reason : $\because XY = XZ$

$$\therefore m(\angle Y) = m(\angle Z)$$

$$\therefore m(\angle Z) = \frac{180^\circ - 100^\circ}{2} = 40^\circ$$

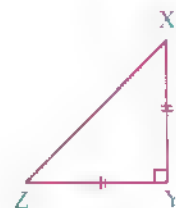


3 (b) The reason : $\because XY = YZ$

$$\therefore m(\angle X) = m(\angle Z)$$

$$\because m(\angle Y) = 90^\circ$$

$$\therefore m(\angle Z) = \frac{180^\circ - 90^\circ}{2} = 45^\circ$$



4 (a) The reason : $\because \triangle LMN$ is an isosceles triangle

\therefore Each of the base angles is acute

$\therefore \angle N$ is acute



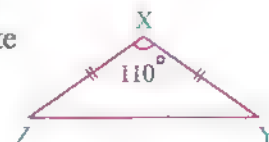
5 (b) The reason : $\because \triangle XYZ$ is an isosceles triangle

\therefore Each of the base angles is acute

$\therefore \angle X$ is the vertex angle

$$\therefore m(\angle Y) = m(\angle Z)$$

$$\therefore m(\angle Y) = \frac{180^\circ - 110^\circ}{2} = 35^\circ$$

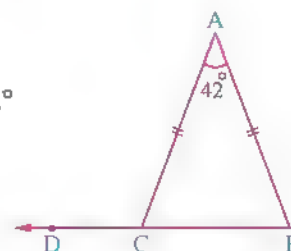


Example 2 In the opposite figure :

ABC is a triangle in which $AB = AC$, $m(\angle A) = 42^\circ$

and $D \in \overrightarrow{BC}$

Find : $m(\angle ACD)$



Solution

Given $AB = AC$, $m(\angle A) = 42^\circ$ and $D \in \overrightarrow{BC}$

R.T.F. $m(\angle ACD)$

Proof \therefore The sum of measures of the interior angles in $\triangle ABC = 180^\circ$

$$, m(\angle A) = 42^\circ$$

$$\therefore m(\angle B) + m(\angle ACB)$$

$$= 180^\circ - 42^\circ = 138^\circ$$

$$, \therefore AB = AC \text{ (given)}$$

$$\therefore m(\angle B) = m(\angle ACB) = \frac{138^\circ}{2} = 69^\circ$$

$$\therefore \angle ACD \text{ is an exterior angle of } \triangle ABC$$

$$\therefore m(\angle ACD) = m(\angle A) + m(\angle B)$$

$$= 42^\circ + 69^\circ = 111^\circ$$

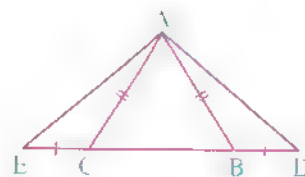
**Remember that**

The measure of any exterior angle of a triangle is equal to the sum of measures of the two non-adjacent interior angles.

Example 3 In the opposite figure :

$B \in \overrightarrow{DE}$, $C \in \overrightarrow{DE}$, $AB = AC$ and $BD = CE$

Prove that : $AD = AE$

**Solution**

Given $AB = AC$ and $BD = CE$

R.T.P. $AD = AE$

Proof $\therefore AB = AC$ (given)

$$\therefore m(\angle ABC) = m(\angle ACB)$$

$$\therefore \angle ABD \text{ supplements } \angle ABC$$

$$, \angle ACE \text{ supplements } \angle ACB$$

$$\therefore m(\angle ABD) = m(\angle ACE)$$

$$\therefore \text{In } \triangle ABD, \triangle ACE :$$

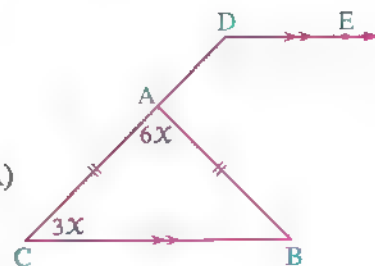
$$\begin{cases} AB = AC & \text{(given)} \\ BD = CE & \text{(given)} \\ m(\angle ABD) = m(\angle ACE) & \text{(by proof)} \end{cases}$$

$$\therefore \triangle ABD \cong \triangle ACE, \text{ then we deduce that } AD = AE$$

(Q.E.D.)

**Remember that**

The supplementaries of the equal angles in measures are equal in measures.

Example 4 In the opposite figure : $AB = AC$, $m(\angle BAC) = 6x$, $m(\angle C) = 3x$ and $\overline{BC} \parallel \overrightarrow{DE}$ **Find :** 1 The value of x 2 $m(\angle EDA)$ **Solution****Given** $AB = AC$, $m(\angle BAC) = 6x$, $m(\angle C) = 3x$ and $\overline{BC} \parallel \overrightarrow{DE}$ **R.T.F.**1 The value of x 2 $m(\angle EDA)$ **Proof** $\therefore AB = AC$ $\therefore m(\angle B) = m(\angle C) = 3x$ \therefore the sum of measures of the interior angles of the triangle $= 180^\circ$ $\therefore 6x + 3x + 3x = 180^\circ$ $\therefore 12x = 180^\circ$ $\therefore x = \frac{180^\circ}{12} = 15^\circ$

(First req.)

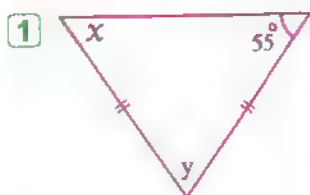
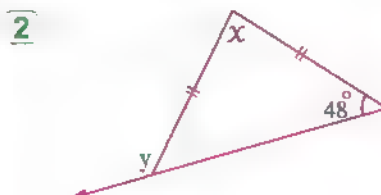
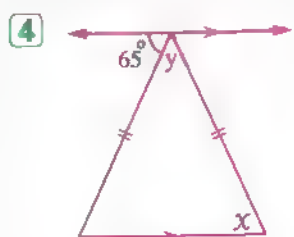
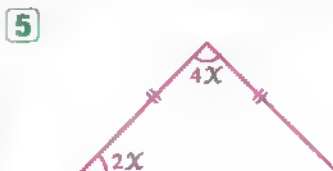
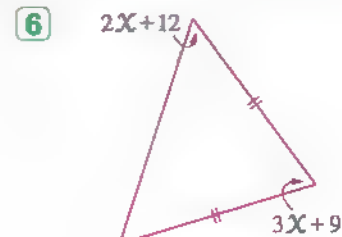
 $\therefore m(\angle C) = 45^\circ$ $\therefore \overline{BC} \parallel \overrightarrow{DE}$ and \overrightarrow{CD} is a transversal $\therefore m(\angle EDA) + m(\angle C) = 180^\circ$ (two interior angles on the same side of the transversal) $\therefore m(\angle EDA) = 180^\circ - 45^\circ = 135^\circ$

(Second req.)

TRY
by Our Self

1

In each of the following figures, find the values of the symbols used as measures for the angles :

 $x = \dots^\circ$, $y = \dots^\circ$  $x = \dots^\circ$, $y = \dots^\circ$  $x = \dots^\circ$, $y = \dots^\circ$, $z = \dots^\circ$  $x = \dots^\circ$, $y = \dots^\circ$  $x = \dots^\circ$  $x = \dots^\circ$

Corollary

If the triangle is equilateral, then it is equiangular where each angle measure is 60°

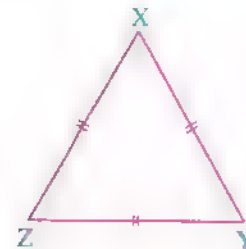


For example:

In the opposite figure :

If XYZ is a triangle in which $XY = YZ = ZX$

, then $m(\angle X) = m(\angle Y) = m(\angle Z) = 60^\circ$



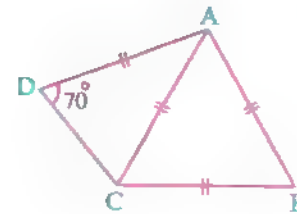
Example 5 In the opposite figure :

$AB = BC = CA = AD$

and $m(\angle D) = 70^\circ$

Find : 1 $m(\angle BCD)$

2 $m(\angle BAD)$



Solution

Given $AB = BC = CA = AD$ and $m(\angle D) = 70^\circ$

R.T.F. 1 $m(\angle BCD)$

2 $m(\angle BAD)$

Proof $\because \triangle ABC$ is an equilateral triangle.

$\therefore m(\angle BCA) = 60^\circ$

In $\triangle ACD$: $\because AC = AD$

$\therefore m(\angle ACD) = m(\angle D) = 70^\circ$

$\therefore m(\angle BCD) = m(\angle BCA) + m(\angle ACD)$

$$= 60^\circ + 70^\circ = 130^\circ$$

(First req.)

\because The sum of measures of the interior angles of the quadrilateral $ABCD = 360^\circ$

, $m(\angle B) = 60^\circ$

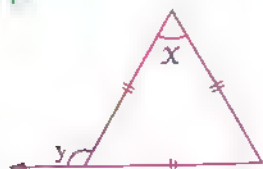
$\therefore m(\angle BAD) = 360^\circ - (60^\circ + 130^\circ + 70^\circ) = 100^\circ$

(Second req.)

TRY
by yourself **2**

In each of the following figures, find the values of the symbols used as measures for the angles :

1



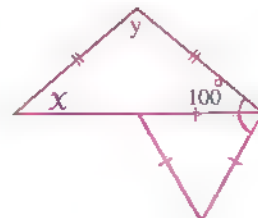
$x = \dots^\circ, y = \dots^\circ$

2



$x = \dots^\circ, y = \dots^\circ$

3



$x = \dots^\circ, y = \dots^\circ$

- | | | | | | |
|---|-----------|---|-----------|---|----------------|
| 1 | 55°, 70° | 2 | 84°, 132° | 3 | 138°, 96°, 42° |
| 2 | 60°, 120° | 4 | 65°, 50° | 5 | 22½° |
| 6 | 21° | 7 | 40°, 100° | | |

Answers / of try by yourself

4

The converse of the isosceles triangle theorem



Theorem

If two angles of a triangle are congruent, then the two sides opposite to these two angles are congruent and the triangle is isosceles.

Given ABC is a triangle in which $\angle B \cong \angle C$

R.T.P. $\overline{AB} \cong \overline{AC}$

Construction Bisect $\angle BAC$ by \overline{AD} to intersect \overline{BC} at D

Proof $\therefore \angle B \cong \angle C$

$\therefore m(\angle B) = m(\angle C)$

$\therefore \overline{AD}$ bisects $\angle BAC$

$\therefore m(\angle BAD) = m(\angle CAD)$

\therefore The sum of measures of the interior angles of the triangle $= 180^\circ$

$\therefore m(\angle ADB) = m(\angle ADC)$

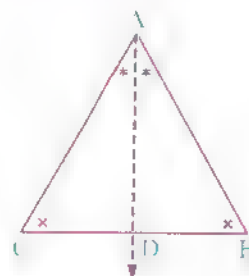
\therefore In $\triangle ABD$ and $\triangle ACD$:

$\left\{ \begin{array}{l} \overline{AD} \text{ is a common side} \\ m(\angle BAD) = m(\angle CAD) \text{ (const.)} \\ m(\angle ADB) = m(\angle ADC) \text{ (by proof)} \end{array} \right.$

$\therefore \triangle ABD \cong \triangle ACD$, then we deduce that :

$\overline{AB} \cong \overline{AC}$, then $\triangle ABC$ is an isosceles triangle.

(Q.E.D.)



Example 1 ABC is a triangle in which $m(\angle A) = 2m(\angle B) = 72^\circ$

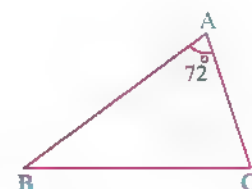
Prove that : ΔABC is an isosceles triangle.

Solution

Given $m(\angle A) = 2m(\angle B) = 72^\circ$

R.T.P. ΔABC is an isosceles triangle.

Proof In $\Delta ABC : \because 2m(\angle B) = 72^\circ \quad \therefore m(\angle B) = \frac{72^\circ}{2} = 36^\circ$
 $\therefore m(\angle A) = 72^\circ \quad \therefore m(\angle C) = 180^\circ - (36^\circ + 72^\circ) = 72^\circ$
 $\therefore m(\angle A) = m(\angle C) \quad \therefore BC = BA$
 $\therefore \Delta ABC$ is an isosceles triangle. (Q.E.D.)

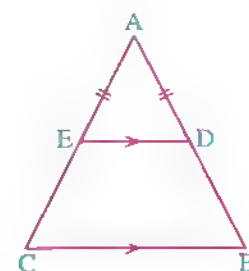


Example 2 In the opposite figure :

$D \in \overline{AB}$ and $E \in \overline{AC}$

where $AD = AE$ and $\overline{DE} \parallel \overline{BC}$

Prove that : $DB = EC$



Solution

Given $AD = AE$ and $\overline{DE} \parallel \overline{BC}$

R.T.P. $DB = EC$

Proof In $\Delta ADE : \because AD = AE \quad \therefore m(\angle ADE) = m(\angle AED)$ (1)

$\therefore \overline{DE} \parallel \overline{BC}$ and \overleftrightarrow{AB} is a transversal

$\therefore m(\angle B) = m(\angle ADE)$ (corresponding angles) (2)

Similarly $\because \overline{DE} \parallel \overline{BC}$ and \overleftrightarrow{AC} is a transversal

$\therefore m(\angle C) = m(\angle AED)$ (corresponding angles) (3)

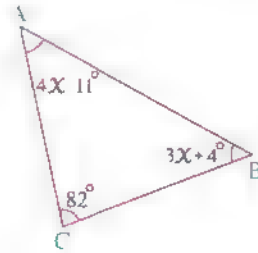
From (1), (2) and (3) : $\therefore m(\angle B) = m(\angle C)$

$\therefore AB = AC$, $\because AD = AE$

Subtracting : $\therefore AB - AD = AC - AE \quad \therefore DB = EC$ (Q.E.D.)

Example 3 In the opposite figure :

- If $m(\angle A) = 4x - 11^\circ$
 $\therefore m(\angle B) = 3x + 4^\circ$
 $\therefore m(\angle C) = 82^\circ$
 \therefore prove that : $\triangle ABC$ is an isosceles triangle.



Solution

Given | $m(\angle A) = 4x - 11^\circ$, $m(\angle B) = 3x + 4^\circ$, $m(\angle C) = 82^\circ$

R.T.P. | $\triangle ABC$ is an isosceles triangle.

Proof \therefore The sum of measures of the interior angles of the triangle = 180°

$$\therefore 4x - 11^\circ + 3x + 4^\circ + 82^\circ = 180^\circ \quad \therefore 7x + 75^\circ = 180^\circ$$

$$\therefore 7x = 105^\circ$$

$$\therefore x = 15^\circ$$

$$\therefore m(\angle A) = 4 \times 15^\circ - 11^\circ = 49^\circ$$

$$\therefore m(\angle B) = 3 \times 15^\circ + 4^\circ = 49^\circ$$

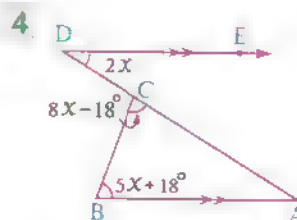
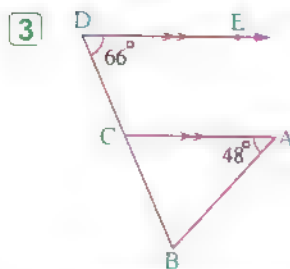
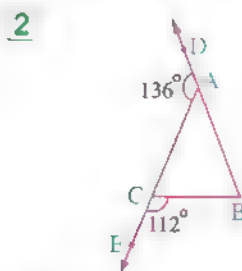
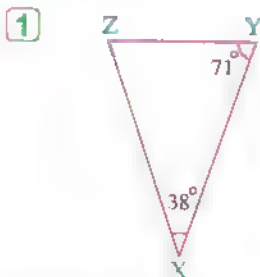
$$\therefore m(\angle A) = m(\angle B)$$

$\therefore BC = AC \quad \therefore \triangle ABC$ is an isosceles triangle.

(Q.E.D.)

TRY 1

In each of the following figures , write the equal sides in length :



Corollary

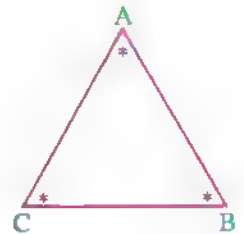
If the angles of a triangle are congruent, then the triangle is equilateral.

For example:

If $\triangle ABC$ is a triangle in which :

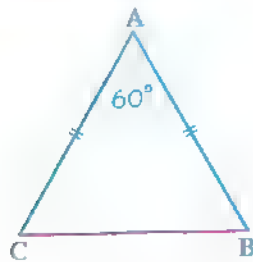
$\angle A \cong \angle B \cong \angle C$, then $AB = BC = CA$

i.e. $\triangle ABC$ is an equilateral triangle.

**Remark**

The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle.

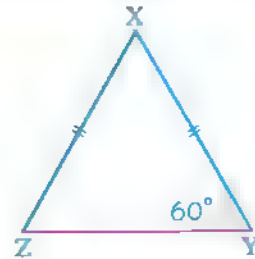
• In the following figure :



If $AB = AC$ and $m(\angle A) = 60^\circ$
 , then $m(\angle B) = m(\angle C) = \frac{180^\circ - 60^\circ}{2}$
 $= 60^\circ$

$\therefore \triangle ABC$ is an equilateral triangle.

• In the following figure :



If $XY = XZ$ and $m(\angle Y) = 60^\circ$
 , then $m(\angle Z) = 60^\circ$
 , $m(\angle X) = 180^\circ - (60^\circ + 60^\circ) = 60^\circ$

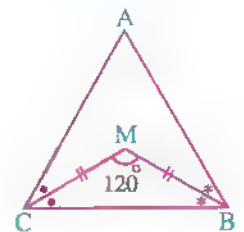
$\therefore \triangle XYZ$ is an equilateral triangle.

Example 4 In the opposite figure :

\overline{BM} bisects $\angle B$, \overline{CM} bisects $\angle C$,

$MB = MC$ and $m(\angle BMC) = 120^\circ$

Prove that : $\triangle ABC$ is an equilateral triangle.



Solution

Given

\overline{BM} bisects $\angle B$, \overline{CM} bisects $\angle C$, $MB = MC$ and $m(\angle BMC) = 120^\circ$

R.T.P.

$\triangle ABC$ is an equilateral triangle.

Proof

In $\triangle MBC$: $\because MB = MC$ and $m(\angle BMC) = 120^\circ$

$$\therefore m(\angle MBC) = m(\angle MCB) = \frac{180^\circ - 120^\circ}{2} = 30^\circ$$

$$\therefore \overrightarrow{BM} \text{ bisects } \angle B \quad \therefore m(\angle ABC) = 2 m(\angle MBC) = 60^\circ$$

$$\therefore \overrightarrow{CM} \text{ bisects } \angle C \quad \therefore m(\angle ACB) = 2 m(\angle MCB) = 60^\circ$$

$$\therefore \text{In } \triangle ABC : m(\angle BAC) = 180^\circ - (60^\circ + 60^\circ) = 60^\circ$$

$$\therefore m(\angle ABC) = m(\angle ACB) = m(\angle BAC) = 60^\circ$$

$\therefore \triangle ABC$ is an equilateral triangle.

(Q.E.D.)

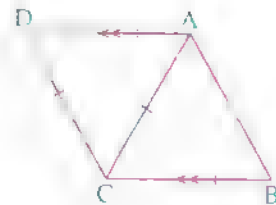
TRY 2

In the opposite figure :

ABCD is a quadrilateral in which :

$AD = DC = CB = CA$, $\overline{AD} \parallel \overline{BC}$.

Prove that : $\triangle ABC$ is an equilateral triangle.



2 Prove by yourself. (Hint : Prove that $m(\angle ACB) = 60^\circ$)

3 \overline{AB} , \overline{AC}

4 \overline{AB} , \overline{AC}

1 \overline{XY} , \overline{XZ}

2 \overline{AB} , \overline{AC}

Answers / of try by yourself

Corollaries of the isosceles triangle theorems



Corollary 1

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.

In the opposite figure :

ABC is a triangle in which $AB = AC$ and \overline{AD} is a median , then :

1 \overline{AD} bisects $\angle BAC$

i.e. $m(\angle BAD) = m(\angle CAD)$

2 $\overline{AD} \perp \overline{BC}$



Corollary 2

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

In the opposite figure :

ABC is a triangle in which $AB = AC$ and \overline{AD} bisects $\angle BAC$, then :

1 D is the midpoint of \overline{BC}

i.e. $BD = CD$

2 $\overline{AD} \perp \overline{BC}$



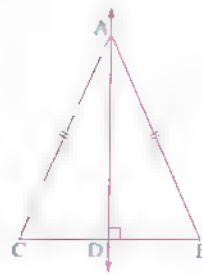
Corollary 3

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.

In the opposite figure :

ABC is a triangle in which $AB = AC$ and $\overline{AD} \perp \overline{BC}$, then :

- 1 D is the midpoint of \overline{BC} i.e. $BD = CD$
- 2 $m(\angle BAD) = m(\angle CAD)$

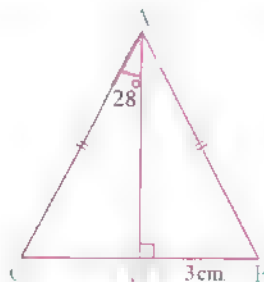


Notice that :

The previous three corollaries can be proved using the congruence of $\triangle ABD$ and $\triangle ACD$

Example 1 In the opposite figure :

- ABC is an isosceles triangle where
 $AB = AC$, $D \in \overline{BC}$ such that $\overline{AD} \perp \overline{BC}$,
 $m(\angle CAD) = 28^\circ$ and $BD = 3$ cm. Find :
- 1 $m(\angle BAC)$
 - 2 The length of \overline{BC}



Solution

Given $AB = AC$, $m(\angle CAD) = 28^\circ$, $BD = 3$ cm. and $\overline{AD} \perp \overline{BC}$

R.T.F. 1 $m(\angle BAC)$ 2 The length of \overline{BC}

Proof In $\triangle ABC$: $\because AB = AC$ and $\overline{AD} \perp \overline{BC}$

$\therefore \overline{AD}$ bisects each of the vertex angle BAC and the base \overline{BC}

$\therefore m(\angle BAC) = 2 m(\angle CAD) = 2 \times 28^\circ = 56^\circ$ (First req.)

, $BC = 2 BD = 2 \times 3 = 6$ cm. (Second req.)

Example 2 Choose the correct answer from those given :

- 1 In $\triangle ABC$, if $AB = AC$, \overline{AD} is a median, $m(\angle BAC) = 100^\circ$, then $m(\angle BAD) = \dots\dots\dots$
 - (a) 100°
 - (b) 50°
 - (c) 90°
 - (d) 40°
- 2 In $\triangle XYZ$, if $XY = XZ$, \overline{XD} bisects $\angle YXZ$, then $\triangle XYD$ is $\dots\dots\dots$
 - (a) acute-angled.
 - (b) right-angled.
 - (c) obtuse-angled.
 - (d) isosceles.
- 3 In $\triangle LMN$, $NM = NL$, $D \in \overline{LM}$ where $\overline{ND} \perp \overline{LM}$, if $LM = 10$ cm., then $LD = \dots\dots\dots$ cm.
 - (a) 20
 - (b) 10
 - (c) 5
 - (d) 2.5

- 4 ABC is a triangle in which $AB = AC$, \overline{AX} is a median, if $BX = 5$ cm, $m(\angle BAX) = 30^\circ$, then the perimeter of $\triangle ABC = \dots\dots\dots$ cm.

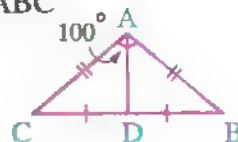
(a) 10 (b) 15 (c) 25 (d) 30

Solution

- 1 (b) The reason : $\because AB = AC$, \overline{AD} is a median in $\triangle ABC$

$$\therefore m(\angle BAD) = m(\angle CAD)$$

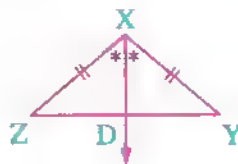
$$\therefore m(\angle BAD) = \frac{100^\circ}{2} = 50^\circ$$



- 2 (b) The reason : $\because XY = XZ$, \overline{XD} bisects $\angle YXZ$

$$\therefore \overline{XD} \perp \overline{YZ}$$

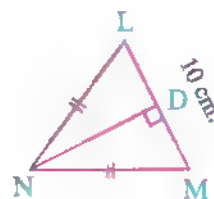
$$\therefore \triangle XYD \text{ is right-angled.}$$



- 3 (c) The reason : $\because NM = NL$, $\overline{ND} \perp \overline{LM}$

$$\therefore D \text{ is the midpoint of } \overline{LM}$$

$$\therefore LD = \frac{10}{2} = 5 \text{ cm.}$$



- 4 (d) The reason : $\because AB = AC$, \overline{AX} is a median

$$\therefore \overline{AX} \text{ bisects } \angle BAC$$

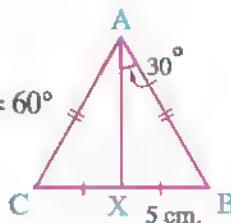
$$\therefore m(\angle BAC) = 2 m(\angle BAX) \\ = 2 \times 30^\circ = 60^\circ$$

$$\therefore \triangle ABC \text{ is isosceles, } m(\angle BAC) = 60^\circ$$

$$\therefore \triangle ABC \text{ is equilateral}$$

$$\therefore BC = 2 BX = 10 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle ABC = 3 \times 10 = 30 \text{ cm.}$$



TRY
yourself

In the opposite figure :

ABDC is a quadrilateral in which :

$AB = AC$, $BD = CD$, $\overline{AD} \perp \overline{BC}$,

$\overline{AD} \cap \overline{BC} = \{E\}$, $m(\angle BAD) = 30^\circ$,

$m(\angle BDC) = 80^\circ$ and $BE = 4$ cm.

Complete the following :

1 $m(\angle DAC) = \dots\dots\dots^\circ$

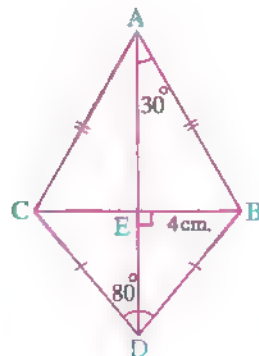
2 $m(\angle BDE) = \dots\dots\dots^\circ$

3 $m(\angle ACB) = \dots\dots\dots^\circ$

4 $EC = \dots\dots\dots$ cm.

5 $AC = \dots\dots\dots$ cm.

6 $AE = \dots\dots\dots$ cm.



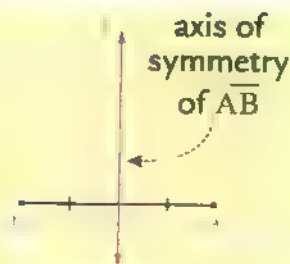
Axis of symmetry of a line segment

Definition

The straight line perpendicular to a line segment at its middle is called the axis of symmetry for that line segment, in brief it is known as the axis of a line segment.

In the opposite figure :

If the straight line $L \perp \overline{AB}$ and $C \in$ the straight line L where C is the midpoint of \overline{AB} , then the straight line L is called the axis of \overline{AB}



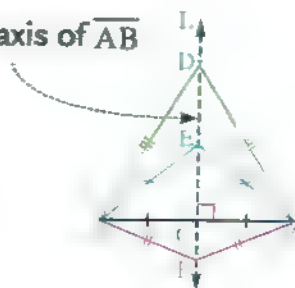
Property

Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).

In the opposite figure :

If the straight line L is the axis of \overline{AB} ,
 $D \in L$, $E \in L$ and $F \in L$, then
 $DA = DB$, $EA = EB$ and $FA = FB$

L is the axis of \overline{AB}

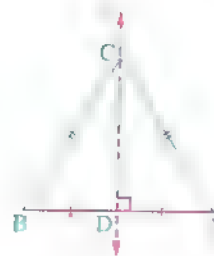


The converse of the previous property is true :

i.e. If a point is at equal distances from the two terminals of a line segment, then this point lies on the axis of this line segment.

In the opposite figure :

If C is a point such
 that $CA = CB$, then
 the point C lies on the axis of \overline{AB}



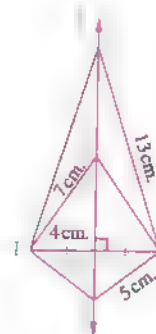
Example

In the opposite figure :

The straight line L is the axis of \overline{AB}

If the points C , D and E belong to the straight line L ,
 $L \cap \overline{AB} = \{M\}$ where $AC = 13$ cm.,
 $DB = 7$ cm., $AE = 5$ cm. and $MB = 4$ cm.

Find the length of each of : \overline{CB} , \overline{DA} , \overline{EB} and \overline{MA}



Solution

Given The straight line L is the axis of \overline{AB} , C , D and E belong to the straight line L , $L \cap \overline{AB} = \{M\}$

$AC = 13$ cm., $DB = 7$ cm., $AE = 5$ cm. and $MB = 4$ cm.

R.T.F. The lengths of : \overline{CB} , \overline{DA} , \overline{EB} and \overline{MA}

Proof $\because C$, D and E belong to L (the axis of \overline{AB})

$\therefore CB = CA = 13$ cm., $DA = DB = 7$ cm.,

$EB = EA = 5$ cm., $MA = MB = 4$ cm.

(The req.)

Example 4 $\triangle ABC$ is an isosceles triangle where $AB = AC$, \overrightarrow{BX} bisects $\angle ABC$ and intersects \overline{AC} at X , \overrightarrow{CY} bisects $\angle ACB$ and intersects \overline{AB} at Y . If $\overrightarrow{BX} \cap \overrightarrow{CY} = \{M\}$, prove that : $\overrightarrow{AM} \perp \overline{BC}$

Solution

Given $AB = AC$, \overrightarrow{BX} bisects $\angle ABC$ and \overrightarrow{CY} bisects $\angle ACB$

R.T.P. $\overrightarrow{AM} \perp \overline{BC}$

Proof $\because AB = AC$

$$\therefore m(\angle ABC) = m(\angle ACB) \quad (1)$$

$$\because \overrightarrow{BX} \text{ bisects } \angle ABC \quad \therefore m(\angle MBC) = \frac{1}{2} m(\angle ABC) \quad (2)$$

Similarly

$$\because \overrightarrow{CY} \text{ bisects } \angle ACB \quad \therefore m(\angle MCB) = \frac{1}{2} m(\angle ACB) \quad (3)$$

From (1), (2) and (3), we deduce that :

$$m(\angle MBC) = m(\angle MCB) \quad \therefore MB = MC$$

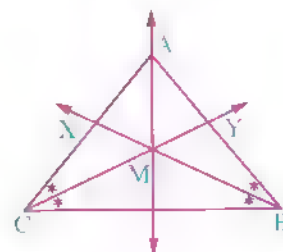
i.e. M is at equal distances from B and C

$$\therefore M \in \text{the axis of } \overline{BC} \quad (4)$$

$\because AB = AC$ i.e. A is at equal distances from B and C

$$\therefore A \in \text{the axis of } \overline{BC} \quad (5)$$

$$\text{From (4) and (5): } \therefore \overrightarrow{AM} \text{ is the axis of } \overline{BC} \quad \therefore \overrightarrow{AM} \perp \overline{BC} \quad (\text{Q.E.D.})$$

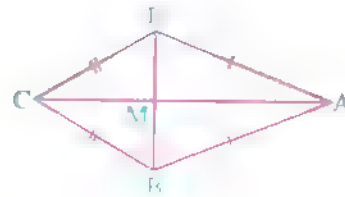


TRY 2

In the opposite figure :

$\overline{BD} \cap \overline{AC} = \{M\}$, $AB = AD$ and $BC = DC$

Prove that : M is the midpoint of \overline{BD}



Axis of symmetry of the isosceles triangle

The isosceles triangle has one axis of symmetry.

It is the straight line drawn from the vertex angle perpendicular to its base.

For example:

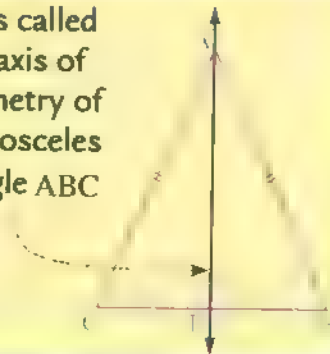
If ABC is an isosceles triangle where

$AB = AC$ and $\overleftrightarrow{AD} \perp \overline{BC}$, then

\overleftrightarrow{AD} is called the axis of symmetry

of the isosceles triangle ABC

\overleftrightarrow{AD} is called
the axis of
symmetry of
the isosceles
triangle ABC



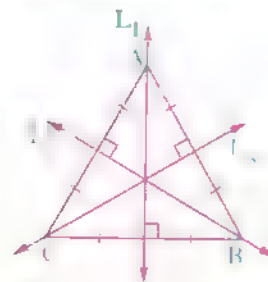
Remarks

- The equilateral triangle has three axes of symmetry , they are the three perpendiculars drawn from its vertices to the opposite sides.

In the opposite figure :

The straight lines L_1 , L_2 and L_3 are the axes of symmetry of the equilateral triangle ABC

- The scalene triangle has no axes of symmetry.



2 Prove by yourself. (Hint : Prove that \overline{AC} is the axis of \overline{BD})

- 1 30°
- 2 40°
- 3 60°
- 4 4
- 5 8
- 6 $4\sqrt{3}$



UNIT

5

Inequality

■ Lessons of the unit :

1. Inequality.
2. Comparing the measures of angles in a triangle.
3. Comparing the lengths of sides in a triangle.
4. Triangle inequality.

► Use your smart phone or tablet to scan the QR Code and enjoy watching videos.



■ Unit Objectives :

By the end of this unit, student should be able to :

- recognize the concept of inequality.
- recognize the axioms of the inequality relation.
- compare between the measures of angles in the triangle.
- deduce the relation between the measures of two angles in a triangle when the two opposite sides to these angles are not equal in length.
- compare side lengths in the triangle.
- deduce the relation between the lengths of two sides in a triangle when the two opposite angles to these sides are not equal in measure.
- recognize the triangle inequality.
- use the axioms of the inequality relation and the triangle inequality in solving problems in geometry.



The concept of inequality

- Through our study of the sets of numbers, we had shown the relation of inequality that is used for comparing two different numbers, we expressed that by using one of the two signs $>$ that is read \rightarrow « is greater than » **or** $<$ that is read \rightarrow « is smaller than »
- Since the lengths of line segments and measures of angles are numbers, then we can use the relation of inequality to compare between the lengths of two line segments or between the measures of two angles.

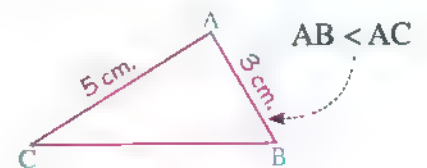
For example:

In $\triangle ABC$:

If $AC = 5$ cm. and $AB = 3$ cm., then we deduce that :

The length of \overline{AC} is greater than the length of \overline{AB} , then we write $AC > AB$

or the length of \overline{AB} is smaller than the length of \overline{AC} , then we write $AB < AC$



Similarly in the figure DEFL:

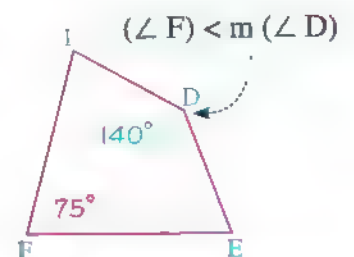
If $m(\angle D) = 140^\circ$ and $m(\angle F) = 75^\circ$, then we deduce that :

$m(\angle D)$ is greater than $m(\angle F)$,

then we write : $m(\angle D) > m(\angle F)$

or $m(\angle F)$ is smaller than $m(\angle D)$

, then we write : $m(\angle F) < m(\angle D)$



In the following, you will be given the axioms of inequality relation that you studied before.

Axioms of inequality relation

For any four numbers a, b, c and d :

- | | |
|--|-------------------------------------|
| 1 If $a > b$, then $a + c > b + c$ | 2 If $a > b$, then $a - c > b - c$ |
| 3 If $a > b, c > 0$, then $ac > bc$ | 4 If $a > b, b > c$, then $a > c$ |
| 5 If $a > b, c > d$, then $a + c > b + d$ | |

Example 1 In the opposite figure :

If B and C belong to \overline{AD} such that $AB > CD$

, prove that : $AC > BD$

**Solution**

Given B and C belong to \overline{AD} and $AB > CD$

R.T.P. $AC > BD$

Proof $\because AB > CD$ (given) and adding BC to both sides

$$\therefore AB + BC > CD + BC$$

$$\therefore AC > BD$$

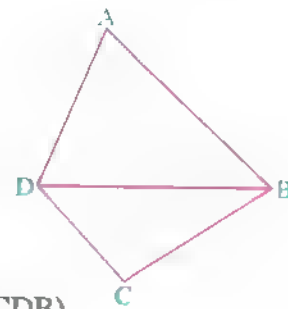
(Q.E.D.)

Example 2 In the opposite figure :

If $m(\angle ADB) > m(\angle ABD)$ and

$m(\angle CBD) < m(\angle CDB)$

, prove that : $m(\angle ADC) > m(\angle ABC)$

**Solution**

Given $m(\angle ADB) > m(\angle ABD)$ and $m(\angle CBD) < m(\angle CDB)$

R.T.P. $m(\angle ADC) > m(\angle ABC)$

Proof $\because m(\angle CBD) < m(\angle CDB)$ (given)

$$\therefore m(\angle CDB) > m(\angle CBD) \quad (1)$$

$$\because m(\angle ADB) > m(\angle ABD) \text{ (given)} \quad (2)$$

Adding (1) and (2) :

$$\therefore m(\angle CDB) + m(\angle ADB) > m(\angle CBD) + m(\angle ABD)$$

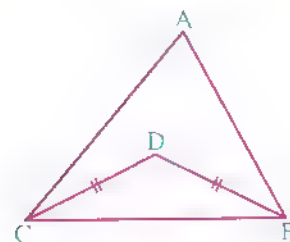
$$\therefore m(\angle ADC) > m(\angle ABC) \quad (\text{Q.E.D.})$$

Example 3 In the opposite figure :

If $m(\angle ABC) > m(\angle ACB)$

and $BD = DC$

, prove that : $m(\angle ABD) > m(\angle ACD)$



Solution

Given $m(\angle ABC) > m(\angle ACB)$ and $BD = DC$

R.T.P. $m(\angle ABD) > m(\angle ACD)$

Proof $\because DB = DC \quad \therefore m(\angle DBC) = m(\angle DCB)$ (1)

, $\because m(\angle ABC) > m(\angle ACB)$ (given) (2)

Subtracting (1) from (2) :

$\therefore m(\angle ABC) - m(\angle DBC) > m(\angle ACB) - m(\angle DCB)$

$\therefore m(\angle ABD) > m(\angle ACD)$ (Q.E.D.)

Remember that

The measure of any exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

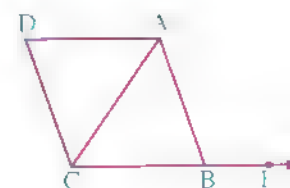
TRY
by yourself

In the opposite figure :

ABCD is a parallelogram and $E \in \overrightarrow{CB}$

Prove that :

$m(\angle ABE) > m(\angle ACD)$



, $m(\angle BAC) = m(\angle ACD)$ (alt. angles)

[Hint : $m(\angle ABE) > m(\angle BAC)$ (exterior angle of $\triangle ABC$)

Prove by yourself

of try by yourself

Answer

Comparing the measures of angles in a triangle



From your study of the previous unit, you learnt that if two sides of a triangle are congruent, then the opposite angles to these sides are equal in measure.

In the following, you shall study the relation between the measures of two angles of a triangle when the two opposite sides to these angles are not equal in length.

Theorem

In a triangle, if two sides have unequal lengths, then the longer is opposite to the angle of the greater measure.

Given | $\triangle ABC$ is a triangle in which $AB > AC$

R.T.P. | $m(\angle ACB) > m(\angle ABC)$

Construction | Take $D \in \overline{AB}$ such that $AD = AC$

Proof | In $\triangle ACD$:

$$\because AD = AC \quad \therefore m(\angle ADC) = m(\angle ACD)$$

$\because \angle ADC$ is an exterior angle of $\triangle DBC$

$$\therefore m(\angle ADC) > m(\angle B)$$

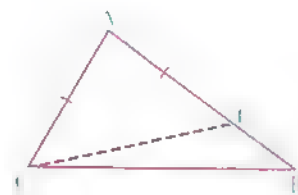
From (1) and (2) :

$$\therefore m(\angle ACD) > m(\angle B)$$

$$\therefore m(\angle ACB) > m(\angle ACD)$$

$$\therefore m(\angle ACB) > m(\angle ABC)$$

(Q.E.D.)



(1)

(2)

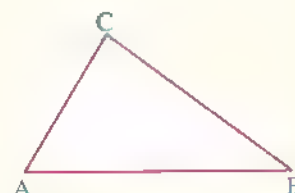
! Remark

The greatest angle in measure of the triangle is opposite to the longest side of the triangle and its measure is greater than 60° and the smallest angle in measure of the triangle is opposite to the shortest side of the triangle and its measure is less than 60°

i.e. In $\triangle ABC$: If $AB > BC > AC$,

then $m(\angle C) > m(\angle A) > m(\angle B)$

, $m(\angle C) > 60^\circ$ and $m(\angle B) < 60^\circ$



Example 1 ABCD is a quadrilateral in which $AB = 5$ cm. , $BC = 2$ cm. , $CD = 3$ cm. and $DA = 4$ cm.

Prove that : $m(\angle DCB) > m(\angle DAB)$

Solution

Given $AB = 5$ cm. , $BC = 2$ cm. , $CD = 3$ cm. and $DA = 4$ cm.

R.T.P. $m(\angle DCB) > m(\angle DAB)$

Construction Draw \overline{AC}

Proof In $\triangle ACD$:

$\therefore AD = 4$ cm. and $CD = 3$ cm.

$\therefore AD > CD \qquad \therefore m(\angle ACD) > m(\angle CAD) \qquad (1)$

In $\triangle ABC$:

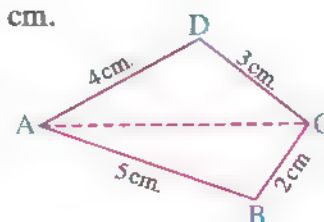
$\therefore AB = 5$ cm. and $CB = 2$ cm.

$\therefore AB > CB \qquad \therefore m(\angle ACB) > m(\angle CAB) \qquad (2)$

Adding (1) and (2) :

$\therefore m(\angle ACD) + m(\angle ACB) > m(\angle CAD) + m(\angle CAB)$

$\therefore m(\angle DCB) > m(\angle DAB) \qquad (Q.E.D.)$



TRY 1

Choose the correct answer from those given :

- 1 In $\triangle XYZ$, $XZ > XY$, then $m(\angle Z) \dots\dots\dots m(\angle Y)$
 (a) $>$ (b) $<$ (c) $=$ (d) \geq
- 2 In $\triangle ABC$, $AB = 8$ cm. , $AC = 10$ cm. , then
 (a) $m(\angle A) > m(\angle B)$ (b) $m(\angle B) > m(\angle C)$
 (c) $m(\angle B) < m(\angle C)$ (d) $m(\angle B) > m(\angle A)$
- 3 In $\triangle XYZ$, $XY = 4$ cm. , $YZ = 8$ cm. , $XZ = 6$ cm. , then
 (a) $m(\angle Z) > m(\angle Y)$ (b) $m(\angle Z) > m(\angle X)$
 (c) $m(\angle X) < m(\angle Y)$ (d) $m(\angle Z) < m(\angle Y)$
- 4 In $\triangle ABC$, $AB = 3$ cm. , $BC = 5$ cm. , $AC = 4$ cm. , then the ascending order of the measures of the angles of $\triangle ABC$ is
 (a) $\angle C, \angle B, \angle A$ (b) $\angle C, \angle A, \angle B$
 (c) $\angle A, \angle B, \angle C$ (d) $\angle B, \angle A, \angle C$

Example 2 ABC is a triangle in which $AB > AC$ and $\angle BAC$ is bisected by \overline{AD} which intersects \overline{BC} at D

Prove that : $\triangle ABD$ is an obtuse-angled triangle.

Solution

Given | ABC is a triangle in which $AB > AC$ and \overline{AD} bisects $\angle BAC$

R.T.P. | $\triangle ABD$ is an obtuse-angled triangle.

Proof | In $\triangle ABC$:

$$\therefore AB > AC \qquad \therefore m(\angle 1) > m(\angle 2)$$

$$\therefore \overline{AD} \text{ bisects } \angle BAC$$

$$\therefore m(\angle 3) = m(\angle 4)$$

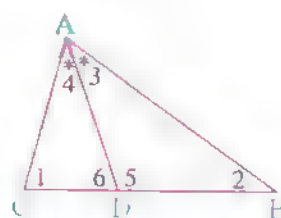
$$\therefore m(\angle 1) + m(\angle 4) > m(\angle 2) + m(\angle 3)$$

$$\text{but } m(\angle 1) + m(\angle 4) = m(\angle 5)$$

because $\angle 5$ is an exterior angle of $\triangle ADC$

$$\therefore m(\angle 5) > m(\angle 2) + m(\angle 3)$$

$$\therefore \triangle ABD \text{ is an obtuse-angled triangle.}$$



Remember that

If the measure of an angle in a triangle is greater than the sum of measures of the two other angles, then this angle is an obtuse angle.

(Q.E.D.)

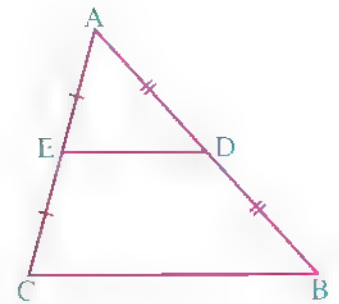
TRY
by yourself **2**

In the opposite figure :

ABC is a triangle in which $AB > AC$

• D and E are the midpoints
of \overline{AB} and \overline{AC} respectively.

Prove that : $m(\angle AED) > m(\angle ADE)$



Answers of try by yourself

1 1 b

2 b

3 d

4 a

2 Prove by yourself (Hint : $AD > AE$ because : $AB > AC$)

3

Comparing the lengths of sides in a triangle



From your previous study , you learnt that : if two angles are equal in measure in a triangle , then the two opposite sides to these angles are equal in length.

In the following , you shall study the relation between the lengths of two sides in a triangle when the two opposite angles are not equal in measure.

Theorem

In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.

Given | ABC is a triangle in which $m(\angle C) > m(\angle B)$

R.T.P. | $AB > AC$

Proof | $\therefore \overline{AB}$ and \overline{AC} are two line segments.

\therefore One of the following cases should be verified :

1 $AB > AC$ **2** $AB = AC$ **3** $AB < AC$

Unless $AB > AC$, then either $AB = AC$ or $AB < AC$

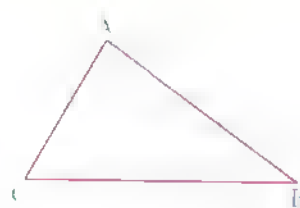
• If $AB = AC$, then $m(\angle C) = m(\angle B)$ and this contradicts the given where $m(\angle C) > m(\angle B)$

• If $AB < AC$, then $m(\angle C) < m(\angle B)$ according to the previous theorem.

Again this contradicts the given where $m(\angle C) > m(\angle B)$

\therefore It should be that $AB > AC$

(Q.E.D.)

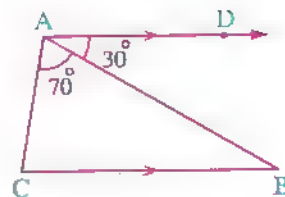


Example 1 In the opposite figure :

ABC is a triangle in which $m(\angle BAC) = 70^\circ$,

$\overrightarrow{AD} \parallel \overrightarrow{BC}$ and $m(\angle DAB) = 30^\circ$

Prove that : $AB > AC$



Solution

Given $\overrightarrow{AD} \parallel \overrightarrow{BC}$, $m(\angle BAC) = 70^\circ$ and $m(\angle DAB) = 30^\circ$

R.T.P. $AB > AC$

Proof $\because \overrightarrow{AD} \parallel \overrightarrow{BC}$ and \overrightarrow{AB} is a transversal to them.

$\therefore m(\angle B) = m(\angle DAB) = 30^\circ$ (alternate angles)

\therefore In $\triangle ABC$: $m(\angle C) = 180^\circ - (30^\circ + 70^\circ) = 80^\circ$

$\therefore m(\angle C) > m(\angle B) \quad \therefore AB > AC$

(Q.E.D.)

Corollaries

Corollary 1

In the right-angled triangle , the hypotenuse is the longest side.



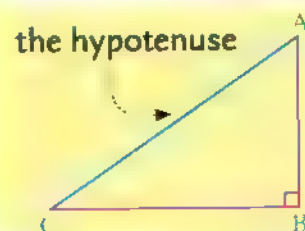
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In the opposite figure :

If $\triangle ABC$ is right-angled at B , then $m(\angle B) > m(\angle A)$,

$m(\angle B) > m(\angle C)$ because $\angle B$ is a right angle and each of $\angle A$ and $\angle C$ is acute , so we find that :

$AC > BC$ and $AC > AB$ (according to the previous theorem).



Notice that :

In the obtuse-angled triangle , the side opposite to the obtuse angle is the longest side in the triangle.

Corollary 2

The length of the perpendicular line segment drawn from a point outside a straight line to this line is shorter than any line segment drawn from this point to the given straight line.

In the opposite figure :

If $C \notin \overleftrightarrow{AB}$ and $D \in \overleftrightarrow{AB}$ such that $\overline{CD} \perp \overleftrightarrow{AB}$,

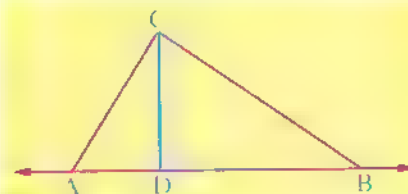
then \overline{CB} is the hypotenuse in $\triangle CBD$

which is right-angled at D ,

\overline{CA} is the hypotenuse in $\triangle CDA$ which is right-angled at D and so on ...

According to corollary 1 , we find that $CB > CD$, $CA > CD$ and so on ...

i.e. $CD < CB$ and $CD < CA$



Definition

The distance between any point and a given straight line is the length of the perpendicular line segment drawn from this point to the given line.

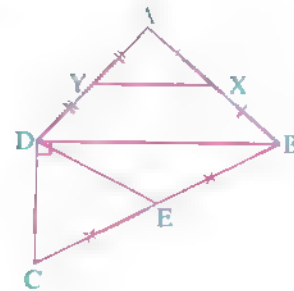
In the previous figure :

The distance between the point C and the straight line \overleftrightarrow{AB} is the length of \overline{CD}

Example 2 In the opposite figure :

ABCD is a quadrilateral X, Y and E are the midpoints of \overline{AB} , \overline{AD} and \overline{BC} respectively and $m(\angle BDC) = 90^\circ$

Prove that : $DE > XY$



Solution

Given X is the midpoint of \overline{AB} , Y is the midpoint of \overline{AD} , E is the midpoint of \overline{BC} and $m(\angle BDC) = 90^\circ$

R.T.P. | $DE > XY$

Proof In $\triangle ABD$: \because X is the midpoint of \overline{AB} and Y is the midpoint of \overline{AD}

$$\therefore XY = \frac{1}{2} BD \quad (1)$$

In $\triangle DBC$: $\because m(\angle BDC) = 90^\circ$ and E is the midpoint of \overline{BC}

$$\therefore DE = \frac{1}{2} BC \quad (2)$$

$\because \overline{BC}$ is the hypotenuse of $\triangle BDC \quad \therefore BC > BD$

$$\therefore \frac{1}{2} BC > \frac{1}{2} BD \quad (3)$$

From (1), (2) and (3) : $\therefore DE > XY$ (Q.E.D.)

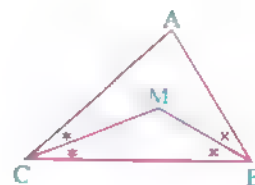


In the opposite figure :

ABC is a triangle in which $AC > AB$,

\overline{BM} bisects $\angle ABC$ and \overline{CM} bisects $\angle ACB$

Prove that : $MC > MB$



Prove by yourself [Hint : $m(\angle ABC) > m(\angle ACB)$]

Answers of try by yourself

Triangle Inequality



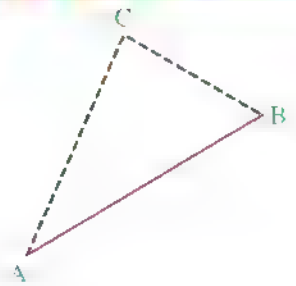
We know that the shortest distance between two points is the length of the line segment joining them.

For example:

In the opposite figure :

The shortest distance between A and B is the length of \overline{AB}

So , for any point $C \notin \overleftrightarrow{AB}$, then $AB < AC + CB$



Generally

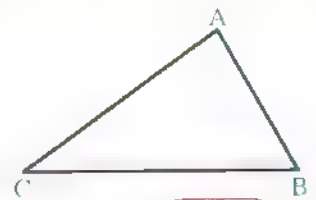
In any triangle , the sum of the lengths of any two sides is greater than the length of the third side.

i.e. In any triangle such as $\triangle ABC$

, we get : $AB + BC > AC$

, $BC + CA > AB$

, $CA + AB > CB$



Corollary

The length of any side in a triangle is greater than the difference between the lengths of the other two sides and less than their sum.



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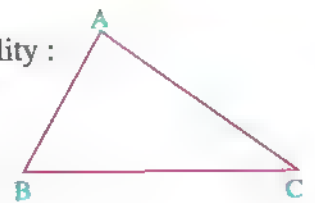
And you can prove that as follows :

In the opposite figure ABC is a triangle and from the triangle inequality :

$$AC + AB > BC \quad (1)$$

$$\therefore AB + BC > AC \quad \text{i.e. } BC > AC - AB \quad (2)$$

From (1) and (2) , we deduce that : $AC - AB < BC < AC + AB$



! Remark

To check the possibility that three lengths can be side lengths of a triangle, do as follows :

Compare the greatest length with the sum of the other two lengths :

- If the greatest length is greater than or equal to the sum of the other two lengths, you deduce that the three given lengths couldn't be lengths of the three sides of a triangle. (i.e. No triangle could be drawn with these side lengths).
- If the greatest length is less than the sum of the other two lengths, you deduce that the three given lengths could be lengths of the three sides of a triangle. (i.e. A triangle could be drawn with these side lengths).

Example 1 Is it possible to draw a triangle whose side lengths are as follows (giving reason) :

- | | | |
|--------------------------|--|--------------------------|
| 1 5 cm. , 7 cm. , 12 cm. | | 2 4 cm. , 6 cm. , 11 cm. |
| 3 14 cm. , 9 cm. , 7 cm. | | 4 8 cm. , 18 cm. , 8 cm. |

- Solution**
- 1 $\because 5 + 7 = 12$
 \therefore It is not possible to draw a triangle of side lengths 5 cm. , 7 cm. and 12 cm.
 - 2 $\because 4 + 6 < 11$
 \therefore It is not possible to draw a triangle of side lengths 4 cm. , 6 cm. and 11 cm.
 - 3 $\because 9 + 7 > 14$
 \therefore It is possible to draw a triangle of side lengths 14 cm. , 9 cm. and 7 cm.
 - 4 $\because 8 + 8 < 18$
 \therefore It is not possible to draw a triangle of side lengths 8 cm. , 18 cm. and 8 cm.

Example 2 Find the interval to which the length of the third side of each of the following triangles belongs if the lengths of the other two sides are :

- | | | |
|-------------------------------------|--|---------------------|
| 1 4 cm. , 3 cm. | | 2 4.5 cm. , 7.5 cm. |
| 3 $2\sqrt{5}$ cm. , $2\sqrt{5}$ cm. | | |

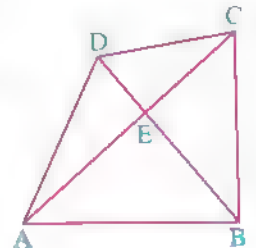
Solution \therefore The length of any side in a triangle is greater than the difference between the lengths of the other two sides and less than their sum, and let the length of the third side be l cm., then

$$\begin{array}{lll} 1 & 4-3 < l < 4+3 & \therefore 1 < l < 7 \quad \therefore l \in]1, 7[\\ 2 & 7.5-4.5 < l < 7.5+4.5 & \therefore 3 < l < 12 \quad \therefore l \in]3, 12[\\ 3 & 2\sqrt{5}-2\sqrt{5} < l < 2\sqrt{5}+2\sqrt{5} & \therefore 0 < l < 4\sqrt{5} \quad \therefore l \in]0, 4\sqrt{5}[\end{array}$$

Example 3 In the opposite figure :

ABCD is a quadrilateral whose diagonals intersect at E

Prove that : $AC + BD > BC + AD$



Solution

Given ABCD is a quadrilateral whose diagonals intersect at E

R.T.P. $AC + BD > BC + AD$

Proof In $\triangle EBC$: $EC + EB > BC$ (triangle inequality) (1)

In $\triangle EAD$: $EA + ED > AD$ (triangle inequality) (2)

Adding (1) and (2) : $\therefore EC + EA + EB + ED > BC + AD$

$\therefore EC + EA = AC$, $EB + ED = BD$ $\therefore AC + BD > BC + AD$ (Q.E.D.)

TRY
yourself

1 Put (\checkmark) in the space in front of each group of the following lengths which can be side lengths of a triangle :

1 2 cm., 3 cm., 4 cm. ()

2 3 cm., 6 cm., 2 cm. ()

3 10 cm., 3 cm., 7 cm. ()

4 12 cm., 5 cm., 7.5 cm. ()

2 Find the interval to which the length of the third side of each of the following triangles belongs if the lengths of the other two sides are :

1 6 cm., 5 cm.

2 7.5 cm., 7.5 cm.

$]1, 15[$ (2)

$]1, 11[$ (1) **2**

$]1, 4[$ (1) **1**

Answers of try by yourself



EL MOASSER

By a group of supervisors

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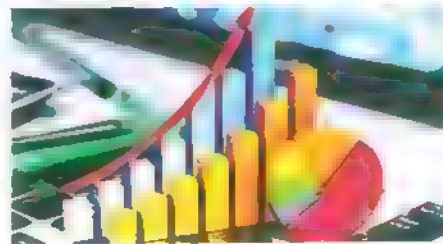
Relation between Two Variables.



Unit

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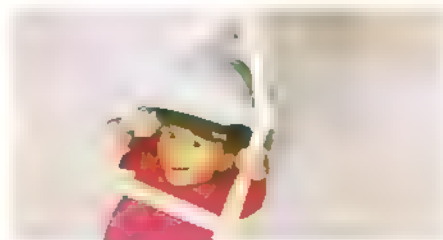
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Accumulative Basic skills
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Revision

 From the school book

- 1**  Complete by writing the following numbers in the form $\frac{a}{b}$ where a and b are two integers and there aren't common factors between them, $b \neq 0$

1 $0.2 = \dots\dots\dots$

2 $0.3 = \dots\dots\dots$

3 $25\% = \dots\dots\dots$

4 $|-0.75| = \dots\dots\dots$

5 $-6 = \dots\dots\dots$

6 $1\frac{1}{4} = \dots\dots\dots$

- 2** Choose the correct answer from those given :

1 $\mathbb{Z}_+ \cup \{0\} = \dots\dots\dots$

(a) \mathbb{Z}

(b) \mathbb{N}

(c) \mathbb{C}

(d) \mathbb{Q}


2  $|-2| + |-4| + |6| = \dots\dots\dots$

(a) Zero

(b) $|-12|$

(c) -12

(d) 6

3  $\sqrt{a^2} = \dots\dots\dots$

(a) a

(b) $-a$

(c) $|a|$

(d) $\pm a$

4  The solution set of the equation : $x + 5 = |-5|$ in \mathbb{N} is $\dots\dots\dots$

(a) $\{0\}$

(b) $\{10\}$

(c) $\{-10\}$

(d) \emptyset

5  Which of the following rational numbers lies between $\frac{1}{5}$, $\frac{2}{5}$?

(a) $\frac{2}{10}$

(b) $\frac{1}{10}$

(c) 0.3

(d) -0.3

6  The product of the rational number $\frac{a}{b}$ by its additive inverse equals $\dots\dots\dots$

(a) zero

(b) $-\frac{a}{b}$

(c) $\frac{a^2}{b^2}$

(d) $-\frac{a^2}{b^2}$

7 $3^{10} + 3^{10} + 3^{10} = \dots\dots\dots$

(a) 3^{10}

(b) 3^{30}

(c) 9^{10}

(d) 3^{11}

8 If $a^{-1} = \frac{2}{3}$, then $a = \dots\dots\dots$

(a) $-\frac{2}{3}$

(b) $\frac{3}{2}$

(c) $-\frac{3}{2}$

(d) 1

9 The multiplicative inverse of 5^{-1} is $\dots\dots\dots$

(a) $\frac{1}{5}$

(b) 5

(c) -5

(d) $-\frac{1}{5}$

REVISION


3 Complete the following :

1  $\sqrt{25 + 144} = \dots\dots\dots$

2  $\sqrt{0.25} = \dots\dots\dots$

3 $-|-5| - |2| = \dots\dots\dots$


4 $-\sqrt{25} + |-5| = \dots\dots\dots$

5  $\sqrt{0.16} + |-0.6| = \dots\dots\dots$

6 $\frac{\sqrt{25-9}}{\sqrt{25}-\sqrt{9}} = \dots\dots\dots$

7  The standard form of the number 0.00015 is $\dots\dots\dots$

8 The standard form of the number 421×10^3 is $\dots\dots\dots$

9  $2^0 + 2^1 + 2^2 + 2^3 = \dots\dots\dots$

10  The sum of the two square roots of the number $2\frac{1}{4}$ equals $\dots\dots\dots$

11 $\left(\frac{2}{3}\right)^2 \times \sqrt{\frac{81}{16}} \times \left(\frac{7}{9}\right)^{\text{zero}} = \dots\dots\dots$

4 Find the value of x which satisfies each of the following equations :

1 $5x + 3 = 20$

2 $7x + 11 = 12$

3 $3x + 5 = 1$

4 $x + 3 = 7$

5 Find the solution set of each of the following equations , where $x \in \mathbb{Q}$:

1 $x^2 + 12 = 21$

2 $2x^2 - 1 = -9$

3 $|x| = 2$

4 $\sqrt{x^2} = 4$



UNIT 1

Real Numbers

Exercises of the unit :

1. The cube root of a rational number.
2. The set of irrational numbers \mathbb{Q}
3. The set of real numbers \mathbb{R}
- Ordering numbers in \mathbb{R}
4. Intervals.
5. Operations on the real numbers.
 - ⊗ Summary of the first part of unit one.
 - ⊗ Exams on the first part of unit one.
6. Operations on the square roots.
7. The two conjugate numbers.

8. Operations on the cube roots.
9. Applications on the real numbers.
10. Solving equations and inequalities of the first degree in one variable in \mathbb{R}
 - ⊗ Summary of the second part of unit one.
 - ⊗ Exams on the second part of unit one.



A research project on unit one



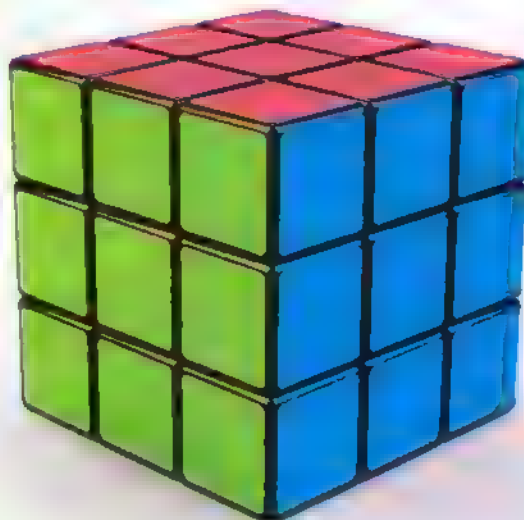
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interactive
test on each
lesson

The cube root of a rational number



Interactive test

From the school book



Remember

Understand

Apply

Problem Solving

1 Complete the following table :

Number a	8	125	-27	$3\frac{3}{8}$	$-\frac{8}{125}$
$\sqrt[3]{a}$	-10	6	-4

2 Complete :

1 $\sqrt[3]{216} = \dots$

3 $\sqrt[3]{0.001} = \dots$

5 $\sqrt[3]{8} + \sqrt[3]{-8} = \dots$

7 $\sqrt[3]{27} - \sqrt[3]{-27} = \dots$

9 $-\sqrt[3]{-1} - \sqrt[3]{1} = \dots$

11 $\sqrt[3]{a^3} = \dots$

13 $\sqrt[3]{\dots} = 4$

15 $|\sqrt[3]{-125}| = \sqrt{\dots}$

2 $\sqrt[3]{-343} = \dots$

4 $\sqrt[3]{-\frac{8}{27}} = \dots$

6 $\sqrt[3]{27} - \sqrt[3]{64} = \dots$

8 $\sqrt[3]{9} + \sqrt[3]{-8} = \dots$

10 $\frac{-\sqrt[3]{64}}{\sqrt[3]{64}} = \dots$

12 $\sqrt[3]{-27 a^6} = \dots$

14 $\sqrt[3]{16} = \sqrt[3]{\dots}$

16 $\sqrt[3]{64 + \dots} = 5$

3 Choose the correct answer from those given :

1 $\sqrt[3]{(-8)^2} = \dots$

- (a) 2 (b) -2 (c) 4 (d) -4

2 $\sqrt[3]{\left(\frac{1}{8}\right)^2} = \dots$

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

3 $\sqrt[3]{-64} + \sqrt{16} = \dots$

- (a) zero (b) 8 (c) -8 (d) ± 8

4 $\sqrt{25} - \sqrt[3]{-125} = \dots$

- (a) 10 (b) zero (c) 5 (d) ± 5

5 $\sqrt{(-2)^2} + \sqrt[3]{(-2)^3} = \dots$

- (a) -4 (b) 8 (c) 4 (d) zero

6 $\sqrt[3]{3\frac{3}{8}} + \sqrt{0.25} = \dots$

- (a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) 2 (d) -2

7 $\sqrt[3]{0.001 \times \frac{1}{8}} = \dots$

- (a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{20}$ (d) 20

8 $\sqrt[3]{1000} \times \sqrt[3]{-0.008} = \dots$

- (a) $\frac{1}{2}$ (b) 10 (c) 2 (d) -2

9 $\sqrt[3]{-27} + \sqrt{12\frac{1}{4}} + \sqrt[3]{0.125} = \dots$

- (a) 1 (b) zero (c) -1 (d) $\frac{11}{2}$

10 If $-\sqrt{25} = \sqrt[3]{y}$, then $y = \dots$

- (a) 5 (b) -5 (c) 125 (d) -125

11 If $x^3 = 64$, then $\sqrt{x} = \dots\dots\dots$

(a) 4

(b) -4

(c) 2

(d) -2

12 If $x^3 = 27$, then $x^2 = \dots\dots\dots$

(a) 3

(b) 6

(c) 9

(d) 81

13 $\sqrt[3]{x^6} = \sqrt{\dots\dots\dots}$

(a) x^3

(b) x^2

(c) x

(d) x^4

14 If $\frac{x}{3} = \frac{9}{x^2}$, then $x = \dots\dots\dots$

(a) 1

(b) 3

(c) 9

(d) 27

4 Find the value of x in each of the following :

1 $\sqrt[3]{x} = 5$

2 $\sqrt[3]{x} = -\frac{1}{4}$

3 $\sqrt[3]{x} = -\sqrt{4}$

4 $\sqrt[3]{x} - 3 = -1$

5 $x^3 = -8$

6 $x^3 = 64$

7 $x^3 + 5 = 32$

8 $2x^3 = 54$

9 $\frac{1}{5}x^3 = -200$

5 Find the S.S. of each of the following equations in \mathbb{Q} :

1 $x^3 + 27 = 0$

2 $8x^3 + 7 = 8$

3 $x^3 + 16 = \frac{3}{8}$

4 $2x^3 - 5 = x^3 + 3$

5 $(x+3)^3 = 343$

6 $(3x+1)^3 = -8$

7 $(2x+1)^3 - 7 = 20$

8 $(5x-2)^3 + 10 = 18$

6 Find each of the following :

1 $\sqrt[3]{2\frac{1}{4} \div \frac{2}{3}}$

2 $-\sqrt[3]{2^9 \times 3^6}$

3 $\sqrt[3]{\sqrt[3]{729}}$

4 $\sqrt[3]{\sqrt[3]{512}}$

5 $\sqrt{27\sqrt[3]{27}}$

Applications

7 A cube of volume 27 cm^3 . Find the area of one face.

« 9 cm^2 »

8 Find the total area of a cube whose volume is 216 cm^3 .

« 216 cm^2 »

9 If the half of the cube of a number equals 32 , find this number. « 4 »

10 Find the inner edge length of a cube vessel with capacity of one litre. « 10 cm »

11 Find the diameter length of a sphere whose volume is $\frac{1372}{81} \pi$ cube unit. $\frac{14}{3}$ length unit »

12 Find the length of the diameter of a sphere whose volume is 113.04 cm^3 ($\pi = 3.14$) « 6 cm »

For excellent pupils

13 Find the S.S. of each of the following equations in \mathbb{Q} :

1) $(x^2 + 6)^3 = 1000$

2) $(x^3 - 14)^2 = 169$

3) $\sqrt[3]{(x-1)^2} = \sqrt[3]{25}$

4) $\sqrt[3]{(x-2)(x^2 - 4x + 4)} = 3$

14 If $\sqrt[3]{\sqrt{x} + 19} = 3$, find the value of $\sqrt[3]{x}$ « 4 »

15 A man was asked about the age of his father and the age of each of his three sons.

His answer was as follows :

My age is half the age of my father. The age of my eldest son is the square root of the age of my father and the age of my middle son is the cube root of the age of my father and the age of my youngest daughter is the quotient of the age of my eldest son by the age of my middle son. Given that the age of my eldest son is twice the age of my middle son.

What is the age of each of his father and his three sons ?

« 64 , 8 , 4 , 2 »

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Remember

Understand

Apply

Problem Solving

1 In each of the following, show which of them is a rational number and which of them is an irrational number :

1 -5

2 $2\frac{2}{3}$

3 2.06

4 2.3×10^5

5 $-\sqrt{36}$

6 $\sqrt[3]{36}$

7 $\sqrt{7}$

8 zero

9 $|-5|$

10 $\sqrt[3]{-\frac{64}{81}}$

11 $\sqrt{\frac{25}{16}}$

12 $\sqrt{\frac{1}{3}}$

13 $\sqrt[3]{3\frac{3}{8}}$

14 $\sqrt[3]{0.343}$

15 $\frac{\pi}{2}$

16 $(-5)^{\text{zero}}$

17 $\frac{\text{zero}}{3}$

18 $\frac{\sqrt{9}}{\sqrt{4}}$

19 $\sqrt{9} + \sqrt{16}$

20 $\sqrt{4} - \sqrt{11}$

2 Find an approximated value for each of the following numbers :

1 $\sqrt{11}$ "to the nearest hundredth".

2 $\sqrt[3]{7}$ "to the nearest tenth".

3 $\sqrt[3]{-9}$ "to the nearest tenth".

3 Find two consecutive integers for each of the following numbers to be included between them :

1 $\sqrt{5}$

2 $\sqrt{12}$

3 $\sqrt[3]{10}$

4 $\sqrt[3]{-20}$

4 If X is an integer, find the value of X in each of the following cases :

1 $X < \sqrt{2} < X + 1$	« 1 »	2 $X < \sqrt[3]{80} < X + 1$	« 8 »
3 $X < \sqrt[3]{5} < X + 1$	« 1 »	4 $X < \sqrt[3]{50} < X + 1$	« 3 »
5 $X < \sqrt[3]{-100} < X + 1$	« -5 »	6 $X < -\sqrt{35} < X + 1$	« 5 »

5 Find an approximated value for each of the following numbers, then check your answer using the calculator :

1 $\sqrt{20}$ 2 $\sqrt[3]{17}$ 3 $\sqrt{5} + 1$ 4 $\sqrt[3]{9} - 1$

6 Choose the correct answer from the given ones :

- 1 The irrational number in the following numbers is

(a) $\sqrt{\frac{1}{4}}$ (b) $\sqrt[3]{8}$ (c) $\sqrt{\frac{4}{9}}$ (d) $\sqrt{2}$

If $X = \sqrt{2}$, $y = 2$, then which of the following does not represent a rational number ?

(a) $X^2 + y$ (b) $X + y^2$ (c) $\sqrt{X^2 y}$ (d) $\sqrt{2} X y$

- 3 The irrational number located between 2 and 3 is .

(a) $\sqrt{10}$ (b) $\sqrt{7}$ (c) 2.5 (d) $\sqrt{3}$

- 4 The irrational number located between -2 and -1 is

(a) -3 (b) $-1\frac{1}{2}$ (c) $-\sqrt{3}$ (d) $\sqrt{2}$

- 5 $\sqrt{10} \approx$

(a) 2.99 (b) 3.71 (c) 3 (d) -3.2

- 6 The nearest integer to $\sqrt[3]{25}$ is

(a) 5 (b) 3 (c) 2 (d) 12.5

- 7 If $n \in \mathbb{Z}_+$, $n < \sqrt{26} < n + 1$, then $n =$

(a) 25 (b) 5 (c) -5 (d) 24

- 8 The side length of a square whose area is 6 cm^2 is

- (a) a natural number. (b) an integer.
(c) a rational number. (d) an irrational number.

- 9 | The area of a square whose side length is $\sqrt{3}$ cm. is cm^2

(a) $4\sqrt{3}$ (b) 9 (c) 3 (d) 6

- 10 | The square whose area is 10 cm^2 , its side length is cm.

(a) 5 (b) -5 (c) $\sqrt{10}$ (d) $\sqrt{10}$

- 11 | The S.S. of the equation : $(x - \sqrt{5})(x + \sqrt{3}) = 0$ in \mathbb{Q} is

(a) $\{\sqrt{5}\}$ (b) $\{-\sqrt{3}\}$ (c) $\{-\sqrt{5}, \sqrt{3}\}$ (d) $\{\sqrt{5}, -\sqrt{3}\}$

7 Find the value of x in each of the following cases and determine whether

$x \in \mathbb{Q}$ or $x \in \mathbb{Q}$:

1 $5x^2 = 10$

« $\pm\sqrt{2}$ »

2 $4x^2 = 9$

« $\pm\frac{3}{2}$ »

3 $x^3 = 125$

« 5 »

4 $3x^3 = 27$

« $\sqrt[3]{9}$ »

5 $0.1x^2 = 10$

« ± 10 »

5 $0.001x^3 = -8$

« 20 »

7 $(x-1)^2 = 4$

« 3 or -1 »

8 $(x-5)^3 = 1$

« 6 »

8 Find in \mathbb{Q} the S.S. of each of the following equations :

1 $x^2 = 13$

2 $x^3 = 16$

3 $\frac{2}{5}x^2 = \frac{25}{2}$

4 $\frac{5}{4}x^3 = -2$

5 $125x^3 - 7 = 20$

6 $\frac{1}{4}x^2 + 2 = 66$

7 $(x^3 + 5)(x^2 - 3) = \text{zero}$

8 $(x + \sqrt{7})(x^3 - 6) = \text{zero}$

9 Prove that :

1 $\sqrt{2}$ is included between 1.4 and 1.5

2 $\sqrt{11}$ is included between 3.31 and 3.32

3 $\sqrt[3]{2}$ is included between 1.2 and 1.3

4 $\sqrt[3]{15}$ is included between 2.4 and 2.5

5 $\sqrt[3]{-17}$ is included between -2.6 and -2.5

6 $\sqrt{3} + 1$ is included between 2.7 and 2.8

10 Determine the point that represents each of the following numbers on the number line :

1 $\sqrt{3}$

2 $-\sqrt{11}$

3 $\sqrt{10}$

4 $\sqrt{5} + 1$

5 $2 - \sqrt{7}$

11 Draw the number line and label point A which represents $\sqrt{2}$

• Label point B which represents $1 + \sqrt{2}$

• Label point C which represents $1 - \sqrt{2}$

12 Draw the right-angled triangle ABC at B where $AB = 1$ cm. and $BC = 3$ cm. , then use the figure to determine the points that represent the following numbers on the number line :

1 $\sqrt{10}$

2 $-\sqrt{10}$

3 $2 + \sqrt{10}$

4 $3 - \sqrt{10}$

13 Calculate the side length and the diagonal length of a square whose area equals 10 cm^2

« $\sqrt{10} \text{ cm.}$, $\sqrt{20} \text{ cm.}$ »

Life Application


14 A tree is 3 metres long. Its upper part was broken because of the wind and it made an angle with the surface of the ground. If the length of the left part of the tree is 1 metre , find the distance between the base of the tree and the point of touching of its top with the ground.



« $\sqrt{3} \text{ metres}$ »

For excellent pupils


15 Without using the calculator , prove that $\sqrt{3} + \sqrt{2}$ is included between 3 and 4



EL MOASSER

Notebook

Free part



- Accumulative tests.

- Final revision.

- Final examinations.

The set of real numbers \mathbb{R} and ordering numbers in \mathbb{R}



interactive test

From the school book

● Remember

● Understand

● Apply

● Problem Solving

- 1 Complete the following table by placing (\checkmark) in the suitable place as shown in the first case :

The number	Natural	Integer	Rational	Irrational	Real
-5	x	\checkmark	\checkmark	x	\checkmark
$\sqrt{2}$					
$1\frac{1}{2}$					
$\sqrt[3]{9}$					
$ -2 $					
$\sqrt{4}$					
$\frac{5}{2}$					
0.3					
$\sqrt{-1}$					

- 2 If $x \in \mathbb{R}$, state whether x is positive or negative or anything else in each of the following cases :

1 $x > 0$

2 $x < 0$

3 $x > |-4|$

4 $|-5| < x < 7$

5 $-2 < x < 0$

6 $|-1| < x < |-7|$

3 Put the suitable sign ($>$, $<$ or $=$) :

1 $\sqrt{5} \dots 2$

2 $\sqrt{7} \dots 2.6$

3 $\sqrt[3]{24} \dots 3$

4 $\sqrt[3]{-24} \dots -2$

5 $3 - \sqrt{5} \dots \sqrt[3]{-1}$

6 $\sqrt[3]{8} \dots \sqrt{4}$

7 $1 + \sqrt{3} \dots \sqrt{5}$

8 $\sqrt[3]{3} - 1 \dots 0.2$

9 $\sqrt{2} - 1 \dots 1 - \sqrt{2}$

4 Choose the correct answer from those given :

1 $\mathbb{R} = \dots$

(a) $\mathbb{Q} \cup \mathbb{Q}$

(b) $\mathbb{Z}_+ \cup \mathbb{Z}_-$

(c) $\mathbb{R}_+ \cup \mathbb{R}_-$

(d) $\mathbb{N} \cup \mathbb{R}_-$

2 $\mathbb{Q} \cap \mathbb{Q} = \dots$

(a) \mathbb{Q}

(b) \mathbb{Q}

(c) \mathbb{R}

(d) \emptyset

3 $\mathbb{Q} \cup \mathbb{Q} = \dots$

(a) \emptyset

(b) \mathbb{R}

(c) \mathbb{Q}

(d) \mathbb{Q}

4 $\mathbb{R}_+ \cap \mathbb{R} = \dots$

(a) \emptyset

(b) \mathbb{R}

(c) \mathbb{R}_+

(d) \mathbb{R}_-

5 $\mathbb{R}_+ \cup \mathbb{R}_- = \dots$

(a) \mathbb{R}

(b) \emptyset

(c) \mathbb{R}_+

(d) \mathbb{R}^*

6 $\mathbb{R} - \mathbb{Q} = \dots$

(a) \mathbb{R}

(b) \emptyset

(c) \mathbb{Q}

(d) $\{0\}$

7 $\mathbb{R} - \mathbb{Q} = \dots$

(a) \mathbb{Q}

(b) \mathbb{R}

(c) \emptyset

(d) $\{0\}$

8 $\mathbb{R}_+ \cap \{-1, 0, 1\} = \dots$

(a) $\{0, 1\}$

(b) $\{1\}$

(c) $\{0\}$

(d) \emptyset

9 $\{x : x \in \mathbb{R}, x < 0\} = \dots$

(a) \mathbb{R}_+

(b) \mathbb{R}_-

(c) \mathbb{R}^*

(d) \mathbb{R}

10 If x is a negative real number, then which of the following numbers is positive?

(a) x^2

(b) x^3

(c) $2x$

(d) $\frac{x}{2}$

11 If $\frac{1}{a}$ and $\frac{a}{\sqrt{5}}$ are two real numbers included between 0 and 1, then $a = \dots$

(a) -2

(b) 1

(c) $\sqrt{5}$

(d) 2

12 If $x \in \mathbb{R}_+$, $y \in \mathbb{R}_+$ and $x^2 > y^2$, then \dots

(a) $x > y$

(b) $x < y$

(c) $x = y$

(d) $x \leq y$

- 13 $\sqrt{(2-\pi)^2} \dots (2-\pi)$ (where π is the ratio between the circumference of the circle and its diameter length)

(a) = (b) < (c) > (d) \leq

- 14 The S.S. of the equation : $x^2 + 1 = 0$ in \mathbb{R} is

(a) $\{-1\}$ (b) $\{1, -1\}$ (c) $\{1\}$ (d) \emptyset

- 5 Arrange the following numbers ascendingly :

1 $\sqrt{8}$, $-\sqrt{3}$, $\sqrt{15}$, $\sqrt{5}$, $-\sqrt{7}$ and $-\sqrt{11}$

2 $\sqrt{27}$, $-\sqrt{45}$, $\sqrt{20}$, 0.6 and $\sqrt[3]{-1}$

- 6 Arrange the following numbers descendingly :

1 $\sqrt{62}$, 8, $-\sqrt{50}$ and $\sqrt{70}$

2 $\sqrt{6}$, 9, $-\sqrt{10}$, $-\sqrt{7}$, $-\sqrt{50}$ and $\sqrt{101}$

- 7 Write three positive irrational numbers less than 2

- 8 Write three negative irrational numbers greater than $-\sqrt{6}$

- 9 Write four irrational numbers included between 15 and 17

- 10 Prove that $\sqrt{3}$ is between 1.7 and 1.8, then represent $\sqrt{3}$, 1.7 and 1.8 on the number line.

- 11 Solve the following equations to the nearest hundredth given $x \in \mathbb{R}$:

1 $x^2 - 6 = 0$

2 $\frac{3}{4}x^2 = 24$

3 $\frac{1}{2}x^2 - 5 = 0$

4 $5x^3 + 3 = 2$

5 $\frac{3}{4}x^2 + 2 = -11$

6 $\frac{2}{x^3} + 5 = 21$ ($x \neq 0$)

7 $(x^2 - 9)(x^3 - 5) = 0$

8 $(2x^3 - 5)(x^2 + 1) = 0$

Geometric Applications

- 12 Find the side length of a square whose area is 5 cm^2 . Is the side length a rational number ? $\ll \sqrt{5} \text{ cm.} \gg$

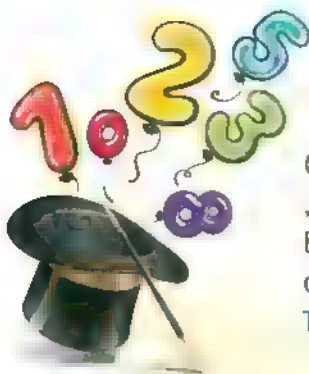
- 13 Find the edge length of a cube whose volume is 1.728 cm^3 . Is the edge length a rational number ? $\ll \frac{6}{5} \text{ cm.} \gg$

- 14 A cube whose total area is 13.5 cm^2 . Find its edge length. Is the edge length a rational number ? $\ll 1.5 \text{ cm.} \gg$

- 15 A square is of side length 6 cm. Find its diagonal length. $\llcorner \sqrt{72} \text{ cm.} \lrcorner$
- 16 A square is of area 32 cm^2 . Find its side length and its diagonal length. $\llcorner \sqrt{32} \text{ cm.}, 8 \text{ cm.} \lrcorner$
- 17 An isosceles right-angled triangle, the length of one side of its right-angle = 5 cm.
Find the length of its hypotenuse. $\llcorner \sqrt{50} \text{ cm.} \lrcorner$
- 18 A rectangle with dimensions 5 cm. and 7 cm. Find the length of its diagonal. And if its area equals the area of a square, then find the side length of the square and its diagonal length. $\llcorner \sqrt{74} \text{ cm.}, \sqrt{35} \text{ cm.}, \sqrt{70} \text{ cm.} \lrcorner$

For excellent pupils

- 19 Without using the calculator, prove that : $\sqrt[3]{3} > \sqrt{2}$
- 20 Two real numbers, the sum of their squares is 7 and the greater number is 2
Find the other number. $\llcorner \sqrt{3} \text{ or } -\sqrt{3} \lrcorner$



Wonders of numbers

Choose a number from 1 to 9, multiply it by 3, add 3 to the product, and multiply the result by 3 once again "use calculator" Find the sum of the digits of the product.
The answer is always 9.



From the school book



Remember

Understand

Apply

Problem Solving

1 Complete the following table :

The interval	Expression by description method	Its representation on the number line
1 $[-1, 2]$	$\{x : -1 \leq x \leq 2, x \in \mathbb{R}\}$	
2 $[1, 3[$
3 	$\{x : 0 < x \leq 3, x \in \mathbb{R}\}$
4	
5 $] -\infty, 1]$
6	
7	$\{x : x < 4, x \in \mathbb{R}\}$
8 $[-2, \infty[$

2 Choose the correct answer from the given ones :

1 $\mathbb{R} = \dots$

(a) $\mathbb{R}_+ \cap \mathbb{R}_-$

(b) $\mathbb{R}_+ \cup \mathbb{R}_-$

(c) $] -\infty, \infty[$

(d) $\mathbb{Q} \cap \mathbb{Q}$

2 $\mathbb{R}_+ = \dots$

(a) $]0, \infty[$

(b) $] -\infty, 0[$

(c) $[0, \infty[$

(d) $] -\infty, 0]$

3 $\mathbb{R}_- =$ _____

- (a) $]0, \infty[$ (b) $] -\infty, 0[$ (c) $[0, \infty[$ (d) $] -\infty, 0]$

4 The set of non-negative real numbers = _____

- (a) $]0, \infty[$ (b) $] -\infty, 0[$ (c) $[0, \infty[$ (d) $] -\infty, 0]$

5 The set of non-positive real numbers = _____

- (a) $]0, \infty[$ (b) $] -\infty, 0[$ (c) $[0, \infty[$ (d) $] -\infty, 0]$

3 Complete each of the following using one of the symbols \in or \notin :

1 $3 \quad [3, 5]$

2 $-2 \quad] -2, 1]$

3 $0 \quad] -1, 4[$

4 $| -3 | \quad [2, \infty[$

5 $\sqrt{9} \quad] -3, \infty[$

6 $\sqrt[3]{-1} \quad] -\infty, 1[$

7 $1.3 \times 10^{-5} \quad \mathbb{R}_+$

8 $\sqrt{2} \quad [2, 5]$

9 $5 \quad] \sqrt{5}, \sqrt{23}[$

10 $\sqrt[3]{-125} \quad] -\sqrt{25}, \sqrt{25}[$

4 If $X = [2, 5[$ and $Y = [-1, 3[$, find using the number line:

1 $X \cup Y$

2 $X \cap Y$

3 $X - Y$

4 $Y - X$

5 \bar{X}

6 \bar{Y}

5 If $X =] -\infty, 3]$ and $Y = [-4, \infty[$, find using the number line:

1 $X \cup Y$

2 $X \cap Y$

3 $X - Y$

4 $Y - X$

5 \bar{X}

6 \bar{Y}

6 If $X = [-1, 4]$, $Y = [3, \infty[$ and $Z = \{3, 4\}$, find the following using the number line:

1 $X \cup Y$

2 $X \cap Y$

3 $X - Y$

4 $X \cap Z$

5 $Y \cap Z$

6 $Y - X$

7 \bar{X}

8 \bar{Y}

7 Find using the number line:

1 $[-1, 4] \cap [2, 5]$

2 $[-1, 4] \cup [2, 5]$

3 $] -2, 3] \cap] 0, 1[$

4 $] -2, 3] \cup] 0, 1[$

5 $[2, 6] - [-1, 3[$

6 $[-1, 3[- [2, 6]$

7 $[-3, 0[\cup] 0, 2]$

8 $[-3, 0] \cap] 0, 2]$

9 $[1, 2] - [-2, 4]$

10 $[-2, 4] - [1, 2]$

11 $[-1, 4] \cap [5, 7[$

12 $[-1, 5] -] -1, 5[$

8 Find using the number line :

1 $[-1, \infty[\cup [-3, 4]$

3 $]-\infty, 3] \cap [-4, \infty[$

5 $]-\infty, 3] - [-1, \infty[$

7 $]-\infty, 2] -]-\infty, 0]$

2 $[2, \infty[\cap]-2, 3[$

4 $[2, \infty[\cup]-\infty, 3]$

6 $]-\infty, -3] - [-3, 1]$

8 $]-\infty, 3[\cup]4, \infty[$

9 Complete the following :

1 $[3, 5] \cup \{3, 5\} = \dots\dots\dots$

3 $[3, 5] \cap \{3, 5\} = \dots\dots\dots$

5 $[3, 5] - \{3, 5\} = \dots\dots\dots$

7 $\{3, 5\} - [3, 5] = \dots\dots\dots$

9 $]3, 5[\cup \{3\} = \dots\dots\dots$

11 $]2, 5[\cap \{-2, 3, 4\} = \dots\dots\dots$

2 $]3, 5[\cup \{3, 5\} = \dots\dots\dots$

4 $]3, 5[\cap \{3, 5\} = \dots\dots\dots$

6 $]3, 5[- \{3, 5\} = \dots\dots\dots$

8 $\{3, 5\} -]3, 5[= \dots\dots\dots$

10 $[3, 5] - \{5\} = \dots\dots\dots$

12 $] - 3, 5] \cup \{-2, 3, 4\} = \dots\dots\dots$

10 Complete the following :

1 $]1, 7[\cup]3, 5[= \dots\dots\dots$

3 $[3, 4[\cup]3, 4] = \dots\dots\dots$

5 $[3, 5] - [3, 5[= \dots\dots\dots$

7 $[2, 7] -]2, 7[= \dots\dots\dots$

9 If $X \cap [2, 7] = [3, 4[$, then $X = \dots\dots\dots$

10 If x is a positive real number, then $x > x^2$ when $x \in] \dots\dots\dots, \dots\dots\dots [$

2 $] - 3, 2] - [0, 2] = \dots\dots\dots$

4 $]2, 5] \cap [2, 5[= \dots\dots\dots$

6 $[3, 7] - [4, 7] = \dots\dots\dots$

8 $[-2, 4] \cap [4, 6] = \dots\dots\dots$

11 Choose the correct answer from the given ones :

1 $[-3, 4] - \{-3, 5\} = \dots\dots\dots$

(a) $] - 3, 4[$

(b) $] - 3, 4]$

(c) $] - 3, 5[$

(d) $] - 3, 5]$

2 If $x \in [-3, \infty[$, then $\dots\dots\dots$

(a) $x < -3$

(b) $x \leq -3$

(c) $x > -3$

(d) $x \geq -3$

3 If $X = \{x : x \in \mathbb{R}, 2 < x \leq 5\}$, then $[3, 4] \dots\dots\dots X$

- (a) \in (b) \notin (c) \subset (d) $\not\subset$

4 $\{3\} \cap [3, 6] = \dots\dots\dots$

- (a) \emptyset (b) $\{3\}$ (c) $]3, 6]$ (d) $\{6\}$

5 $\{8, 9, 10\} -]8, 10[= \dots\dots\dots$

- (a) \emptyset (b) $\{8, 10\}$ (c) $\{9\}$ (d) \mathbb{N}

6 The sum of all real numbers in $[-75, 75]$ is $\dots\dots\dots$

- (a) -75 (b) 75 (c) 150 (d) zero

12 Complete the following :

1 $\mathbb{R} \cap [-3, 3] = \dots\dots\dots$

2 $\mathbb{R} \cup]-1, 4] = \dots\dots\dots$

3 $\mathbb{R} - [-1, \infty[= \dots\dots\dots$

4 $\mathbb{R} - [-3, 1] = \dots\dots\dots$

5 $] -2, 5] - \mathbb{R}_+ = \dots\dots\dots$

6 $[-2, 2] - \mathbb{R}_- = \dots\dots\dots$

7 $] -3, 2] \cap \mathbb{Z}_+ = \dots\dots\dots$






8 $\mathbb{N} \cap [-5, 2[= \dots\dots\dots$

9 $\mathbb{Z} \cap [-1, 3[= \dots\dots\dots$

10 $\mathbb{R}_+ \cap [0, 5] = \dots\dots\dots$

11 $\mathbb{R}_- \cap [-3, 2] = \dots\dots\dots$

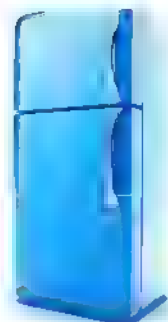
13 Choose from column (B) the suitable interval which represents the figure in column (A) :

(A)	(B)
1 	$\mathbb{R} =]-3, 1]$
2 	$\mathbb{R} - [-3, 1[$
3 	$\mathbb{R} -]-3, 1[$
4 	$[-3, 3[- \{1\}$
5 	$[-3, 1[$
	$] -3, 1[$

Life Application

- 14** Two kinds of food, the first kind needs to be kept in a temperature between -3 and 4 degrees, and the other kind needs to be kept in a temperature between 2 and 10 degrees.

What is the temperature needed to keep the two kinds altogether at the same place?



For excellent pupils

- 15** Choose the correct answer from the given ones :

- 1** In the opposite figure :

If X is a real number, then $X \in$



- (a) \mathbb{R}_- (b) \mathbb{R}_+ (c) $]-\infty, -1]$ (d) $]-\infty, -1[$
- 2** If $X \in [-3, 4]$, then $X^2 \in$
 (a) $[9, 16]$ (b) $[0, 9]$ (c) $[0, 16]$ (d) $[-9, 0]$
- 3** If $X \in [-5, 4]$, then $X^2 \in$
 (a) $[0, 16]$ (b) $[16, 25]$ (c) $[0, 25]$ (d) $[-5, 0]$
- 4** If $X \in [1, 16]$, then $-\sqrt{X} \in$
 (a) $[1, 4]$ (b) $[-1, 4]$ (c) $[-4, -1]$ (d) $[-4, 0]$
- 5** If $X \subset \mathbb{R}$, $[2, 5] - X =]2, 5[$, then $X =$
 (a) $[2, 5]$ (b) $\{2, 5\}$ (c) $[2, 5[$ (d) $]2, 5]$
- 6** If $X \subset \mathbb{R}$, $]4, 7] \cup X = [1, 7]$, then $X =$
 (a) $[1, 3[$ (b) $[1, 3]$ (c) $[1, 4[$ (d) $[1, 5]$
- 7** If $M \subset \mathbb{R}$, $M \cap [3, 8[= [3, 8[$, then $M =$
 (a) $]3, 8[$ (b) $]3, 8]$ (c) $[3, 9]$ (d) $[3, 7]$
- 8** If $]-\infty, k[\cap [-2, 5] = [-2, 3[$, then $k =$
 (a) -2 (b) 5 (c) 3 (d) zero
- 9** If $[-1, X] \cap [y, 5] = [2, 3]$, then $X^y =$
 (a) 8 (b) $\frac{1}{5}$ (c) 9 (d) -1

- 16** If $X \cap Y = [4, 7]$, $X \cup Y = [3, 7]$ and $X \subset Y$, find : X , Y and $Y - X$

Operations on
the real numbers

Interactive test

From the school book



Remember

Understand

Apply

Problem Solving

1 Find each of the following in the simplest form :

1 $\sqrt{3} + 2\sqrt{3}$

2 $2\sqrt{5} - 3\sqrt{5} + \sqrt{5}$

3 $5^3\sqrt{7} - 8^3\sqrt{7} + 2^3\sqrt{7}$

4 $4\sqrt{5} - 2\sqrt{5} + 5\sqrt{5} - \sqrt{5}$

2 Find each of the following in the simplest form :

1 $\sqrt{5} - \sqrt{3} + 2\sqrt{5} + \sqrt{3}$

2 $2\sqrt{3} + 5 + \sqrt{3} - 6$

3 $2\sqrt{7} - 3\sqrt{2} + \sqrt{7} + 5\sqrt{7}$

4 $2\sqrt{2} - 3^3\sqrt{2} + 5\sqrt{2} + \sqrt[3]{2}$

5 $\frac{1}{4}\sqrt{2} + \frac{2}{7}\sqrt{5} + \frac{3}{4}\sqrt{2} - \frac{2}{7}\sqrt{5}$

6 $8\sqrt{\frac{1}{4}} + 2^3\sqrt{3} - \sqrt[3]{64} - 5^3\sqrt{3}$

3 Find the result of each of the following :

1 $\sqrt{3} \times \sqrt{3}$

2 $-2\sqrt{5} \times 3\sqrt{5}$

3 $2 \times 3\sqrt{2}$

4 $\frac{1}{3}\sqrt{3} \times \sqrt{3}$

5 $(\sqrt[3]{5})^3 \times 3\sqrt{3}$

6 $2\sqrt{3} \times \frac{2\sqrt{7}}{7} \div \frac{20\sqrt{3}}{5\sqrt{7}}$

4 Find the result of each of the following in the simplest form :

1 $2(\sqrt{2} + \sqrt{5})$

2 $\sqrt{2}(5 + \sqrt{2})$

3 $\sqrt{7}(\sqrt{7} + 2)$

4 $-\sqrt{3}(-5 - \sqrt{3})$

5 $-2\sqrt{5}(3 - \sqrt{5})$

6 $\sqrt{7}\left(\frac{2}{\sqrt{7}} - \sqrt{7} + 3\right)$

7 $-3(8 + 2\sqrt{3}) + 6\sqrt{3}$

8 $\sqrt{5}(3 - \sqrt{5}) - 2(1 + \sqrt{5})$

5 Find the result of each of the following operations :

1 $(\sqrt{2} + 1)(\sqrt{2} - 1)$

2 $(4 - 3\sqrt{2})(4 + 3\sqrt{2})$

3 $(\sqrt{5} - 1)^2$

4 $(2\sqrt{3} + 4)^2$

5 $(\sqrt{3} + 2)(\sqrt{3} - 1)$

6 $(5 - \sqrt{3})^2 - 28$

6 Make the denominator in each of the following an integer :

1 $\frac{3}{\sqrt{3}}$

2 $\frac{10}{\sqrt{5}}$

3 $-\frac{6}{\sqrt{3}}$

4 $\frac{8}{\sqrt{6}}$

5 $\frac{2}{3\sqrt{2}}$

8 $\frac{6}{2\sqrt{3}}$

7 $\frac{25}{2\sqrt{10}}$

8 $\frac{\sqrt{2} + 3}{\sqrt{2}}$

9 $\frac{\sqrt{5} - 15}{2\sqrt{5}}$

7 Choose the correct answer from those given :

1 $\sqrt{7} + \sqrt{7} = \dots\dots\dots$

(a) 7

(b) 14

(c) $2\sqrt{7}$

(d) $\sqrt{14}$

2 $\sqrt{3} + (-\sqrt{3}) = \dots\dots\dots$

(a) $2\sqrt{3}$

(b) $2\sqrt{6}$

(c) $\sqrt{6}$

(d) zero

3 $2\sqrt{3} + 3\sqrt{3} = \dots\dots\dots$

(a) $5\sqrt{6}$

(b) $5\sqrt{3}$

(c) $6\sqrt{3}$

(d) $5^3\sqrt{3}$

4 $5 + 7\sqrt{2} - 4 + \sqrt{2} = \dots\dots\dots$

(a) 15

(b) $1 + 7\sqrt{2}$

(c) $1 + 8\sqrt{2}$

(d) $1 + 6\sqrt{2}$

5 $-2\sqrt{3} \times \sqrt{3} = \dots\dots\dots$

(a) -6

(b) $-2\sqrt{3}$

(c) $2\sqrt{3}$

(d) 6

6 $(2^3\sqrt{5})^3 = \dots\dots\dots$

(a) 10

(b) 20

(c) $4^3\sqrt{5}$

(d) 40

7 The additive inverse of the number $\frac{6}{\sqrt{2}}$ is

(a) $-2\sqrt{3}$

(b) $2\sqrt{3}$

(c) $-3\sqrt{2}$

(d) $3\sqrt{2}$

8 The additive inverse of the number $(\sqrt{2} - \sqrt{5})$ is

(a) $\sqrt{2} + \sqrt{5}$

(b) $\sqrt{5} - \sqrt{2}$

(c) $\sqrt{2} - \sqrt{5}$

(d) $-\sqrt{2} - \sqrt{5}$

9 The multiplicative inverse of the number $\sqrt{5}$ is

- (a) -5 (b) $\frac{-1}{5}$ (c) $\frac{5}{\sqrt{5}}$ (d) $\frac{\sqrt{5}}{5}$

10 The multiplicative inverse of the number $\frac{\sqrt{2}}{6}$ is

- (a) $\sqrt{3}$ (b) $3\sqrt{2}$ (c) $\sqrt{6}$ (d) $\frac{\sqrt{2}}{2}$

11 $(\sqrt{5} + 3\sqrt{5}) \div \sqrt{5} = \dots\dots\dots$

- (a) $3\sqrt{5}$ (b) 3 (c) 5 (d) 4

12 If $x = \sqrt{2} + 10$, $y = \sqrt{2} - 10$, then $(x + y)^2 = \dots\dots\dots$

- (a) 4 (b) 6 (c) 8 (d) $4\sqrt{2}$

8 Complete the following :

1 The multiplicative neutral in \mathbb{R} is and the additive neutral in \mathbb{R} is

2 The additive inverse of the number $1 - \sqrt{2}$ is

3 The multiplicative inverse of the number $\frac{2\sqrt{3}}{5}$ is $\frac{\dots}{6}$

4 The multiplicative inverse of the number $\frac{3}{\sqrt{3}}$ is $\frac{\dots}{\sqrt{3}}$

5 $7 + \sqrt{3} = 5 + (\dots + \dots)$

If $a = \frac{\sqrt{2}}{\sqrt{3}}$, $b = \frac{\sqrt{3}}{\sqrt{2}}$, then $\frac{a}{b} = \dots\dots\dots$

$(\sqrt{3} - 2)^2 = 7 - \dots\dots\dots$

6 If $\sqrt{x} = \sqrt{2} + 1$, then $x = \dots\dots\dots$

7 If $x^2 = (2\sqrt{3} - \sqrt{7})(2\sqrt{3} + \sqrt{7})$, then $x = \dots\dots\dots$

8 If $x^2 - y^2 = 16$, $x - y = \sqrt{2}$, then $x + y = \dots\dots\dots$

9 If the side length of a square is ℓ cm. and its area is 15 cm^2 , then the area of the square of side length 2ℓ cm. is $\dots\dots\dots$

10 If $a \in \mathbb{R}$ and $b \in \mathbb{R}$, then $a + b$ means the sum of the number a and $\dots\dots\dots$ of the number b

11 If $a \in \mathbb{N}$, $b \in \mathbb{Q}$ and $c \in \mathbb{R}$, then $a + b + c \in \dots\dots\dots$

9 If $x = \sqrt{5} - 2$ and $y = \sqrt{5} + 2$, find the value of each of the following :

1 $x + y$

2 $x - y$

3 xy

4 $x^2 - y^2$

5 $x^2 + 2xy + y^2$

6 $x^2 - 2xy + y^2$

10 If $\frac{a}{2\sqrt{2}+2} = \frac{b}{2\sqrt{2}-2} = 1$ Prove that : $a \times b = a - b$

11 If $x = \sqrt{15} + 2$ and $y = 4 - \sqrt{25}$, estimate the value of each of the following :

1 x, y

2 $x \times y$

3 $x + y$

Check the reasonability of each value using your calculator.

Geometric Application

12 A rectangle is of dimensions $(6 + \sqrt{5})$ cm. and $(6 - \sqrt{5})$ cm.

Calculate its perimeter and its area.

« 24 cm. , 31 cm² »



For excellent pupils

13 If $a - b = 2\sqrt{3}$, find the value of : $a(a - b)^3 + b(b - a)^3$

« 144 »

14 If the multiplicative inverse of the number $\sqrt{a} - 1$ is $\frac{\sqrt{a} + 1}{4}$, find the numerical value of a

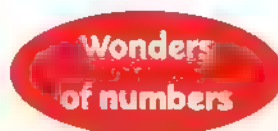
« 5 »

15 If $x = 2, y = 4, z = \sqrt{2}$, find the value of : $x^2 + 2y^2 + 4z^2$

« 3 $\frac{1}{2}$ »

16 If the number y is the additive inverse of x and $\frac{1}{2}(y - x) = 1 - \sqrt{2}$

Prove that : $xy - 2\sqrt{2} = -3$



1 $\times 1 = 1$

11 $\times 11 = 121$

111 $\times 111 = 12321$

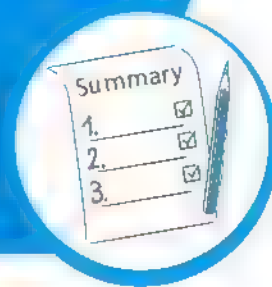
1111 $\times 1111 = 1234321$

What happens when you multiply 11111×11111 ?



Summary of the first part of Unit I

"From lesson 1 to lesson 5"



- ★ The cube root of the number "a" is the number whose cube equals a
For example : $\sqrt[3]{64} = 4$, $\sqrt[3]{-64} = -4$
- ★ The cube root of the positive number is positive , and the cube root of the negative number is negative.
- ★ $\sqrt[3]{a^3} = a$ For example : $\sqrt[3]{(-5)^3} = -5$
 $\sqrt[3]{a^n} = a^{\frac{n}{3}}$ where $n \in \mathbb{Z}$
For example : $\sqrt[3]{a^6} = a^{\frac{6}{3}} = a^2$
- ★ If "a" is a perfect cube number , then the equation : $x^3 = a$ has a unique solution in \mathbb{R} , which is $\sqrt[3]{a}$
- ★ Each irrational number lies between two rational numbers and can be represented by a point on the number line.
- ★ The set of rational numbers \mathbb{Q} and the set of irrational numbers \mathbb{Q}^c are disjoint sets.
i.e. $\mathbb{Q} \cap \mathbb{Q}^c = \emptyset$
- ★ $\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}^c$, $\mathbb{R} - \mathbb{Q} = \mathbb{Q}^c$, $\mathbb{R} - \mathbb{Q}^c = \mathbb{Q}$
- ★ $\mathbb{R} = \mathbb{R}_+ \cup \{0\} \cup \mathbb{R}_- =]-\infty, \infty[$, $\mathbb{R}_+ \cap \mathbb{R}_- = \emptyset$
 $\mathbb{R}_+ =]0, \infty[$, $\mathbb{R}_- =]-\infty, 0[$
 , the set of non-negative real numbers $= \mathbb{R}_+ \cup \{0\} = [0, \infty[$
 , the set of non-positive real numbers $= \mathbb{R}_- \cup \{0\} =]-\infty, 0]$
 $\mathbb{R}^* = \mathbb{R} - \{0\} = \mathbb{R}_+ \cup \mathbb{R}_-$
- ★ It is possible to carry out the operations of intersection , union , difference and complement on the intervals.
- ★ The set of real numbers is closed under addition , subtraction and multiplication operations and is not closed under division operation.
- ★ Each of addition and multiplication operation in \mathbb{R} is commutative and associative , but each of subtraction and division operation in \mathbb{R} is not commutative and associative.
- ★ Zero is the additive neutral in \mathbb{R} , and one is the multiplicative neutral in \mathbb{R}
- ★ For every real number "a" , there is an additive inverse which is the real number "- a" , and for every real number "a" where $a \neq 0$, there is a multiplicative inverse which is the real number $\frac{1}{a}$
- ★ The multiplication in the set of real numbers is distributed on the addition and the subtraction from right and from left.

Exams on the first part of unit on
from lesson (1) to lesson (5)



Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1 $[-2, 5] - \{-2, 5\} = \dots\dots\dots$

- (a) $\{-2, 5\}$ (b) $[-2, 5[$ (c) $] -2, 5[$ (d) $] -2, 5]$

2 $\mathbb{Q} \cap \mathbb{Q} = \dots\dots\dots$

- (a) \mathbb{Z} (b) \mathbb{R} (c) \mathbb{Q} (d) \emptyset

3 $\sqrt{x^4} = \sqrt[3]{\dots\dots\dots}$

- (a) x^6 (b) x^4 (c) x^2 (d) x

4 The irrational number included between 3 and 4 is $\dots\dots\dots$

- (a) $\sqrt{7}$ (b) $\sqrt{10}$ (c) $\sqrt[3]{12}$ (d) $3\frac{1}{4}$

5 The multiplicative inverse of the number $\sqrt{3}$ is $\dots\dots\dots$

- (a) $\frac{3}{\sqrt{3}}$ (b) -3 (c) 3 (d) $\frac{\sqrt{3}}{3}$

6 If $\frac{X}{4} = \frac{16}{X^2}$, then $X = \dots\dots\dots$

- (a) 2 (b) 4 (c) 8 (d) 16

2 Complete the following :

1 $\sqrt[3]{4 + \dots\dots\dots} = 3$

2 The square whose side length is $\sqrt{5}$ cm., its area is $\dots\dots\dots$ cm²

3 $\mathbb{R} - \mathbb{Q} = \dots\dots\dots$

4 $[-2, 7] \cap] -2, 7[= \dots\dots\dots$

5 The additive inverse of the number $5 - \sqrt{3}$ is $\dots\dots\dots$

3 [a] If $X = [-2, 3]$ and $Y = [1, 5[$, find using the number line each of :

- 1 $X \cap Y$ 2 $X \cup Y$ 3 $Y - X$

[b] Find the solution set in \mathbb{R} of the equation : $(x^2 - 4)(x^3 - 7) = 0$

4 [a] Prove that: $\sqrt{12}$ is included between 3.4 and 3.5

[b] A square of side length 5 cm. , find its diagonal length.

5 [a] Determine the point which represents the number $\sqrt{5}$ on the number line.

[b] Find the result of each of the following operations :

1 $(\sqrt{3} + 1)(\sqrt{3} - 1)$

2 $(\sqrt{7} + 2)(\sqrt{7} - 1)$



Answer the following questions :

1 Choose the correct answer from those given :

$5 \in \dots$

(a) $]3, 7]$

(b) $]5, 7]$

(c) $(-5, 5)$

(d) $\{55\}$

2 The irrational number from the following numbers is

(a) $\sqrt{\frac{4}{25}}$

(b) $\sqrt[3]{1}$

(c) $\sqrt{\frac{27}{8}}$

(d) $\sqrt[3]{\frac{1}{64}}$

3 If $-\sqrt{4} = \sqrt[3]{x}$, then $x = \dots$

(a) 8

(b) -8

(c) 4

(d) 16

4 $\sqrt{4} - \sqrt[3]{-8} = \dots$

(a) -2

(b) 4

(c) -4

(d) 8

5 $\{2, 5, 7\} -]2, 7] = \dots$

(a) $\{2\}$

(b) $\{2, 5\}$

(c) $]2, 5[$

(d) $[2, 5]$

6 $(\sqrt[3]{-3})^3 = \dots$

(a) 3

(b) -3

(c) ± 3

(d) $\sqrt[3]{-9}$

2 Complete the following :

1 The additive inverse of the number $\sqrt{7} - \sqrt{2}$ is

2 $\mathbb{R}_+ \cup \mathbb{R}_- = \dots$

3 $[-4, 6[- \mathbb{R}_+ = \dots$

4 The sum of the real numbers in the interval $[-3, 3[$ is ...

5 The solution set of the equation $x^2 + 25 = 0$ in \mathbb{R} is

- 3 [a] Find the result of the following in the simplest form :

$$2\sqrt{7} - 5\sqrt{2} + \sqrt{7} + 5\sqrt{2}$$

- [b] If $X =]-\infty, 1[$ and $Y = [2, 4[$, using the number line find in the form of an interval each of the following :

1 $X \cup Y$

2 $X \cap Y$

3 \bar{X}

- 4 [a] Find in \mathbb{Q} the solution set of each of the following equations :

1 $\frac{1}{2}x^2 - 3 = 7$

2 $125x^3 - 3 = 5$

- [b] Prove that: $\sqrt[3]{17}$ is included between 2.57 and 2.58

- 5 [a] Simplify to the simplest form :

1 $(2\sqrt{3} - 5)^2$

2 $\sqrt{5}(\sqrt{5} + 2)$

- [b] Write four irrational numbers included between 11 and 12

Wonders of numbers

Pick a number , add 3 to it , multiply the sum by 2 , add 4 to the product , divide the sum by 2 and subtract the original number from the quotient the result is always 5



Operations on
the square roots

Interactive test

From the school book



Remember

Understand

Apply

Problem Solving

- 1 Put each of the following in the form $a\sqrt{b}$ where a and b are two integers, b is the least possible value :

1 $\sqrt{12}$

2 $\sqrt{28}$

3 $2\sqrt{72}$

4 $\frac{2}{5}\sqrt{1000}$

5 $2\sqrt{\frac{1}{2}}$

6 $6\sqrt{\frac{2}{3}}$

- 2 Simplify each of the following to the simplest form :

1 $\sqrt{50} + \sqrt{8}$

« $7\sqrt{2}$ »

2 $\sqrt{20} - \sqrt{45}$

« $-\sqrt{5}$ »

3 $3\sqrt{2} + \sqrt{8} - \sqrt{18}$

« $2\sqrt{2}$ »

4 $\sqrt{98} - \sqrt{128} - \sqrt{18} + 4\sqrt{2}$

« zero »

5 $2\sqrt{18} + \sqrt{50} + \frac{1}{3}\sqrt{162}$

« $11\sqrt{3}$ »

6 $\sqrt{98} + \sqrt{50} - \frac{1}{2}\sqrt{200} - \sqrt{2}$

« $6\sqrt{2}$ »

7 $\sqrt{27} + 5\sqrt{18} - \sqrt{300}$

« $15\sqrt{2} - 7\sqrt{3}$ »

- 3 Put each of the following in the simplest form :

1 $2\sqrt{5} + 4\sqrt{20} - 5\sqrt{\frac{1}{5}}$

« $9\sqrt{5}$ »

2 $\sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}}$

« $\sqrt{2}$ »

3 $2\sqrt{5} + 6\sqrt{\frac{1}{3}} - \sqrt{12} - 5\sqrt{\frac{1}{5}}$

« $\sqrt{5}$ »

4 $\sqrt{3} + \frac{3}{\sqrt{3}} - \sqrt{2} \times \sqrt{6}$

« zero »

5 $\sqrt{18} - \frac{\sqrt{12}}{\sqrt{6}}$

« $2\sqrt{2}$ »

6 $\sqrt{(-5)^2} + \sqrt{18} - \frac{6}{\sqrt{2}}$

« 5 »

4 Simplify each of the following to the simplest form :

1] $2\sqrt{3} \times 5\sqrt{2}$	« $10\sqrt{6}$ »	2] $2\sqrt{18} \times 3\sqrt{2}$	« 36 »
3] $\sqrt{5} \times 2\sqrt{10}$	« $10\sqrt{2}$ »	4] $\sqrt{\frac{2}{7}} \times \sqrt{\frac{7}{2}}$	« 1 »
5] $\frac{3\sqrt{15}}{\sqrt{5}}$	« $3\sqrt{3}$ »	6] $12\sqrt{\frac{2}{3}} \times \sqrt{54}$	« 72 »

5 Simplify each of the following to the simplest form :

1] $\sqrt{6}(\sqrt{3} - \sqrt{2})$	2] $5\sqrt{2}(2\sqrt{2} + \sqrt{12})$
3] $(3\sqrt{5} - \sqrt{7})(3\sqrt{5} + \sqrt{7})$	4] $(\sqrt{3} - \sqrt{2})^2$
5] $(\sqrt{3} + \sqrt{5})^2 - \sqrt{60}$	6] $\sqrt{18} - \frac{12}{\sqrt{6}} + \sqrt{2}(2\sqrt{3} - 3)$

6 Write each of the following such that the denominator is an integer :

1] $\frac{\sqrt{3}}{\sqrt{2}}$	2] $\sqrt{\frac{5}{3}}$	3] $\frac{5\sqrt{3}}{\sqrt{5}}$	4] $\frac{4\sqrt{3} - \sqrt{2}}{2\sqrt{3}}$
--------------------------------	-------------------------	---------------------------------	---

7 Choose the correct answer from those given :

1] $\frac{\sqrt{63}}{\sqrt{7}} = \dots\dots\dots$	(a) 3	(b) $\sqrt{3}$	(c) 9	(d) ± 3
2] $\sqrt{8} - \sqrt{2} = \dots\dots\dots$	(a) $\sqrt{6}$	(b) $\sqrt{2}$	(c) 2	(d) 1
3] $(\sqrt{8} + \sqrt{2})^2 = \dots\dots\dots$	(a) $\sqrt{10}$	(b) 10	(c) 18	(d) $\sqrt{18}$
4] $(\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5}) = \dots\dots\dots$	(a) 2	(b) 12	(c) $2\sqrt{7}$	(d) $-2\sqrt{5}$
5] $\sqrt{5} + \sqrt{5} = \dots\dots\dots$	(a) $\sqrt{10}$	(b) $\sqrt{20}$	(c) 5	(d) 10
6] $\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{2}} = \dots\dots\dots$	(a) 1	(b) $\sqrt{\frac{1}{4}}$	(c) $\sqrt{2}$	(d) $\frac{\sqrt{2}}{2}$

7 $\frac{\sqrt[3]{27}}{\sqrt{3}} \div \frac{\sqrt[3]{72}}{\sqrt{2}} = \dots\dots\dots$

- (a) $\frac{1}{2}$ (b) 2 (c) -2 (d) 4

8 The multiplicative inverse of the number $\sqrt{50}$ is $\dots\dots\dots$

- (a) $\frac{\sqrt{2}}{10}$ (b) $\frac{-\sqrt{2}}{10}$ (c) $-5\sqrt{2}$ (d) $5\sqrt{2}$

9 If $X = \frac{\sqrt[3]{6}}{\sqrt{2}}$, then $X^{-1} = \dots\dots\dots$

- (a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt[3]{3}}{3}$ (d) $2\sqrt{3}$

10 If $X = \sqrt{7} + \sqrt{3}$ and $y = \sqrt{28} + \sqrt{12}$, then $X = \dots\dots\dots$

- (a) y (b) $\frac{1}{2}y$ (c) $2y$ (d) y^2

8 Complete the following :

$\frac{3\sqrt{2}}{2\sqrt{18}} = \dots\dots\dots$

2 $\sqrt{3} \times \sqrt{6} = 3 \times \dots\dots\dots$

3 $\frac{1}{2}\sqrt{48} = 2 \times \dots\dots\dots$

4 $\sqrt[3]{3\frac{3}{8}} = \frac{3}{2}\sqrt{\dots\dots\dots}$

5 If $2\sqrt{27} - 2\sqrt{48} = X\sqrt{3}$, then $X = \dots\dots\dots$

6 $\sqrt{5}, \sqrt{20}, \sqrt{45}, \sqrt{80}, \dots\dots\dots$ (in the same pattern).

7 If $X^2 = \frac{8}{9}$, then X in the simplest form = $\dots\dots\dots$

8 If $X^2 = 5$, then $(X + \sqrt{5})^2 = \dots\dots\dots$ or $\dots\dots\dots$

9 Find the value of each of $X + y$, $X \times y$ in each of the following cases :

1 $X = 3 + \sqrt{5}$, $y = 1 - \sqrt{5}$ « 4, -2-2 $\sqrt{5}$ »

2 $X = \sqrt{3} - \sqrt{2}$, $y = \sqrt{3} + \sqrt{2}$ « 2 $\sqrt{3}$, 1 »

3 $X = 5 - 3\sqrt{2}$, $y = 5 - 3\sqrt{2}$ « 10-6 $\sqrt{2}$, 43-30 $\sqrt{2}$ »

10 If $X = \frac{\sqrt{2}}{\sqrt{3}}$ and $y = \frac{\sqrt{3}}{\sqrt{2}}$, find the value of : 6 (X + y) « 5 $\sqrt{6}$ »

11 If $X = \frac{10}{\sqrt{5}}$, $y = \sqrt{45} + \sqrt{2}$ and $z = \sqrt{8} + \sqrt{5}$, find in the simplest form the value of the expression $(X - y + z)^2$ « 2 »

12 If $x = 2\sqrt{5} + \sqrt{2}$, $y = 2\sqrt{5} - \sqrt{2}$

, find the value of the expression : $x^2 + 2xy + y^2$

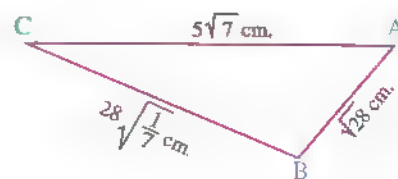
« 80 »

13 If $x = \sqrt{7} + \frac{1}{2}\sqrt{12}$ and $y = \frac{1}{3}\sqrt{63} - \sqrt{3}$, prove that : $x^2 y^2 = 16$

Geometric Applications

14 In the opposite figure :

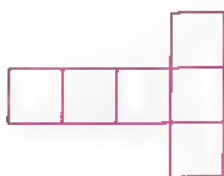
Find the perimeter of $\triangle ABC$ in the simplest form.



« $11\sqrt{7}$ cm. »

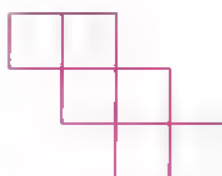
15 Each of the following figures consists of squares equal in area. Find the perimeter of each figure in the simplest form if its area is known :

1



Its area = 300 cm^2

2



Its area = 72 cm^2

3



Its area = 40 cm^2

« $70\sqrt{2}$ cm. , $28\sqrt{3}$ cm. , $24\sqrt{2}$ cm. »



For excellent pupils

16 If $a^x = 6$ and $a^{-y} = \sqrt{3}$, find the value of : a^{x+y}

« $2\sqrt{3}$ »

17 Simplify each of the following to the simplest form :

1 $\frac{(\sqrt{5})^3 \times (\sqrt{5})^5}{(\sqrt{10})^6}$

« $\frac{5}{8}$ »

2 $\frac{2\sqrt{2} \times (\sqrt{6})^{-3}}{(\sqrt{3})^{-3}}$

« 1 »

18 If $\sqrt{27} + 2\sqrt{\frac{1}{2}} + \sqrt{18} + \sqrt{12} - \sqrt{50} = x\sqrt{2} + y\sqrt{3}$

, find the value of each of x and y where x and y are two rational numbers.

« 1 , 5 »

The two conjugate numbers



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Apply

Problem Solving

1 Write the conjugate number of each of the following numbers :

1 $\sqrt{5} + \sqrt{3}$

2 $5 - 2\sqrt{7}$

3 $\sqrt{5} + \frac{2}{\sqrt{2}}$

2 Make the denominator of each of the following a rational number :

1 $\frac{5}{\sqrt{7} - \sqrt{2}}$

2 $\frac{\sqrt{3}}{2\sqrt{3}}$

3 $\frac{\sqrt{7} + 3}{\sqrt{7} - 3}$

3 If $x = \frac{2}{\sqrt{7} - \sqrt{5}}$ and $y = \sqrt{7} - \sqrt{5}$, find the value of : $(x + y)^2$

« 28 »

4 If $x = \frac{4}{\sqrt{7} - \sqrt{3}}$ and $y = \frac{4}{\sqrt{7} + \sqrt{3}}$, find the value of : $x^2 y^2$

« 16 »

5 If $x = \sqrt{5} + \sqrt{3}$, prove that : $\frac{4}{x} + 2x = 4\sqrt{5}$ 6 If $a = \sqrt{3} + \sqrt{2}$ and $b = \frac{1}{\sqrt{3} + \sqrt{2}}$, find the value of : $a^2 - b^2$ in its simplest form. « $4\sqrt{6}$ »7 If $x = \sqrt{5} - \sqrt{3}$ and $y = \frac{2}{\sqrt{5} - \sqrt{3}}$, find the value of : $x^2 + 2xy + y^2$

« 20 »

8 If $x = \sqrt{5} - \sqrt{2}$ and $y = \frac{3}{\sqrt{5} - \sqrt{2}}$, prove that x and y are conjugate numbers, thenfind the value of : $x^2 - 2xy + y^2$

« 8 »

- 9 If $X = 3 + \sqrt{5}$ and $y = \frac{4}{3 + \sqrt{5}}$, prove that X and y are conjugate numbers, then find :

1 Their product.

2 $X^2 + y^2$

« 4, 28 »

- 10 If $X = \frac{2}{\sqrt{5} - \sqrt{3}}$ and $y = \frac{2}{\sqrt{5} + \sqrt{3}}$, find the value of : $X^2 - Xy + y^2$

« 14 »

- 11 If $X = \sqrt{5} + \sqrt{2}$ and $y = \sqrt{5} - \sqrt{2}$, find the value of : $\frac{X+y}{Xy-1}$ in its simplest form. « $\sqrt{5}$ »

- 12 If $a = \frac{4}{\sqrt{7} - \sqrt{3}}$ and $b = \frac{4}{\sqrt{7} + \sqrt{3}}$, find the value of : $\frac{a-b}{ab}$

« $\frac{\sqrt{3}}{2}$ »

- 13 If $X = 2\sqrt{2} - \sqrt{3}$ and $y = \frac{5}{\sqrt{8} \cdot \sqrt{3}}$

, prove that X and y are conjugate numbers and calculate : $\frac{X+y}{Xy}$

« $\frac{1+\sqrt{2}}{5}$ »

- 14 If $X = \frac{5\sqrt{2} + 3\sqrt{5}}{\sqrt{5}}$ and $y = \frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{2}}$

, find the value of each of : 1 $X^2 + y^2$

2 Xy

« 38, 1 »

, then prove that : $X^2 + y^2 = 38Xy$

- 15 If $X = \frac{1}{2 + \sqrt{3}}$ and $y = \frac{12}{\sqrt{3}}$, find the value of : $X^2 + y$

« 7 »

- 16 If $X = \frac{1}{\sqrt{3} - \sqrt{2}}$ and y is the multiplicative inverse of X

, find y , then prove that : $(X+y)^2 = 12$

« $\sqrt{3} - \sqrt{2}$ »

- 17 If $X = \sqrt{13} + \sqrt{6}$, $Xy = 1$, find the value of : $X^2 - 49y^2$

« $4\sqrt{78}$ »

- 18 If $X = \frac{4}{\sqrt{7} - \sqrt{3}}$ and $y^{-1} = \frac{1}{\sqrt{7} - \sqrt{3}}$

(Remember that $y^{-1} = \frac{1}{y}$)

, prove that X and y are conjugate numbers, then find the value of : $X^2 y^2$

« 16 »

19 If $x = \sqrt{7} + \sqrt{5}$ and $y = \frac{2}{x}$, find the value of $\frac{x+y}{xy}$ in its simplest form.

20 If $x = \frac{\sqrt{6} + \sqrt{5}}{\sqrt{6} - \sqrt{5}}$, prove that $x + \frac{1}{x} = 22$

21 Complete the following :

1) $(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3}) = \dots$

2) If $x = 3 + \sqrt{2}$, then its conjugate is and the product of multiplying x by its conjugate is

3) The conjugate number of the number $\frac{2}{\sqrt{5} - \sqrt{3}}$ is

4) The conjugate number of the number $1 + \frac{7}{\sqrt{7}}$ in the simplest form is

5) The multiplicative inverse for $(\sqrt{3} + \sqrt{2})$ in its simplest form is

6) If $x = 2 + \sqrt{5}$ and y is the conjugate number of x , then $(x - y)^2 = \dots$

7) If $\frac{x}{5 - \sqrt{5}} = 5 + \sqrt{5}$, then the value of x in its simplest form is

8) If $\frac{1}{x} = \sqrt{5} - 2$, then the value of x in its simplest form is

9) If $x = \sqrt{3} + 2$, $y = \sqrt{3} - 2$, then $(xy, x + y) = \dots$

10) $(\sqrt{2} + \sqrt{3})^{-9}(\sqrt{2} - \sqrt{3})^{-9} = \dots$

22 In each of the following, if a and b are two integers, find the value of each of them :

1) $\frac{11}{2\sqrt{5} + 3} = a\sqrt{5} + b$ « 2, -3 »

2) $\frac{3}{2\sqrt{2} - \sqrt{5}} = a\sqrt{2} + b\sqrt{5}$ « 2, 1 »

3) $\frac{7}{\sqrt{8} + 1} = a + b\sqrt{2}$ « -1, 2 »

23 Simplify each of the following :

1 $\frac{4}{\sqrt{5}+\sqrt{3}} + \frac{4}{\sqrt{5}-\sqrt{3}}$

« $4\sqrt{5}$ »

2 $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$

« $-4\sqrt{30}$ »

3 $\sqrt{75} - \sqrt[3]{125} + \frac{10}{\sqrt{3}-1}$

« $10\sqrt{3}$ »

Geometric Application

24 ΔABC is a right-angled triangle at B ,

$AB = (\sqrt{28} + 2)$ cm. and

$BC = (\sqrt{7} - 1)$ cm.

Find the area of ΔABC

« 6 cm^2 »



For excellent pupils

25 If $x = \sqrt{4+\sqrt{7}}$, $y = \sqrt{4-\sqrt{7}}$
 , find in the simplest form : $(x+y)^2$

« 14 »

26 If $x = \sqrt{5} + 1$ and $y = \sqrt{5} - 1$, find the value of : $xy^{-1} + yx^{-1}$

« 3 »

27 If $x = \sqrt{7} + \sqrt{6}$ and $y = \sqrt{7} - \sqrt{6}$, find the value of : $\frac{x^8 y^9 y}{(x+y)^5}$

« zero »

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Problem Solving

- 1 Put each of the following in the form $a\sqrt[3]{b}$ where a and b are two integers, b is the least possible positive value :

1 $\sqrt[3]{16}$

2 $\sqrt[3]{-54}$

3 $2\sqrt[3]{250}$

4 $\frac{2}{3}\sqrt[3]{-135}$

5 $3\sqrt[3]{\frac{1}{3}}$

6 $-10\sqrt[3]{\frac{2}{5}}$

- 2 Find the result of each of the following in its simplest form :

1 $\sqrt[3]{2} \times \sqrt[3]{32}$

2 $\frac{\sqrt[3]{72}}{\sqrt[3]{9}}$

3 $\frac{4\sqrt[3]{-54}}{2\sqrt[3]{-2}}$

4 $\frac{1}{2}\sqrt[3]{10} \times 6\sqrt[3]{100}$

5 $3\sqrt[3]{\frac{2}{5}} \times 3\sqrt[3]{\frac{4}{25}}$

6 $3\sqrt[3]{\frac{3}{4}} \div 3\sqrt[3]{\frac{2}{9}}$

- 3 Find the result of each of the following in its simplest form :

1 $\sqrt[3]{16} - \sqrt[3]{2}$

« $\sqrt[3]{2}$ »

2 $\sqrt[3]{125} - \sqrt[3]{24}$

« $5 - 2\sqrt[3]{3}$ »

3 $\sqrt[3]{81} + \sqrt[3]{-24}$

« $\sqrt[3]{3}$ »

4 $\sqrt[3]{54} + \sqrt[3]{16} - \sqrt[3]{250}$

« zero »

5 $2\sqrt[3]{54} - 5\sqrt[3]{2} + \sqrt[3]{16}$

« $\sqrt[3]{2}$ »

6 $\sqrt[3]{16} - \frac{1}{3}\sqrt[3]{54} + \sqrt[3]{-2}$

« zero »

7 $\sqrt[3]{16} + \sqrt[3]{10} \times \sqrt[3]{25}$

« $\sqrt[3]{10}$ »

8 $\sqrt[3]{24} - 6\sqrt[3]{13\frac{8}{9}}$

« $\sqrt[3]{3}$ »

4 Prove that :

1) $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = \text{zero}$

2) $\sqrt[3]{54} \times \sqrt[3]{16} \div (\sqrt[3]{4} \times 6) = 1$

5 Simplify each of the following to the simplest form :

1) $\sqrt[3]{81} + \sqrt[3]{-24} - 3\sqrt[3]{\frac{1}{9}}$ « zero »

2) $\sqrt[3]{54} + 8\sqrt[3]{-\frac{1}{4}} + 5\sqrt[3]{16}$ « $9\sqrt[3]{2}$ »

3) $\sqrt[3]{108} - 2\sqrt[3]{4} - \sqrt[3]{\frac{1}{2}}$ « $\frac{1}{2}\sqrt[3]{4}$ »

4) $\sqrt[3]{3} - \sqrt[3]{4} \times \sqrt[3]{6} + 3\sqrt[3]{\frac{1}{9}}$ « zero »

6 Simplify each of the following to its simplest form :

1) $\frac{7}{3}\sqrt[3]{18} + \sqrt[3]{54} - 7\sqrt[3]{2} + \sqrt[3]{16}$ « $5\sqrt[3]{2}$ »

2) $\sqrt[3]{27} + \frac{1}{3}\sqrt[3]{27} - 9\sqrt[3]{\frac{1}{3}} - 1$ « zero »

3) $\sqrt[3]{-16} + \frac{14}{\sqrt[3]{7}} - \sqrt[3]{28} + \sqrt[3]{54}$ « $\sqrt[3]{2}$ »

4) $\sqrt[3]{18} + \sqrt[3]{54} - \frac{\sqrt[3]{216}}{\sqrt[3]{12}} - \sqrt[3]{16}$ « $\sqrt[3]{2}$ »

5) $5\sqrt[3]{2} - \frac{1}{2}\sqrt[3]{200} + (\sqrt[3]{5} \times \sqrt[3]{25})$ « 5 »

7 Simplify the following to its simplest form :

$2\sqrt[3]{16} (3\sqrt[3]{4} + 5\sqrt[3]{32} - 2\sqrt[3]{\frac{1}{2}})$ « 96 »

8 Choose the correct answer from those given :

1) $\sqrt[3]{54} + \sqrt[3]{-2} = \dots\dots\dots$

(a) $\sqrt[3]{52}$

(b) $\sqrt[3]{2}$

(c) $2\sqrt[3]{2}$

(d) $4\sqrt[3]{2}$

2) $\sqrt[3]{-64} + \sqrt[3]{16} = \dots\dots\dots$

(a) zero

(b) 8

(c) -8

(d) ± 8

3) $\frac{\sqrt[3]{16}}{\sqrt[3]{2}} = \dots\dots\dots$

(a) 8

(b) -2

(c) 2

(d) $2\sqrt[3]{2}$

$$\sqrt[3]{2} + \sqrt[3]{2} = \dots$$

$$(a) \sqrt[3]{2}$$

$$(b) \sqrt[3]{4}$$

$$(c) \sqrt[3]{8}$$

$$(d) \sqrt[3]{16}$$

$$\sqrt[3]{\frac{2}{9}} = \dots$$

$$(a) \frac{\sqrt[3]{6}}{3}$$

$$(b) \sqrt[3]{\frac{1}{6}}$$

$$(c) \sqrt[3]{6}$$

$$(d) \sqrt[3]{2}$$

9 Complete the following :

$$\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{-12} = \dots$$

$$3 \sqrt[3]{54} - \sqrt[3]{-16} = \sqrt[3]{\dots}$$

$$5 \text{ If } x = 2, y = \sqrt[3]{-16}, \text{ then } \left(\frac{x}{y}\right)^3 = \dots$$

$$\sqrt[3]{3} \times \sqrt[3]{9} = \sqrt{\dots}$$

$$4 \frac{1}{2} \sqrt[3]{56} - \sqrt[3]{\frac{7}{27}} = \dots$$

$$6 \frac{\sqrt[3]{250} \sqrt[3]{16}}{\sqrt[3]{54}} = \dots$$

10 If $a = \sqrt[3]{5} + 1$, $b = \sqrt[3]{5} - 1$, find the value of each of the following :

$$1 (a - b)^5$$

$$2 (a + b)^3$$

« 32 , 40 »

11 If $x = 3 + \sqrt[3]{6}$, $y = 3 - \sqrt[3]{6}$, find the value of : $\left(\frac{x-y}{x+y}\right)^3$

12 Find the result of the following in its simplest form :

$$\sqrt[3]{32} + 4\sqrt[3]{\frac{1}{2}} - (2\sqrt[3]{-2})^2 + (\sqrt[3]{2})^{\text{zero}} - \left(\frac{2}{\sqrt[3]{2}}\right)^2$$

« -1 »

Life Application

13 The opposite figure represents a number of cubic boxes , the volume of each one is 24 dm^3 .

Find the area of the ground for putting the boxes.



« $20\sqrt[3]{9} \text{ dm}^2$ »

For excellent pupils

14 If $x = \sqrt[3]{2} + 1$, $y = \sqrt[3]{2} - 1$, prove that : $x^2 + y^2 = 2\sqrt[3]{4} + 2$

15 Make the denominator of $\frac{2}{\sqrt[3]{2}}$ a rational number.



● Remember

● Understand

● Apply

● Problem Solving

The cube

1 Complete the following :

- 1 If the edge length of a cube is 5 cm. , then its volume = cm^3
- 2 The edge length of a cube is 4 cm. , then its total area = cm^2
- 3 The lateral area of a cube whose edge length is l cm. is cm^2
- 4 The cube whose volume is $l^3 \text{ cm}^3$, its total area = cm^2
- 5 The cube whose edge length is $2l$ cm. , then its volume = cm^3

2 A cube whose lateral area is 36 cm^2 . Find :

- 1 Its total area.
- 2 Its volume. « 54 cm^2 , 27 cm^3 »

3 The perimeter of one face of a cube is 12 cm. Find :



- 1 Its volume.
- 2 Its lateral area. « 27 cm^3 , 36 cm^2 »

4 The sum of lengths of all edges of a cube is 60 cm. Find :

- 1 Its volume.
- 2 Its total area. « 125 cm^3 , 150 cm^2 »

5 Choose the correct answer from those given :

- 1 The volume of a cube is 1 cm^3 , then the sum of its edge lengths = cm.
(a) 1 (b) 6 (c) 8 (d) 12

- 2  The volume of a cube is 64 cm^3 , then its lateral area = cm^2
 (a) 4 (b) 8 (c) 64 (d) 96
- 3 A cube of volume 27 cm^3 , then its total area = cm^2
 (a) 9 (b) 27 (c) 36 (d) 54
- 4 If the total area of a cube is 96 cm^2 , then the area of one face = cm^2
 (a) 16 (b) 64 (c) 24 (d) 48
- 5 A cube of total area 150 cm^2 , then its lateral area = cm^2
 (a) 25 (b) 100 (c) 125 (d) 150
- 6 If the area of the six faces of a cube = 54 cm^2 , then its volume = cm^3
 (a) 54 (b) 44 (c) 72 (d) 27
- 7 If the volume of a cube = 64 cm^3 , then the length of a diagonal of one face = cm.
 (a) 16 (b) $4\sqrt{2}$ (c) 32 (d) 64
- 8 The volume of a cube is 5 cm^3 . If the edge length became twice the first, then its volume = cm^3
 (a) 10 (b) 20 (c) 30 (d) 40
- 9  The edge length of a cube whose volume is $2\sqrt{2} \text{ cm}^3$ is cm.
 (a) $\sqrt{2}$ (b) 2 (c) 8 (d) 1.5

The cuboid

- 6 The dimensions of the base of a cuboid are 9 cm. and 10 cm. and its height is 5 cm. Find :

1 Its volume. 2 Its lateral area. 3 Its total area.

« 450 cm^3 , 190 cm^2 , 370 cm^2 »

- 7 The dimensions of a cuboid are $\sqrt{2} \text{ cm}$, $\sqrt{3} \text{ cm}$. and $\sqrt{6} \text{ cm}$. Find its volume. « 6 cm^3 »

- 8 The dimensions of the base of a cuboid are $\sqrt{3} \text{ cm}$. and $(\sqrt{3}-1) \text{ cm}$. and its height equals $(3+\sqrt{3}) \text{ cm}$. Calculate its volume. « 6 cm^3 »

- 9 The lateral area of a cuboid is 480 cm^2 and its base is in the shape of a square whose side length is 10 cm. Calculate its height. « 12 cm. »

- 10  Find the total area of a cuboid whose volume is 720 cm^3 and its height is 5 cm. with a squared-shape base. « 528 cm^2 »

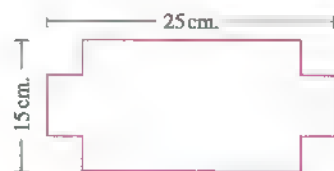
11 Which is more in size :

A cube whose total area is 294 cm^2 or a cuboid with dimensions $7\sqrt{2} \text{ cm}$, $5\sqrt{2} \text{ cm}$ and 5 cm ?

12 In the opposite figure :

A rectangular piece of cardboard has a length of 25 cm , and a width of 15 cm . A square whose side length = 4 cm , was cut from each of its four corners, then the projected parts were folded to form a basin in the shape of a cuboid.

Find the volume and the total area of that cuboid.



« 476 cm^3 , 311 cm^2 »

The circle

Consider $\pi = \frac{22}{7}$ if there are not any other values given.

13 A circle is of radius length 10.5 cm . Find each of its circumference and its area.

« 66 cm , 346.5 cm^2 »

14 The area of a circle is 154 cm^2 . Find its circumference and its diameter length.

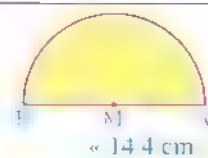
« 44 cm , 14 cm »

15 A circle whose area is $64\pi \text{ cm}^2$. Find the length of its radius, then find its circumference approximating it to the nearest integer. ($\pi = 3.14$)

« 8 cm , 50 cm »

16 In the opposite figure :

AB is a diameter of the semicircle. If the area of this region is 12.32 cm^2 , find the perimeter of the figure.

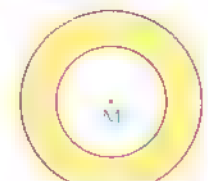


« 14.4 cm »

17 In the opposite figure :

These are two concentric circles at M and their radii lengths are 3 cm and 5 cm .

Find the area of the shaded part in terms of π



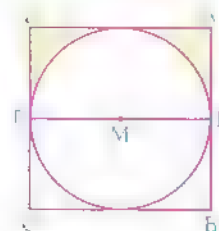
« $16\pi \text{ cm}^2$ »

18 In the opposite figure :

The circle M is inside the square ABCD

If the area of the shaded part = $10\frac{5}{7} \text{ cm}^2$,

find the perimeter of this part.



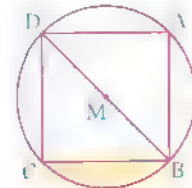
« $35\frac{5}{7} \text{ cm}$ »

19 In the opposite figure :

The square ABCD is inside the circle M

If the radius length of the circle M is 7 cm. ,

find the area of the shaded part and its perimeter.



$$\ll 14 \text{ cm}^2, (11 + 7\sqrt{2}) \text{ cm} \gg$$

The right circular cylinder

Consider $\pi = \frac{22}{7}$ if there are not any other values given.

20 A right circular cylinder , the radius length of its base is 14 cm. and its height is 20 cm.

Find the volume and the total area of the cylinder.

$$\ll 12320 \text{ cm}^3, 2992 \text{ cm}^2 \gg$$

21 Find the lateral area for a right circular cylinder of volume 924 cm^3 and of a height 6 cm.

$$\ll 264 \text{ cm}^2 \gg$$

22 Find the total area of a right circular cylinder of volume 7536 cm^3 and its height is 24 cm.

$$(\pi = 3.14)$$

$$\ll 2135.2 \text{ cm}^2 \gg$$

23 Which is more in volume :

A right circular cylinder with base radius length 7 cm. and its height = 10 cm.
or a cube whose edge length is equal to 11 cm. ?

24 Complete the following :

- 1 A right circular cylinder whose base radius length is r cm. and its height = h cm.
 , then its lateral area = cm^2 and its volume = cm^3
- 2 A right circular cylinder with volume $40 \pi \text{ cm}^3$ and its height = 10 cm. , then its base radius length =
- 3 A right circular cylinder with volume $500 \pi \text{ cm}^3$ and its base radius length = 5 cm. ,
 then its height =
- 4 A right circular cylinder with volume $\pi r^3 \text{ cm}^3$, then its height =
- 5 If the lateral area of a right circular cylinder is $2 \pi r^2 \text{ cm}^2$, then its height =

25 The circumference of the base of a right circular cylinder is 44 cm. and its height = 25 cm.

Find its volume.

$$\ll 3850 \text{ cm}^3 \gg$$

- 26 The lateral area of a right circular cylinder is 52 cm^2 and the length of the diameter of its base is 8 cm. Find its volume. « 104 cm^3 . »

- 27 A right circular cylinder of volume $36 \pi \text{ cm}^3$ and height 4 cm, the radius length of its base equals the edge length of a cube.

Find : The total area of the cube.

« 54 cm^2 . »

- 28 Find the height of a right circular cylinder whose height is equal to its base radius length and its volume is $72 \pi \text{ cm}^3$. « $2\sqrt[3]{9} \text{ cm}$. »

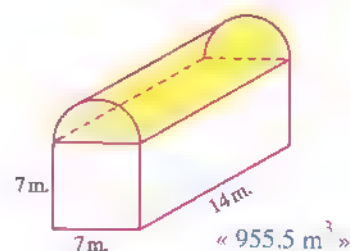
- 29 In the opposite figure :

A cuboid-shaped water tank with dimensions 7 m.

, 7 m. and 14 m. , and the upper part of it is in the

form of half of a right circular cylinder.

Calculate the volume of the tank in m^3 .



- 30 A piece of paper has a shape of a rectangle ABCD in which $AB = 10 \text{ cm}$. and $BC = 44 \text{ cm}$. It was folded to form a right circular cylinder such that \overline{AB} is coincident to \overline{DC}

Find the volume of the resulted cylinder.

« 1540 cm^3 . »

The sphere

Consider $\pi = \frac{22}{7}$ if there are not any other values given.

- 31 Find the volume and the surface area of a sphere if the length of its diameter is 4.2 cm.

« $38\,808 \text{ cm}^3$, 55.44 cm^2 . »

- 32 The volume of a sphere is 4188 cm^3 . Find its radius length. ($\pi = 3.141$) « 10 cm . »

- 33 The volume of a sphere is $562.5 \pi \text{ cm}^3$.

Find its surface area in terms of π

« $225 \pi \text{ cm}^2$. »

- 34 Choose the correct answer from those given :

- 1 The volume of the sphere =

(a) $4 \pi r^2$

(b) $\frac{4}{3} \pi r^3$

(c) $\frac{3}{4} \pi r^3$

(d) $\frac{4}{3} \pi r^2$

- 2 The sphere whose radius length is $\sqrt[3]{3} \text{ cm}$. , its volume = cm^3 .

(a) 4π

(b) $4\sqrt{3} \pi$

(c) $\frac{4}{3} \pi$

(d) $\frac{9}{4} \pi$

- 3 The volume of the sphere whose diameter length is 6 cm. equals cm^3 .





(a) 288

(b) 12π


(c) 36π

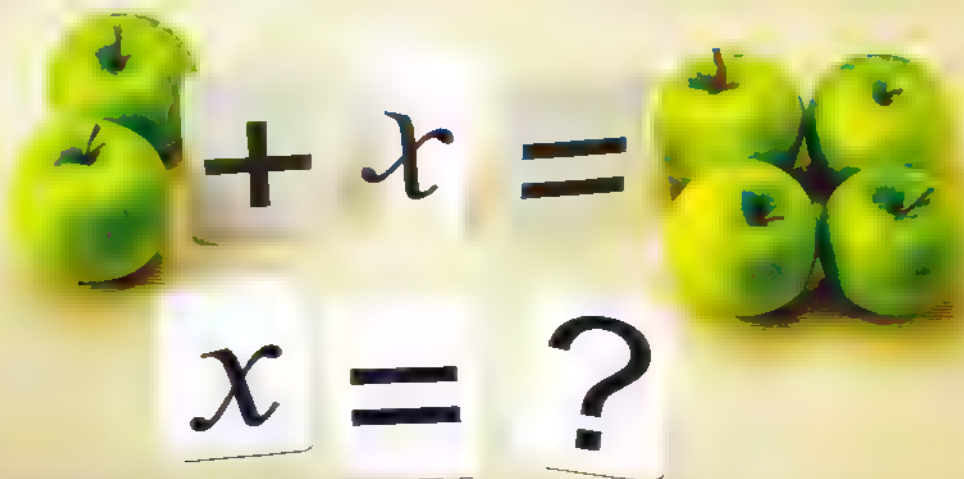
(d) 288π

- If the volume of a sphere = $\frac{9}{16} \pi \text{ cm}^3$, then its radius length = cm.
 (a) 3 (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$
- If the surface area of a sphere is $9 \pi \text{ cm}^2$, then its diameter length = cm.
 (a) 9 (b) 3 (c) 1.5 (d) 6
- If three quarters of the volume of a sphere equals $8 \pi \text{ cm}^3$, then the length of its radius equals cm.
 (a) 64 (b) 8 (c) 4 (d) 2
- If the radius length of a sphere is $r \text{ cm}$, then which of the following represents the ratio between the area of the sphere and its volume?
 (a) $\frac{4}{r}$ (b) $\frac{3}{r}$ (c) $\frac{r}{4}$ (d) $\frac{r}{\pi}$

- 35 Find the radius length of a sphere if its volume equals the volume of a right circular cylinder whose height is 18 cm, and its base radius length is 4 cm. « 6 cm »
- 36 Find the volume of a sphere if its radius length equals the radius length of a right circular cylinder with volume 7536 cm^3 and height 24 cm. ($\pi = 3.14$) « $4186 \frac{2}{3} \text{ cm}^3$ »
- 37  A lead cuboid is of dimensions 77 cm, 24 cm, and 21 cm. It was melted to make a sphere. Find the radius length of that sphere. « 21 cm. »
- 38  A metallic sphere, with diameter length 6 cm has got melt and changed into a right circular cylinder with base radius length 3 cm. Find its height. « 4 cm »
- 39  A sphere with volume $36 \pi \text{ cm}^3$ is placed inside a cube. If the sphere touches the six faces of the cube, find :
 1 The radius length of the sphere.
 2 The volume of the cube. « 3 cm, 216 cm^3 »
- 40 A metallic sphere is of radius length 16.8 cm. It is melted and it is converted to 8 small spheres which are equal in volume. Find the radius length of each small sphere. « 8.4 cm »
- 41  A right circular cylinder has a height of 20 cm. Find its base radius length if its volume equals $\frac{4}{9}$ of the volume of a sphere with a diameter length of 30 cm. « 10 cm »

 For excellent pupils

- 42 A cuboid has a square-shaped base whose height = 3 cm. If the sum of lengths of its edges is 52 cm, find its volume. « 75 cm^3 »
- 43  A hollow metal sphere is with internal radius length 2.1 cm, and external radius length 3.5 cm. Find its mass approximated to the nearest gram taking into consideration that the mass of a cubic centimetre of such a metal is 20 gm. « 2817 gm. »



Remember

Understand

Apply

Problem Solving

- 1** Find the solution set for each of the following equations in \mathbb{R} , then represent the solution on the number line :

1 $x + 5 = 0$

2 $5x + 6 = 1$

3 $2x + 4 = 3$

4 $2x - 3 = 4$

5 $4x - 1 = |-2|$

6 $\sqrt{5}x - 1 = 4$

7 $x - 1 = \sqrt{3}$

8 $2 - \sqrt{6}x = |-8|$

9 $x + 2\sqrt{3} = 3$

- 2** Choose the correct answer from those given :

- 1 The figure represents the solution set of the inequality in \mathbb{R}

(a) $x > -3$

(b) $x \geq -3$

(c) $x < -3$

(d) $x \leq -3$

- 2 The figure represents the solution set of the inequality in \mathbb{R}

(a) $-6 < x < 6$

(b) $-6 \leq x < 6$

(c) $-6 < x \leq 6$

(d) $-6 \leq x \leq 6$

- 3 If $x \in]3, \infty[$, then

(a) $x < 3$

(b) $x \leq 3$

(c) $x > 3$

(d) $x \geq 3$

- 4 The S.S. of the inequality : $x > 7$ in \mathbb{R} is

(a) $] -7, \infty[$

(b) $[7, \infty[$

(c) $] -\infty, 7[$

(d) $]7, \infty[$

5 The S.S. of the inequality : $-1 < x \leq 5$ in \mathbb{R} is

- (a) $]-1, 5]$ (b) $[-1, 5]$ (c) $\{-1, 5\}$ (d) $[-1, 5[$

6 The S.S. of the inequality : $-x > 3$ in \mathbb{R} is

- (a) $\{-3\}$ (b) $]3, \infty[$ (c) $]-\infty, 3[$ (d) $]-\infty, -3[$

3 Find the solution set for each of the following inequalities in \mathbb{R} in the form of an interval, then represent the solution on the number line :

1) $2x > 6$

2) $-7x \geq -14$

3) $x + 3 \leq 5$

4) $5 - x > 3$

5) $2x + 5 \geq 3$

6) $1 - 5x < 6$

7) $\frac{1}{2}x + 1 \leq 2$

8) $3 - 2x \leq 7$

4 Find the solution set for each of the following inequalities in \mathbb{R} in the form of an interval, then represent the solution on the number line :

1) $3 < x + 2 \leq 6$

2) $-5 < x + 3 < 9$

3) $-3 \leq -x < 3$

4) $1 < 5 - x \leq 3$

5) $\sqrt[3]{-8} \leq x + 1 \leq \sqrt{9}$

6) $5 < 3 - x \leq 3^2$

7) $-8 \leq 3x + 1 \leq 4$

8) $|-3| < 2x - 1 < 5$

9) $-3 < \frac{1}{2}x - 2 \leq \text{zero}$

10) $0 \leq \frac{-2x+6}{3} < 4$

5 Find the solution set for each of the following inequalities in \mathbb{R} in the form of an interval, then represent the solution on the number line :

1) $3x < 2x + 4$

2) $7x - 9 \geq 4x$

3) $5x - 3 < 2x + 9$

4) $7x - 12 \geq 5x - 8$

5) $x - 1 \leq 3 - x$

6) $1 - x \geq -2x - 3$

6 Find the solution set for each of the following inequalities in \mathbb{R} in the form of an interval, then represent the solution on the number line :

1) $x + 3 \geq 2x \geq x - 2$

2) $-x < x < 4 - x$

3) $4x \leq 5x + 2 < 4x + 3$

4) $x - 1 < 3x - 1 \leq x + 1$

5) $2 + 2x \leq 3x + 3 < 5 + 2x$

6) $\frac{3x-4}{6} < x + 1 < \frac{x+3}{2}$

7 Complete the following :

1) If $x - 3 \geq 0$, then x

2) If $5x < 15$, then x

3) If $1 - x > 4$, then x

4) If $-2x \leq 3$, then x

- 5 If $\sqrt{2}x \leq 4$, then x
- 6 The S.S. of the inequality : $4 < 2x < 8$ in \mathbb{R} is
- 7 The S.S. of the inequality : $-5 \leq -x < 2$ in \mathbb{R} is
- 8 The S.S. of the inequality : $2 - x < 0$ in \mathbb{R} is
- 9 If $-3 < x < 3$ where $x \in \mathbb{R}$, then $2x \in]-6, \dots [$

8 Choose the correct answer from those given :

- 1 The S.S. of the inequality : $x + 3 < 3$ in \mathbb{R} is
 (a) $] -\infty, 0[$ (b) $] -\infty, 0]$ (c) $[0, \infty[$ (d) $[0, \infty[$
- 2 The S.S. of the inequality : $1 > x - 5 > -1$ in \mathbb{R} is
 (a) $[4, 6]$ (b) $]4, 6[$ (c) $]4, 6]$ (d) $[4, 6[$
- 3 If $x > 5$, then $-x$
 (a) < -9 (b) ≥ -5 (c) < -5 (d) > -5
- 4 If $-2 < x < 2$, then $2x + 3$ belongs to
 (a) $[-1, 7]$ (b) $] -1, 5[$ (c) $] -1, 7[$ (d) $] -4, 6[$
- 5 The number 5 belongs to the S.S. of the inequality
 (a) $x > 5$ (b) $x < 5$ (c) $-x \geq -5$ (d) $-x \geq 5$

Life Application

- 9 A lift for carrying goods can carry 2200 kg. as a maximum weight. If we have 60 boxes of cans and the weight of one box is 45 kg., what is the maximum number of boxes can the lift carry in one time without carrying any person ?
 « 48 boxes »



For excellent pupils

- 10 Prove that $\sqrt{3}$ belongs to the S.S. of the inequality : $0 < 4 - 2x < 6$ in \mathbb{R}
- 11 If $[4, 7]$ is the S.S. of the inequality : $a \leq x - 3 \leq b$, find the value of each of a and b « 1, 4 »
- 12 If $[m, m + n]$ is the S.S. of the inequality : $\frac{1}{5} \leq \frac{2x+1}{5} \leq 1$, find the value of n « 2 »
- 13 If $5 \leq \frac{2x}{3} + 1 \leq 7$, find the smallest value of the expression : $x - 2$ « 4 »
- 14 Find in \mathbb{R} the S.S. of the inequality : $\frac{x}{\sqrt{3}-\sqrt{5}} \geq \sqrt{3} + \sqrt{5}$

Summary of the second part of Unit I

"From lesson 6 to lesson 10"



★ If a and b are two non-negative real numbers, then :

$$\bullet \sqrt{a} \times \sqrt{b} = \sqrt{a b}$$

$$\bullet \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \text{ (where } b \neq 0 \text{)}$$

★ If a and b are two real numbers, then :

$$\bullet \sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{a b}$$

$$\bullet \frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\frac{a}{b}} \text{ (where } b \neq 0 \text{)}$$


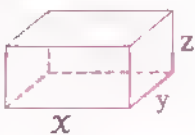


★ If a and b are two positive rational numbers, then each of the two numbers $(\sqrt{a} + \sqrt{b})$ and $(\sqrt{a} - \sqrt{b})$ is conjugate to the other one and :

$$\bullet \text{ Their sum } = 2\sqrt{a}$$

$$\bullet \text{ Their product } = a - b$$

★ If we have a real number whose denominator is written in the form $(\sqrt{a} + \sqrt{b})$ or $(\sqrt{a} - \sqrt{b})$, we put it in the simplest form by multiplying both the numerator and the denominator by the conjugate of the denominator.

★ The following table summarizes the rules of areas and volumes of some solids :

The solid		The lateral area	The total area	The volume
The cube		$4 \ell^2$	$6 \ell^2$	ℓ^3
The cuboid		$2 (X + y) \times z$	$2 (X y + y z + z x)$	$x y z$
The cylinder		$2 \pi r h$	$2 \pi r h + 2 \pi r^2$ $= 2 \pi r (h + r)$	$\pi r^2 h$
The sphere		-	$4 \pi r^2$	$\frac{4}{3} \pi r^3$

Remember that : The circumference of the circle $= 2 \pi r$, the area of the circle $= \pi r^2$

★ Solving the equation or the inequality is finding the values of the unknown which satisfy this equation or inequality.

★ The solution set of the inequality of the first degree in one variable in \mathbb{R} is written in the form of an interval.

Exams on the second part of unit one from lesson (6) to lesson (10)



Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1 The volume of the sphere of diameter length 3 cm, equals cm^3

- (a) 4.5π (b) 36π (c) 288π (d) 4.5

2 If $X > 3$, then $-X$

- (a) < 3 (b) > -3 (c) < -3 (d) $< -\frac{1}{3}$

3 $\sqrt{20} - \sqrt{5} = \dots\dots\dots$

- (a) $\sqrt{15}$ (b) $\sqrt{5}$ (c) $\sqrt{10}$ (d) 15

4 A cube, its volume is 125 cm^3 , then its total area equals cm^2

- (a) 30 (b) 25 (c) 100 (d) 150

5 If $X = \sqrt{7} + \sqrt{3}$ and $y = \sqrt{7} - \sqrt{3}$, then $XY = \dots\dots\dots$

- (a) 4 (b) 10 (c) 40 (d) 58

6 $\frac{\sqrt[3]{24}}{\sqrt[3]{3}} = \dots\dots\dots$

- (a) 8 (b) 3 (c) 2 (d) $\sqrt[3]{2}$

2 Complete the following :

1 The multiplicative inverse of the number $(\sqrt{3} - \sqrt{2})$ in the simplest form is

2 $\sqrt{2} \times \sqrt{12} = 2 \times \dots\dots\dots$

3 $\sqrt[3]{54} - \sqrt[3]{2} = \dots\dots\dots$ (in the simplest form)

4 A right circular cylinder, its volume is $500 \pi \text{ cm}^3$ and the diameter length of its base is 10 cm., then its height is

5 If $1 - X > 5$, then X

3 [a] Find in the simplest form the value of the expression :

$$\sqrt{18} + \sqrt[3]{54} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{16}$$

[b] A sphere, its volume is $36 \pi \text{ cm}^3$. Calculate its area.

- 4 [a] Find in \mathbb{R} the solution set of the inequality :

$-3 < 2x + 1 < 7$, then represent it on the number line.

- [b] A right circular cylinder , its height equals the radius length of its base and its volume is $27\pi \text{ cm}^3$. Find the radius length of its base.

- 5 [a] Simplify to the simplest form : $\sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}}$

[b] If $x = \sqrt{5} - \sqrt{2}$ and $y = \frac{3}{\sqrt{5} - \sqrt{2}}$

Prove that : x and y are conjugate , then find : $x^2 + 2xy + y^2$



Answer the following questions :

- 1 Choose the correct answer from those given :

1 $\sqrt{3\frac{3}{8}} = \frac{3}{2}\sqrt{\dots\dots\dots}$

(a) $\frac{3}{8}$

(b) $\frac{3}{2}$

(c) $\frac{27}{8}$

(d) $\frac{729}{64}$

- 2 The number $(1 - \sqrt{3})(1 + \sqrt{3})$ is number.

(a) a natural

(b) a rational

(c) an irrational

(d) a prime

3 $\sqrt{3} + \sqrt{3} = \dots\dots\dots$

(a) 3

(b) $\sqrt{6}$

(c) $2\sqrt{6}$

(d) $2\sqrt{3}$

- 4 A sphere , its volume is $\frac{4}{3}\pi \text{ cm}^3$, then its diameter length is cm.

(a) 0

(b) 1

(c) 2

(d) $\frac{4}{3}$

- 5 A cube , its volume is $2\sqrt{2} \text{ cm}^3$, then its edge length equals cm.

(a) $\sqrt{2}$

(b) 2

(c) 8

(d) 4

6 $\sqrt[3]{2} \times \sqrt[3]{2} = \dots\dots\dots$

(a) 2

(b) 4

(c) $\sqrt[3]{4}$

(d) $\sqrt{2}$

- 2 Complete the following :

1 If $x = \frac{1}{\sqrt{8} - \sqrt{5}}$ and $xy = 1$, then $y = \dots\dots\dots$

2] The solution set of the inequality : $4 > -2x$ in \mathbb{R} is

3] $\frac{\sqrt[3]{32}}{\sqrt[3]{4}} = \dots\dots\dots$

4] $(\sqrt{8} + \sqrt{2})^2 = \dots\dots\dots$

5] A right circular cylinder , its volume is $90\pi \text{ cm}^3$, and its height is 10 cm. , then the radius length of its base equals cm.

3 [a] Prove that : $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = 0$

[b] If $a = \frac{4}{\sqrt{7}-\sqrt{3}}$ and $b = \sqrt{7}-\sqrt{3}$
 , find in the simplest form : $\frac{a-b}{ab}$

4 [a] Simplify :

$$2\sqrt{5} + 9\sqrt{\frac{1}{3}} - \sqrt{27} - 5\sqrt{\frac{1}{5}}$$

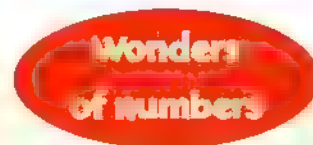
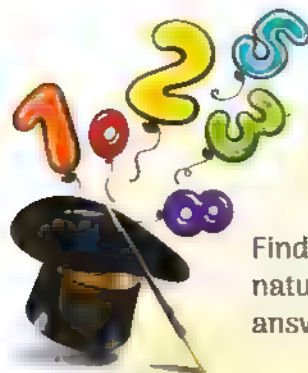
[b] A right circular cylinder with volume $36\pi \text{ cm}^3$ and its height is 4 cm. , and the radius length of its base equals the edge length of a cube. Find the total area of the cube.

5 [a] A right circular cylinder , its volume is 231 cm^3 , and its height is 6 cm.

Calculate its lateral area $(\pi = \frac{22}{7})$

[b] Find in \mathbb{R} the solution set of the inequality :

$$5x - 3 < 2x + 9 \text{ and represent it on the number line.}$$



Find the product of the number 99 by the natural numbers from 1 to 10 write your answers each time. Where do you stand ?

A Research Project

On Unit One



Project aims:

- Performing arithmetic operations on real numbers.
- Finding volumes as applications on operations on real numbers.
- Associating mathematics with science.

Do a research project on the following topic :

The solar system consists of the sun and the planets that revolve around it. The planets are Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune.

Discuss the following points using available resources :

- 1 State the names of the solar system planets.
- 2 Find the radius length of each planet in the solar system and calculate its volume.
- 3 Arrange the solar system planets in a descending order according to their volumes.
- 4 Find out the weight of a body on the surface of the Earth in the simplest form if its weight on the surface of the moon is $30\sqrt{18}$ kg.



UNIT

2

Relation between Two Variables

Exercises of the unit :

- 11. Relation between two variables.
- 12. Slope of straight line.
- 13. Real life applications on the slope.
 - ★ Summary of unit two.
 - ★ Unit exams.
- 🔍 A research project on unit two



Scan the
QR code
to solve an
interactive
test on each
lesson



Remember Understand Apply Problem Solving

1 Complete the following ordered pairs which satisfy the relation : $y = 3x - 1$

(5 ,), (2 ,), (0 ,), (-3 ,)

2 Show which of the following ordered pairs satisfies the relation : $y - 4x = 7$

1 (1 , 2)

2 (3 , -5)

3 (-1 , 3)

3 Find four ordered pairs satisfying each of the following relations :

1 $2x - y = 5$

2 $y = \frac{1}{2}x + 5$

3 $y = 2$

4 $2x = 5$

4 Using the linear relations , complete the following tables :

1 $4x - y = -1$

x	0	1	2	3
y

2 $y = 5x + 15$

x	-4	-3	2
y

3 $a - b = 4$

a	1
b	0	-1

4 $a - 3b = 5$

a	2	1
b	0

5 If $y - 2x = 1$, find :

1) y at $x = 3$

3) x at $y = 1$

2) y at $x = -5$

4) x at $y = -1$

6 If $(3, 6)$ satisfies the relation : $y = kx$, find the value of k

« 2 »

7 If $(3, 1)$ satisfies the relation : $y - 3x = a$, find the value of a

« -8 »

8 Find the value of b , where $(-3, 2)$ satisfies the relation : $3x + by = 1$

« 5 »

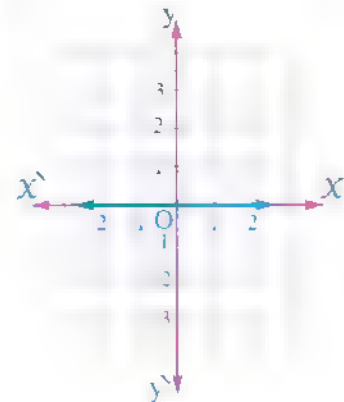
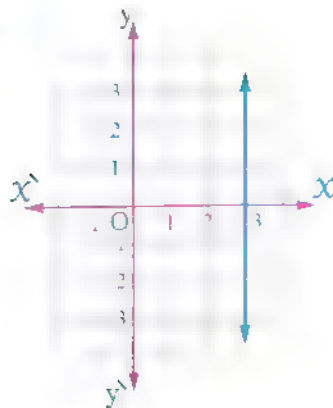
9 If $(3, a)$ satisfies the relation : $y - 2x = 4$, find the value of a

« 10 »

10 Find the value of k , where $(k, 2k)$ satisfies the relation : $x + y = 15$

« 5 »

11 Find the relation that is represented by the line in each figure below :



12 Represent graphically each of the following relations :

1) $x + y = 2$

3) $x + 2y = 3$

5) $y = -2x$

7) $2x = 5$

2) $x - y = 3$

4) $y - 3x = 1$

6) $y - 2x = -1$

8) $y + 1 = 0$

13 Graph the relation : $2x + 3y = 6$, if the straight line representing this relation

intersects the x -axis at the point A and the y -axis at the point B

, find the area of the triangle OAB where O is the origin point.

« 3 square units »

- 14 If the straight line which represents the relation : $2x - y = a$ intersects the x -axis at the point $(3, b)$, find a and b

« 6, 0 »

15 Choose the correct answer from those given :

- 1 Which of the following ordered pairs satisfies the relation : $2x + y = 5$?
 (a) $(-1, 3)$ (b) $(1, 3)$ (c) $(3, 1)$ (d) $(2, 2)$
- 2 $(3, 2)$ does not satisfy the relation
 (a) $y + x = 5$ (b) $3y - x = 3$ (c) $y + x = 7$ (d) $x - y = 1$
- 3 The relation : $5x = 7y$ is represented by a straight line passes through the point
 (a) $(5, 7)$ (b) $(0, 0)$ (c) $(5, 0)$ (d) $(0, 7)$
- 4 The point $(3, 5)$ lies on the straight line which represents the relation
 (a) $y = 3x - 5$ (b) $2x - y = 1$ (c) $3x + y = 1$ (d) $y = 3x - 1$
- 5 If $(2, -5)$ satisfies the relation : $3x - y + c = 0$, then $c =$
 (a) 1 (b) -1 (c) 11 (d) -11
- 6 If $(-1, 5)$ satisfies the relation : $3x + ky = 7$, then $k =$
 (a) 2 (b) -2 (c) 1 (d) 10
- 7 Which of the following relations is represented by a straight line parallel to the y -axis ?
 (a) $y = -5$ (b) $x = -5$ (c) $x = y$ (d) $x + y = 0$
- 8 Which of the following relations is represented by a straight line parallel to the x -axis ?
 (a) $2y = 6$ (b) $2x = 6$ (c) $x = -y$ (d) $x - y = 0$
- 9 Which of the following relations is represented by a straight line passes through the origin point ?
 (a) $y = 5$ (b) $x = -3$ (c) $y = x + 2$ (d) $y = 3x$
- 10 The relation : $3x + 8y = 24$ is represented by a straight line intersecting the y -axis at the point
 (a) $(0, 8)$ (b) $(8, 0)$ (c) $(0, 3)$ (d) $(3, 0)$
- 11 The relation $2x + 7y = 14$ is represented by a straight line intersecting the x -axis at the point
 (a) $(2, 0)$ (b) $(0, 2)$ (c) $(7, 0)$ (d) $(0, 7)$

- 12 The opposite table represents the relation between X and y , which of the following expresses this relation ?

X	1	2	3	4
y	-2	-5	-8	-11

- (a) $X + y = -1$ (b) $X - y = 3$ (c) $3X + y = 1$ (d) $y = -X - 3$

- 13 The opposite table shows the relation between X and y , which is

X	1	2	3	4	5
y	1	3	5	7	9

- (a) $y = X + 4$ (b) $y = X + 1$
(c) $y = 2X - 1$ (d) $y = 3X - 2$

- 14 The relation which expresses the two ordered pairs $(2, 1)$ and $(4, 3)$ together is

- (a) $y = \frac{1}{2}X$ (b) $y = 2X - 5$ (c) $y = X - 1$ (d) $y = 3X + 3$

- 16 Two even natural numbers, twice the first plus the second equals 12

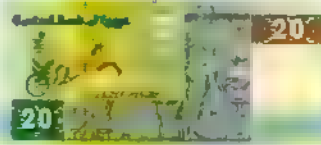
Find the different possibilities of the two numbers.

Geometric Application

- 17 The perimeter of a rectangle is 14 cm. What are the different possibilities of the length and the width given that each of them belongs to \mathbb{Z}_+ ?

Life Applications

- 18 Essam has 10 bills of L.E. 5 and other bills of L.E. 20. He bought some goods from a shopping centre for L.E. 65. Determine the different possibilities to pay this amount of money. Find the relation and graph it.



- 19 The selling price of a computer table is L.E. 100 and its chair is L.E. 50. If the store sells in one week with L.E. 500, what are the represented expectations to the number of sold computer tables and chairs ? Represent the relation graphically.



For excellent pupils

- 20 The perimeter of an isosceles triangle is 19 cm. What are the different possible lengths of its sides given that its sides lengths $\in \mathbb{Z}_+$?

Notice that : The sum of the lengths of any two sides of the triangle is greater than the length of the third side.



Remember Understand Apply Problem Solving

- 1 Classify the slope of the straight line in each of the following figures showing whether it is (positive – negative – zero – undefined) :

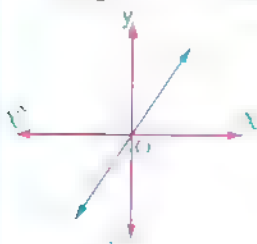


Fig. (1)

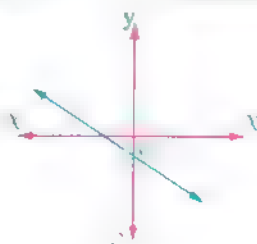


Fig. (2)



Fig. (3)

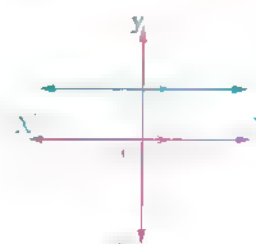
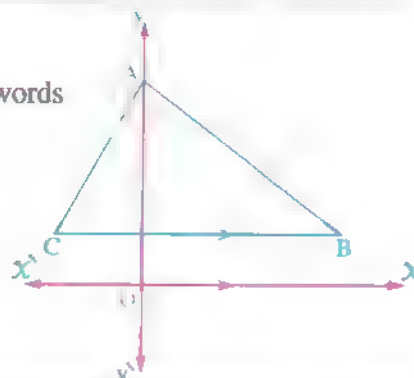


Fig. (4)

- 2 In the opposite figure :

ABC is a triangle. Complete by using one of the following words (positive , negative , zero , undefined)

- 1 The slope of \overrightarrow{AB} is
- 2 The slope of \overrightarrow{BC} is
- 3 The slope of \overrightarrow{AO} is
- 4 The slope of \overrightarrow{AC} is



- 3 Complete the following :

- 1 The slope of any horizontal straight line equals
- 2 The slope of any straight line parallel to y-axis is
- 3 The straight line whose slope = zero is parallel to
- 4 If A , B and C are collinear , then the slope of \overrightarrow{AB} = the slope of

4 Find the slope of the straight line passing through the two points in each of the following :

1 A (1 , 3) , B (3 , 4)

3 A (3 , 2) , B (6 , 5)

5 A (1 , 3) , B (2 , 3)

7 A (3 , -1) , B (3 , 2)

9 A (-1 , 3) , B (2 , 1)

11 E (-3 , -1) , O (0 , 0)

2 A (1 , 2) , B (5 , 0)

4 A (2 , -1) , B (4 , -1)

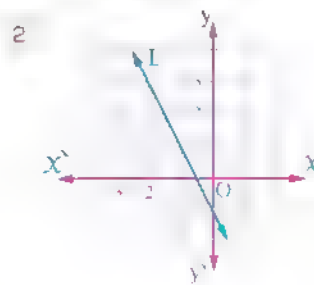
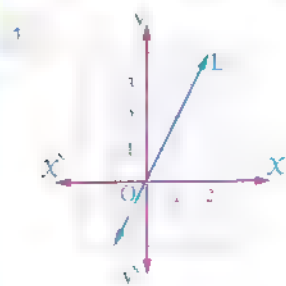
6 A (5 , 2) , B (5 , 4)

8 A (3 , -2) , B (4 , 1)

10 N (4 , -2) , K (-1 , -7)

12 A (-6 , -9) , B (-1 , -1)

5 Find the slope of the straight line L in each of the following graphs :



6 In the opposite figure :

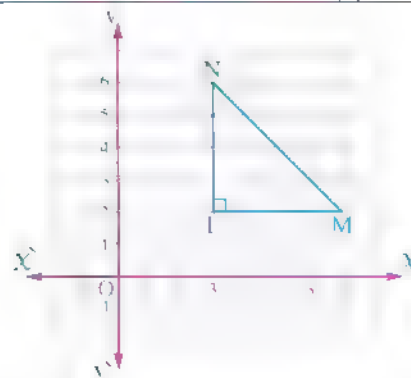
LMN is a right-angled triangle at L

, where $m(\angle M) = 45^\circ$

Given that L (3 , 2) and M (7 , 2)

, find the coordinates of N

and calculate the slope of \overrightarrow{MN}



7 If A (2 , -1) , B (10 , 3) and C (2 , 3) , find the slope of each of \overrightarrow{AB} , \overrightarrow{BC} and \overrightarrow{CA}

Draw the triangle ABC on a square grid , then mention the type of the triangle according to the measures of its angles.

8 If the slope of the straight line which passes through the two points (1 , 3) and (3 , k) equals 3 , find the value of k

« 9 »

9 If the slope of the straight line which passes through the two points (3 , c) and (5 , -2) equals -3 , find the value of c

« 4 »

10 If A (-1 , 4) , B (x , 2) and the slope of \overrightarrow{AB} equals -2

, find the value of x

« zero »

Exercise 12

- 11** If the straight line which passes through the two points $(-2, y)$ and $(3, -1)$ has a slope -0.6 , find the value of y « 2 »
-
- 12** Find the value of k such that the straight line passing through the two points $(3, 4)$ and $(2, k)$ is parallel to X -axis. « 4 »
-
- 13** Find the value of X such that the straight line which passes through the two points $(2X, 3)$ and $(6, 7)$ is parallel to y -axis. « 3 »
-
- 14** Find the value of y such that the straight line passing through the two points $(3, 6)$ and $(-2, 3y)$ is perpendicular to y -axis. « 2 »
-
- 15** Are the points $(-5, 11)$, $(0, 8)$ and $(5, 5)$ collinear ?
-
- 16** Find the slope of each of \overrightarrow{AB} , \overrightarrow{BC} and \overrightarrow{AC} , where $A(2, 1)$, $B(3, 2)$ and $C(4, 5)$ and represent each line graphically. What do you observe ?
-
- 17** In each of the following, prove that the points A , B and C are collinear :
- 1 $A(1, 1)$, $B(2, 2)$, $C(-3, -3)$
 - 2 $A(4, -3)$, $B(-6, 7)$, $C(5, -4)$
 - 3 $A(-2, 12)$, $B(2, 4)$, $C(6, -4)$
-
- 18** In each of the following, prove that the points A , B and C are not collinear :
- 1 $A(2, 1)$, $B(3, 0)$, $C(5, -1)$
 - 2 $A(-1, 2)$, $B(3, 1)$, $C(7, 2)$
 - 3 $A(0, -3)$, $B(2, 2)$, $C(-3, -3)$
-
- 19** Find the slope of the line \overrightarrow{AB} , where $A(-1, 3)$ and $B(2, 5)$
Is the point $C(8, 1) \in \overrightarrow{AB}$? « $\frac{2}{3}$ »
-
- 20** Find the value of y such that the points $(4, 1)$, $(-2, 7)$ and $(3, y)$ are collinear. « 2 »

For excellent pupils

- 21** If the straight line which passes through the points $(3, -1)$, $(X, 1)$ and $(9, y)$ has a slope $= \frac{2}{3}$, find the value of each of X and y « 6, 3 »

Real life applications
on the slope

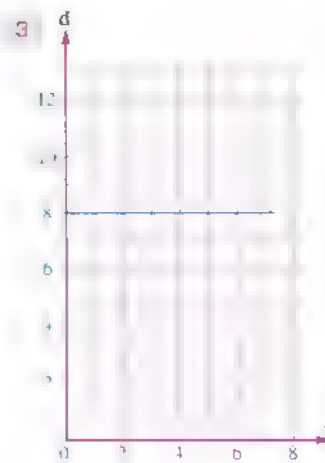
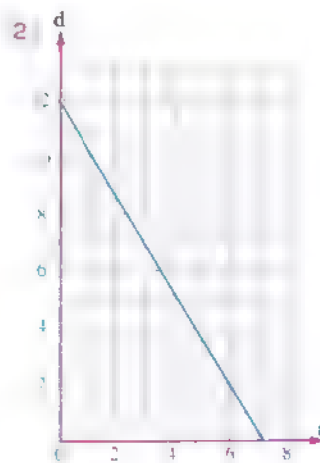
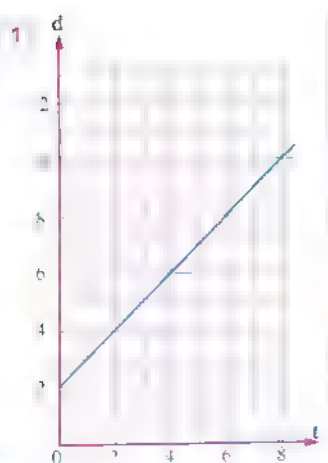
interactive test

From the school book

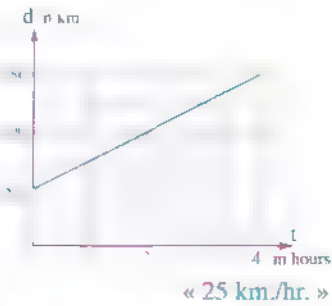


● Remember ● Understand ● Apply ● Problem Solving

- 1 A car moves with uniform velocity such that it covers 180 km. per 3 hours. If the car moves for 5 hours , what is the covered distance ? « 300 km. »
- 2 An irrigation machine consumes 2.47 litres of diesel to work for 3 hours. If the machine works for 10 hours , how many litres of diesel will the machine consume ? « $8\frac{7}{30}$ litres »
- 3 The following diagrams show the relation between the covered distance (in m.) and the elapsed time (in sec.) of an object. Determine the position of the object at the starting of motion and its position after 6 seconds (when $t = 6$ sec.) Find the slope of the line in each case and state what it represents.



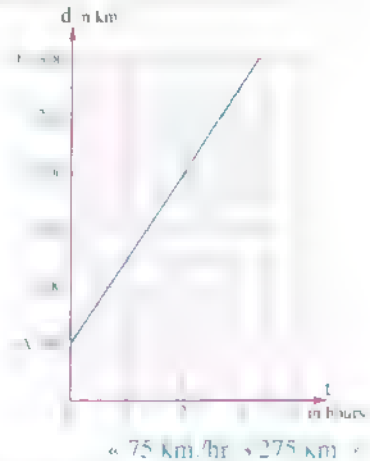
- 4 The opposite graph represents the motion of a car moving with uniform velocity. Determine the velocity of the car.



- 5 Bassem drove his car from the city A to the city B. The opposite graph shows the relation between the distance d in km. and the time t in hours.

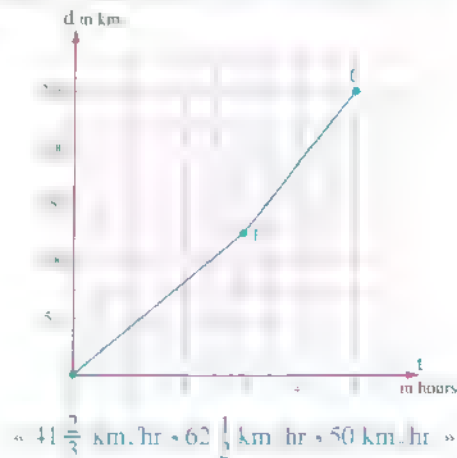
Answer the following :

- 1 What is the uniform velocity of the car of Bassem ?
- 2 Find the distance between the car and the point O after three hours from the moment of beginning.



- 6 The opposite graph represents the motion of a car :

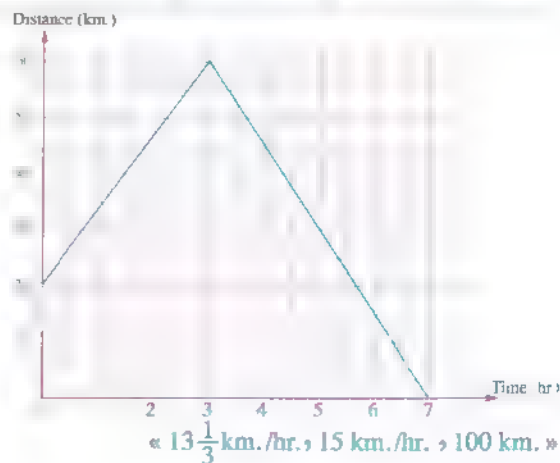
- 1 Find the velocity of the car within the first three hours from the beginning , then find the velocity within the next two hours.
- 2 Find the average velocity of the car within the total time.



- 7 The opposite figure represents the motion of a bicycle measured from a constant point. Find the regular velocity of the bicycle during :

- 1 The first three hours.
- 2 The next four hours.

Find the total distance covered by the bicycle.

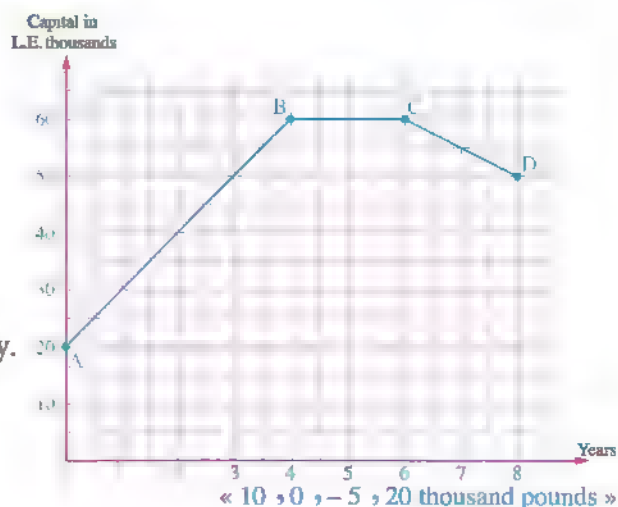


- 8 The opposite figure shows the capital change of a company during 8 years :

- 1 Find the slope of each of \overrightarrow{AB} , \overrightarrow{BC} and \overrightarrow{CD}

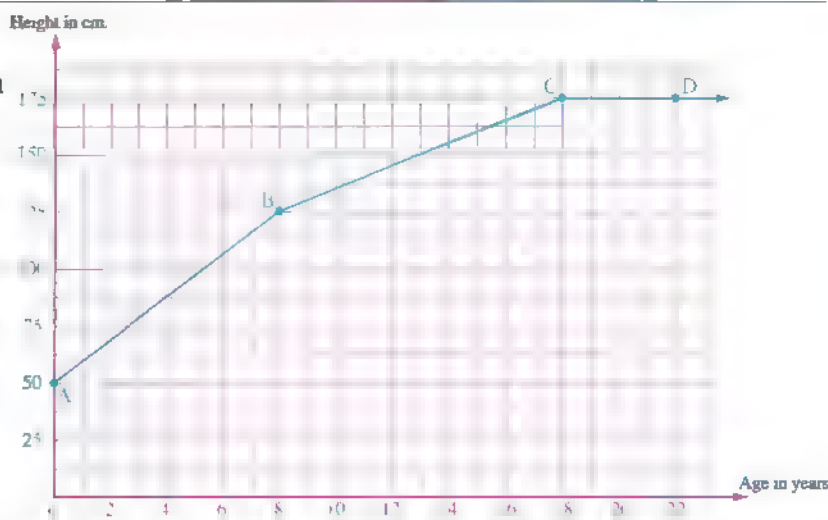
What is the meaning of each ?

- 2 Find the starting capital of the company.



- The opposite figure shows the relation between the height of a person (in cm.) and his age (in years) :

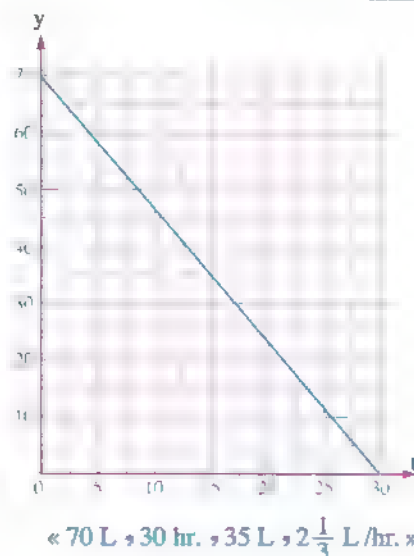
- 1 Find the slope of each of \overrightarrow{AB} , \overrightarrow{BC} and \overrightarrow{CD}
- What is the meaning of each ?



- 2 Calculate the difference between the height of this person when he was 8 years old and his height when he was 30 years old.
- « $9\frac{3}{8}$, 5, 0, 50 cm. »

- 10 Magdi filled the tank of his car by fuel. The opposite figure represents the relation between the time (t) in hours and the amount of remained fuel in the tank (y) in litres :

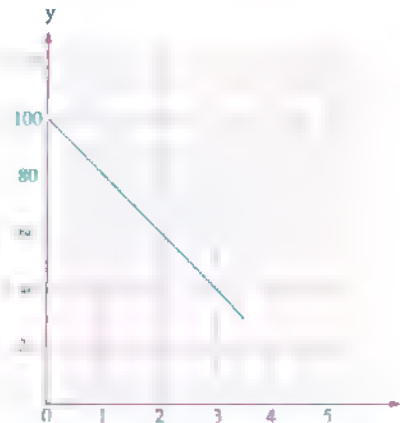
- 1 What is the greatest capacity of the tank ?
- 2 When will the tank become empty ?
- 3 What is the amount of remained fuel after 15 hours ?
- 4 What is the range of consumption of fuel in each hour ?



11 A person read a book.

The opposite graph shows the relation between the time (t) in hours and the number of remained pages (y) :

- 1 How many pages are remained in the beginning ?
- 2 Find the rate of reading pages per hour.
- 3 When does this person finish reading this book ?

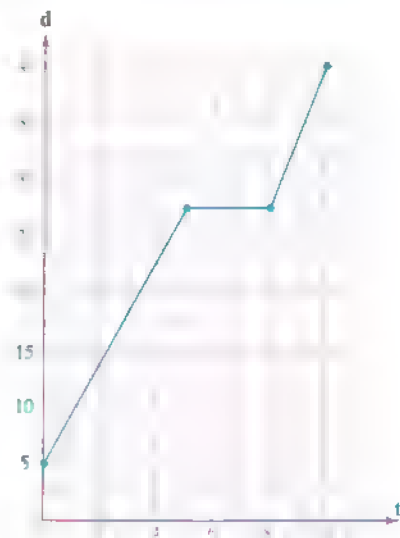


« 100 pages , 20 pages/hr. , after 5 hours »

12 A farmer wanted to complete digging a well in his farm.

He rented a digging machine. The opposite graph shows the depth of the well (d) in metres after time (t) in hours , find :

- 1 The depth of the well before beginning digging.
- 2 The depth of the well after finishing digging.
- 3 The total time which the machine took in digging the well.
- 4 The average of depth of the well which the machine digs within the first five hours.
- 5 The average of the depth of the well within the last two hours of digging.



« 5 m. , 40 m. , 10 hr. , 4.5 m./hr. , 6.25 m./hr. »

13 The opposite graph shows the relation between the distance in km. and the time (t) in hours for a bicycle which moved between two towns A and B going and returning back.

Answer the following :

- 1 What is the uniform velocity during the going trip ?
- 2 What is the average velocity during returning back ?
- 3 What is the meaning of the horizontal line segment in the graph ?



« 20 km./hr. , 12 km./hr. »

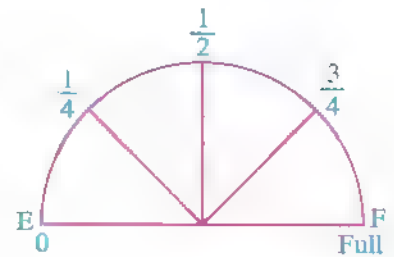
- 14 Hazem filled up the 40 L tank of his car.

After covering a distance of 120 km., the fuel gauge shows that the rest of fuel is $\frac{3}{4}$ of the tank.

Draw a diagram to show the relation between the amount of fuel in the tank and the covered distance
(This relation is linear).

Calculate the covered distance until the tank totally gets empty.

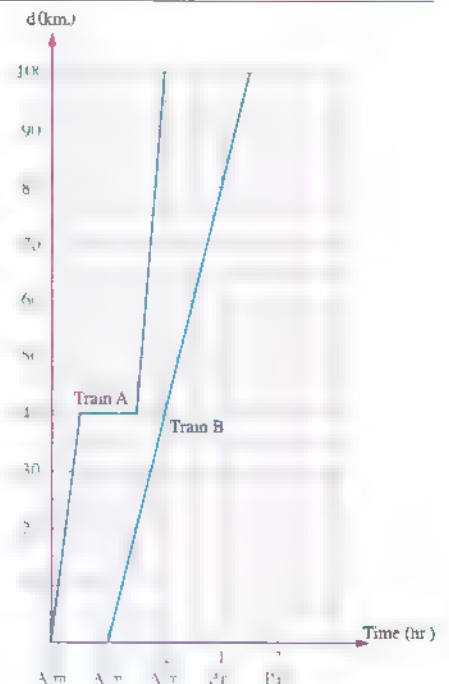
« 480 km »



- 15 The opposite diagram shows the relation between the covered distance (in km.) and the elapsed time (in hr.) for two trains A and B between two railway stations.

Use the diagram to find :

- 1 The distance between the two railway stations.
- 2 The elapsed time of each train.
- 3 The average speed of each train.
- 4 The meaning of the horizontal segment in the diagram of train A

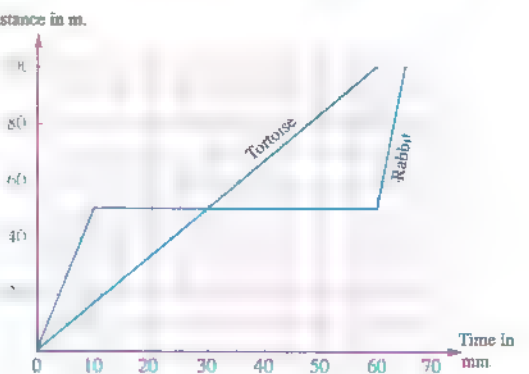


« 100 km., 2 hr., 2.5 hr., 50 km./hr., 40 km./hr. »

- 16 The opposite graph shows the race of 100 metres between a rabbit and a tortoise.

Answer the following :

- 1 Which of them is the winner ?
- 2 What is the velocity of the tortoise ?
- 3 What is the average velocity of the rabbit ?
- 4 What is the meaning of the horizontal line segment in the graph ?



« tortoise, $1\frac{2}{3}$ m./min., $1\frac{7}{13}$ m./min. »

For excellent pupils

- 17 During the motion of a bicycle with a uniform velocity in a straight line , the distances between the bicycle and a fixed point have been registered after periods measured in hours from the moment of beginning the motion in the following table :

The distance between the bicycle and the fixed point	125	150	175	200
The passed time in hours	2	4	6	8

Graph the relation between the distance between the bicycle and the fixed point and the passed time. From the graph , find :

- 1 The velocity of the bicycle in km./hr.
 - The distance between the bicycle and the fixed point after 300 minutes.
 - The time at which the bicycle is at a distance = 187.5 km. from the fixed point.
 - The distance between the starting point of the bicycle and the fixed point.
- « 12.5 km./hr. , 162.5 km. , 7 hr. , 100 km. »

For the next term

Ask for



in
Maths & Science
& English



For all educational stages

Summary of Unit 2



- ★ The linear relation is a relation of the first degree between two variables X and y , it is in the form : $aX + by = c$ where a , b and c are real numbers, a and b are not both equal to zero, and there is an infinite number of ordered pairs which satisfy this relation, and it is represented graphically by a straight line.
- ★ To graph a linear relation, you need to graph at least two ordered pairs satisfying this relation, you can add a third ordered pair to check that the three points lie on the same straight line which is the graphic representation of the relation.
- ★ The relation : $y = 0$ is represented by X -axis.
- ★ The relation : $X = 0$ is represented by y -axis.
- ★ The linear relation $aX + by = 0$ is represented graphically by a straight line passing through the origin point.
- ★ The slope of the straight line = $\frac{\text{the change in } y\text{-coordinates}}{\text{the change in } X\text{-coordinates}} = \frac{\text{the vertical change}}{\text{the horizontal change}}$
i.e. $S = \frac{y_2 - y_1}{x_2 - x_1}$ where $x_1 \neq x_2$
- ★ The slope of the straight line parallel to X -axis equals zero
- ★ The slope of the straight line parallel to y -axis is undefined.



Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1 Which of the following ordered pairs satisfies the relation : $2x + y = 5$?

- (a) $(-3, -1)$ (b) $(3, 1)$ (c) $(1, 3)$ (d) $(2, 2)$

2 If $(2k, k)$ satisfies the relation : $y + 2x = 5$, then $k = \dots\dots\dots$

- (a) 1 (b) 2 (c) 3 (d) 4

The slope of the straight line passing through the two points $(2, 3)$, $(-5, 3)$ is

- (a) 2 (b) 1 (c) 0 (d) undefined.

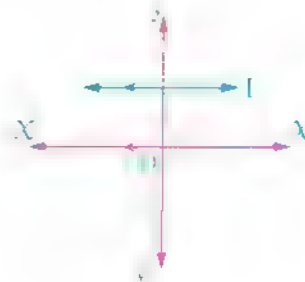
4 The relation : $x - 3 = 0$ is represented by a straight line of slope

- (a) 0 (b) undefined (c) 5 (d) -5

5 In the opposite figure :

The slope of the straight line L is

- (a) positive. (b) negative.
(c) 0 (d) undefined.



6 $(3, 1)$ does not satisfy the relation

- (a) $y + x = 4$ (b) $2x - y = 5$
(c) $3y + x = 4$ (d) $4y + 2x = 10$

2 Complete the following :

1 The relation : $3x + 4y = 12$ is represented by a straight line intersecting the x -axis at the point

2 If the slope of the straight line passing through the two points $(3, y)$, $(5, -2)$ is -3 , then $y =$

3 If $(-1, 5)$ satisfies the relation : $3x + ky = 7$, then $k =$

4 The slope of the straight line that is parallel to the y -axis is

5 If the straight line : $ax + by + c = 0$ passes through the origin point, then $c =$

3 [a] Represent graphically the relation : $2x + y = 4$

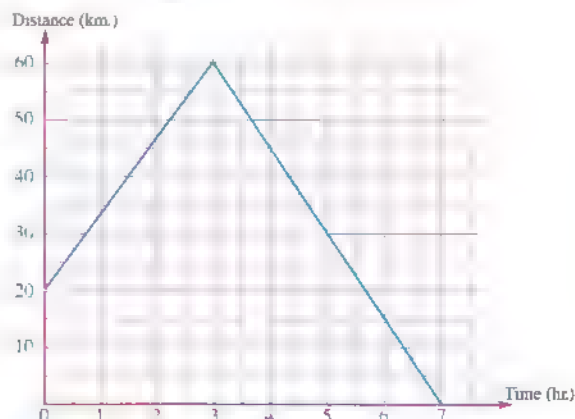
[b] Prove that the points A $(4, 3)$, B $(1, 1)$ and C $(-5, -3)$ are collinear.

- 4** [a] Represent graphically the straight line that represents the relation : $2y - 3x = -6$ and if the straight line intersects the x -axis at the point A and intersects the y -axis at the point B , find the area of ΔOAB where O is the origin point.
- [b] Find the value of y such that the straight line passing through the two points $(4, -1)$, $(-2, 2)$ is perpendicular to the y -axis.

- 5** The opposite figure represents the movement of a bicycle from a fixed point.

Find :

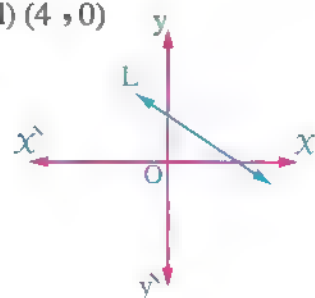
- 1** The velocity of the bicycle during the first three hours.
- 2** The velocity of the bicycle during the next four hours.
- 3** The total distance.



Model 2

Answer the following questions :

- 1** Choose the correct answer from those given :
- 1** The ordered pair which does not satisfy the relation : $y = x + 1$ is
 (a) $(0, 1)$ (b) $(2, 3)$ (c) $(1, 2)$ (d) $(2, 5)$
 - 2** If $(5, 2m)$ satisfies the relation : $y = 3x - 1$, then $m =$
 (a) 2 (b) 7 (c) 10 (d) 14
 - 3** If the slope of the straight line representing the relation $x + my = 5$ is undefined , then $m =$
 (a) 1 (b) - 1 (c) 5 (d) zero.
 - 4** The relation : $2x + 3y = 12$ is represented by a straight line intersecting the y -axis at the point
 (a) $(6, 0)$ (b) $(0, 6)$ (c) $(0, 4)$ (d) $(4, 0)$
 - 5** In the opposite figure :
 The slope of the straight line L is
 (a) positive. (b) negative.
 (c) zero. (d) undefined.



6 The slope of the straight line \overleftrightarrow{yy} is

- (a) zero. (b) undefined. (c) 1 (d) -1

2 Complete the following :

- 1 The slope of the straight line parallel to X -axis is
- 2 If $(2, -1)$ satisfies the relation : $2X + 3y + c = 0$, then $c =$
- 3 The straight line which represents the relation :
 $y = 2X + 5$ intersects X -axis at the point
- 4 The relation : $X - 5 = 0$ is represented by a straight line whose slope is
- 5 If the slope of $\overleftrightarrow{AB} =$ the slope of \overleftrightarrow{BC} , then A , B and C are

3 [a] Represent graphically the relation : $y - 2X + 1 = 0$

[b] If the straight line which represents the relation : $X - 2y - a$ intersects y -axis at the point $(b, 3)$, then find the value of each of a and b

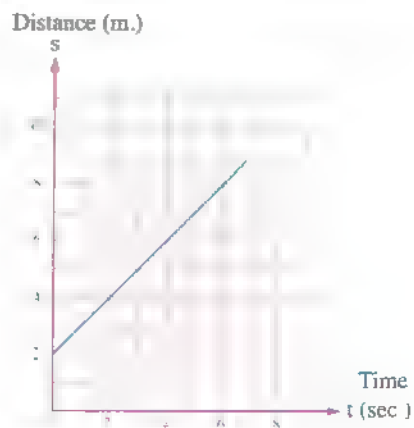
4 [a] If the slope of the straight line which passes through the two points $(3, a)$ and $(5, 4)$ equals 3 , find the value of a

[b] Prove that the points $A(2, -3)$, $B(4, -5)$ and $C(0, -1)$ are collinear.

5 The opposite graph represents the relation between the distance (s) in metres which a particle away from the observer and the elapsed time(t) in seconds.

First : Find the distance between the particle and the observer :

- 1 At beginning the motion.
- 2 After $t = 6$ sec.



Second : Find the slope of the straight line which represents the relation.

A Research Project

On Unit Two



Project aims :

- Recognizing the relation between two variables of the first degree.
- Representing the relation between two variables of the first degree graphically.
- Using algebra to solve life problems.
- Associating mathematics with social studies.

Do a research project on the following topic :

"Doing sports is the start on the road of a more healthy life. Our Arab champs have achieved a lot of important achievements in many world competitions".

Discuss the following points using available resources :

- 1 State some achievements of our champs of Arab countries in the field of sports.
- 2 In football matches, a team gets three points in case of winning and one point in case of a draw. If one team scored 30 points :
 - * Write the mathematical relation between (x) and (y) , where (x) is the number of matches a team wins and (y) is the number of matches, the team are held to a draw. From this relation, write five different methods to score 30 points.
 - * Represent this relation graphically.



UNIT

3

Statistics

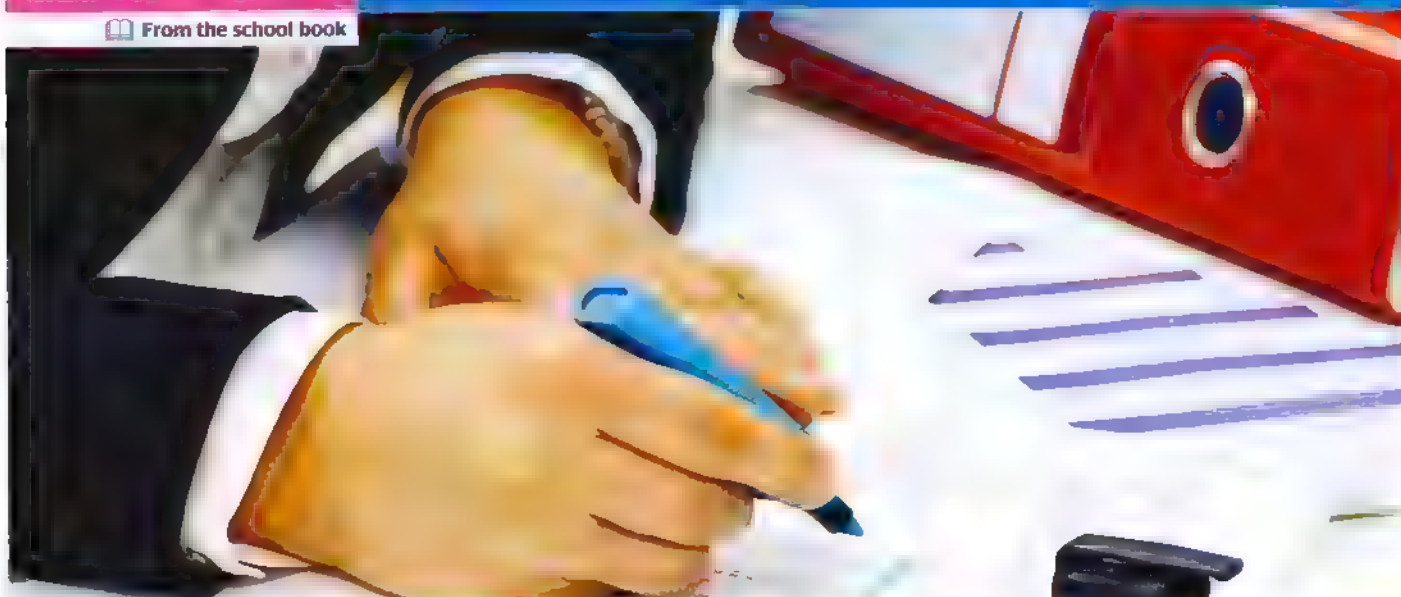
Exercises of the unit :

14. Collecting and organizing data.
 15. The ascending and descending cumulative frequency tables and their graphical representation.
 16. Mean.
 17. Median.
 18. Mode.
- ✧ Summary of unit three.
 - ✧ Unit exams.
 - 🔍 A research project on unit three



Scan the
QR code
to solve an
interactive
test on each
lesson

From the school book



● Remember

● Understand

● Apply

● Problem Solving

- 1 The following are the weights of 40 pupils of one class of the second year preparatory in kg. The required is forming the frequency table with sets. Use the subsets (25–, 30–, 35–,):

36	30	42	37	25	34	35	28	30	28
29	36	38	32	44	39	34	36	35	30
30	35	30	38	27	41	33	39	31	36
36	33	37	31	43	35	40	31	39	45

- 2 The following are the weekly wages of 40 workers in a factory in L.E.:

47	71	36	94	54	64	87	89	62	57
51	61	44	52	70	66	56	32	69	36
79	48	77	90	65	99	96	67	60	55
95	75	81	84	78	38	49	94	48	59

Required: Form a frequency table with sets (use the subsets: 30–, 40–, 50–,, 90–). What is the set with the highest frequency? What is the set with the lowest frequency?

- 3 The following are the scores of 30 students in a monthly math exam:

25	35	40	20	30	37	40	33	22	38
35	36	28	37	39	28	32	26	29	37
23	34	35	36	29	38	40	35	37	31

- 1) Form a frequency table with sets for these scores.
- 2) Find the total number of excellent students. The excellence rate is 36 marks or more.

« 12 students »

The following are the marks of the students in a class in the second year preparatory in algebra exam. Given that their number is 40 students and the full mark is 20 marks :

7	11	7	13	14	3	18	13	10	14
16	8	15	12	5	15	11	12	6	11
8	9	15	8	15	14	7	10	14	19
10	7	2	10	12	4	11	17	13	15

The required is forming a frequency table with sets for the marks of students in algebra using the subsets 0- , 4- , 8- , and so on , then find the percentage of the number of students who obtained 12 marks at least.

5 The following are the heights of 50 persons in centimetres :

155	183	163	181	186	144	199	150	182	166
197	126	188	158	153	130	163	166	154	173
137	163	146	198	164	156	173	177	157	118
138	187	178	173	184	143	147	142	176	160
170	194	154	167	149	112	196	128	126	156

Using the previous data :

- 1 Find the least height in these data and the greatest height and the range in which these two heights lie.
- 2 Form a frequency table using sets of length 10 centimetres for each.

6 In a military camp , the heights of 55 soldiers were measured in centimetres , their measures were as follows :

169	194	200	185	165	188	166	186	181	176	173
177	179	188	170	193	180	173	173	184	192	167
182	168	186	189	171	179	172	175	175	181	166
185	177	175	165	190	172	177	178	184	166	174
178	177	172	174	175	179	195	176	189	187	189

Form a frequency table using the sets (165- , 170- , 175- ,)
From the table , find :

- 1 The number of soldiers whose heights are less than 185 cm.
- 2 The number of soldiers whose heights are 180 cm. at least.

« 39 soldiers »

« 22 soldiers »

The ascending and descending cumulative frequency tables and their graphical representation



Remember

Apply

Problem Solving

Problems on the ascending cumulative frequency curve

- 1 The following table shows the frequency distribution of the scores of 50 students in an experimental math exam :

Sets	2-	6-	10-	14-	18-	22-	26-	Total
Frequency	3	5	9	10	12	7	4	50

Graph the ascending cumulative frequency curve.

- 2 The following frequency table represents the marks of 60 pupils in math :

Sets	10	20-	30-	40	50-	Total
Frequency	9	11	13	17	10	60

Graph the ascending cumulative frequency curve and if the success mark is 30 marks , find the number of failed pupils.

« 20 pupils »

- 3 The following table shows the frequency distribution of 100 factories according to the number of weekly work hours :

Sets of hours	50-	60-	70-	80-	90	100-	Total
Number of factories	5	16	30	22	15	12	100

Graph the ascending cumulative frequency curve of this distribution.

- 1 Graph the ascending cumulative frequency curve of this distribution.
2 From the graph , find the number of factories which work less than 75 hours in the week.

« 37 factories »

- 3 Find the percentage of the number of factories which work less than 75 hours in the week.

« 37 % »

Second Problems on the descending cumulative frequency curve

- 4 The following table shows the frequency distribution of the daily wages of some workers :

Sets	5–	10–	15–	20–	25–	30–	Total
Frequency	10	14	24	30	12	10	100

Graph the descending cumulative frequency curve.

- 5 A class has 50 pupils , the following table shows the distribution of studying hours among them every day :

Sets	1	2 –	3 –	4 –	5 –	6 –	7 –	Total
Freq.	2	3	5	12	15	7	6	50

- Graph the descending cumulative frequency curve of this distribution.
- From the graph , find the number of pupils who study 6 hours or more daily. « 13 pupils »
- Find the percentage of the number of pupils who study 6 hours or more daily. « 26 % »

- 6 The following table shows the frequency distribution of a group of 60 persons according to their weights in kg. :

Sets of weights in kg.	55	60	65	70	75	80 –	85 –	Total
No. of persons	8	12	18	7	3	2	60

Complete the table , then graph the descending cumulative frequency curve of this distribution and from the graph , find the number of persons whose weigh 68 kg. or more for each. « 28 persons »

Third Problems on the two curves together

- 7 Graph the ascending and descending curves for the following frequency distribution :

Sets	8–	12–	16–	20–	24–	28–	32–	36–	40–	Total
Freq.	4	7	12	18	20	19	11	6	3	100

- 8 The following table shows the frequency distribution of the scores of 1000 students in a final year exam :

Percentage	20–	30–	40–	50–	60–	70–	80–	90–	Total
Number of students	30	70	160	260	150	130	110	90	1000

- Graph the ascending and descending cumulative frequency curves.
- Find the number of students whose scores are less than 75% « 740 students »
- Find the number of students whose scores are 85% or more. « 140 students »

- 9 The following are the scores of 100 students in an experimental math exam :

Sets	0 -	10	20–	30–	40–	50–	Total
Frequency	8	14	15	28	23	12	100

- Form both the ascending and descending cumulative frequency tables.
- Graph both the ascending and descending cumulative frequency curves on the same graph paper.
- From the graph , find the number of students who got less than 40 marks and those who got 40 marks or more. « 65 students , 35 students »
- Find the percentage of the number of students who succeeded given that the success mark is 20 marks. « 78 % »
- Find the percentage of the number of students who got 45 marks or more. « 23 % »



For excellent pupils

- 10 A factory has 120 workers whose experiences are from 5 years to 35 years.

The opposite table shows the descending cumulative frequency distribution for those workers according to the years of experience :

- Deduce from the table the frequency table.
- Form the ascending cumulative frequency table.
- Graph the ascending cumulative frequency curve.
- From the graph , deduce the number of workers whose experience years are less than 17.5 years.

Lower boundaries of sets	Descending cumulative frequency
5 and more	120
10 and more	113
15 and more	93
20 and more	64
25 and more	27
30 and more	12
35 and more	0

« 40 workers »



From the school book



● Remember ● Understand ● Apply ● Problem Solving

1 Complete the following :

- 1 The mean of a set of values = $\frac{\dots + \dots + \dots + \dots + \dots}{\dots}$
- 2 The centre of the set = $\frac{\dots + \dots}{2}$
- 3 The arithmetic mean of the values : 5 , 12 , 17 , 6 is
- 4 If the lower limit of a set is 8 and the upper limit of the same set is 14 , then its centre is
- 5 If the lower limit of a set is 4 and its centre is 9 , then its upper limit is
- 6 If the mean of a frequency distribution is 39.4 and the total of frequencies is 100 , then the total of the products of frequencies of the sets by their centres is

2 Choose the correct answer from those given :

- 1 The mean of the values : $2 - a$, 4 , 1 , 5 , $3 + a$ is
 (a) 1 (b) 2 (c) 3 (d) 15
- 2 If the mean of marks of 5 pupils is 20 , then the sum of their marks is marks.
 (a) 4 (b) 15 (c) 25 (d) 100
- 3 The centre of the first set of the sets : $7 -$, $13 -$, $19 -$, $25 -$ is
 (a) 6 (b) 7 (c) 10 (d) 13
- 4 If the upper limit of a set is 14 and its centre is 10 , then its lower limit is
 (a) 5 (b) 6 (c) 20 (d) 24
- 5 If the beginning of a set is 5 and its centre is 7.5 , then the length of the set is
 (a) 5 (b) 7.5 (c) 10 (d) 12.5

- 3 Find the mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	Total
Frequency	6	8	4	2	20

« 21 »

- 4 The following table shows the frequency distribution of marks of 10 students in mathematics :

Sets	10	20 –	30	40 –	50 –	Total
Frequency	1	2	4	2	1	10

- 1 Calculate the mean of marks of students.

- 2 If the mark of success is 30 , calculate the number of failed students.

« 35 marks , 3 students »

- 5 The following table shows the frequency distribution of weekly wages of 100 workers in one factory :

Sets	16 –	20	24 –	28 –	32 –	36	Total
Frequency	10	15	22	25	20	8	100

Calculate the mean.

« 28.16 »

- 6 The following table shows the frequency distribution of extra wages of 30 workers :

Sets	15 –	25 –	35 –	45 –	55 –	65 –	75 –	Total
Freq.	2	3	5	8	6	4	2	30

Find the arithmetic mean.

« 51 »

- 7 The following table shows the frequency distribution of the heights of 120 students in centimetres :

Height (in cm.)	140 –	144 –	148 –	152 –	156 –	160 –	Total
Frequency	12	20	38	22	17	11	120

Find the mean.

« 151.5 cm. »

- 8 The following table shows the frequency distribution of number of daily studying hours of 50 pupils in a class :

Number of hours	1 -	2 -	3 -	4 -	5 -	6 -	7	Total
Number of pupils	2	3	5	12	15	7	6	50

- 1 Calculate the mean of the number of hours of study per day.

Find the number of pupils who study less than 4 hours daily.

« 1 hours : 10 pupils »

- 9 The following table shows the distribution of marks of 40 students in one exam :

Sets	5 -	15 -	35 -	45 -	Total
Number of students	3	12	10	5	40

- 1 Complete the table.

- 2 Calculate the mean.

- 3 Find the number of students whose marks are not less than 35 marks.

« 31 marks : 15 students »

- 10 The following table shows the frequency distribution of the weights of 30 children in kg. :

Weight (kg.)	6	10 -	14	18 -	22	26 -	30 -	Total
Frequency	2	3	8	6	4	2	30

Complete the table , then find the mean of this distribution.

« 20.4 kg. »

- 11 Using the following set frequency table (given that the sets are equal in range) :

Sets	10 -	20	X -	40	50 -	60	Total
Frequency	10	17	20	32	k + 2	4	100

Find :

- 1 The value of each of X and k

- 2 The mean of this distribution.

« X = 30 , k = 15 , 39.1 »

- 12** The following table shows the frequency distribution of weights of 50 pupils in kg. in one school :

Weight in kg.	30 -	35 -	40	45 -	50	55 -	Total
Number of pupils	7	3 k	4 k	10	8	4	50

- 1 Calculate the value of k
- 2 Find the mean of this distribution.

« 3 , 44 kg. »

- 13** The following table shows the frequency distribution of 50 workers days-off :

Sets	2 -	6	10 -	14 -	18 -	22 -	26 -	Total
Frequency	4	5	8	k - 2	7	5	1	50

Find :

- 1 The value of k
- 2 The mean.

« 22 , 15.2 days »

- 14** If the mean of the scores of a student during the first 5 months is 23.8 , what is the score of the 6th month if the mean of his scores is 24 marks ?
- 15** If the mean of marks of Magdi in 4 exams is 16 marks , what is the mark which he should obtain in the fifth exam so that his mean in the five exams will be 18 marks ?

« 26 marks »



For excellent pupils

- 16** The opposite table is for finding the mean of marks of m pupils in one exam :

- 1 Deduce the value of each of : a , b , c , d , e , f , X , y , z and m
- 2 Find the mean of these marks.

Sets	Centres of sets	Frequency	Centres of sets × frequency
0 -	a	5	10
4 -	6	b	90
d -	c	30	300
12 -	e	z	y
16 -	f	10	X
Total		m	1140

Median



interactive tool

From the school book



● Remember

● Understand

● Apply

● Problem Solving

1 Choose the correct answer from those given :

- **1** The median of the values : 9 , 4 , 8 , 1 and 3 is
 (a) 3 (b) 4 (c) 5 (d) 8
- **2** The median of the values : 3 , 7 , 2 , 9 , 5 and 11 is
 (a) 12 (b) 7 (c) 6 (d) 5
- **3** The order of the median of the values : 7 , 6 , 5 , 8 and 4 is
 (a) third. (b) fourth. (c) fifth. (d) sixth.
- **4** If the order of the median of a set of values is the fourth , then the number of these values equals
 (a) 4 (b) 5 (c) 6 (d) 7
- **5** If the median of the values : $k + 1$, $k + 2$, $k + 5$, $k + 4$ and $k + 3$ where k is a positive integer is 13 , then $k =$
 (a) - 10 (b) 10 (c) 13 (d) 16
- **6** The point of intersection of the ascending and descending cumulative frequency curves determines on the set-axis.
 (a) the mean (b) length of the set
 (c) centre of the set (d) the median
- **7** If the the point of intersection of the ascending and descending frequency curves is (30 , 50) , then the sum of frequencies is
 (a) 30 (b) 50 (c) 60 (d) 100

- 2 Using the ascending cumulative frequency curve, find the median of the following frequency distribution :

Sets	0–	2–	4–	6–	Total
Frequency	1	2	2	5	10

« 6 »

- 3 The following table shows the frequency distribution of 40 persons according to the percentage of intelligence of each of them :

Sets of intelligence percentage	40–	50–	60	70	80–	90–	Total
Number of persons	1	3	8	14	10	4	40

Using the ascending cumulative frequency curve, find the median of percentage of intelligence.

« Approximately 75 % »

- 4 The following table shows the frequency distribution of 100 factories according to the number of weekly working hours :

Sets of hours	50	60	70	80	90–	100–	Total
Number of factories	5	8	12	28	33	14	100

Find using the descending cumulative frequency curve the median number of hours of work of these factories.

« 89.5 hours »

- 5 The following table shows the frequency distribution of 50 workers' wages in pounds :

Sets of wages	300–	400–	500–	600–	700–	Total
Number of workers	8	12	18	7	5	50

Graph the descending cumulative frequency curve, then find the median.

« 520 pounds »

- 6 The following table shows the frequency distribution of marks of 60 students in mathematics exam :

Sets of marks	5–	10–	15–	20–	25–	30	35–	Total
Number of students	2	5	14	20	13	5	1	60

Find the median mark.

« 22 marks »

- 7 The following table shows the frequency distribution of weights of 20 children in kg. :

Sets	5–	15–	25–	35–	45–	Total
Frequency	3	4	7	4	2	20

Find the median weight in kg. using the ascending and descending cumulative frequency curves of this distribution.

« 29 kg. »

- 8 The following table shows the distribution of the students of a secondary school in a governorate according to their ages in years :

Sets of ages in years	14	15	16	17	18	19–	Total
Frequency	90	130	110	80	70	20	500

Graph the ascending and descending cumulative frequency curves of this distribution , then find the median age.

« 16.3 years »

- 9 The following table shows the frequency distribution of the marks of 90 students in a monthly exam :

Sets of marks	10–	14	18	22	26	30	34–	Total
Number of students	8	10	24	21	12	9	6	90

Find the median mark using the ascending and descending cumulative frequency curves.

« 22.5 marks »

- 10 The following table shows the frequency distribution for the scores of 50 students in an examination :

Sets	2	6	10	14–	18–	22–	26–	Total
Frequency	3	5	9	10	12	7	4	50

Find : 1 The mean of the student's score.

2 The median.

« 16.8 , 17.6 »

- 11 From the following frequency table with equal sets in range :

Sets	10–	20–	X–	40–	50–	60–	Total
Frequency	10	17	20	32	k + 2	4	100

1 Find the value of each of X and k

« X = 30 , k = 15 »

2 Graph the ascending and descending cumulative curves on one figure , then calculate the median.

« 41 »



From the school book



Remember

Understand

Apply

Problem Solving

1 Choose the correct answer from those given :

- 1 The mode of a set of values is

(a) $\frac{\text{sum of values}}{\text{number of these values}}$

(b) the most common value.

(c) the middle value after rearranging the values ascendingly or descendingly.

(d) the point of intersection of the ascending and descending cumulative frequency curves.

- 2 The mode of the values : 5 , 3 , 8 , 5 , 9 is

(a) 3

(b) 5

(c) 8

(d) 9

- 3 The mode of the values : 8 , 7 , 8 , 7 , 6 , 5 , 8 is

(a) 8

(b) 7

(c) 6

(d) 5

- 4 If the mode of the values : 4 , a , 5 , 3 is 3 , then a =

(a) 5

(b) 4

(c) 3

(d) 6

- 5 If the mode of the values : 12 , 7 , $X + 1$, 7 , 12 is 7 , then $X =$

(a) 12

(b) 11

(c) 7

(d) 6

- 6 If the mode of the values : 4 , 11 , 8 , $2X$ is 4 , then $X =$

(a) 1

(b) 2

(c) 4

(d) 8

- 2** A factory has 600 workers. A sample of 120 workers is taken such that it represents the all groups very well. It is found that the distribution of their ages in years is as the following table :

Age	25–	30–	35–	40–	45–	50–	Total
Number of workers	12	17	18	40	25	8	120

Draw the histogram , then deduce the mode age.

« 43 years »

- 3** The following table shows the frequency distribution of marks of 100 pupils in an exam :

Sets of marks	10–	14–	18–	22–	26–	30–	34–	Total
Number of pupils	2	10	15	40	25	6	2	100

Find the mode mark using the histogram of this distribution.

« 24.5 marks »

- 4** The following is the frequency distribution of 100 workers in one of the factories according to their daily wages :

Sets of wages in pounds	10–	15	20–	25	30–	35–	40	Total
Number of workers	6	12	16	24	20	14	8	100

Draw the histogram of this frequency distribution , then deduce the mode wage of the worker.

« 28.5 pounds »

- 5** Find the mode of the following frequency distribution for the scores of 40 students in an examination :

Sets of marks	30–	40	50–	60	70–	80	Total
Frequency	3	4	12	8	7	6	40

« 57 »

- 6** The following is the frequency distribution of ages of 45 persons :

Sets of ages in years	12–	14–	16–	18–	20–	22	24–	Total
Number of persons	5	7	8	12	6	4	3	45

Find the mode age.

« 18.8 years »

- 7 The following table shows the frequency distribution of the heights of 200 students :

Height in cm.	110–	115–	120–	125	130–	135–	140	Total
Number of students	10	12	28	35	60	40	15	200

Graph the frequency histogram , then find the mode height.

« 132.75 cm »

- 8 The following table shows the frequency distribution of 102 cows according to the weekly amount of milk in gallons :

Sets of milk in gallons	14	16–	18	20–	22–	24	Total
Number of cows	8	16	28	20	18	12	102

Use the histogram of this distribution to find the mode of the weekly amount of milk.

« 19.2 gallons »

- 9 The following table shows the frequency distribution of marks of 100 pupils in mathematics at the end of the year :

Marks	15–	20	25–	30	35–	40–	45–	50–	55–	Total
Number of pupils	4	6	8	12	16	20	22	7	5	100

Graph the histogram of that distribution , then find the mode mark.

« 45.5 marks »

- 10 The following table shows the frequency distribution of the weights of 100 children in kg. :

Weight in kg.	10–	14–	18	22–	26	30–	Total
Frequency	5	15	30	24	17	9	100

Find the mode weight.

« 20.8 kg. »

- 11 The following table shows the frequency distribution of the weights of 50 students in kg. :

Weight in kg.	30–	35–	40–	45–	50–	55–	Total
Number of students	$k + 4$	$3k$	$4k$	$3k + 1$	$3k - 1$	$k + 1$	50

- 1 Find the value of k

« 3 »

- 2 Graph the frequency histogram , then find the mode.

« 43 kg. »

- 12 The following table shows the frequency distribution with equal range sets for the weekly wages of 100 workers in a factory :

Sets of wages in L.E.	70–	80–	90	100	X –	120	130
Number of workers	10	13	$k - 4$	20	16	14	11

- Find : 1 The value of each of X and k « $X = 110$, $k = 20$ »
 2 The mode of wages in L.E. « 105 pounds »

- 13 The following is the frequency distribution of 100 workers of building according to the number of weekly working hours :

Sets of working hours	35	45	55–	65–	75–	85–	Total
Number of workers	15	30	23	20	8	4	100

The required is finding :

- 1 The mean. « 58.8 hours »
 2 The median. « 57.5 hours »
 3 The mode. « 52 hours »

- 14 The following is the frequency distribution of the weekly bonus of 100 workers in a factory :

Bonus in L.E.	20–	30–	40–	50–	60–	70–
No. of workers	10	k	22	26	20	8

- 1 Calculate the value of k « 14 »
 2 Find the mean of this distribution. « 50.6 pounds »
 3 Find the mode value of the weekly bonus using the histogram. « 54 pounds »

- 15 The following table shows the frequency distribution for the weights of 50 students in kg. at a school :

Weight in kg.	30–	35–	40–	45–	50–	55–	Total
Number of students	7	$3k$	$4k$	10	8	4	50

- 1 Find the value of k « 3 »
 2 Calculate the mean. « 44 kg. »
 3 Draw the ascending cumulative frequency curve.
 4 Draw the histogram and find the mode of weights. « 43 kg. »
 5 Find the median. « 43.5 kg. »

Summary of Unit 3



★ You can represent the frequency table with sets by the ascending or the descending cumulative frequency curves.

★ The range is the difference between the greatest value and the smallest value.

★ The mean of a set of values = $\frac{\text{The sum of values}}{\text{Number of values}}$

★ The mean of frequency distribution with sets = $\frac{\text{The sum of } (X \times f)}{\text{The sum of } f}$

where f is frequency and X is the centre of the set and equals $\frac{\text{its lower limit} + \text{its upper limit}}{2}$

★ The median is the middle value in a set of values after arranging it ascendingly or descendingly such that the number of values which are less than it is equal to the number of values which are greater than it ,

if the values number is odd , then the median is the value lying in the middle exactly ,

if the values number is even , then the median = $\frac{\text{The sum of the two values lying in the middle}}{2}$

★ The intersection point of the ascending and the descending cumulative frequency curves determines the median on the sets axis.

★ The mode of a set of values is the most common value in the set , or it is the value which is repeated more than any other values.

Exams on Unit Three



Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1 The median of the values : 15 , 22 , 9 , 11 , 33 is

- (a) 9 (b) 15 (c) 18 (d) 90

2 The arithmetic mean of the values : 19 , 32 , 27 , 6 , 6 is

- (a) 90 (b) 32 (c) 18 (d) 6

3 If the mode of the values : 4 , 5 , a , 3 is 3 , then a =

- (a) 3 (b) 4 (c) 5 (d) 6

4 If the median of the values : $k + 1$, $k + 2$, $k + 5$, $k + 4$, $k + 3$ is 13 , then $k = \dots\dots\dots$

- (a) 2 (b) 5 (c) 10 (d) 13

5 If the arithmetic mean of the marks of five pupils is 30 , then the sum of their marks equals .. . marks.

- (a) 15 (b) 6 (c) 100 (d) 150

2 Complete the following :

1 If the order of the median of a set of values is the fifth , then the number of values equals .. .

2 If the mode of the values : 15 , 9 , $x + 6$, 9 , 15 is 9 , then $x = \dots\dots\dots$

3 The point of intersection of the ascending and descending cumulative curves determines on the horizontal axis.

4 If the arithmetic mean of the values : 1 , 6 , 4 , 4 , 5 k is 7 , then $k = \dots\dots\dots$

5 The centre of the set whose lower boundary is 2 and its upper boundary is 6 , is .. .

- 3 The following table shows the frequency distribution of marks of 10 students in a mathematics exam :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	1	2	4	2	1	10

- 1 Find the arithmetic mean of marks.
- 2 If the mark of success is 30 , find the number of failure students.

- 4 Find using the following frequency distribution :

Sets	0 –	2 –	4 –	6 –	k –	Total
Frequency	m	5	8	7	6	30

- 1 The values of k and m.
- 2 The median using the ascending cumulative frequency curve.

- 5 Find the mode of the following frequency distribution of marks of 40 students in an exam :

Sets of marks	30 –	40	50 –	60 –	70 –	80 –	Total
Frequency	4	8	12	7	5	4	40

Model 2

Answer the following questions :

- 1 Choose the correct answer from those given :

- 1 The order of the median of the values : 4 , 5 , 6 , 7 and 8 is the
 (a) third. (b) fourth. (c) fifth. (d) sixth.
- 2 If the arithmetic mean of the values : 18 , 23 , 29 , $2k - 1$ and k is 18 , then $k = \dots\dots\dots$
 (a) 1 (b) 7 (c) 29 (d) 90
- 3 The mode of the values : 14 , 11 , 10 , 11 , 14 , 15 and 11 is
 (a) 14 (b) 10 (c) 11 (d) 15
- 4 The arithmetic mean of the values : $3 - a$, 5 , 1 , 4 and $2 + a$ equals
 (a) 1 (b) 2 (c) 3 (d) 15
- 5 If the centre of a set is 10 and its lower boundary is 4 , then its upper boundary is
 (a) 10 (b) 4 (c) 7 (d) 16

2 Complete the following :

- 1 The point of intersection of the ascending and the descending cumulative frequency curves determines on the vertical axis.
- 2 The most common value of a set of values is called
- 3 If the arithmetic mean of a frequency distribution is 35.7 and the total of frequencies is 200 , then the total of the products of frequencies of each set by its centre is
- 4 If the order of the median of a set of values is the ninth , then the number of these values is
- 5 If the mode of the values : 9 , 8 , 9 , y , 8 is 8, then $\sqrt[3]{y} = \dots$

3 Find the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	3	10	12	10	5	40

4 Find using the following frequency distribution :

Sets .	0 –	2 –	k –	6 –	8 –	Total
Frequency	3	4	7	m + 2	1	20

- 1 The values of k and m
- 2 The median using the descending cumulative frequency curve of this distribution.

5 600 workers at a factory , a sample of 120 workers is chosen such that it represents the society completely to be found that their ages are distributed as the following table :

Sets of ages	25 –	30 –	35 –	40 –	45 –	50 –	Total
Number of workers	12	16	18	40	25	9	120

Graph the histogram , then find the mode age.

A Research Project

On Unit Three



Project aims:

- Organizing data in frequency tables with sets.
- Forming the ascending cumulative frequency table and graphing it.
- Finding the mean and the mode of some data organized in a frequency table with sets.
- Finding the median of a frequency distribution with sets.
- Appreciating the role of statistics in practical life.

Do a research project on the following topic :

"Statisticians use several measurement tools to measure the central tendency, as the mean, the median and the mode".

Discuss the following points using available resources :

- 1 Define the mean, the median and the mode.
- 2 Record the marks of your mates in class in a test of mathematics, then do the following :
 - * Organize this data in a tally table, then form the frequency table with sets.
 - * From the frequency table with sets, calculate the mean of the marks of your mates.
 - * Using the frequency table with sets, draw the histogram and then find the mode mark.
 - * Form the ascending cumulative frequency table, then represent it by the ascending cumulative frequency curve. At last, find the median mark.

SKILLS

TIMSS Problems

Accumulative basic skills

1 Complete the following :

1 A turtle covers 80 metres per hour , then it covers 8 metres in minutes.

2 The sum of the real numbers in the interval $[-12, 12]$ equals

3 If $\bigcirc + \square = 20$, $\bigcirc + \bigcirc + \square = 35$, then $\bigcirc =$

In three games of bowling , Sara gained 139 , 143 , 144 points , then the number of points she needs in the 4th game so that the mean of points is 145 , is . . .

Two boxes of apples , the sum of their weights is 54 kg. The first has 12 kg. more than the second , then the number of kilograms in the second box is kg.

6 $300 \div 200 = 1 \div$

7 $(301 + 302 + 303 + \dots + 325) - (1 + 2 + 3 + \dots + 25) =$

8 If four times a number is 48 , then $\frac{1}{3}$ this number is

Gamal has 3 sisters and 5 brothers , his sister Sara has x sisters and y brothers , then $x \cdot y =$

If $a + b + c = 26$, $a + b = 15$, $b + c = 20$, then $b =$

Three girls can perform a work in 36 hours , then the needed hours for four girls to perform the same work is hours.

12 If

$$\begin{array}{r}
 \square \quad \square \quad \bigcirc \\
 \square \quad \square \quad \triangle \\
 + \quad \square \quad \triangle \quad \square \\
 \hline
 2 \quad 0 \quad 1 \quad 6
 \end{array}$$

, then $\square =$... , $\triangle =$, $\bigcirc =$

2 Choose the correct answer from the given ones :

1 The number 3.015 lies on the number line between

- (a) $\frac{5}{2}$, 3 (b) $\frac{7}{2}$, $\frac{11}{3}$ (c) 3, $\frac{16}{5}$ (d) 3.12, 3.15

2 Which of the following numbers lies between 0.07, 0.08 ?

- (a) 0.00075 (b) 0.0075 (c) 0.075 (d) -0.75

3 Which of the following is different in value ?

- (a) $1 \div 9 + 9 - 1$ (b) $1 + 9 \div 9 - 1$ (c) $1 - 9 + 9 \times 1$ (d) $1 \times 9 - 9 + 1$

4 If X is a negative number, which of the following is a positive number ?

- (a) X^2 (b) X^3 (c) $2X$ (d) $\frac{X}{2}$

5 The greatest number of the following is

- (a) -1.25 (b) -0.125 (c) -0.0125 (d) -0.00125

6 The best estimation to the number opposite to X is



- (a) 1.1 (b) 1.2 (c) 1.5 (d) 1.7

7 If 10% of X equals y , then $X =$

- (a) $0.1y$ (b) y (c) $9y$ (d) $10y$

8 If $X = (-2)^4$, $y = -2^4$, then

- (a) $X = y$ (b) $X > y$ (c) $X < y$ (d) $X \leq y$

9 $\sqrt[4]{81 \times 81 \times 81 \times 81} =$

- (a) 3 (b) 9 (c) 27 (d) 81

10 For any number k , then $k + k + (k \times k \times k)$ can be written as

- (a) $2k^2 + 3k$ (b) $5k$ (c) k^5 (d) $2k + k^3$

11 A machine produces two kinds of rods, one is red and of length (10 ± 0.5) cm. and the other is white and of length (6 ± 0.5) cm.



If we put two rods as shown in the opposite figure, then the smallest difference between their lengths may be

- (a) 4 cm. (b) 5 cm. (c) 3 cm. (d) 8.5 cm.

12 All numbers divisible by 4 and 15 are divisible by

- (a) 6 (b) 8 (c) 24 (d) 45

Second

Geometry

Revision 104

Unit

4

Medians of Triangle –
Isosceles Triangle. 108

Unit

5

Inequality. 149

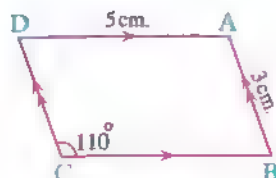
Accumulative Basic skills
"TIMSS Problems" 173



Revision

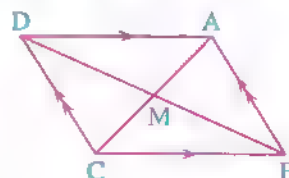
1 Complete the following using the given data of each figure :

1



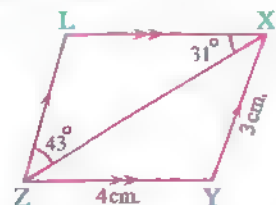
$BC = \dots\dots\dots \text{ cm.}$, $CD = \dots\dots\dots \text{ cm.}$,
 $m(\angle A) = \dots\dots\dots^\circ$ and $m(\angle D) = \dots\dots\dots^\circ$

2 $AC = 8 \text{ cm.}$ and $BM = 7 \text{ cm.}$



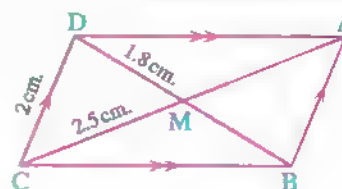
$AM = \dots\dots\dots \text{ cm.}$ and
 $BD = \dots\dots\dots \text{ cm.}$

3



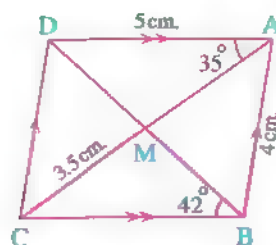
$m(\angle Y) = \dots\dots\dots^\circ$
 , the perimeter of $\square XYZL = \dots\dots\dots \text{ cm.}$

4



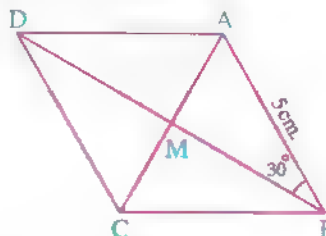
The perimeter of $\triangle ABM = \dots\dots\dots \text{ cm.}$

5



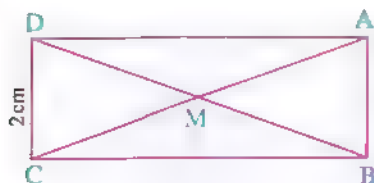
The perimeter of $\triangle ABC = \dots\dots\dots \text{ cm.}$,
 $m(\angle AMB) = \dots\dots\dots^\circ$

6 ABCD is a rhombus



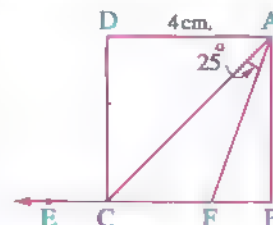
$AD = \dots\dots\dots \text{ cm.}$,
 $m(\angle BAM) = \dots\dots\dots^\circ$

7 ABCD is a rectangle and $AC = 6 \text{ cm.}$



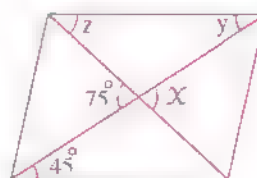
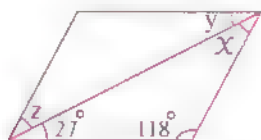
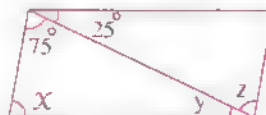
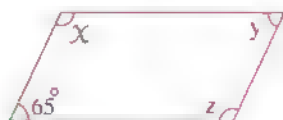
$AB = \dots\dots\dots \text{ cm.}$, $DM = \dots\dots\dots \text{ cm.}$,
 the perimeter of $\triangle ABM = \dots\dots\dots \text{ cm.}$

8 ABCD is a square , $E \in \overrightarrow{BC}$

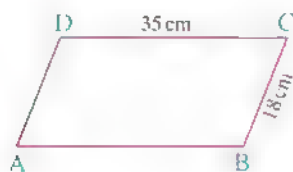


The perimeter of the
 square ABCD = $\dots\dots\dots \text{ cm.}$,
 $m(\angle ACE) = \dots\dots\dots^\circ$,
 $m(\angle AFC) = \dots\dots\dots^\circ$

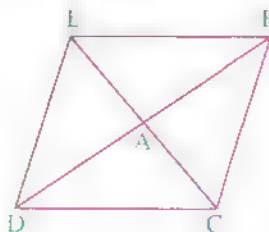
2 Find the values of x , y and z in each of the following parallelograms :



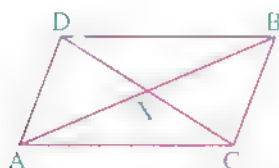
3 Find the length of AB in each of the following parallelograms :



$BD = 15\text{ cm}$.



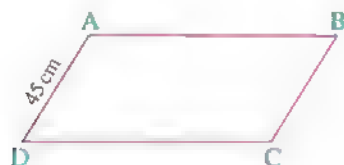
$AX = 7\text{ cm}$.



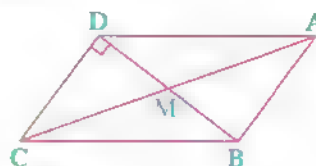
$AB = \frac{1}{3} AD$



5 $BC = \frac{1}{2} AB$



6 $BC = 15\text{ cm}$
and $BM = 6\text{ cm}$.



REVISION

4 Find the values of x , y and z in each of the following figures :

Rectangle



Fig. (1)

Square

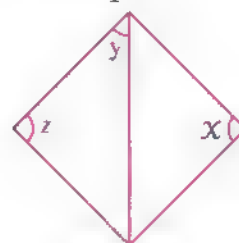


Fig. (2)

Rhombus

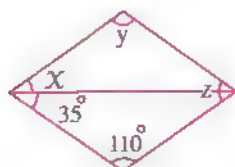


Fig. (3)

Rectangle

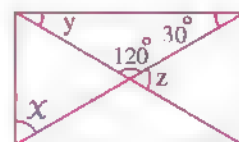
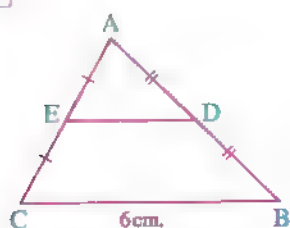


Fig. (4)

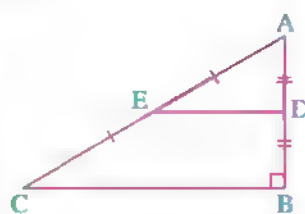
5 Complete the following :

1



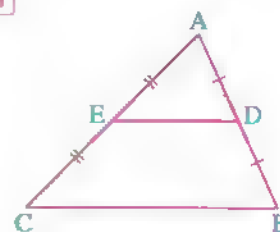
If D and E are the midpoints of \overline{AB} and \overline{AC} respectively ,
 $BC = 6 \text{ cm.}$, then
 $DE = \dots\dots\dots \text{ cm.}$

2



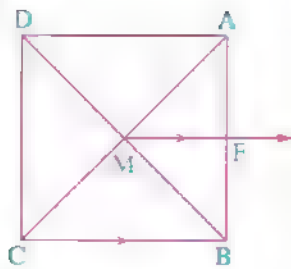
If $m(\angle B) = 90^\circ$,
 D and E are the midpoints of \overline{AB} and \overline{AC} respectively ,
 then $m(\angle ADE) = \dots\dots\dots^\circ$

3



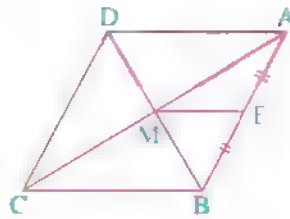
If D and E are the midpoints of \overline{AB} and \overline{AC} respectively , and the perimeter of $\triangle ABC = 24 \text{ cm.}$, then the perimeter of $\triangle ADE = \dots\dots\dots \text{ cm.}$

4



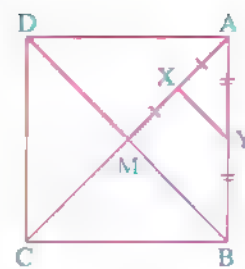
If the perimeter of the square $ABCD = 20$ cm. ,
 $\overline{MF} \parallel \overline{CB}$,
 then $AF = \dots\dots\dots$ cm.

5



If $ABCD$ is a rhombus
 its perimeter is 24 cm. ,
 E is the midpoint of \overline{AB} ,
 then $ME = \dots\dots\dots$ cm.

6

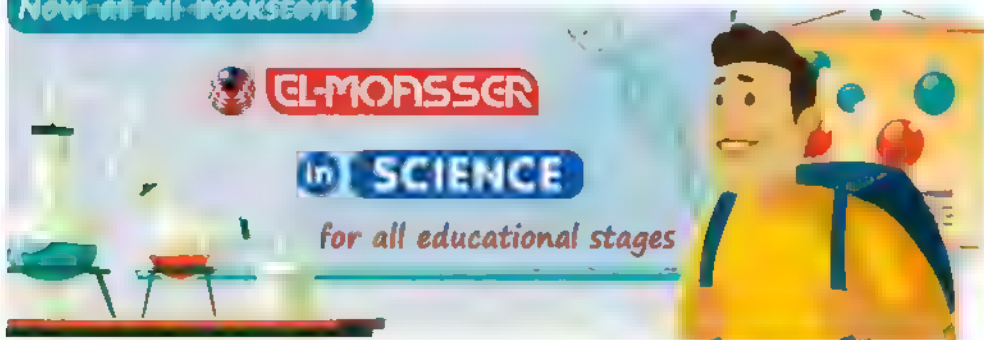


If $ABCD$ is a square ,
 X and Y are the midpoints
 of \overline{AM} and \overline{AB} respectively ,
 $AC = 12$ cm. ,
 then $XY = \dots\dots\dots$ cm. ,
 $m(\angle AYX) = \dots\dots\dots^\circ$

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UNIT

4

Medians of Triangle – Isosceles Triangle

Exercises of the unit :

1. Medians of triangle.
 2. Medians of triangle "Follow".
 3. The isosceles triangle.
 4. The converse of the isosceles triangle theorem.
 5. Corollaries of the isosceles triangle theorems.
- Summary of unit four.
 - Unit exams.

 A research project on unit four



Scan the
QR code
to solve an
interactive
test on each
lesson .



● Remember

● Understand

○ Apply

● Problem Solving

1 Complete the following :

- 1 In $\triangle ABC$, if D is the midpoint of \overline{BC} , then \overline{AD} is called
- 2 The number of medians of the triangle is
- 3 The medians of the triangle intersect at
- The point of concurrence of the medians of the triangle divides each median in the ratio : from its base.
- The point of concurrence of the medians of the triangle divides each median in the ratio : from the vertex.
- The point of intersection of the medians of the triangle divides each of them in the ratio 2 : from the base.
- The point of intersection of medians of the triangle divides each of them in the ratio : 8 from the vertex.

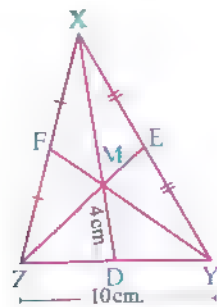
2 Choose the correct answer from those given :

- 1 The number of medians of the obtuse-angled triangle is
(a) zero (b) 1 (c) 2 (d) 3
- If \overline{YD} is a median in $\triangle XYZ$, M is the point of intersection of medians, then $MD = \dots\dots\dots YM$
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$
- If M is the point of intersection of medians of $\triangle ABC$, \overline{BD} is a median, then $BD : MD = \dots\dots\dots$
(a) 2 : 3 (b) 1 : 3 (c) 3 : 2 (d) 3 : 1

- 4 If \overline{AD} is a median in $\triangle ABC$, M is the point of intersection of medians, then $AD = \dots\dots\dots AM$
 (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$
- 5 If \overline{AD} is a median in $\triangle ABC$ of length 9 cm., M is the point of intersection of medians, then $DM = \dots\dots\dots$ cm.
 (a) 3 (b) 4.5 (c) 6 (d) 9
- 6 If M is the point of intersection of the medians of $\triangle ABC$, \overline{AD} is a median of length 6 cm., then $AM = \dots\dots\dots$ cm.
 (a) 1 (b) 2 (c) 3 (d) 4
- 7 If M is the point of intersection of the medians of $\triangle ABC$, D is the midpoint of \overline{BC} , then $AD = \dots\dots\dots$
 (a) 2 AM (b) $\frac{2}{3}$ MD (c) $\frac{3}{2}$ AM (d) 4 MD

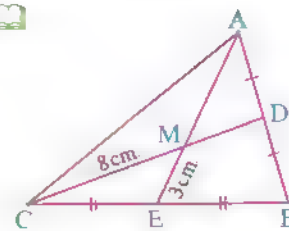
3 Using data given for each of the following figures, find the required below each figure :

1



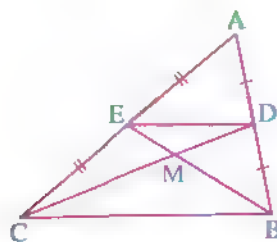
$XM = \dots\dots\dots$ cm. and
 $YD = \dots\dots\dots$ cm.

2



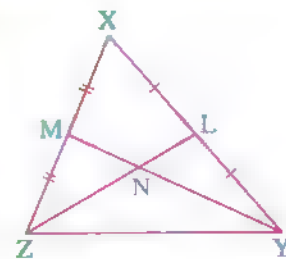
$MA = \dots\dots\dots$ cm.,
 $MD = \dots\dots\dots$ cm.,
 $ME = \dots\dots\dots$ AE
 and $MC = \dots\dots\dots$ CD

3



If $BC = 12$ cm., $BE = 9$ cm.
 and $MC = 8$ cm.,
 then $DE = \dots\dots\dots$ cm.,
 $ME = \dots\dots\dots$ cm. and
 $MD = \dots\dots\dots$ cm.

4



If $LZ = 15$ cm., $YM = 18$ cm.
 and $XY = 20$ cm.,
 then $NL = \dots\dots\dots$ cm.,
 $NY = \dots\dots\dots$ cm. and the perimeter of
 $\triangle NLY = \dots\dots\dots$ cm.

Exercise 1

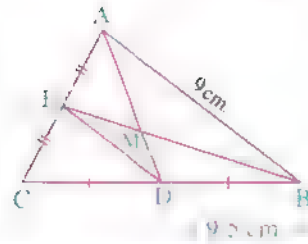
4 In the opposite figure :

ABC is a triangle in which D is the midpoint of \overline{BC}

, E is the midpoint of \overline{AC} and $\overline{AD} \cap \overline{BE} = \{M\}$

If $AD = 6$ cm. and $AB = BE = 9$ cm.

Calculate : The perimeter of $\triangle MDE$



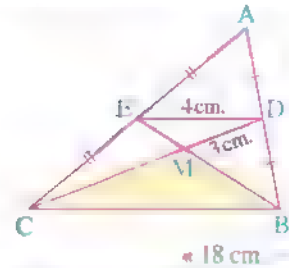
5 In the opposite figure :

If D is the midpoint of \overline{AB} , E is the midpoint of \overline{AC}

and $\overline{BE} \cap \overline{DC} = \{M\}$, $DE = 4$ cm. ,

$DM = 3$ cm. and $BE = 6$ cm.

Find : The perimeter of $\triangle BMC$



6 In the opposite figure :

ABC is a triangle , X is the midpoint of \overline{AB} ,

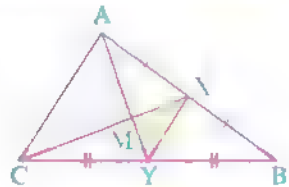
Y is the midpoint of \overline{BC} , $XY = 5$ cm. and $\overline{XC} \cap \overline{AY} = \{M\}$

where $CM = 8$ cm. , $YM = 3$ cm. **Find :**

1 The perimeter of $\triangle MX Y$

2 The perimeter of $\triangle MAC$

« 12 cm. » 24 cm. »



7 In $\triangle ABC$, $BC = 8$ cm. , F and E are the midpoints of \overline{AB} and \overline{AC} respectively and

$\overline{BE} \cap \overline{CF} = \{M\}$ If $BM = 4$ cm. and $CM = 6$ cm. **Find :** The perimeter of $\triangle MFE$

« 9 cm »

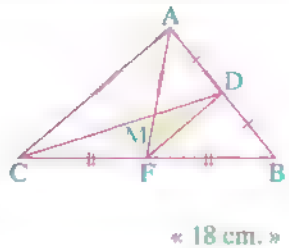
8 In the opposite figure :

\overline{AF} and \overline{CD} are two medians in $\triangle ABC$,

$\overline{AF} \cap \overline{CD} = \{M\}$

If the perimeter of $\triangle AMC = 36$ cm.

Find : The perimeter of $\triangle MFD$



« 18 cm. »

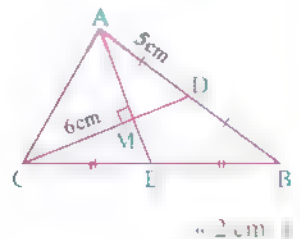
9 In the opposite figure :

M is the point of concurrence of the medians

of $\triangle ABC$, $\overline{AM} \perp \overline{CD}$

, $MC = 6$ cm. , $AD = 5$ cm.

Find : The length of \overline{ME}



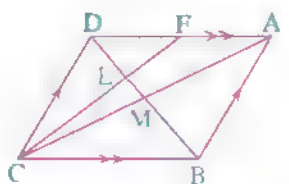
« 2 cm »

10 In the opposite figure :

ABCD is a parallelogram, its diagonals intersect at M ,

$E \in \overline{DM}$ where $DE = 2 EM$, draw \overline{CE} to cut \overline{AD} at F

Prove that : $AF = FD$



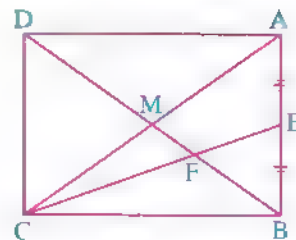
11 In the opposite figure :

ABCD is a rectangle , its diagonals intersect at M ,

E is the midpoint of \overline{AB} , $\overline{CE} \cap \overline{BD} = \{F\}$

1 Prove that : F is the intersection point of the medians of the triangle ABC

2) If $BF = 4$ cm. , find : The length of \overline{AM}



« 6 cm. »

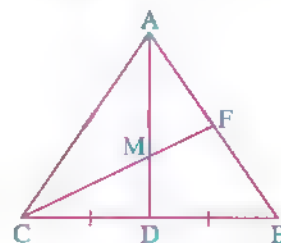
12 In the opposite figure :

ABC is a triangle in which D is the midpoint of \overline{BC} ,

$AB = AC$, $M \in \overline{AD}$ where $AM = \frac{2}{3} AD$ and

$\overline{CM} \cap \overline{AB} = \{F\}$

Prove that : $BF = \frac{1}{2} AC$



13 ABC is a triangle where point D is the midpoint of \overline{BC} and point $M \in \overline{AD}$, $AM = 2 MD$

Draw \overline{CM} to intersect \overline{AB} at point E If $EC = 12$ cm. , then find : The length of \overline{EM} « 4 cm »

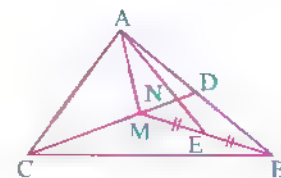
14 In the opposite figure :

$M \in \overline{CD}$, M is the point of concurrence of the medians

of $\triangle ABC$, $N \in \overline{DM}$ where $ND = (x - 1)$ cm.

, $MN = (x + 3)$ cm. , \overline{AN} is drawn to intersect \overline{BM} at E which is the midpoint of \overline{BM}

Find : The length of \overline{MC}



« 24 cm. »

15 ABCD is a parallelogram whose diagonals intersect at M , E is the midpoint of \overline{BC} ,

\overline{DE} intersects \overline{AC} at F

Prove that : 1 \overline{BF} bisects \overline{CD}

2 $CF = \frac{1}{3} AC$



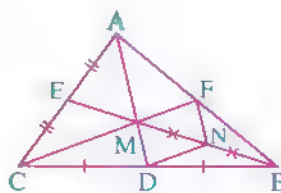
For excellent pupils

16 In the opposite figure :

\overline{AD} and \overline{BE} are medians in the triangle ABC intersecting at M ,

$\overline{CM} \cap \overline{AB} = \{F\}$, if N is the midpoint of \overline{MB}

Prove that : The figure FNDM is a parallelogram.



17 In the opposite figure :

- ABC is a triangle in which D is the midpoint of \overline{BC}
 , $M \in \overline{AD}$ where $AM = 2 MD$
 , $\overline{BM} \cap \overline{AC} = \{E\}$
 , $ME = 2 \text{ cm.}$, draw $\overline{DF} \parallel \overline{BE}$ and cut \overline{AC} at F

Find : The length of \overline{DF}



18 In the opposite figure :

- ABC is a triangle in which D is the midpoint of \overline{BC}
 and E is the midpoint of \overline{BD} , draw $\overline{DE} \parallel \overline{AC}$
 and cut \overline{AE} at M and \overline{AB} at F

Prove that : $DM = \frac{1}{3} AC$



19 ABC is a triangle , D is the midpoint of \overline{AB} and E is the midpoint of \overline{AC}

If $\overline{CD} \cap \overline{BE} = \{M\}$ Draw \overline{AM} to intersect \overline{BC} at F

Prove that : The figure DBFE is a parallelogram.



EL-MOFASSER

Notebook

Free part



- Accumulative tests.
- Final revision.
- Final examinations.



From the school book



Remember

Understand

Apply

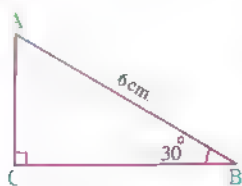
Problem Solving

1 Complete the following :

- 1 The number of medians in the right-angled triangle is
- 2 The length of the median from the vertex of the right angle in the right-angled triangle equals
- 3 If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is
- 4 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals
- 5 The length of the hypotenuse in thirty and sixty triangle equals the length of the side opposite to the angle whose measure is 30°
- 6 The length of the hypotenuse in the right-angled triangle equals the length of the median drawn from the vertex of the right angle.

2 Using data given for each of the following figures , find the required below each figure :

1



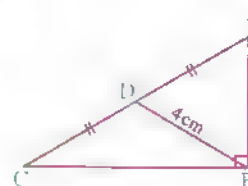
AC = cm.

2



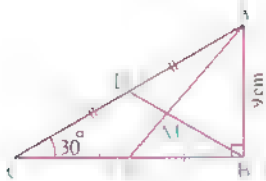
XZ = cm.

3



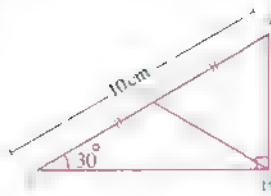
AC = cm.

4



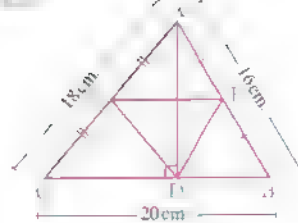
AC = cm. ,
BD = cm. ,
MD = BD
and MD = cm.

5



BD = cm. ,
AB = cm.
and the perimeter of
 $\triangle ABD$ = cm.

6



DF = cm. ,
DE = cm. ,
FE = cm.
and the perimeter of
 $\triangle DEF$ = cm.

3 Choose the correct answer from those given :

- 1 In the right-angled triangle , the ratio between the length of the median drawn from the vertex of the right angle and the length of the hypotenuse is
(a) 2 : 1 (b) 1 : 2 (c) 2 : 3 (d) 3 : 2
- 2 In the thirty-sixty triangle , the ratio between the length of the hypotenuse and the length of the side opposite to the angle of measure 30° is
(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3
- 3 In the thirty-sixty triangle , the ratio between the length of the median drawn from the vertex of the right angle and the length of the side opposite to the angle of measure 30° is
(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 2 : 3
- 4 ABC is a right-angled triangle at B , D is the midpoint of \overline{AC} , then $BD = \dots\dots\dots$
(a) $\frac{1}{2} AC$ (b) AC (c) $\frac{1}{2} BC$ (d) AB
- 5 ABC is a triangle in which $m(\angle A) = 90^\circ$, $AC = \frac{1}{2} BC$, then $m(\angle C) = \dots\dots\dots$
(a) 30° (b) 60° (c) 90° (d) 120°
- 6 In $\triangle ABC$, $m(\angle B) = 90^\circ$, if $2AB - AC = 0$, then $m(\angle C) = \dots\dots\dots$
(a) 30° (b) 60° (c) 90° (d) 120°

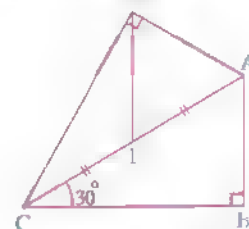
4 In the opposite figure :

$m(\angle ABC) = m(\angle ADC) = 90^\circ$,

$m(\angle ACB) = 30^\circ$ and

E is the midpoint of \overline{AC}

Prove that : $AB = DE$



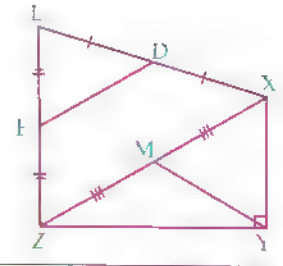
5 In the opposite figure :

$m(\angle XYZ) = 90^\circ$, D is the midpoint of \overline{XL} ,

E is the midpoint of \overline{ZL} and

M is the midpoint of \overline{XZ}

Prove that : $DE = YM$



6 In the opposite figure :

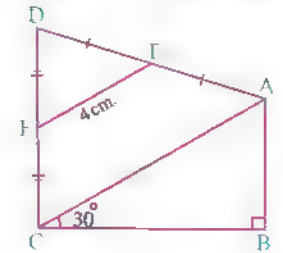
ABCD is a quadrilateral in which $m(\angle B) = 90^\circ$,

E is the midpoint of \overline{AD} , F is the midpoint of \overline{CD} ,

$m(\angle ACB) = 30^\circ$ and $EF = 4$ cm.

Find by proof : The length of \overline{AB}

« 4 cm. »



7 In the opposite figure :

$m(\angle BAC) = m(\angle CBE) = 90^\circ$

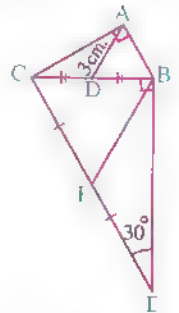
, $m(\angle BEC) = 30^\circ$

, D and F are the midpoints

of \overline{BC} and \overline{CE} respectively and $AD = 3$ cm.

Find : The length of \overline{BF}

« 6 cm. »



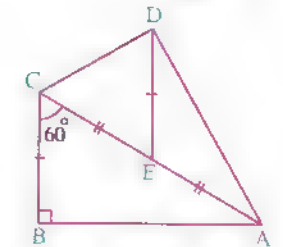
8 In the opposite figure :

ABC is a right-angled triangle at B, $m(\angle ACB) = 60^\circ$,

E is the midpoint of \overline{AC} and

$DE = BC$

Prove that : $m(\angle ADC) = 90^\circ$



9 In the opposite figure :

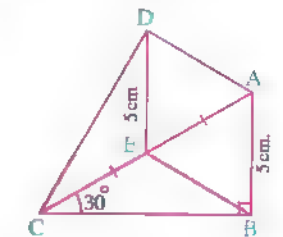
ABC is a right-angled triangle at B,

$m(\angle ACB) = 30^\circ$, $AB = 5$ cm. and

E is the midpoint of \overline{AC}

If $DE = 5$ cm.,

prove that : $m(\angle ADC) = 90^\circ$



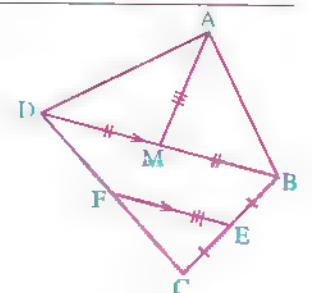
10 In the opposite figure :

ABD is a triangle, M is the midpoint of \overline{BD} ,

E is the midpoint of \overline{BC} ,

$F \in \overline{CD}$, $\overline{EF} \parallel \overline{BD}$ and $AM = EF$

Prove that : $m(\angle BAD) = 90^\circ$



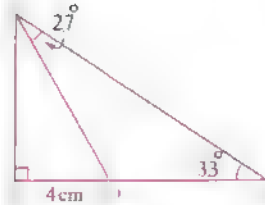
Exercise 2

11 In the opposite figure :

ABC is a triangle in which $m(\angle B) = 33^\circ$
 $m(\angle C) = 90^\circ$, $D \in \overline{BC}$ where $CD = 4$ cm.
 $m(\angle BAD) = 27^\circ$

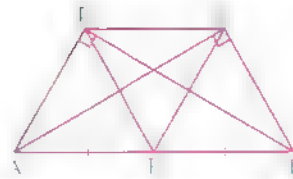
Find : The length of \overline{AD}

« 8 cm »



12 In the opposite figure :

ADB is a right-angled triangle at D ,
 ACB is a right-angled triangle at C and E is the midpoint of \overline{AB}
Prove that : $\triangle CED$ is an isosceles triangle.

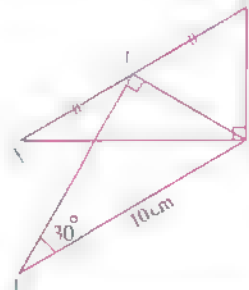


13 In the opposite figure :

$m(\angle YLE) = 90^\circ$, $m(\angle E) = 30^\circ$, $YE = 10$ cm.,
 $m(\angle XYZ) = 90^\circ$ and
 L is the midpoint of \overline{XZ}

Find by proof : The length of \overline{XZ}

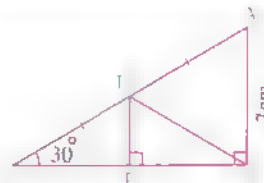
« 10 cm. »



14 In the opposite figure :

ABC is a right-angled triangle at B , D is the midpoint of \overline{AC} , $\overline{DE} \perp \overline{BC}$, $AB = 7$ cm. and $m(\angle C) = 30^\circ$

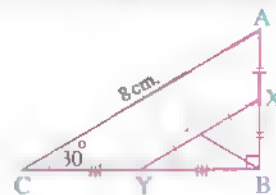
Find the length of each of : \overline{BD} and \overline{DE}



15 In the opposite figure :

ABC is a triangle in which $m(\angle ABC) = 90^\circ$, $m(\angle C) = 30^\circ$,
 X , Y and Z are the midpoints of \overline{AB} , \overline{BC} and \overline{XY}
 respectively and $AC = 8$ cm.

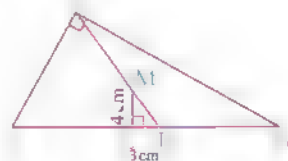
Find the length of each of : \overline{AB} , \overline{XY} and \overline{BZ}



16 In the opposite figure :

ABC is a right-angled triangle at A
 M is the point of concurrence of its medians
 $E \in \overline{DC}$ where $\overline{ME} \perp \overline{DC}$, $DE = 3$ cm.
 and $ME = 4$ cm.

Find : The length of \overline{BC}

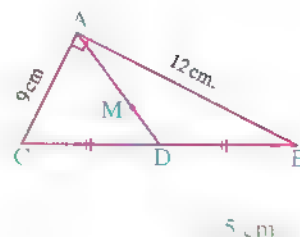


17 In the opposite figure :

$m(\angle BAC) = 90^\circ$, $AB = 12$ cm., $AC = 9$ cm.

\overline{AD} is a median of $\triangle ABC$ and M is the point of concurrence of the medians of $\triangle ABC$

Find : The length of \overline{AM}



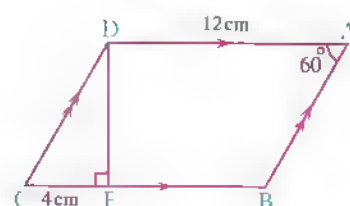
18 In the opposite figure :

ABCD is a parallelogram in which

$m(\angle A) = 60^\circ$, $\overline{DE} \perp \overline{BC}$

, $AD = 12$ cm. and $EC = 4$ cm.

Find : The perimeter of the parallelogram ABCD

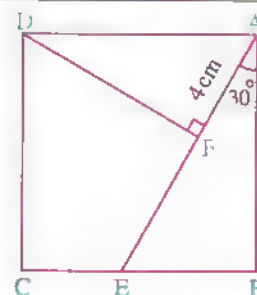


19 In the opposite figure :

ABCD is a square, $E \in \overline{BC}$ where $m(\angle BAE) = 30^\circ$ and

$\overline{DF} \perp \overline{AE}$ If $AF = 4$ cm.

Calculate : The area of the square ABCD



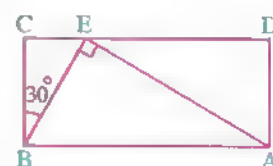
20 In the opposite figure :

ABCD is a rectangle, $E \in \overline{DC}$

where $m(\angle CBE) = 30^\circ$

and $m(\angle AEB) = 90^\circ$

Prove that : $CE = \frac{1}{4} AB$



21 In the opposite figure :

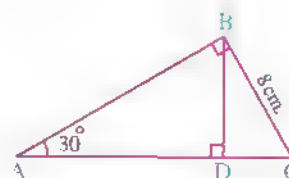
ABC is a right-angled triangle at B ,

$m(\angle A) = 30^\circ$,

$D \in \overline{AC}$ such that $\overline{BD} \perp \overline{AC}$

If $BC = 8$ cm.

Find : The length of \overline{AD}



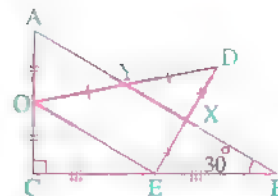
22 In the opposite figure :

ABC is a right-angled triangle at C in which $m(\angle B) = 30^\circ$

, E, O, X, Y are the midpoints of \overline{BC} , \overline{AC}

, \overline{DE} , \overline{DO} respectively

Prove that : $XY = \frac{1}{2} AC$



- 23** ABC is a triangle in which $AB = AC$ and \overline{AD} is drawn to be perpendicular to \overline{BC} where $\overline{AD} \cap \overline{BC} = \{D\}$ If E and F are the two midpoints of \overline{AB} and \overline{AC} respectively, **prove that :** $DE + DF = AB$

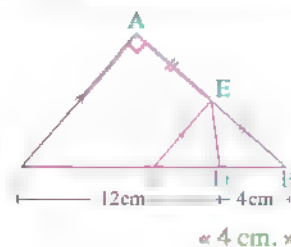
24 In the opposite figure :

ABC is a right-angled triangle at A

, E is the midpoint of \overline{AB} , $O \in \overline{BC}$

where $\overline{EO} \parallel \overline{AC}$, $D \in \overline{BO}$ where $BD = 4 \text{ cm}$, $DC = 12 \text{ cm}$.

Find : The length of \overline{DE}

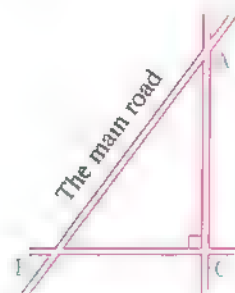


« 4 cm. »

Life Application

- 25** The opposite figure is a sketch for three towns A, B and C such that the distance between the towns A and C is 40 km. and the distance between the towns B and C is 30 km.

If we want to build a service station lying on the main road at the half-way between the towns A and B, also we want to build a road linking this station to the town C, then how long will this road be ?



« 25 km. »

Now try these problems

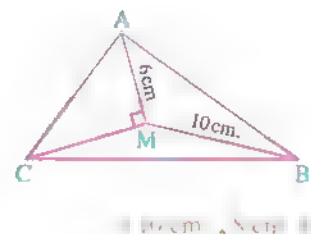
26 In the opposite figure :

M is the point of concurrence of the medians of $\triangle ABC$

, $AM = 6 \text{ cm}$, $BM = 10 \text{ cm}$.

, $m(\angle AMC) = 90^\circ$

Find by proof : 1 The length of \overline{AC} 2 The length of \overline{MC}



- 27** ABCD is a parallelogram, X is an interior point in it such that \overline{DX} bisects $\angle ADC$, \overline{CX} bisects $\angle DCB$, if the point Y is the midpoint of \overline{DC} , **prove that :** $XY = YC$



From the school book



Remember Understand Apply Problem Solving

1 In each of the following, find the value of the symbol used for the measure of the angle :

1



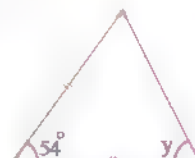
$$x = \dots\dots\dots^\circ$$

2



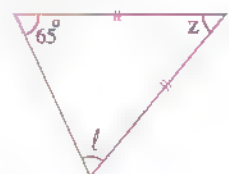
$$x = \dots\dots\dots^\circ$$

3



$$y = \dots\dots\dots^\circ$$

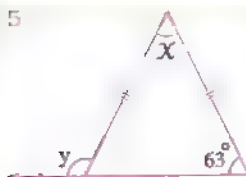
4



$$l = \dots\dots\dots^\circ,$$

$$z = \dots\dots\dots^\circ$$

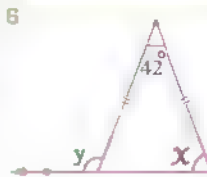
5



$$x = \dots\dots\dots^\circ,$$

$$y = \dots\dots\dots^\circ$$

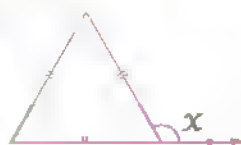
6



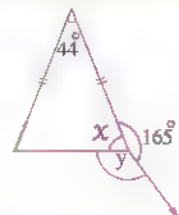
$$x = \dots\dots\dots^\circ,$$

$$y = \dots\dots\dots^\circ$$

7



$$x = \dots\dots\dots^\circ$$



$$x = \dots\dots\dots^\circ,$$

$$y = \dots\dots\dots^\circ$$

2 Complete the following :

- 1 The base angles of the isosceles triangle are
- The measure of each angle in the equilateral triangle equals $\dots\dots\dots^\circ$
- In $\triangle DEF$, if $DE = DF$, then $m(\angle E) = m(\angle \dots\dots\dots)$
- In the isosceles triangle, if the measure of one of the two base angles is 65° , then the measure of its vertex angle equals $\dots\dots\dots^\circ$

- In the isosceles triangle, if the measure of the vertex angle equals 40° , then the measure of one of the two base angles equals $^\circ$
- An isosceles triangle, the measure of its vertex angle is 80° , if the measure of one of its base angles is $(X + 30^\circ)$, then $X =$

3 Choose the correct answer from those given :

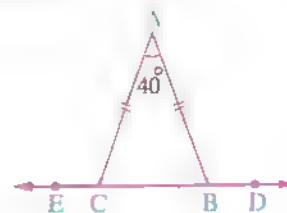
- 1 In $\triangle XYZ$, if $XY = YZ = XZ$, then $m(\angle X) =$
(a) 30° (b) 60° (c) 90° (d) 180°
 - 2 The measure of the exterior angle of the equilateral triangle equals
(a) 60° (b) 90° (c) 120° (d) 180°
 - 3 LMN is a triangle in which $LM = MN$, $m(\angle M) = 70^\circ$, $m(\angle N) =$
(a) 20° (b) 35° (c) 55° (d) 70°
 - 4 In $\triangle ABC$, $AB = AC$, $m(\angle C) = 65^\circ$, then $m(\angle A) =$
(a) 30° (b) 50° (c) 55° (d) 130°
 - 5 In $\triangle XYZ$, $ZY = ZX$, $m(\angle Z) = 120^\circ$, then $m(\angle X) =$
(a) 30° (b) 60° (c) 90° (d) 120°
 - 6 If $\triangle ABC$ is right-angled at A and $AB = AC$, then $m(\angle B) =$
(a) 30° (b) 45° (c) 60° (d) 90°
 - 7 XYZ is an isosceles triangle in which, $m(\angle Y) = 100^\circ$, then $m(\angle Z) =$
(a) 100° (b) 80° (c) 50° (d) 40°
- If the measure of one of the two base angles in the isosceles triangle is 30° , then the triangle is
- (a) obtuse-angled. (b) acute-angled.
(c) right-angled. (d) equilateral.
- 9 In $\triangle ABC$, $AB = AC$, $m(\angle B) = 6X^\circ$, $m(\angle A) = 3X^\circ$, then $X =$
(a) 30° (b) 12° (c) 60° (d) 90°
 - 10 In $\triangle XYZ$, if $XY = XZ$, then the exterior angle at the vertex Z is
(a) acute. (b) obtuse. (c) right. (d) reflex.

4 In the opposite figure :

ABC is an isosceles triangle in which $AB = AC$,
 $m(\angle A) = 40^\circ$ and $D \in \overrightarrow{CB}$, $E \in \overrightarrow{BC}$

1 Find : $m(\angle ABC)$

2 Prove that : $\angle ABD \equiv \angle ACE$



« 70° »

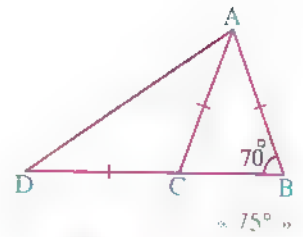
5 In the opposite figure :

$$AB = AC = CD$$

$$\text{and } m(\angle B) = 70^\circ$$

Find by proof :

$$m(\angle BAD)$$



6 In the opposite figure :

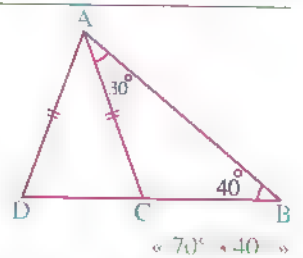
$$m(\angle B) = 40^\circ, m(\angle BAC) = 30^\circ$$

$$\text{and } AC = AD$$

Find by proof :

$$1) m(\angle D)$$

$$2) m(\angle CAD)$$

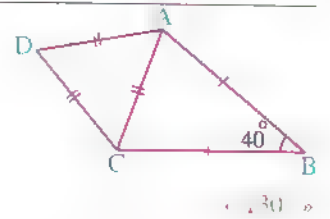


7 In the opposite figure :

$$AD = DC = AC, AB = BC$$

$$\text{and } m(\angle ABC) = 40^\circ$$

Find : $m(\angle BAD)$



8 In the opposite figure :

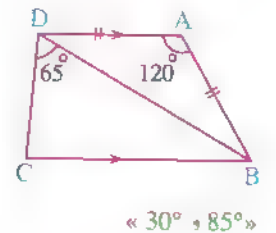
$$AB = AD, \overline{AD} \parallel \overline{BC},$$

$$m(\angle BAD) = 120^\circ \text{ and } m(\angle BDC) = 65^\circ$$

Find :

$$1) m(\angle ADB)$$

$$2) m(\angle C)$$

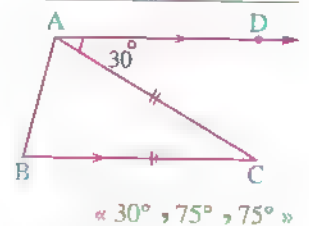


9 In the opposite figure :

ABC is a triangle in which $AC = BC$,

$$\overline{AD} \parallel \overline{BC} \text{ and } m(\angle DAC) = 30^\circ$$

Find : The measures of the angles of $\triangle ABC$

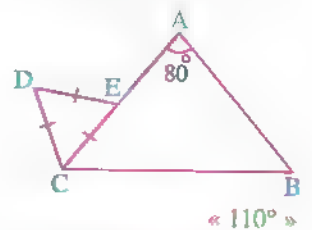


10 In the opposite figure :

$$AB = AC, m(\angle BAC) = 80^\circ$$

$$\text{and } CE = ED = CD$$

Find by proof : $m(\angle BCD)$

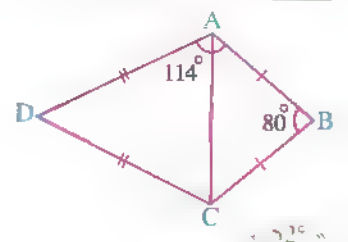


11 In the opposite figure :

$$AB = BC, AD = CD, m(\angle BAD) = 114^\circ$$

$$\text{and } m(\angle B) = 80^\circ$$

Find : $m(\angle ADC)$



Exercise 3

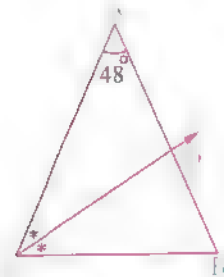
12 In the opposite figure :

$AB = AC$, $m(\angle BAC) = 48^\circ$, \overrightarrow{CD} bisects $\angle BCA$
and intersects \overline{AB} at D

Find :

1 $m(\angle B)$

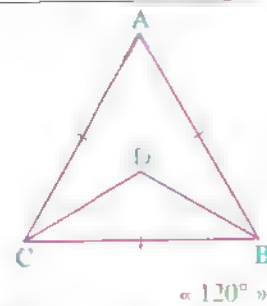
2 $m(\angle BCD)$



13 In the opposite figure :

ABC is an equilateral triangle and the two bisectors of $\angle B$ and $\angle C$ intersect together at D

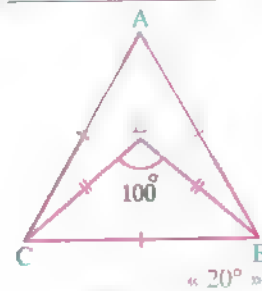
Find : $m(\angle BDC)$



14 In the opposite figure :

ABC is an equilateral triangle , $DB = DC$
and $m(\angle BDC) = 100^\circ$

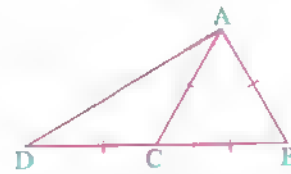
Find by proof : $m(\angle ABD)$



15 In the opposite figure :

ABC is an equilateral triangle.
 $D \in \overline{BC}$ such that $BC = CD$

Prove that : $\overline{BA} \perp \overline{AD}$

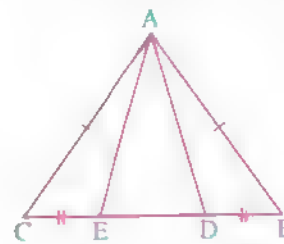


16 In the opposite figure :

ABC is an isosceles triangle in which $AB = AC$, $D \in \overline{BC}$
and $E \in \overline{BC}$, such that $BD = EC$

Prove that : 1 $\triangle ADE$ is an isosceles triangle.

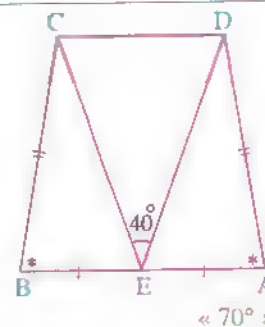
2 $\angle AED \equiv \angle ADE$



17 In the opposite figure :

E is the midpoint of \overline{AB} , $AD = BC$, $m(\angle A) = m(\angle B)$
and $m(\angle DEC) = 40^\circ$

Find : $m(\angle EDC)$

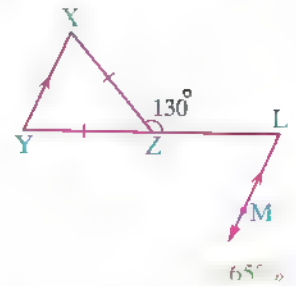


18 In the opposite figure :

$Z \in \overline{LY}$, $XZ = YZ$, $m(\angle LZX) = 130^\circ$

and $\overline{LM} \parallel \overline{XY}$

Find : $m(\angle MLY)$

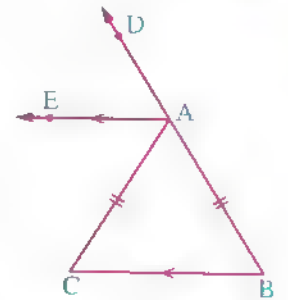


19 In the opposite figure :

$A \in \overline{BD}$, $AB = AC$ and $\overline{AE} \parallel \overline{BC}$

Prove that :

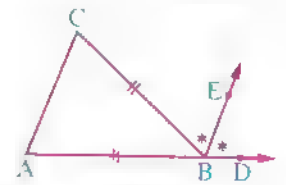
\overline{AE} bisects $\angle DAC$



20 In the opposite figure :

$AB = BC$ and \overline{BE} bisects $\angle CBD$

Prove that : $\overline{BE} \parallel \overline{AC}$



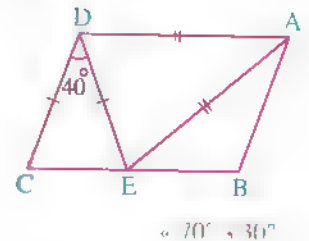
21 In the opposite figure :

$ABCD$ is a parallelogram, $E \in \overline{BC}$,

where $AE = AD$, $DE = DC$ and $m(\angle EDC) = 40^\circ$

Find : 1 $m(\angle AED)$

2 $m(\angle BAE)$



22 In the opposite figure :

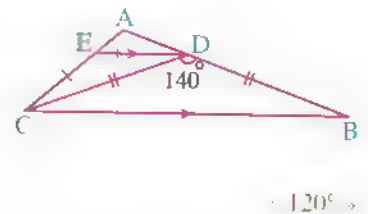
ABC is a triangle in which

$D \in \overline{AB}$, $E \in \overline{AC}$

where $\overline{DE} \parallel \overline{BC}$, $DE = EC$

, $DB = DC$ and $m(\angle BDC) = 140^\circ$

Find : $m(\angle A)$

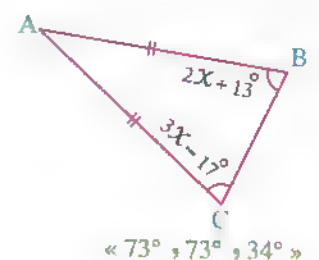


23 In the opposite figure :

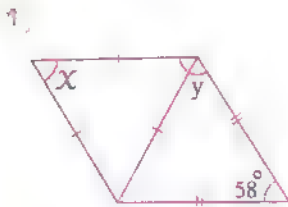
$AB = AC$, $m(\angle B) = 2x + 13^\circ$

and $m(\angle C) = 3x - 17^\circ$

Find : The measures of the angles of $\triangle ABC$

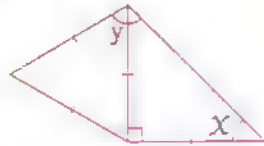


24 In each of the following figures, find the value of the symbol used for the measure of the angle :



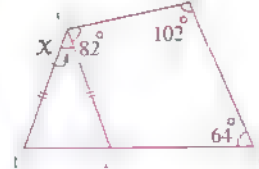
$$x = \dots^\circ,$$

$$y = \dots^\circ$$

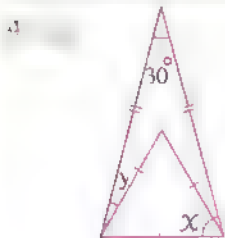


$$x = \dots^\circ,$$

$$y = \dots^\circ$$

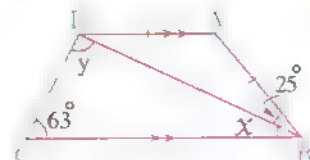


$$x = \dots^\circ$$



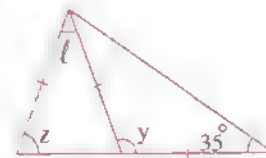
$$x = \dots^\circ,$$

$$y = \dots^\circ$$



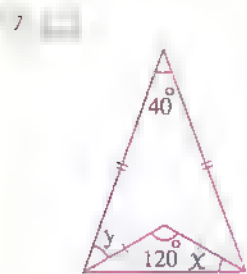
$$x = \dots^\circ,$$

$$y = \dots^\circ$$



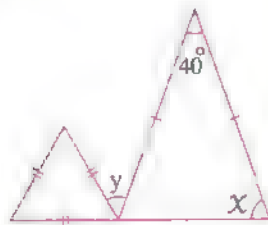
$$y = \dots^\circ, l = \dots^\circ,$$

$$z = \dots^\circ$$



$$x = \dots^\circ,$$

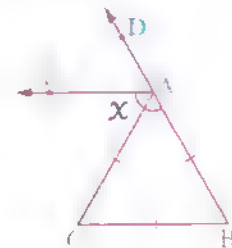
$$y = \dots^\circ$$



$$x = \dots^\circ,$$

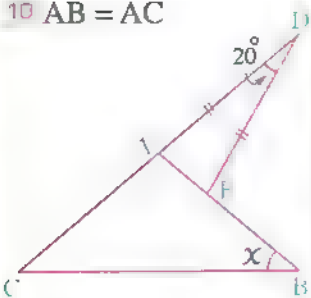
$$y = \dots^\circ$$

9) \overrightarrow{AE} bisects $\angle CAD$

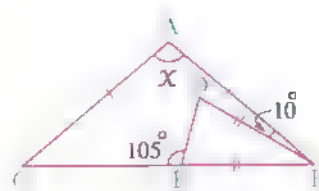


$$x = \dots^\circ$$

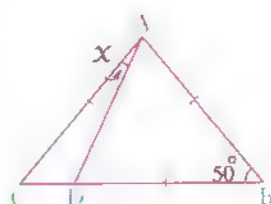
10 $AB = AC$



$$x = \dots^\circ$$



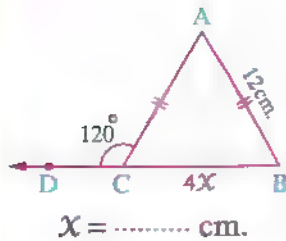
$$x = \dots^\circ$$



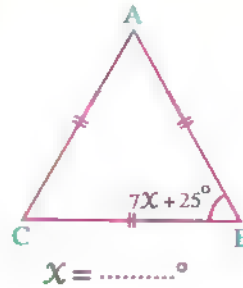
$$x = \dots^\circ$$

25 Find the value of X in each of the following figures :

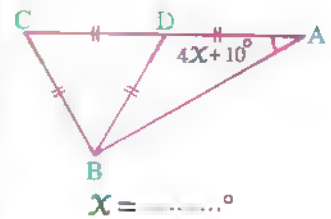
1



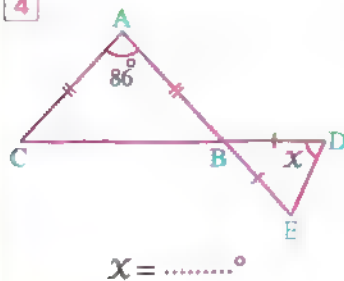
2



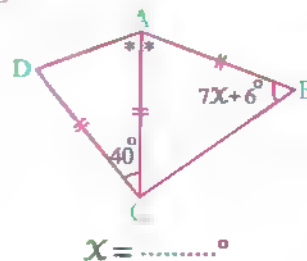
3



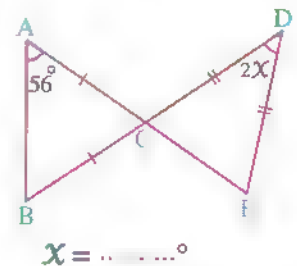
4



5



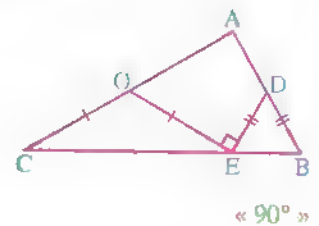
6



26 In the opposite figure :

ABC is a triangle in which $D \in \overline{AB}$, $E \in \overline{BC}$, $O \in \overline{AC}$
where $m(\angle DEO) = 90^\circ$, $DB = DE$ and $OE = OC$

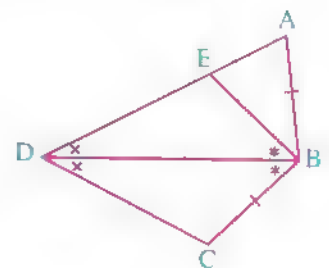
Find : $m(\angle A)$



27 In the opposite figure :

$BA = BC$, $E \in \overline{AD}$
and \overleftrightarrow{BD} bisects each
of $\angle CBE$ and $\angle CDE$

Prove that : $m(\angle A) + m(\angle C) = 180^\circ$

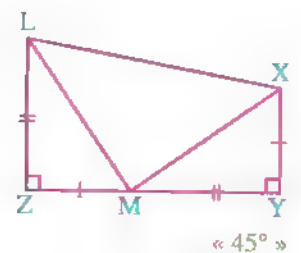


For excellent pupils

28 In the opposite figure :

$m(\angle Y) = m(\angle Z) = 90^\circ$
, $XY = MZ$ and $YM = ZL$

Find by proof : $m(\angle MXL)$



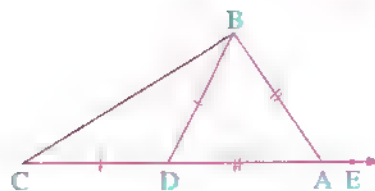
Exercise 3

29 In the opposite figure :

ABC is a triangle , $D \in \overline{AC}$ such that $BD = DC$

$AD = AB$ and $E \in \overrightarrow{CA}$

Prove that : $m(\angle BAE) = 4 m(\angle BCD)$

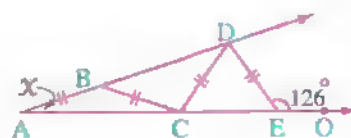


30 In the opposite figure :

$m(\angle A) = x^\circ$, $AB = BC = CD = DE$

and $m(\angle DEO) = 126^\circ$

Find : The value of x



« 18° »

Wonders of numbers

➤ Pick any positive 2-digit number, add the two digits, and subtract the sum from the original number.

➤ Is the difference divisible by 9? 😊

Try other numbers.



4

The converse of the isosceles triangle theorem



From the school book



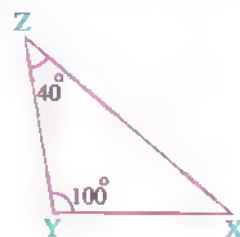
Remember Understand Apply Problem Solving

1 In each of the following figures, write the equal sides in length :

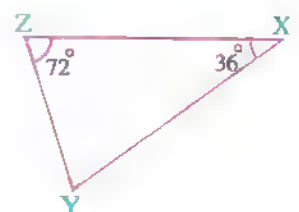
1



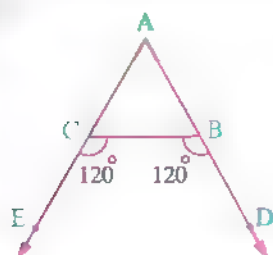
2



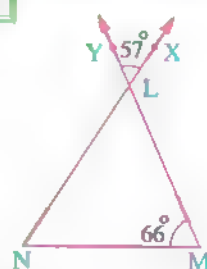
3



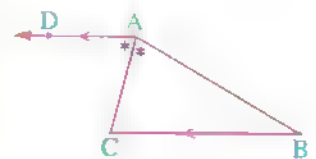
4



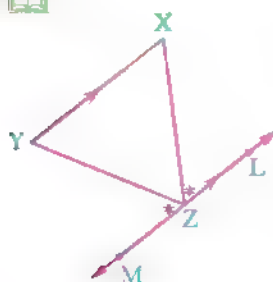
5



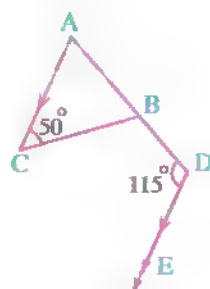
6



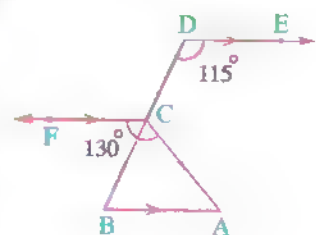
7



8



9



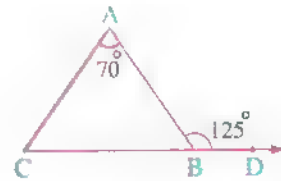
2 Complete the following :

- If two angles in the triangle are congruent , then the two sides opposite to these two angles are and the triangle is
- If the three angles in the triangle are congruent , then the triangle is
- In $\triangle ABC$, if $m(\angle A) = 50^\circ$ and $m(\angle B) = 80^\circ$, then the triangle is
- If the measure of one angle in the right-angled triangle is 45° , then the triangle is
- If the measure of one angle of an isosceles triangle is 60° , then the triangle is
- 6 ABC is a triangle in which $AB = AC$ and $m(\angle A) = 60^\circ$
If its perimeter = 18 cm. , then $BC = \dots\dots\dots$ cm.
- 7 In $\triangle ABC$, $CA = CB$, $m(\angle C) = m(\angle A)$, then $m(\angle B) = \dots\dots\dots^\circ$

3 In the opposite figure :

$D \in \overrightarrow{CB}$, $m(\angle ABD) = 125^\circ$
and $m(\angle A) = 70^\circ$

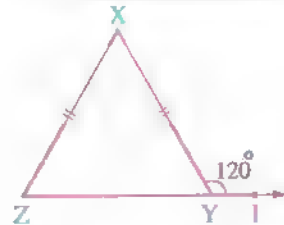
Prove that : $\triangle ABC$ is an isosceles triangle.



4 In the opposite figure :

$XY = XZ$, $m(\angle XYL) = 120^\circ$
and $L \in \overrightarrow{ZY}$

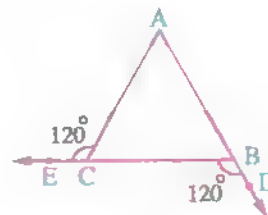
Prove that : $\triangle XYZ$ is an equilateral triangle.



5 In the opposite figure :

$D \in \overrightarrow{AB}$, $E \in \overrightarrow{BC}$ and
 $m(\angle CBD) = m(\angle ACE) = 120^\circ$

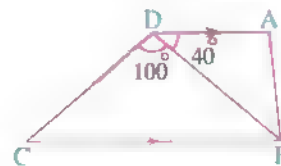
Prove that : $\triangle ABC$ is an equilateral triangle.



6 In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle ADB) = 40^\circ$
and $m(\angle BDC) = 100^\circ$

Prove that : $\triangle DBC$ is an isosceles triangle.





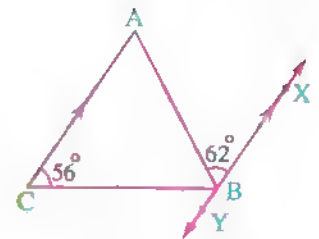
In the opposite figure :

$B \in \overrightarrow{XY}$, $\overrightarrow{XY} \parallel \overrightarrow{AC}$

, $m(\angle ABX) = 62^\circ$ and

$m(\angle C) = 56^\circ$

Prove that : $AC = BC$



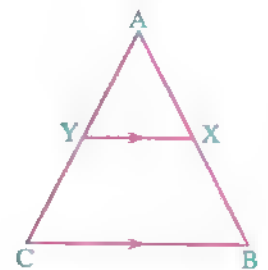
In the opposite figure :

ABC is a triangle in which $AB = AC$, $X \in \overrightarrow{AB}$,

$Y \in \overrightarrow{AC}$ and $\overrightarrow{XY} \parallel \overrightarrow{BC}$

Prove that : 1 $\triangle AXY$ is an isosceles triangle.

2 $XB = YC$



ABC is a triangle in which $D \in \overrightarrow{AB}$ and $E \in \overrightarrow{BC}$ such that $BD = BE$

So if $\overrightarrow{DE} \parallel \overrightarrow{AC}$, prove that : $AB = BC$

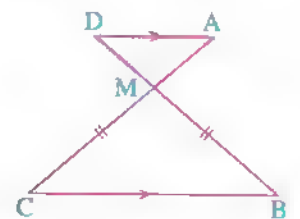


In the opposite figure :

$\overrightarrow{AC} \cap \overrightarrow{BD} = \{M\}$,

$MB = MC$ and $\overrightarrow{AD} \parallel \overrightarrow{BC}$

Prove that : $MA = MD$

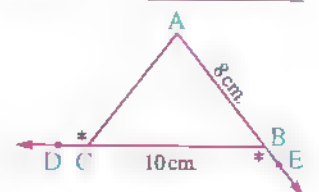


In the opposite figure :

$B \in \overrightarrow{AE}$, $C \in \overrightarrow{BD}$, $AB = 8$ cm. ,

$BC = 10$ cm. and $m(\angle EBC) = m(\angle ACD)$

Find : The perimeter of $\triangle ABC$



« 26 cm. »



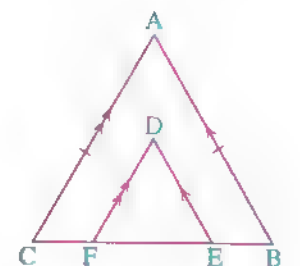
In the opposite figure :

$AB = AC$, $\overrightarrow{DE} \parallel \overrightarrow{AB}$ and $\overrightarrow{DF} \parallel \overrightarrow{AC}$

Prove that :

1 $DE = DF$

2 $m(\angle BAC) = m(\angle EDF)$



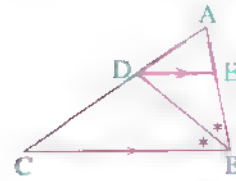
Exercise 4

13 In the opposite figure :

ABC is a triangle

, \overline{BD} bisects $\angle ABC$ and $\overline{ED} \parallel \overline{BC}$ where $E \in \overline{AB}$

Prove that : $\triangle EBD$ is an isosceles triangle.

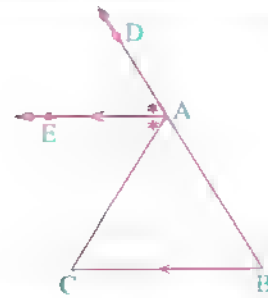


14 In the opposite figure :

$A \in \overline{BD}$, $\overline{AE} \parallel \overline{BC}$

and \overline{AE} bisects $\angle CAD$

Prove that : $AB = AC$

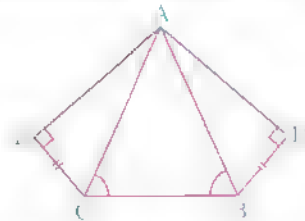


15 In the opposite figure :

$BD = CE$, $m(\angle ABC) = m(\angle ACB)$

and $m(\angle D) = m(\angle E) = 90^\circ$

Prove that : $m(\angle DAB) = m(\angle CAE)$



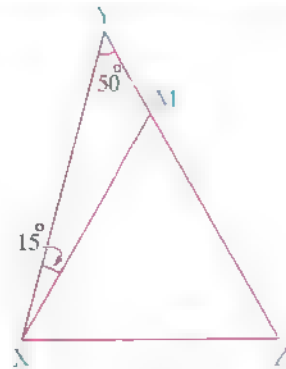
16 In the opposite figure :

YZX is a triangle in which $YZ = YX$

, $m(\angle Y) = 50^\circ$

and $m(\angle YXM) = 15^\circ$

Prove that : $\triangle MZX$ is an isosceles triangle.

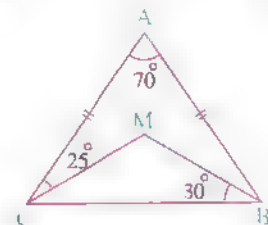


17 In the opposite figure :

ABC is a triangle in which $AB = AC$, $m(\angle A) = 70^\circ$

, $m(\angle MCA) = 25^\circ$ and $m(\angle MBC) = 30^\circ$

Prove that : $\triangle MBC$ is an isosceles triangle.

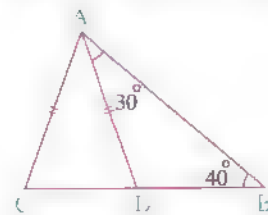


18 In the opposite figure :

$AD = AC$, $m(\angle B) = 40^\circ$

and $m(\angle BAD) = 30^\circ$

Prove that : $AB = CB$



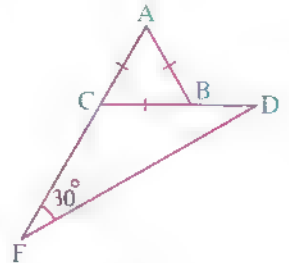
- 19 ABC is a triangle in which $AB = AC$, \overrightarrow{BD} bisects $\angle ABC$ and \overrightarrow{CD} bisects $\angle ACB$

Prove that : $\triangle DBC$ is an isosceles triangle.

- 20 In the opposite figure :

ABC is an equilateral triangle, $F \in \overline{AC}$,
 $D \in \overline{CB}$ and $m(\angle DFC) = 30^\circ$

Prove that : $\triangle DCF$ is an isosceles triangle.

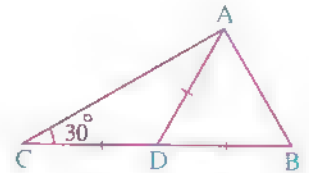


- 21 In the opposite figure :

$D \in \overline{BC}$ such that $DA = DB = DC$
 and $m(\angle C) = 30^\circ$

Prove that :

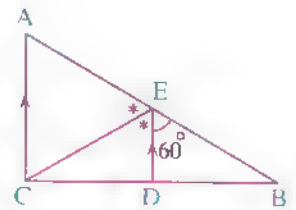
- 1 $\triangle ABD$ is an equilateral triangle.
- 2 $\triangle ABC$ is a right-angled triangle.



- 22 In the opposite figure :

ABC is a triangle in which $E \in \overline{AB}$,
 $\overline{ED} \parallel \overline{AC}$, $m(\angle BED) = 60^\circ$
 and \overrightarrow{EC} bisects $\angle AED$

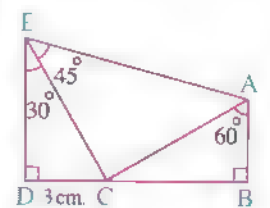
Prove that : $\triangle AEC$ is an equilateral triangle.



- 23 In the opposite figure :

$C \in \overline{BD}$, $m(\angle B) = m(\angle D) = 90^\circ$,
 $m(\angle CED) = 30^\circ$
 $m(\angle AEC) = 45^\circ$, $m(\angle BAC) = 60^\circ$
 and $CD = 3$ cm.

Find : The length of \overline{AC}



« 6 cm »

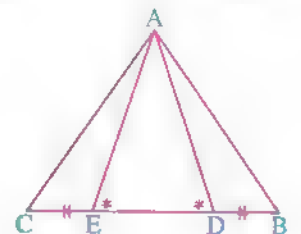
- 24 In the opposite figure :

$\angle ADE \cong \angle AED$

, B, D, E, C are collinear

and $BD = CE$

Prove that : $\triangle ABC$ is an isosceles triangle.

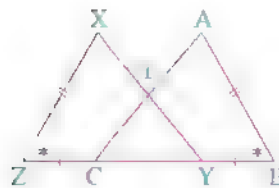


25 In the opposite figure :

$Y \in \overline{BZ}$, $C \in \overline{BZ}$, $AB = XZ$,

$BY = CZ$, $\overline{XY} \cap \overline{AC} = \{E\}$ and $m(\angle B) = m(\angle Z)$

Prove that : $\triangle EYC$ is an isosceles triangle.

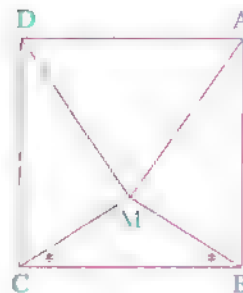


26 In the opposite figure :

ABCD is a square.

M is a point inside it such that : $m(\angle MBC) = m(\angle MCB)$

Prove that : $\triangle AMD$ is an isosceles triangle.



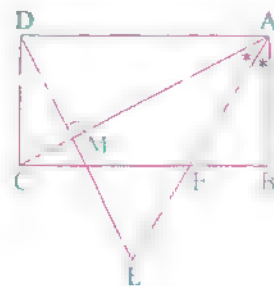
27 In the opposite figure :

ABCD is a rectangle in which

\overline{AC} is a diagonal, \overline{AE} bisects $\angle BAC$

and $\overline{DE} \perp \overline{AC}$ where $\overline{AE} \cap \overline{DE} = \{E\}$, $\overline{AC} \cap \overline{DE} = \{M\}$

Prove that : $DA = DE$



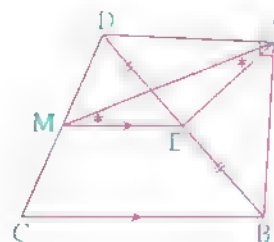
28 In the opposite figure :

ABCD is a quadrilateral in which

$m(\angle BAD) = 90^\circ$, E is the midpoint of \overline{BD} and $M \in \overline{DC}$

such that $\overline{EM} \parallel \overline{BC}$ and $m(\angle EAM) = m(\angle EMA)$

Prove that : $BD = BC$

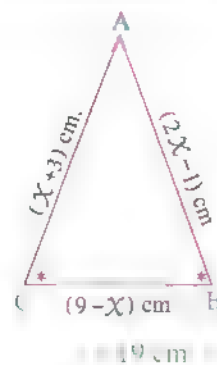


29 In the opposite figure :

ABC is a triangle in which :

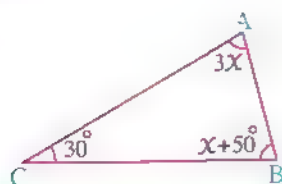
$m(\angle B) = m(\angle C)$

Find : The perimeter of the triangle.

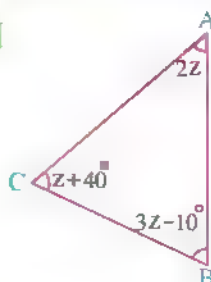


30 In each of the following figures, write the equal sides in length showing the steps of solution :

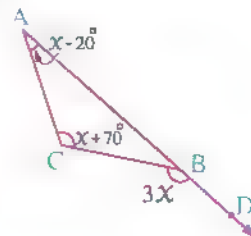
1



2



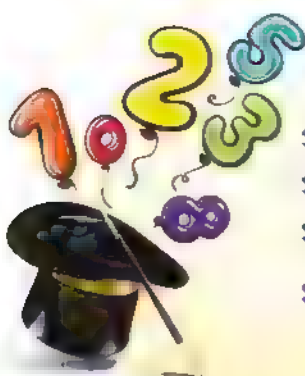
3



For excellent pupils

31 Choose the correct answer from those given :

- 1 If the sum of measures of two congruent angles in a triangle = $\frac{2}{3}$ the sum of measures of its angles, then the triangle is
 (a) right-angled. (b) isosceles. (c) equilateral. (d) scalene.
- 2 ABC is a triangle in which $m(\angle A) = 30^\circ$ and $m(\angle B) : m(\angle C) = 1 : 4$, then ΔABC is
 (a) right-angled. (b) isosceles. (c) equilateral. (d) scalene.



Wonders of numbers

- Pick any positive 2-digit number.
- Interchange the two digits to get a new number.
- Subtract the smaller number from the bigger number.
- Is the difference divisible by 9? 😊

Do the exercise again using different numbers.



From the school book

Remember Understand Apply Problem Solving

1 Complete the following :

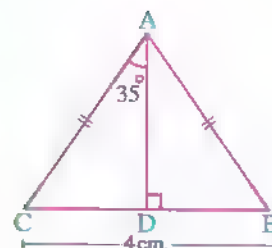
- The straight line drawn from the vertex of the isosceles triangle perpendicular to the base is called
- The number of axes of symmetry in the equilateral triangle equals
- 3 The number of axes of symmetry in the isosceles triangle equals
- 4 The number of axes of symmetry in the scalene triangle equals
- 5 The median of the isosceles triangle drawn from the vertex angle
- 8 The bisector of the vertex angle of the isosceles triangle
- The straight line passing through the vertex angle of the isosceles triangle perpendicular to its base
- 8 The axis of the line segment is
- Any point belonging to the axis of a line segment is ... from its two terminals.
- In $\triangle ABC$, if $m(\angle A) = m(\angle B) = 60^\circ$, then the number of axes of symmetry of $\triangle ABC$ is
- 1 In $\triangle ABC$, if $m(\angle A) = m(\angle B) \neq 60^\circ$, then the number of axes of symmetry of $\triangle ABC$ is
- 12 In $\triangle ABC$, if $AB = AC$, $m(\angle A) = 60^\circ$, then the number of axes of symmetry of $\triangle ABC$ is

2 In the opposite figure :

If $AB = AC$, $\overline{AD} \perp \overline{BC}$, $BC = 4$ cm. and

$m(\angle DAC) = 35^\circ$, complete the following :

- | | |
|---|--|
| 1. $m(\angle BAD) = \dots\dots\dots^\circ$ | 2. $m(\angle BAC) = \dots\dots\dots^\circ$ |
| 3. $m(\angle B) = \dots\dots\dots^\circ$ | 4. $BD = \dots\dots\dots$ cm. |
| 5. The axis of symmetry of $\triangle ABC$ is | |

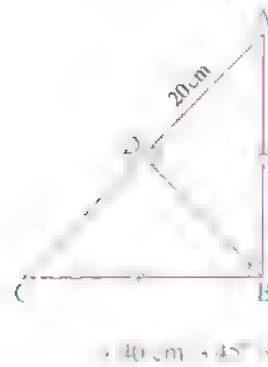
**3 Choose the correct answer from those given :**

1. If $C \in$ the axis of symmetry of \overline{AB} , then $AC - BC = \dots\dots\dots$
 (a) zero (b) 1 (c) 2 (d) 4
2. In $\triangle XYZ$, $XY = XZ$, \overline{XE} is a median, if $m(\angle YXE) = 30^\circ$, then $m(\angle YXZ) = \dots\dots\dots$
 (a) 15° (b) 30° (c) 60° (d) 90°
3. In $\triangle LMN$, $LM = LN$, $E \in \overline{MN}$ where $\overline{LE} \perp \overline{MN}$, if $ME = 4$ cm., then $MN = \dots\dots\dots$ cm.
 (a) 12 (b) 8 (c) 4 (d) 2
4. If the measure of one angle in the right-angled triangle is 45° , then the number of axes of symmetry of the triangle is
- (a) zero (b) 1 (c) 2 (d) 3
5. In $\triangle ABC$, $m(\angle A) = 40^\circ$, $m(\angle C) = 100^\circ$, then the number of axes of symmetry of the triangle is
- (a) 1 (b) 2 (c) 3 (d) infinite number.
- The triangle in which the measures of two angles in it are 45° , 65° , then the number of axes of symmetry of the triangle is
- (a) zero (b) 3 (c) 2 (d) 1
- An isosceles triangle, the measure of one of its angles is 60° , then the number of its axes of symmetry is
- (a) 4 (b) 3 (c) 2 (d) 1
6. If $\triangle ABC$ has 1 axis of symmetry, $m(\angle ABC) = 120^\circ$, $m(\angle A) = \dots\dots\dots$
 (a) 30° (b) 60° (c) 90° (d) 120°

Exercise 5

4 In the opposite figure :

ABC is a right-angled triangle at B and it is also an isosceles triangle , $\overline{BD} \perp \overline{AC}$ and $AD = 20$ cm. Find the length of \overline{AC} and $m(\angle DBC)$, then deduce that $\triangle BDC$ is an isosceles triangle.

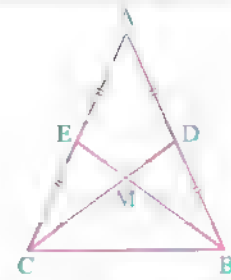


5 In the opposite figure :

$AB = AC$, D and E are the midpoints of \overline{AB} and \overline{AC} respectively and $\overline{BE} \cap \overline{CD} = \{M\}$

Prove that :

1. $\overline{AM} \perp \overline{BC}$
2. \overline{AM} bisects $\angle BAC$

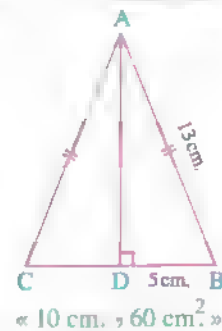


6 In the opposite figure :

In $\triangle ABC$, $AB = AC$, $\overline{AD} \perp \overline{BC}$, $AB = 13$ cm. and $BD = 5$ cm.

Find :

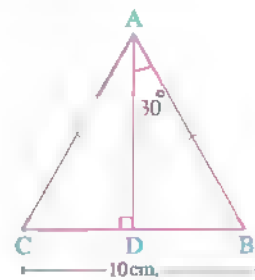
1. The length of \overline{BC}
2. The area of $\triangle ABC$



7 In the opposite figure :

$AB = AC$, $BC = 10$ cm. , $m(\angle BAD) = 30^\circ$ and $\overline{AD} \perp \overline{BC}$

1. Find the length of each of : \overline{BD} and \overline{AD}
2. How many axes of symmetry are there at $\triangle ABC$?
3. Find the area of $\triangle ABC$



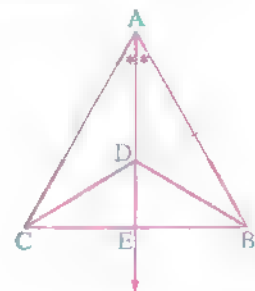
« 5 cm. , $5\sqrt{3}$ cm. , $25\sqrt{3}$ cm². »

8 In the opposite figure :

ABC is a triangle in which $AB = AC$, \overline{AE} bisects $\angle BAC$, $\overline{AE} \cap \overline{BC} = \{E\}$ and $D \in \overline{AE}$

Prove that :

1. $BE = \frac{1}{2} BC$
2. $BD = CD$



9 In the opposite figure :

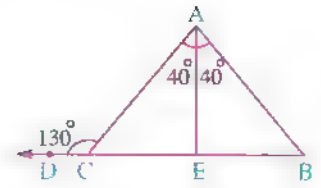
$C \in \overline{BD}$, $m(\angle ACD) = 130^\circ$

and $m(\angle BAE) = m(\angle CAE) = 40^\circ$

Prove that :

1 $\overline{AE} \perp \overline{BC}$

2 E is the midpoint of \overline{BC}



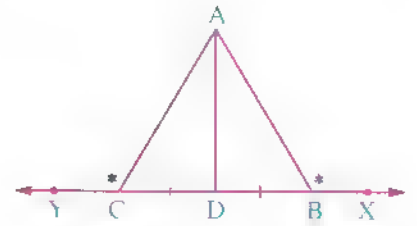
10 In the opposite figure :

X, B, C, D and Y are collinear points,

\overline{AD} is a median of $\triangle ABC$ and

$m(\angle ABX) = m(\angle ACY)$

Prove that : $\overline{AD} \perp \overline{BC}$



11 In the opposite figure :

ABCD is a quadrilateral in which

$\overline{AD} \parallel \overline{BC}$, \overline{BD} bisects $\angle ABC$ and

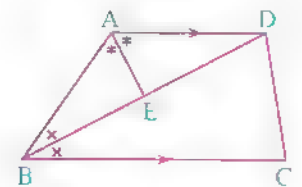
\overline{AE} bisects $\angle BAD$

Prove that :

1 $AB = AD$

2 $\overline{AE} \perp \overline{BD}$

3 $BE = ED$



12 In the opposite figure :

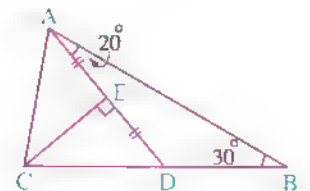
ABC is a triangle in which

$m(\angle B) = 30^\circ$, $D \in \overline{BC}$

where $m(\angle BAD) = 20^\circ$

, E is the midpoint of \overline{AD} and $\overline{CE} \perp \overline{AD}$

Find : $m(\angle ACE)$



40° »

13 In the opposite figure :

ABC is a triangle in which

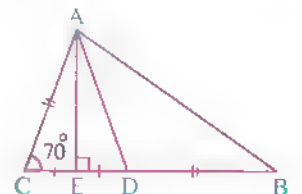
$m(\angle C) = 70^\circ$, $D \in \overline{BC}$

where $BD = AC$

, E is the midpoint of \overline{DC}

and $\overline{AE} \perp \overline{DC}$

Find : $m(\angle B)$



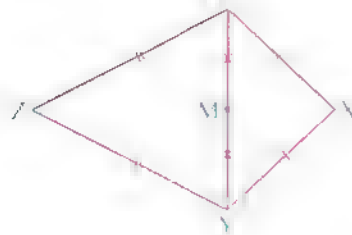
35° »

14 In the opposite figure :

$XY = XL$, $ZY = ZL$ and $LM = YM$

Prove that :

X , M and Z are on the same straight line.

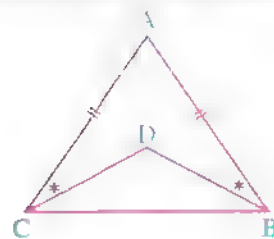


15 In the opposite figure :

ABC is a triangle , D is a point inside it such that

$m(\angle ABD) = m(\angle ACD)$ and $AB = AC$

Prove that : \overleftrightarrow{AD} is the axis of symmetry of \overline{BC}



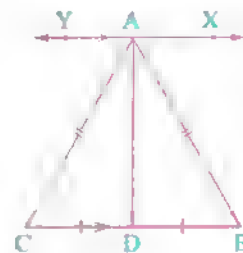
16 In the opposite figure :

ABC is a triangle in which $AB = AC$,

D is the midpoint of \overline{BC} and \overleftrightarrow{XY} passes through the vertex A

such that $\overleftrightarrow{XY} \parallel \overline{BC}$

Prove that : $\overline{AD} \perp \overleftrightarrow{XY}$

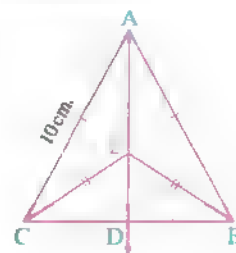


17 In the opposite figure :

$AB = AC = 10$ cm. , $EB = EC$ and $\overleftrightarrow{AE} \cap \overline{BC} = \{D\}$

Prove that : $BD = DC$ and if $BC = 6$ cm.

Find the length of each of : \overline{CD} and \overline{AD}



« 3 cm. , $\sqrt{91}$ cm. »

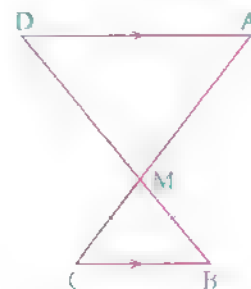
18 In the opposite figure :

$\overline{AC} \cap \overline{BD} = \{M\}$, $\overline{AD} \parallel \overline{BC}$ and $MB = MC$

Prove that :

1 ΔAMD is an isosceles triangle.

2 The axis of symmetry of ΔAMD is the same of ΔBMC



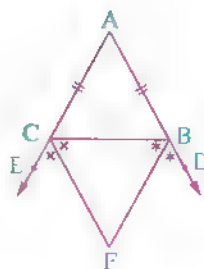
19 In the opposite figure :

$AB = AC$, $D \in \overrightarrow{AB}$, $E \in \overrightarrow{AC}$,

\overrightarrow{BF} bisects $\angle DBC$ and \overrightarrow{CF} bisects $\angle BCE$

Prove that :

- 1 $\triangle BFC$ is an isosceles triangle.
- 2 \overrightarrow{AF} is the axis of symmetry of \overline{BC}



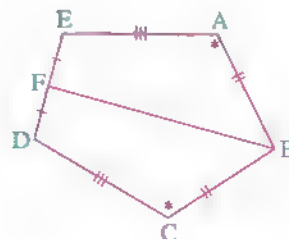
20 In the opposite figure :

$AB = BC$, $AE = CD$,

$m(\angle BAE) = m(\angle BCD)$

and F is the midpoint of \overline{DE}

Prove that : $\overline{BF} \perp \overline{DE}$



21 Choose the correct answer from those given :

If ABCD is a quadrilateral in which $AB = AD$ and $BC = DC$, then \overrightarrow{AC} is \overline{BD}

- | | |
|-----------------------------|------------------|
| (a) parallel to | (b) equal to |
| (c) the axis of symmetry of | (d) congruent to |

22 The triangle whose sides lengths are 2 cm. , $(X + 3)$ cm. and 5 cm. becomes an isosceles triangle when $X =$ cm.

- | | | | |
|-------|-------|-------|-------|
| (a) 1 | (b) 2 | (c) 3 | (d) 4 |
|-------|-------|-------|-------|

23 If the length of any side in a triangle = $\frac{1}{3}$ of the perimeter of the triangle , then the number of axes of symmetry of the triangle equals

- | | | | |
|-------|-------|-------|----------|
| (a) 1 | (b) 2 | (c) 3 | (d) zero |
|-------|-------|-------|----------|

24 If \overrightarrow{XY} is the axis of symmetry of \overline{AB} , then

- | | | | |
|---------------|---------------|---------------|---------------|
| (a) $AX = BY$ | (b) $AX = BX$ | (c) $BY = XY$ | (d) $AY = BX$ |
|---------------|---------------|---------------|---------------|

25 In the rhombus ABCD , the axis of symmetry of \overline{AC} is

- | | | | |
|---------------------------|---------------------------|---------------------------|---------------------------|
| (a) \overrightarrow{BD} | (b) \overrightarrow{AB} | (c) \overrightarrow{AD} | (d) \overrightarrow{CD} |
|---------------------------|---------------------------|---------------------------|---------------------------|

26 In the square ABCD , \overrightarrow{BD} is the axis of symmetry of

- | | | | |
|---------------------|---------------------|---------------------|---------------------|
| (a) \overline{AB} | (b) \overline{AC} | (c) \overline{AD} | (d) \overline{CD} |
|---------------------|---------------------|---------------------|---------------------|

For excellent pupils

22 In the opposite figure :

ABCD is a quadrilateral in which

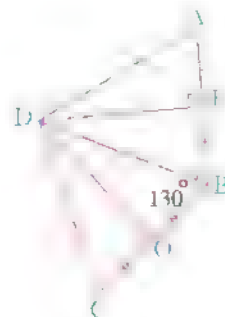
$$m(\angle ABC) = 130^\circ$$

, E is the midpoint of \overline{AB}

, O is the midpoint of \overline{BC}

, $\overline{DE} \perp \overline{AB}$ and $\overline{DO} \perp \overline{BC}$

Find : $m(\angle ADC)$

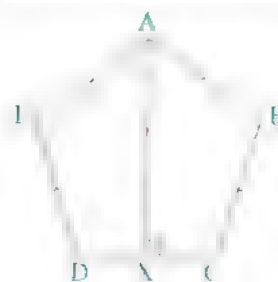


« 100° »

23 In the opposite figure :

ABCDE is a regular pentagon and $\overline{AX} \perp \overline{CD}$

Find : $m(\angle DAX)$



« 18° »

Wonders
of numbers

Choose an integer from 1 to 9, multiply it by 9, then multiply the product by 123456789.

Where do you stand ?



Summary of Unit 4



- ★ The median of a triangle is the line segment drawn from any vertex of this triangle to the midpoint of the opposite side of this vertex.
- ★ The medians of a triangle are concurrent.
- ★ The point of concurrence of the medians of the triangle divides each median in the ratio of 1 : 2 from its base or in the ratio of 2 : 1 from the vertex.
- ★ The point which divides the median in a triangle in the ratio of 1 : 2 from the base is the point of intersection of the medians of this triangle.
- ★ In the right-angled triangle, the length of the median from the vertex of the right angle equals half the length of the hypotenuse.
- ★ If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right.
- ★ The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.
- ★ The base angles of the isosceles triangle are congruent. (i.e. equal in measure)
- ★ If two angles of a triangle are congruent, then the two sides opposite to these two angles are congruent and the triangle is isosceles.
- ★ If the triangle is equilateral, then it is equiangular where each angle measure is 60°
- ★ If the angles of a triangle are congruent, then the triangle is equilateral.
- ★ The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle.
- ★ The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.
- ★ The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

- ✧ The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.
- ✧ The axis of symmetry of a line segment is the straight line perpendicular to it from its midpoint.
- ✧ Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).
- ✧ If a point is at equal distances from the two terminals of a line segment, then this point lies on the axis of this line segment.
- ✧ The isosceles triangle has one axis of symmetry which is the straight line perpendicular from its vertex to its base.
- ✧ The equilateral triangle has three axes of symmetry.
- ✧ The scalene triangle has no axes of symmetry.

Exams on Unit Four



Model 1

Answer the following questions :

1 Choose the correct answer from those given :

- 1 If M is the point of intersection of the medians in $\triangle ABC$ and \overline{AD} is a median of length 6 cm. , then $AM = \dots\dots\dots$
 (a) 1 cm. (b) 4 cm. (c) 3 cm. (d) 2 cm.
- 2 If the measure of a base angle of an isosceles triangle is 40° , then the measure of the vertex angle is
 (a) 40° (b) 50° (c) 80° (d) 100°
- 3 The measure of the exterior angle of the equilateral triangle equals
 (a) 30° (b) 60° (c) 90° (d) 120°
- 4 If C \in the axis of symmetry of \overline{AB} , then $AC - BC = \dots\dots\dots$
 (a) zero (b) 1 (c) 2 (d) 3
- 5 If ABC is a right-angled triangle at A and $AB = AC$, then $m(\angle B) = \dots\dots\dots$
 (a) 30° (b) 45° (c) 60° (d) 90°
- 6 The number of medians of the isosceles triangle is
 (a) 0 (b) 1 (c) 2 (d) 3

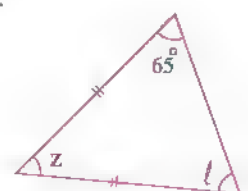
2 Complete the following :

- 1 The point of intersection of the medians of the triangle divides each of them in the ratio : 2 from the vertex.
- 2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals
- 3 The median of the isosceles triangle drawn from the vertex ,
- 4 If the length of the median of the triangle which is drawn from one of its vertices equals half the length of the opposite side to this vertex , then

5 In the opposite figure :

$$l = \dots\dots\dots^\circ$$

$$z = \dots\dots\dots^\circ$$

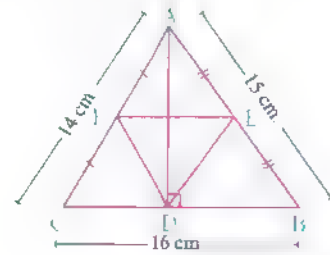


3 [a] In the opposite figure :

$\overline{AD} \perp \overline{BC}$, E is the midpoint of \overline{AB}

and F is the midpoint of \overline{AC}

Find : The perimeter of $\triangle DEF$

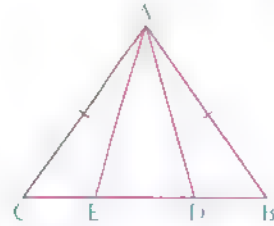


[b] In the opposite figure :

$m(\angle BAE) = m(\angle CAD)$

and $AB = AC$

Prove that : $AE = AD$



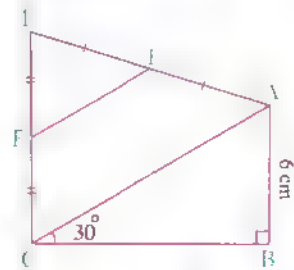
4 [a] In the opposite figure :

$m(\angle B) = 90^\circ$, $m(\angle ACB) = 30^\circ$

$AB = 6$ cm., E is the midpoint of \overline{AD}

and F is the midpoint of \overline{DC}

Find : The length of \overline{EF}

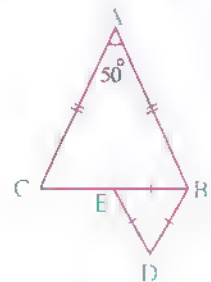


[b] In the opposite figure :

$AB = AC$, $m(\angle A) = 50^\circ$

and $\triangle BDE$ is an equilateral triangle.

Find : $m(\angle ABD)$



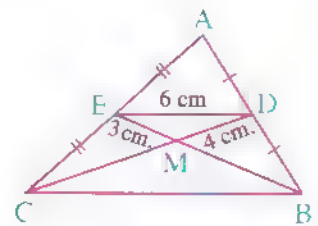
5 [a] In the opposite figure :

\overline{BE} and \overline{CD} are two medians of $\triangle ABC$

intersecting at M, $ME = 3$ cm.

, $MD = 4$ cm. and $DE = 6$ cm.

Find : The perimeter of $\triangle MBC$



[b] In the opposite figure :

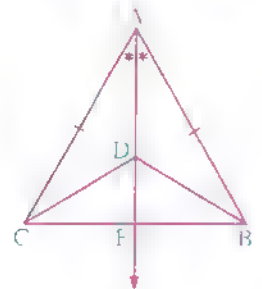
ABC is a triangle in which : $AB = AC$

, \overline{AE} bisects $\angle BAC$

, $\overline{AE} \cap \overline{BC} = \{E\}$ and $D \in \overline{AE}$

Prove that : 1 $BE = \frac{1}{2} BC$

2 $BD = CD$



Model 2

Answer the following questions :

1 Choose the correct answer from those given :

- 1 The base angles of the isosceles triangle are
 (a) complementary. (b) supplementary. (c) congruent. (d) straight.
- 2 If M is the point of intersection of the medians of $\triangle ABC$, D is the midpoint of \overline{BC} , then $AD = \dots\dots\dots$
 (a) $2 AM$ (b) $\frac{2}{3} MD$ (c) $\frac{3}{2} AM$ (d) $4 MD$
- 3 If the measure of the vertex angle of an isosceles triangle is 50° , then the measure of each of the base angles is
 (a) 40° (b) 65° (c) 70° (d) 130°
- 4 ABC is a right-angled triangle at B, D is the midpoint of \overline{AC} , then $BD = \dots\dots\dots$
 (a) $\frac{1}{2} AC$ (b) AC (c) $\frac{1}{2} BC$ (d) AB
- 5 The triangle which has three axes of symmetry is
 (a) isosceles. (b) equilateral. (c) right-angled. (d) obtuse-angled.
- 6 In $\triangle ABC$, if $AB = AC$, $m(\angle A) = 2 m(\angle B)$, then $m(\angle C) = \dots\dots\dots$
 (a) 30° (b) 45° (c) 60° (d) 90°

2 Complete the following :

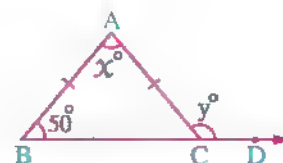
- 1 The bisector of the vertex angle of an isosceles triangle is ,
- 2 Any point on the axis of symmetry of a line segment is at distances from its two terminals.
- 3 ABC is a right-angled triangle at B, $m(\angle C) = 30^\circ$, $AB = 4$ cm. , then $AC = \dots\dots\dots$ cm.

4 In the opposite figure :

$$AB = AC, D \in \overrightarrow{BC}$$

, then $x = \dots\dots\dots$

, $y = \dots\dots\dots$

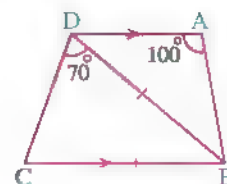


3 [a] In the opposite figure :

$$\overline{AD} \parallel \overline{BC}, BD = BC$$

$$, m(\angle A) = 100^\circ \text{ and } m(\angle BDC) = 70^\circ$$

Prove that : $\triangle ABD$ is isosceles.

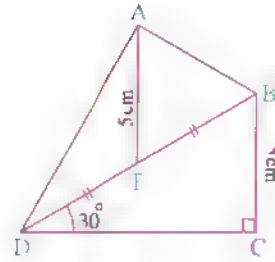


[b] In the opposite figure :

$m(\angle C) = 90^\circ$, \overline{AF} is a median in $\triangle ABD$, $m(\angle BDC) = 30^\circ$
and $BC = AF = 5$ cm.

1. Find : The length of \overline{BD}

2. Prove that : $m(\angle BAD) = 90^\circ$



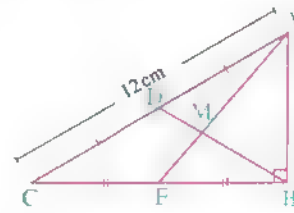
4 [a] In the opposite figure :

$m(\angle ABC) = 90^\circ$

, $AD = DC$ and $BE = EC$

If $AC = 12$ cm.

Find the length of each of : \overline{BD} and \overline{MD}



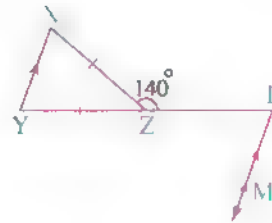
[b] In the opposite figure :

$Z \in \overline{LY}$, $XZ = YZ$

, $m(\angle LZX) = 140^\circ$

and $\overline{LM} \parallel \overline{XY}$

Find : $m(\angle MLY)$

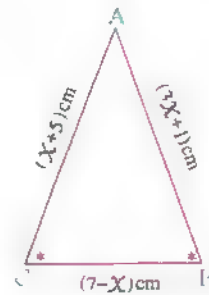


5 [a] In the opposite figure :

ABC is a triangle in which

$m(\angle B) = m(\angle C)$

Find : The perimeter of $\triangle ABC$



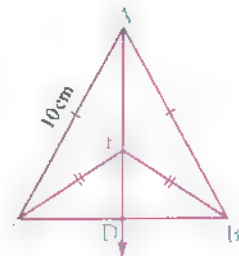
[b] In the opposite figure :

$AB = AC = 10$ cm.

, $EB = EC$ and $\overline{AE} \cap \overline{BC} = \{D\}$

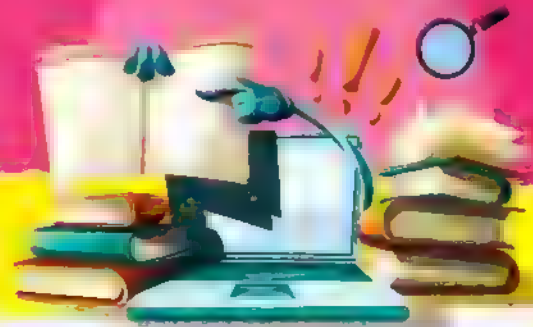
1. Prove that : $BD = DC$

2. If $BC = 6$ cm., find the length of each of : \overline{CD} and \overline{AD}



A Research Project

On Unit Four



Project aims :

- Using geometrical instruments to make art designs.
- Using the properties of the equilateral triangle.
- Calculating the area of an equilateral triangle.
- Calculating the area of a square.
- Calculating the area of a geometrical shape consisting of a group of shapes.
- Associating geometry with arts.
- Associating geometry with science.

Do a research project on the following topic :

"Geometry is used in many fields of life. One of these is making art designs".

Discuss the following points using available resources :

- 1 Using geometrical instruments, design a logo of a fossil museum which consists of a square. On each side, draw an equilateral triangle.
- 2 Calculate the area of the resulted shape.
- 3 Decorate the logo with colours of your choice and stick a picture of one fossil inside the square.
- 4 Write a short note on the kinds of fossils and how to be formed, and mention an example of each kind.



UNIT

5

Inequality

Exercises of the unit :

- 6. Inequality.
- 7. Comparing the measures of angles in a triangle.
- 8. Comparing the lengths of sides in a triangle.
- 9. Triangle inequality.
- Summary of unit five.
- Unit exams.



A research project on unit five



Scan the
QR code
to solve an
interactive
test on each
lesson



● Remember

● Understand

● Apply

● Problem Solving

1 Complete each of the following using $>$ or $<$:

1 In the opposite figure :

If C and B belong to \overleftrightarrow{AD} such that $DC < BA$, then AC DB



2 In the opposite figure :

If B and C belong to \overleftrightarrow{AD} where $AB > CD$, then AC BD



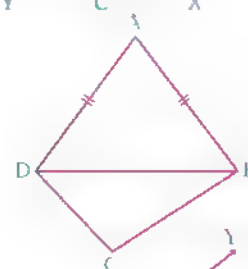
3 In the opposite figure :

If $C \in \overleftrightarrow{XY}$, $m(\angle ACX) = 35^\circ$
and $m(\angle BCY) = 45^\circ$, then $m(\angle XCB)$ $m(\angle ACY)$



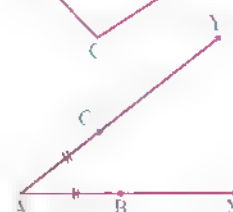
4 In the opposite figure :

$AB = AD$, $m(\angle DBC) < m(\angle CDB)$
, then $m(\angle ABC)$ $m(\angle ADC)$



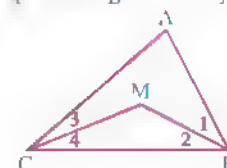
5 In the opposite figure :

If $AB = AC$ and $AY > AX$, then BX CY



6 In the opposite figure :

$m(\angle 1) > m(\angle 3)$, $m(\angle 2) > m(\angle 4)$
, then $m(\angle ABC)$ $m(\angle ACB)$



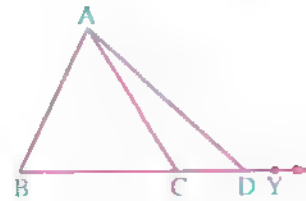
Exercise 6

7 In the opposite figure :

ABC is a triangle , $C \in \overline{BD}$ and $Y \in \overline{CD}$

, then $m(\angle ADY) \dots\dots\dots m(\angle DAC)$

, $m(\angle ABC) \dots\dots\dots m(\angle ADY)$



2 Use the opposite figure to arrange the given measures ascendingly , where B, C, D and E are collinear :

1) $m(\angle 1), m(\angle 3)$

2) $m(\angle 2), m(\angle 4)$

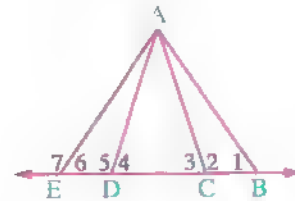
3) $m(\angle 5), m(\angle 3)$

4) $m(\angle 2), m(\angle 6)$

5) $m(\angle 3), m(\angle 1), m(\angle 5)$

6) $m(\angle 3), m(\angle 5), m(\angle 7)$

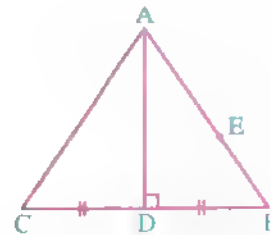
7) $m(\angle 3), m(\angle 1), m(\angle 7), m(\angle 5)$



3 In the opposite figure :

$E \in \overline{AB}$, $\overline{AD} \perp \overline{BC}$ and D is the midpoint of \overline{BC}

Prove that : $AC > AE$



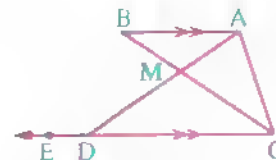
4 In the opposite figure :

$\overline{AB} \parallel \overline{CD}$, $\overline{AD} \cap \overline{BC} = \{M\}$, $E \in \overline{CD}$ and $E \notin \overline{CD}$

Prove that :

1) $m(\angle ACD) > m(\angle ABC)$

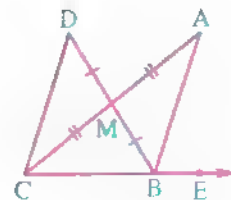
2) $m(\angle ADE) > m(\angle ABC)$



5 In the opposite figure :

$E \in \overline{CB}$ and M is the midpoint of each of \overline{AC} and \overline{BD}

Prove that : $m(\angle ABE) > m(\angle ACD)$

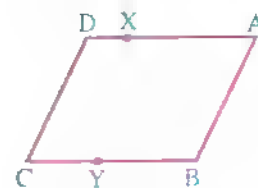


6 In the opposite figure :

$ABCD$ is a parallelogram , $X \in \overline{AD}$ and $Y \in \overline{BC}$

such that $DX < BY$

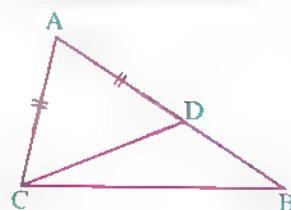
Prove that : $AX + AB > CY + CD$



7 In the opposite figure :

$D \in \overline{AB}$ where $AD = AC$

Prove that : $m(\angle ACB) > m(\angle B)$

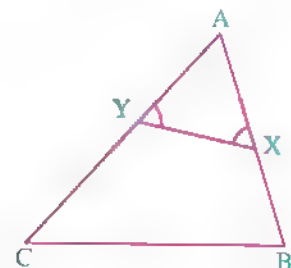


8 In the opposite figure :

ABC is a triangle in which : $AC > AB$, $X \in \overline{AB}$

and $Y \in \overline{AC}$ where $m(\angle AXY) = m(\angle AYX)$

Prove that : $YC > XB$

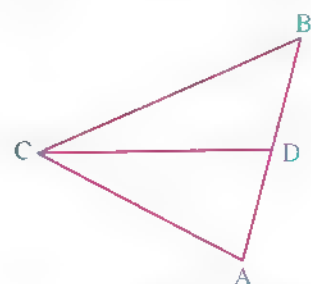


9 In the opposite figure :

ABC is a triangle in which :

$\overline{AB} \cong \overline{AC}$ and $D \in \overline{AB}$

Prove that : $m(\angle ADC) > m(\angle ACB)$



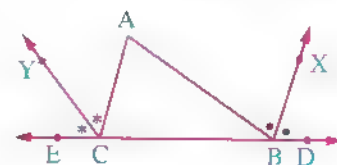
10 In the opposite figure :

$B \in \overleftrightarrow{DE}$, $C \in \overleftrightarrow{DE}$ such that

$m(\angle ACB) > m(\angle ABC)$

, \overrightarrow{BX} bisects $\angle ABD$ and \overrightarrow{CY} bisects $\angle ACE$

Prove that : $m(\angle ABX) > m(\angle ACY)$



11 M is a point inside the triangle ABC

Prove that : $m(\angle AMB) > m(\angle ACB)$

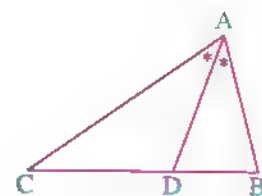


12 In the opposite figure :

ABC is a triangle in which : $m(\angle B) > m(\angle C)$, $D \in \overline{BC}$

such that \overrightarrow{AD} bisects $\angle BAC$

Prove that : $\angle ADC$ is an obtuse angle.

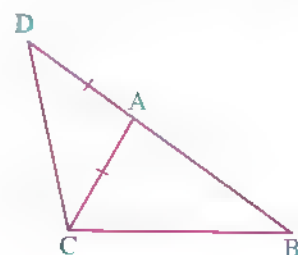


13 In the opposite figure :

ABC is a triangle in which : $m(\angle ACB) > m(\angle ABC)$

, $A \in \overline{BD}$ such that $AC = AD$

Prove that : $\angle BCD$ is an obtuse angle.





Remember

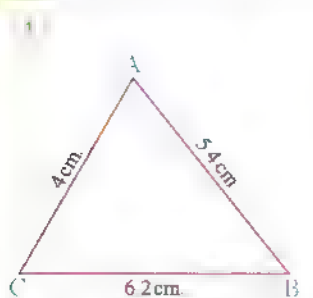
Understand

Apply

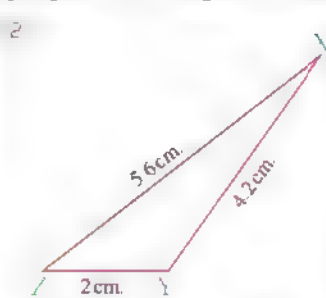
Problem Solving

1 Complete the following :

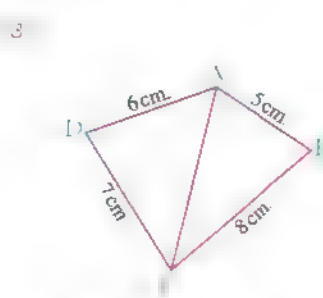
- The lengths of two sides in a triangle are not equal , then the greater side in length is opposite to
- In $\triangle ABC$, $AB = 7$ cm. , $BC = 5$ cm. and $AC = 6$ cm. , then the smallest angle in measure is
- In $\triangle DEF$, if $DE > EF$, then $m(\angle F) > \dots\dots\dots$
- In any triangle ABC , if $AB > AC > BC$, then $m(\angle \dots\dots\dots) < m(\angle \dots\dots\dots) < m(\angle \dots\dots\dots)$

2 In each of the following figures , complete using ($>$ or $<$) :

$m(\angle A) \dots\dots\dots m(\angle B)$
 $m(\angle A) \dots\dots\dots m(\angle C)$
 $m(\angle B) \dots\dots\dots m(\angle C)$



$m(\angle Z) \dots\dots\dots m(\angle Y)$
 $m(\angle X) \dots\dots\dots m(\angle Y)$
 $m(\angle Z) \dots\dots\dots m(\angle X)$



$m(\angle BAC) \dots\dots\dots m(\angle BCA)$
 $m(\angle DAC) \dots\dots\dots m(\angle DCA)$
 $m(\angle BAD) \dots\dots\dots m(\angle BCD)$

3 Arrange the measures of the angles of $\triangle ABC$ in each of the following cases ascendingly :

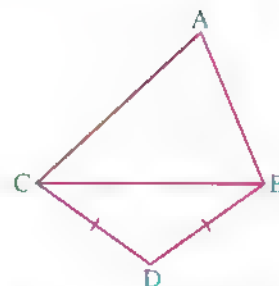
- If $AB = 12$ cm. , $BC = 15$ cm. and $AC = 10$ cm.
- If $AB = 5.7$ cm. , $BC = 8.5$ cm. and $AC = 6$ cm.

4 In the opposite figure :

$AC > AB$ and $DB = DC$

Prove that :

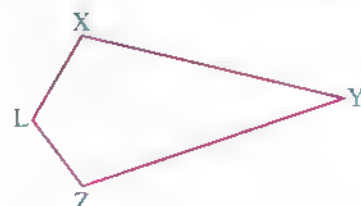
$m(\angle ABD) > m(\angle ACD)$



5 In the opposite figure :

$XY > XL$ and $YZ > ZL$

Prove that : $m(\angle XLZ) > m(\angle XYZ)$

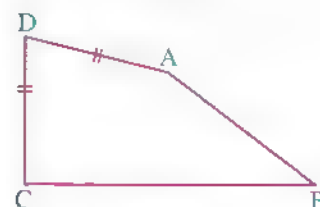


6 In the opposite figure :

ABCD is a quadrilateral in which :

$AD = DC$ and $BC > AB$

Prove that : $m(\angle A) > m(\angle C)$



7 ABCD is a quadrilateral in which : \overline{AB} is the longest side , \overline{CD} is the shortest one

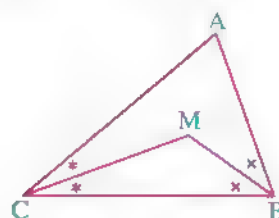
Prove that : $m(\angle BCD) > m(\angle BAD)$

8 In the opposite figure :

ABC is a triangle , \overline{BM} bisects $\angle ABC$ and \overline{CM} bisects $\angle ACB$

If $MC > MB$

, prove that : $m(\angle ABC) > m(\angle ACB)$



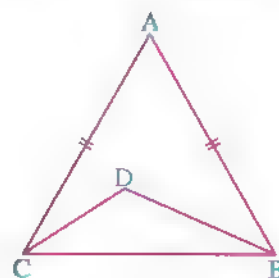
9 In the opposite figure :

ABC is a triangle in which :

$AB = AC$ and $DB > DC$

Prove that :

$m(\angle ABD) > m(\angle ACD)$



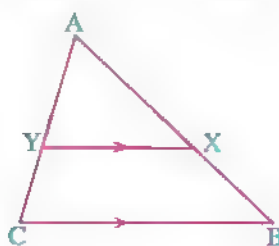
10 In the opposite figure :

ABC is a triangle ,

$AB > AC$ and $\overline{XY} \parallel \overline{BC}$

Prove that :

$m(\angle AYX) > m(\angle AXY)$

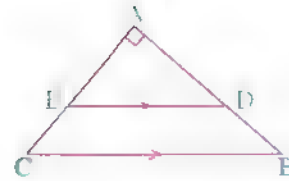


11 In the opposite figure :

ABC is a triangle in which : $m(\angle A) = 90^\circ$, $AB > AC$,

$D \in \overline{AB}$, $E \in \overline{AC}$ and $\overline{DE} \parallel \overline{BC}$

Prove that : $m(\angle AED) > 45^\circ$



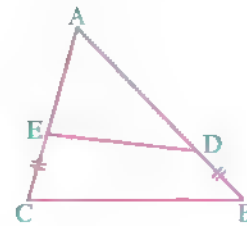
12 In the opposite figure :

ABC is a triangle in which :

$AB > AC$, $D \in \overline{AB}$ and

$E \in \overline{AC}$ where $BD = CE$

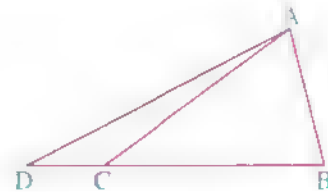
Prove that : $m(\angle AED) > m(\angle ADE)$



13 In the opposite figure :

$C \in \overline{BD}$ such that $AC > AB$

Prove that : $m(\angle ABD) > m(\angle D)$



14 In the opposite figure :

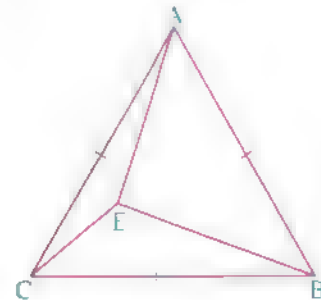
ABC is an equilateral triangle ,

E is a point inside it ,

$m(\angle ECB) > m(\angle EBC)$

Prove that : 1 $m(\angle ABE) > m(\angle ACE)$

2 $m(\angle A) > m(\angle ABE) > m(\angle ACE)$

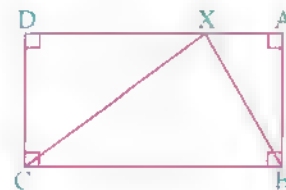


15 In the opposite figure :

ABCD is a rectangle , $X \in \overline{AD}$

such that $XC > XB$

Prove that : $m(\angle ABX) < m(\angle XCD)$



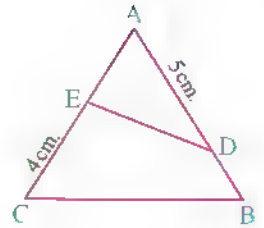
16 In the opposite figure :

ABC is an equilateral triangle

whose side length = 7 cm. , $D \in \overline{AB}$ such that

$AD = 5$ cm. and $E \in \overline{AC}$ such that $CE = 4$ cm.

Prove that : $m(\angle AED) > 60^\circ$



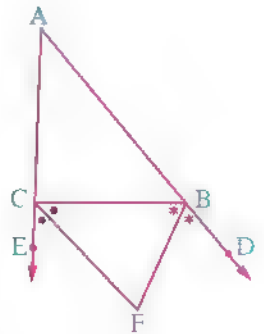
17 In the opposite figure :

ABC is a triangle in which :

$AB > AC$, $D \in \overrightarrow{AB}$, $E \in \overrightarrow{AC}$,

\overrightarrow{BF} bisects $\angle DBC$ and \overrightarrow{CF} bisects $\angle BCE$

Prove that : $m(\angle FBC) > m(\angle BCF)$



18 ABC is a triangle , D is a point inside it. If $DA > DB > DC$

Prove that : $m(\angle ACB) > m(\angle DAC) + m(\angle DBC)$

19 ABC is a triangle , \overline{AD} , \overline{CE} are two medians intersecting at M If $MD > ME$

Prove that : $m(\angle CAM) < m(\angle MCA)$

20 ABC is a triangle in which : $AB > AC$, D is the midpoint of \overline{AB}

Draw $\overline{DE} \parallel \overline{AC}$ to meet \overline{BC} at E

Prove that : $m(\angle CAE) > m(\angle DAE)$

21 In the opposite figure :

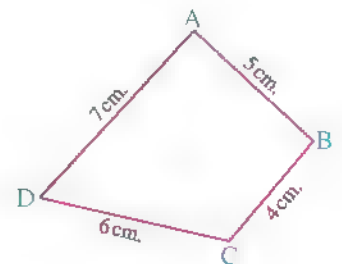
ABCD is a quadrilateral in which :

$AB = 5$ cm. , $BC = 4$ cm. , $CD = 6$ cm. and $DA = 7$ cm.

Prove that : [1] $m(\angle ABC) > m(\angle ADC)$

[2] $m(\angle BCD) > m(\angle BAD)$

[3] $m(\angle B) + m(\angle C) > 180^\circ$



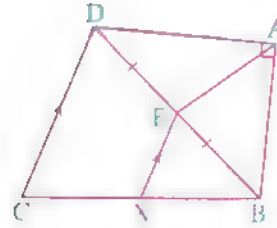
22 In the opposite figure :

ABCD is a quadrilateral in which : $m(\angle A) = 90^\circ$,

\overline{AE} is a median of $\triangle ABD$, $\overline{EX} \parallel \overline{DC}$ and

$\overline{EX} \cap \overline{BC} = \{X\}$ If $AE > EX$

Prove that : $m(\angle C) > m(\angle DBC)$

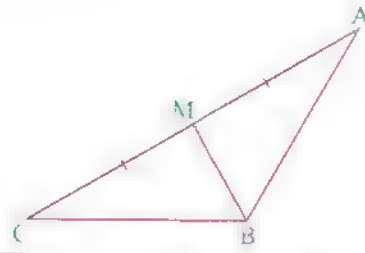


23 In the opposite figure :

\overline{BM} is a median in the

triangle ABC and $BM < AM$

Prove that : $\angle ABC$ is an obtuse angle.

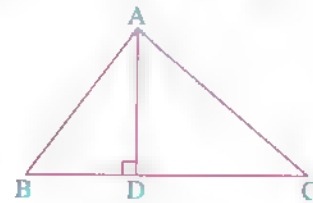


24 In the opposite figure :

ABC is a triangle in which : $AC > AB$, $\overline{AD} \perp \overline{BC}$

and intersects it at D

Prove that : $m(\angle BAD) < m(\angle CAD)$



25 ABC is a triangle , \overline{AD} bisects $\angle A$ and intersects \overline{BC} at D , if $AC > AB$

Prove that : $\angle ADC$ is an obtuse angle.

26 ABCD is a parallelogram in which : $AC > BD$

Prove that : $\angle D$ is an obtuse angle.

For excellent pupils

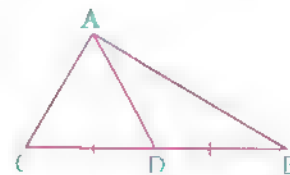
27 ABC is a triangle , D is the midpoint of \overline{BC} , if the perimeter of $\triangle ACD >$ the perimeter of $\triangle ABD$

Prove that : $m(\angle B) > m(\angle C)$

28 In the opposite figure :

$AB > AC$ and $DB = DC$

Prove that : $m(\angle BAD) < m(\angle CAD)$





Remember

Understand

Apply

Problem Solving

1 Complete the following :

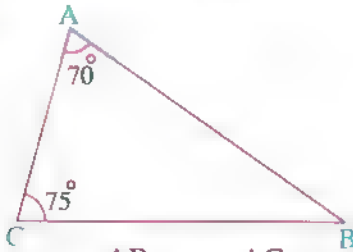
- 1 If two angles in a triangle are unequal in measure , then the greater angle in measure is opposite to and if the two lengths of two sides in a triangle are unequal , then the greater side in length is opposite to the angle which is
- 2 The smallest angle of a triangle (in measure) is opposite to
- 3 The longest side in the right-angled triangle is
- 4 The shortest distance between a given point and a given straight line is
- 5 ABC is a triangle in which : $m(\angle C) = 110^\circ$, then its longest side is
- 6 In $\triangle ABC$: If $m(\angle A) = 50^\circ$, $m(\angle B) = 30^\circ$, then the shortest side in the triangle is
- 7 In $\triangle ABC$: If $m(\angle A) = m(\angle B) + m(\angle C)$, then the longest side in the triangle is

2 Choose the correct answer from those ones :

- 1 In $\triangle ABC$, if $m(\angle B) > m(\angle C)$, then
(a) $AB > AC$ (b) $BC > AC$ (c) $AC > AB$ (d) $AB > BC$
- 2 In $\triangle ABC$, if $m(\angle B) = 90^\circ$, then
(a) $AC > CB$ (b) $AB > AC$ (c) $BC > AC$ (d) $AB = AC$
- 3 In $\triangle ABC$, if $m(\angle A) = 40^\circ$ and $m(\angle B) = 70^\circ$, then
(a) $AB < AC$ (b) $AB > AC$ (c) $\overline{AB} \perp \overline{AC}$ (d) $AB = AC$
- 4 In $\triangle XYZ$, if $m(\angle X) = 110^\circ$, $m(\angle Y) = 40^\circ$, then XY XZ
(a) $<$ (b) $>$ (c) $=$ (d) $//$

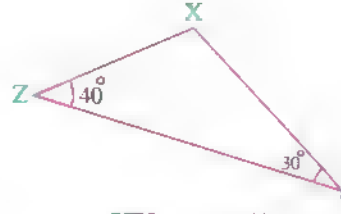
3 In the following figures, complete using $>$, $<$ or $=$:

1



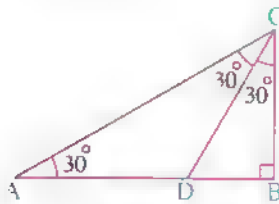
AB AC
AB BC
AC BC

2



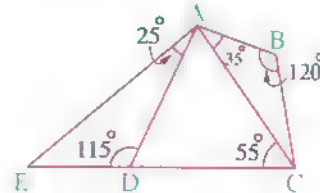
XY XZ
YZ XY
YZ XZ

3



AC BC
BC DB
AC BD
CA DC

4



BC AB
CD CA
AD AE
CD AD

4 XYZ is a triangle in which : $m(\angle X) = 45^\circ$, $m(\angle Y) = 85^\circ$ and $m(\angle Z) = 50^\circ$

Arrange the lengths of the sides of the triangle ascendingly.

5 ABC is a triangle in which : $m(\angle A) = 40^\circ$ and $m(\angle B) = 75^\circ$

Order the lengths of the sides of the triangle descendingly.

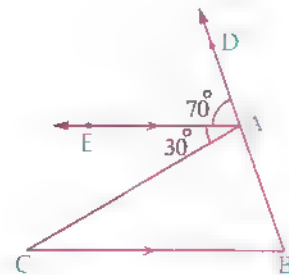
6 In the opposite figure :

$\overrightarrow{AE} \parallel \overrightarrow{BC}$,

$m(\angle DAE) = 70^\circ$

and $m(\angle EAC) = 30^\circ$

Prove that : $AC > AB$



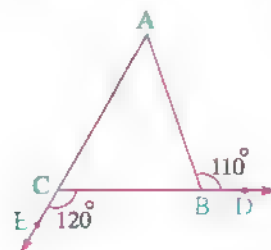
7 In the opposite figure :

ABC is a triangle, $D \in \overrightarrow{CB}$,

$E \in \overrightarrow{AC}$, $m(\angle ABD) = 110^\circ$

and $m(\angle BCE) = 120^\circ$

Prove that : $AB > BC$

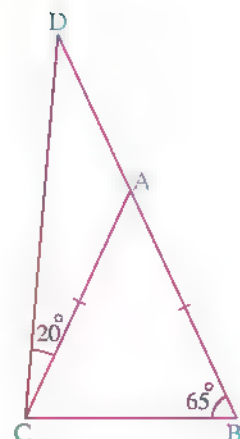


8 In the opposite figure :

$AB = AC$, $m(\angle ABC) = 65^\circ$

, $m(\angle ACD) = 20^\circ$, $A \in \overline{BD}$

Prove that : $AB > AD$

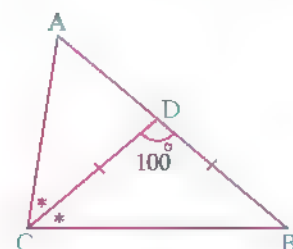


9 In the opposite figure :

ABC is a triangle , \overrightarrow{CD} bisects $\angle C$ and intersects \overline{AB} at point D

, $m(\angle BDC) = 100^\circ$ and $DB = DC$

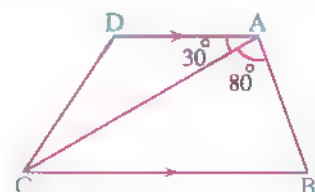
Prove that : $AC > DB$



10 In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle BAC) = 80^\circ$ and $m(\angle DAC) = 30^\circ$

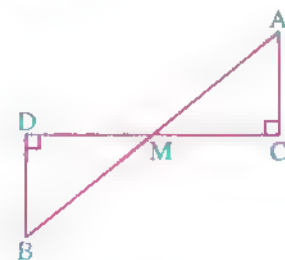
Prove that : $BC > AB$



11 In the opposite figure :

$\overline{AB} \cap \overline{CD} = \{M\}$, $\overline{AC} \perp \overline{CD}$ and $\overline{BD} \perp \overline{CD}$

Prove that : $AB > CD$

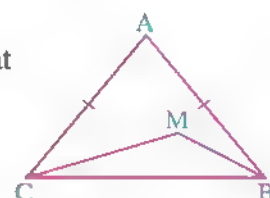


12 In the opposite figure :

ABC is a triangle in which : $AB = AC$, M is a point inside it such that

$m(\angle ABM) < m(\angle ACM)$

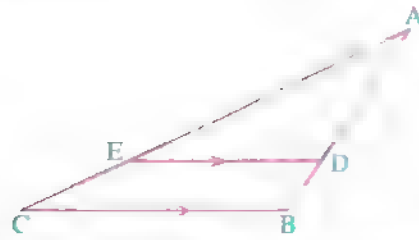
Prove that : $MC > MB$



13 In the opposite figure :

ABC is an obtuse-angled triangle at B
 $\overline{DE} \parallel \overline{BC}$

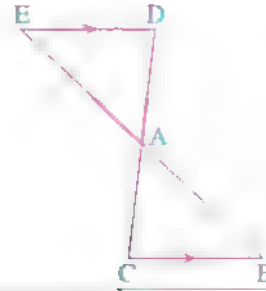
Prove that : $AE > AD$



14 In the opposite figure :

$AB > AC$, $\overline{DE} \parallel \overline{BC}$ and
 $\overline{DC} \cap \overline{BE} = \{A\}$

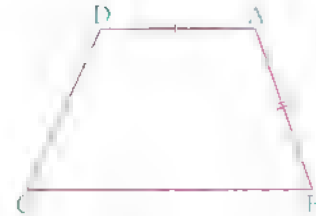
Prove that : $AE > AD$



15 In the opposite figure :

$ABCD$ is a quadrilateral , $AB = AD$
 and $m(\angle D) > m(\angle B)$

Prove that : $BC > CD$



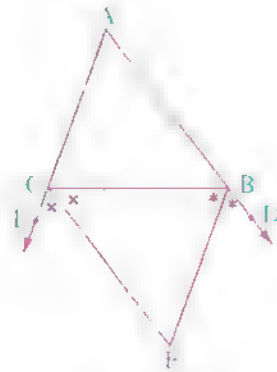
16 In the opposite figure :

ABC is a triangle in which : $AB > AC$, $D \in \overline{AB}$, $E \in \overline{AC}$
 \overline{BF} bisects $\angle DBC$ and \overline{CF} bisects $\angle BCE$
 $\overline{BF} \cap \overline{CF} = \{F\}$

Prove that :

1 $m(\angle FBC) > m(\angle BCF)$

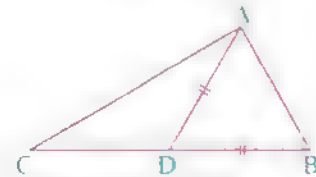
2 $CF > BF$



17 In the opposite figure :

ABC is a triangle and $D \in \overline{BC}$ where $BD = AD$

Prove that : $BC > AC$



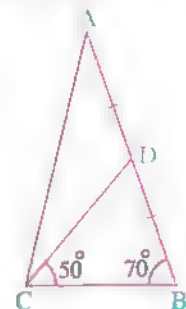
18 In the opposite figure :

D is the midpoint of \overline{AB} , $m(\angle B) = 70^\circ$ and $m(\angle DCB) = 50^\circ$

Prove that :

1 $m(\angle A) > m(\angle ACD)$

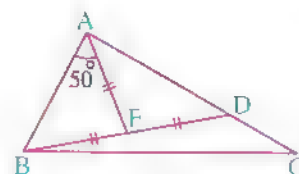
2 $\angle ACB$ is an acute angle.



19 In the opposite figure :

$AF = BF = DF$ and $m(\angle FAB) = 50^\circ$

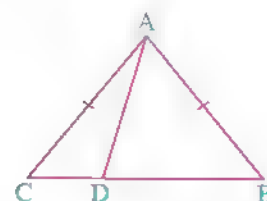
Prove that : 1 $AD > AB$ 2 $BC > AC$



20 In the opposite figure :

ABC is a triangle in which : $AB = AC$ and $D \in \overline{BC}$

Prove that : $AB > AD$

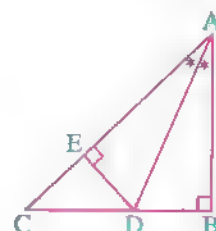


21 In the opposite figure :

$m(\angle B) = 90^\circ$, $\overline{DE} \perp \overline{AC}$ and \overline{AD} bisects $\angle BAE$

Prove that : 1 $BD = DE$

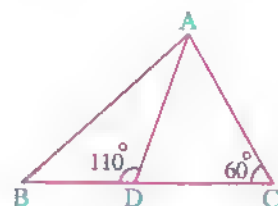
2 $DC > DB$



22 In the opposite figure :

$m(\angle ADB) = 110^\circ$ and $m(\angle C) = 60^\circ$

Prove that : $AB + AC > 2AD$



23 ABC is a right-angled triangle at B

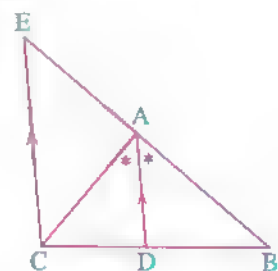
Prove that : $AB + BC < 2AC$

24 In the opposite figure :

ABC is a triangle, \overline{AD} bisects $\angle BAC$

, $\overline{CE} \parallel \overline{DA}$ and cuts \overline{BA} at E

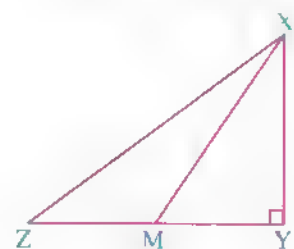
Prove that : $BE > BC$



25 In the opposite figure :

XYZ is a right-angled triangle at Y and $M \in \overline{YZ}$

Prove that : $XZ > XM$

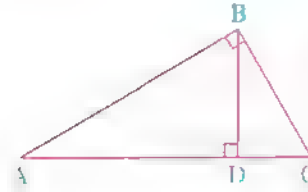


26 In the opposite figure :

$$m(\angle ABC) = 90^\circ, \overline{BD} \perp \overline{AC}$$

and $AB > BC$

Prove that : $AD > BD$



27 ABC is a triangle, \overrightarrow{CD} bisects $\angle C$, $\overrightarrow{CD} \cap \overline{AB} = \{D\}$

Prove that : $BC > BD$

28 ABC is a right-angled triangle at B, $D \in \overline{AC}$ and $E \in \overline{BC}$ where $AD = BE$

Prove that : $m(\angle CED) > m(\angle CDE)$

29 ABC is a triangle in which : $AB = AC$ and $X \in \overline{AC}$, draw \overrightarrow{XY} to cut \overline{AB} at Y and cut \overline{CB} at Z

Prove that : $AY > AX$

30 ABC is a triangle in which : $m(\angle A) = (5x + 2)^\circ$,

$$m(\angle B) = (6x - 10)^\circ \text{ and } m(\angle C) = (x + 20)^\circ$$

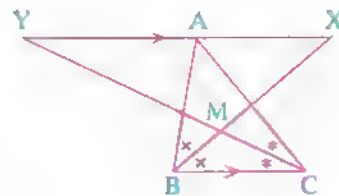
Order the lengths of sides of the triangle ascendingly.

For excellent pupils

31 In the opposite figure :

$AB < AC$, $\angle B$ and $\angle C$ are bisected by two bisectors meeting at M, \overrightarrow{BM} and \overrightarrow{CM} intersect the straight line drawn from A parallel to \overline{BC} at X and Y respectively.

Prove that : $BX < CY$



For the next term Ask for



EL-MORASSER



Maths & Science
& English



For all educational stages

Triangle inequality



Interactive Text

From the school book



Remember

Understand

Apply

Problem Solving

1 Is it possible to draw a triangle whose side lengths are as follows ? Give reasons :

1) 3 cm. , 4 cm. , 9 cm.

2) 5 cm. , 7 cm. , 8 cm.

3) 10 cm. , 6 cm. , 4 cm.

4) 13 cm. , 8 cm. , 6 cm.

5) 5 cm. , 3 cm. , 4 cm.

6) 9 cm. , 9 cm. , 19 cm.

2 Find the interval to which the length of the third side of the triangle belongs in each of the following triangles if the lengths of the two other sides are :

1) 6 cm. , 9 cm.

2) 3 cm. , 3 cm.

3) 2.9 cm. , 3.2 cm.

4) 5.7 cm. , 7.3 cm.

3 Choose the correct answer from those given :

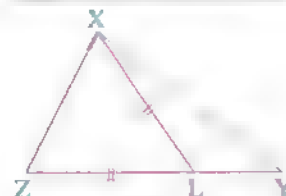
- 1 The sum of lengths of any two sides in a triangle is the length of the third side.
(a) less than (b) greater than (c) equal (d) half
- 2 The length of any side in a triangle the sum of lengths of the other two sides.
(a) $>$ (b) $<$ (c) $=$ (d) twice
- 3 Which of the following numbers cannot be the lengths of sides of a triangle ?
(a) 7 , 7 , 5 (b) 9 , 9 , 9 (c) 3 , 6 , 12 (d) 3 , 4 , 5
- 4 If the lengths of two sides in a triangle are 7 cm. and 4 cm. , then the length of the third side can be
(a) 1 cm. (b) 2 cm. (c) 3 cm. (d) 4 cm.

- 1 If the lengths of two sides of an isosceles triangle are 3 cm. and 7 cm. , then the length of the third side is
- (a) 7 cm. (b) 3 cm. (c) 4 cm. (d) 10 cm.
- 2 A triangle has one axis of symmetry, the lengths of two sides in it are 4 cm. and 8 cm. , then its perimeter =
- (a) 16 cm. (b) 20 cm. (c) 24 cm. (d) 30 cm.
- 3 In $\triangle ABC$, if $AB = 3$ cm. , $BC = 5$ cm. and $AC = x$ cm. , then $x \in$
- (a) $] 3, 5 [$ (b) $] 2, 5 [$ (c) $] 5, 8 [$ (d) $] 2, 8 [$
- 4 If the lengths of two sides of a triangle are 5 cm. and 10 cm. , then the length of the third side belongs to
- (a) $[10, 15 [$ (b) $] 5, 15 [$ (c) $] 5, 10]$ (d) $[10, 15]$
- 5 In $\triangle ABC$: $AB + BC - AC$
- (a) $> \text{zero}$ (b) $< \text{zero}$
(c) $= \text{zero}$ (d) $= \text{the perimeter of the triangle ABC}$
- 6 In $\triangle ABC$, $\frac{AB + BC}{AC}$ 1
- (a) $>$ (b) $<$ (c) $=$ (d) \leq

4 In the opposite figure :

XYZ is a triangle in which $L \in \overline{YZ}$ such that $XL = LZ$

Prove that : $YZ > XY$



5 ABC is a triangle in which \overline{BC} is the longest side , $D \in \overline{BC}$ such that $CD = CA$

Prove that : $AB > BD$

6 ABC is a triangle , \overline{AD} is drawn to cut \overline{BC} at D

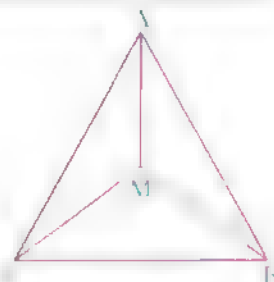
Prove that : $BD + DC + 2 AD > AB + AC$

7 In the opposite figure :

ABC is a triangle in which M is a point inside it.

Prove that :

$MA + MB + MC > \frac{1}{2}$ the perimeter of the triangle ABC

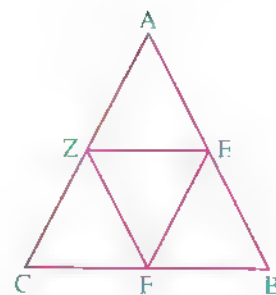


8 In the opposite figure :

ABC is a triangle in which $E \in \overline{AB}$
 $F \in \overline{BC}$ and $Z \in \overline{AC}$

Prove that :

The perimeter of $\triangle ABC >$ the perimeter of $\triangle EFZ$

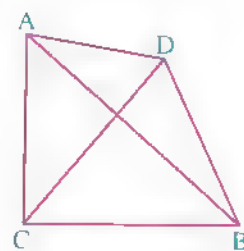


9 In the opposite figure :

ABC is a triangle and D is a point outside it.

Prove that :

The perimeter of $\triangle ABC < 2(DA + DB + DC)$

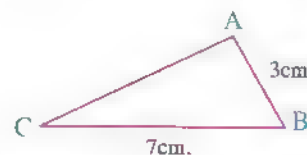


10 In the opposite figure :

ABC is a triangle in which :

$AB = 3 \text{ cm.}$, $BC = 7 \text{ cm.}$


Prove that: $m(\angle C) < m(\angle B)$



11  Prove that the length of any side in a triangle is less than half of the perimeter.

12 ABCD is a quadrilateral.

Prove that : $AB + BC + CD > AD$

13  Prove that the sum of the lengths of two diagonals in a convex quadrilateral is less than its perimeter.

14 Prove that the perimeter of any quadrilateral is less than twice the sum of lengths of its diagonals.

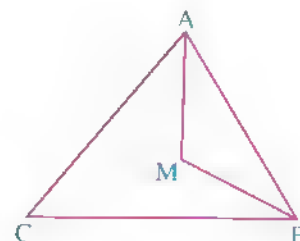


For excellent pupils

15 In the opposite figure :

M is a point inside the triangle ABC

Prove that : $AM + MB < AC + BC$



16 ABC is a triangle and F is the midpoint of \overline{BC} **Prove that :**

1) $AB + AC > 2AF$

2) $AB + AC > AF + B$

Summary of Unit 5



★ Axioms of inequality relation :

For any four numbers a , b , c and d :

$$1 \quad \text{If } a > b, \text{ then } a + c > b + c$$

$$\text{If } a > b, c > 0, \text{ then } ac > bc$$

$$5 \quad \text{If } a > b, c > d, \text{ then } a + c > b + d$$

$$2 \quad \text{If } a > b, \text{ then } a - c > b - c$$

$$\text{If } a > b, b > c, \text{ then } a > c$$

★ In a triangle , if two sides have unequal lengths , then the longer is opposite to the angle of the greater measure.

★ In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.

★ In the right-angled triangle , the hypotenuse is the longest side.

★ The length of the perpendicular line segment drawn from a point outside a straight line to this line is shorter than any line segment drawn from this point to the given straight line.

★ The distance between any point and a given straight line is the length of the perpendicular line segment drawn from this point to the given line.

★ Triangle inequality :

In any triangle , the sum of the lengths of any two sides is greater than the length of the third side.

★ The length of any side in a triangle is greater than the difference between the lengths of the other two sides and less than their sum.

Exams on Unit Five



Model 1

Answer the following questions :

1 Choose the correct answer from those given :

- 1 The sum of lengths of any two sides of a triangle the length of the third side.
(a) is smaller than (b) is greater than (c) equals (d) equals twice
- 2 In $\triangle ABC$, if $m(\angle B) > m(\angle C)$, then
(a) $AB < AC$ (b) $AB = AC$ (c) $AB > AC$ (d) $\overline{AB} \equiv \overline{BC}$
- 3 If the lengths of two sides in an isosceles triangle are 3 cm. and 7 cm. , then the length of the third side equals
(a) 7 cm. (b) 3 cm. (c) 4 cm. (d) 10 cm.
- 4 Which of the following numbers can be lengths of sides of a triangle ?
(a) 2 , 3 , 4 (b) 2 , 3 , 5 (c) 2 , 3 , 6 (d) 2 , 3 , 7
- 5 In $\triangle ABC$, if $m(\angle C) = 65^\circ$ and $m(\angle A) = 75^\circ$, then
(a) $AB > BC$ (b) $AB < AC$ (c) $BC > AB$ (d) $AB = AC$
- 6 In $\triangle ABC$, if $m(\angle B) = 130^\circ$, then its longest side is
(a) \overline{BC} (b) \overline{AC} (c) \overline{AB} (d) its median.

2 Complete the following :

- 1 If two sides in a triangle are unequal in length , then the longer of them is opposite to an angle
- 2 The longest side of the right-angled triangle is
- 3 In $\triangle ABC$, if $AB < BC < AC$, then the smallest angle in measure is
- 4 In the opposite figure :
If B , C belong to \overleftrightarrow{AD} , such that
 $DC > AB$, then AC DB
- 5 ABC is a triangle in which : $AB = 5$ cm. and $BC = 3$ cm. , then $AC \in] \dots , \dots [$



- 3** [a] In $\triangle ABC$: $m(\angle A) = 30^\circ$ and $m(\angle B) = 65^\circ$

Arrange the lengths of the sides of the triangle descendingly.

- [b] ABCD is a quadrilateral in which : AB = 6 cm. , BC = 3 cm. , CD = 4 cm.
and DA = 5 cm.

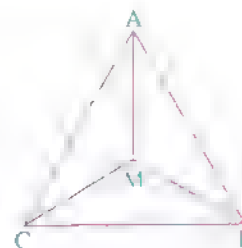
Prove that : $m(\angle DCB) > m(\angle DAB)$

- 4 [a] In the opposite figure :**

ABC is a triangle

and M is a point inside it.

Prove that : $MA + MB + MC > \frac{1}{2}$ the perimeter of ΔABC

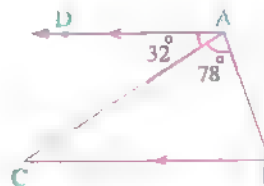


- [b] In the opposite figure :**

$$\overrightarrow{AD} \parallel \overrightarrow{BC}, m(\angle BAC) = 78^\circ$$

and $m(\angle CAD) = 32^\circ$

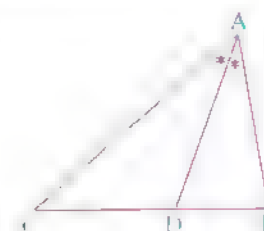
Prove that : $AC > AB$



- 5 [a]** In the opposite figure :

\overrightarrow{AD} bisects $\angle A$

Prove that : $AC > DC$

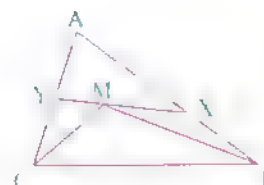


- [b] In the opposite figure :**

ABC is a triangle in which : $X \in \overline{AB}$

$$, Y \in \overline{AC} \text{ and } M \in \overline{XY}$$

Prove that : $AB + AC > MB + MC$



Model 2

Answer the following questions :

1 Choose the correct answer from those given :

1 If the triangle ABC is right-angled at B , then

- (a) $AC < AB$ (b) $AC < BC$ (c) $AB < AC$ (d) $BC = AB$

2 A triangle of two side lengths 4 cm. and 9 cm. , and it has one axis of symmetry , then the length of the third side equals

- (a) 4 cm. (b) 5 cm. (c) 9 cm. (d) 13 cm.

3 The length of any side in a triangle the sum of lengths of the two other sides.

- (a) is smaller than (b) is greater than (c) equals (d) is half

4 $\triangle ABD$ is an obtuse-angled triangle at B , C is the midpoint of \overline{BD} , then the greatest side in length is

- (a) \overline{AB} (b) \overline{AC} (c) \overline{BD} (d) \overline{AD}

5 Which of the following numbers can't be lengths of sides of a triangle ?

- (a) 3 , 4 , 4 (b) 3 , 4 , 5 (c) 3 , 4 , 6 (d) 3 , 4 , 7

6 In $\triangle XYZ$, $XY + YZ - XZ$

- (a) > 0 (b) < 0
(c) $= 0$ (d) = the perimeter of $\triangle XYZ$

2 Complete the following :

1 If two angles are unequal in measure in a triangle , then the greater angle in measure is opposite to

2 In the isosceles triangle ABC , if $AB = AC$, $m(\angle A) = 70^\circ$, then $AB < \dots\dots\dots$

3 In the triangle ABC , if $m(\angle A) = 67^\circ$, $m(\angle B) = 33^\circ$, then $AB > \dots\dots\dots > \dots\dots\dots$

4 If ABC is a triangle in which $m(\angle A) = m(\angle B) + m(\angle C)$, then the greatest side in length is

5 In the opposite figure :

$$M \in \overleftrightarrow{CD}$$

, then $m(\angle CMB) \dots\dots\dots m(\angle AMD)$

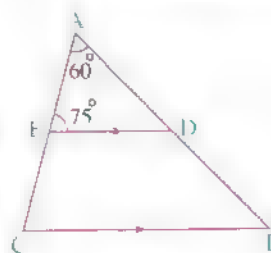


3 [a] In the opposite figure :

$$\overline{ED} \parallel \overline{BC}, m(\angle A) = 60^\circ$$

$$\text{and } m(\angle AED) = 75^\circ$$

Prove that : $AB > AC$

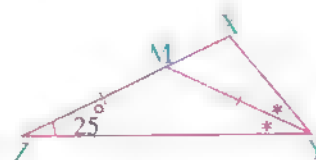


[b] In the opposite figure :

$$\overline{YM} \text{ bisects } \angle XYZ$$

$$, MY = MZ \text{ and } m(\angle Z) = 25^\circ$$

Prove that : $YM > XY$



4 [a] ABC is a triangle in which $AB = 7$ cm.

$$, BC = 4 \text{ cm. and } CA = 5 \text{ cm.}$$

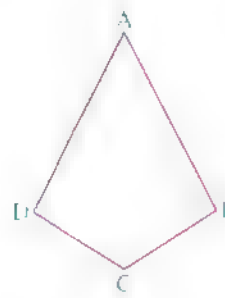
Arrange the angles of the triangle ascendingly due to their measures.

[b] In the opposite figure :

$$AB > BC$$

$$\text{and } AD > DC$$

Prove that : $m(\angle BCD) > m(\angle BAD)$



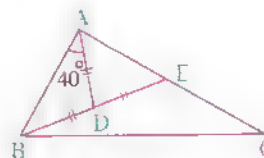
5 [a] In the opposite figure :

$$AD = BD = DE \text{ and } m(\angle DAB) = 40^\circ$$

Prove that :

$$1 \quad AD < AB$$

$$2 \quad BC > AC$$

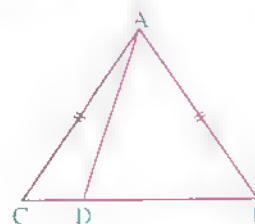


[b] In the opposite figure :

$$AB = AC$$

$$\text{and } D \in \overline{BC}$$

Prove that : $AB > AD$



A Research Project

On Unit Five



Project aims :

- Using the triangle inequality to determine three numbers can be side lengths of a triangle.
- Drawing a triangle knowing the lengths of its sides.
- Recognizing the type of a triangle according to the lengths of its sides.
- Recognizing the type of a triangle according to the measures of its angles.
- Comparing between the measures of angles of a triangle.
- Associating geometry with history.

Do a research project on the following topic :

"Many Arab scientists excelled in the field of geometry".

Discuss the following points using available resources :

- 1 Write a short note about some Arab scientists and their contributions in geometry.
- 2 Select three numbers can be side lengths of a triangle.
- 3 Use a ruler and a compass to draw that triangle.
- 4 Determine the type of that triangle according to the lengths of its sides and according to the measures of its angles.
- 5 Arrange the measures of the angles of that triangle in a descending order.

SKILLS

TIMSS Problems

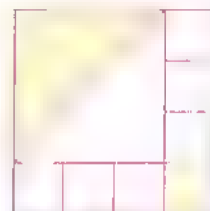
Accumulative basic skills

1 Complete the following :

- 1 A lamppost of height 4.5 metres is 2 metres far from a building of height 10.5 metres , then the distance between the top of the lamppost and the top of the building is metres.
- 2 The ratio between the lateral and the total areas of a cube is
- 3 A cuboid is of lateral area 200 cm^2 , and the dimensions of its base are 8 cm. and 12 cm. , then its height equals cm.
- 4 The measure of the angle between the two hands of the clock at 7 o'clock in degrees is°

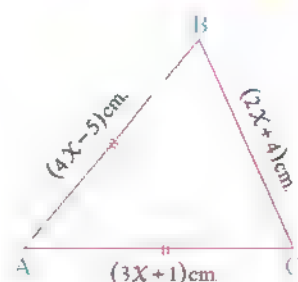
5 In the opposite figure :

A square divided into 7 small congruent squares and two congruent triangles. If the area of the coloured square = 4 cm^2 . , then the area of the coloured triangle is cm^2 .



6 In the opposite figure :

ABC is a triangle in which : $AB = (4x - 5) \text{ cm}$. , $BC = (2x + 4) \text{ cm}$. , $AC = (3x + 1) \text{ cm}$. , $AB = AC$, then the perimeter of $\triangle ABC = \dots\dots\dots \text{ cm}$.



- 7 A rectangle its length is $x \text{ cm}$. , its width is $y \text{ cm}$. and its perimeter is $P \text{ cm}$. , then the relation between x , y and P is $x = \dots\dots\dots$
- 8 If the side length of an equilateral triangle is 10 cm. , then its height is cm.

9 The measure of the angle of the regular pentagon is°

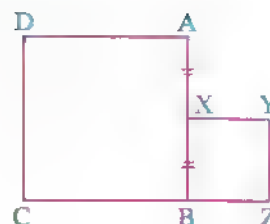
- 10 The opposite figure shows a coloured rectangle inside a parallelogram , then the area of the rectangle equals cm^2 .



11 In the opposite figure :

If the perimeter of the square ABCD = 24 cm.

, then the area of the square XYZB is cm^2 .



12 A cuboid is of total area 148 cm^2 . and its lateral area is 110 cm^2 . , then the area of its base is cm^2 .

2 Choose the correct answer from the given ones :

1 The acute angle supplements angle.

- (a) an acute (b) an obtuse (c) a right (d) a reflex

2 The number of diagonals of the hexagon equals

- (a) 3 (b) 6 (c) 9 (d) 12

3 The number of axes of symmetry of the opposite shape is

- (a) 1 (b) 2 (c) 3



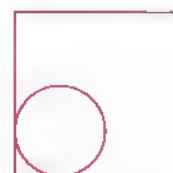
- (d) 4

4 A wire in the shape of an equilateral triangle of side length 4 cm. is reshaped as a square, then the side length of the square is cm.

- (a) 12 (b) 16 (c) 4 (d) 3

5 In the opposite figure :

A circle of radius length 2 cm. touches two sides of a square , then the area of the coloured part is cm^2 .



- (a) $4 - \pi$ (b) $\pi - 2$ (c) $\frac{\pi}{2}$ (d) 2π

6 The ratio between the area of a square region of side length l cm. and the area of a square region of side length $2l$ cm. is

- (a) 1 : 2 (b) l : 4 (c) 1 : 4 (d) 4 : 1

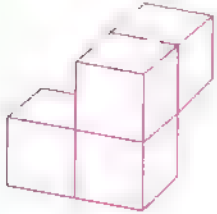
7 On a map , each 1 cm. represents 5 km. If the distance between two places is $\frac{1}{2}$ km. , then the distance between them on the map is

- (a) 0.1 cm. (b) 10 cm. (c) 2.5 cm. (d) 0.4 cm.

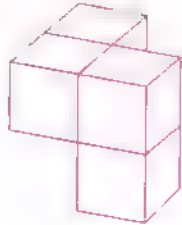
8 If the area of the base of a cuboid is 12 cm^2 . and the areas of two side faces are 6 cm^2 . and 8 cm^2 . , then the volume of the cuboid is cm^3 .

- (a) 9 (b) 576 (c) 24 (d) 32

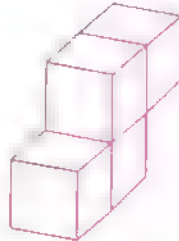
- 9 This solid will be rotated to another position.
Which of the following may be a position of the solid after rotation ?



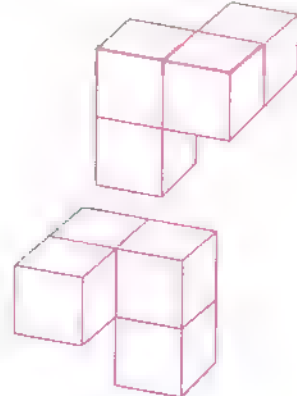
(a)



(b)



(c)

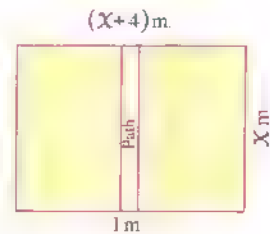


(d)

10 In the opposite figure :

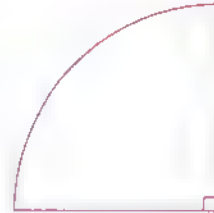
A rectangular garden with a rectangular path of width 1 metre. Which expression shows the area of the coloured part of the garden in square metres ?

- (a) $x^2 + 3x$ (b) $x^2 + 4x$
(c) $x^2 + 4x - 1$ (d) $x^2 + 3x - 1$



- 11 The opposite figure represents a quarter of a circle of radius length 2 cm. , then the perimeter of the figure in centimetres is

- (a) 2π (b) 5π
(c) $\pi + 4$ (d) $4\pi + 4$



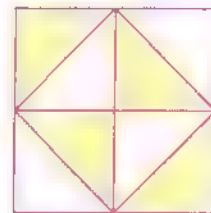
- 12 The area of a square whose side length is an integer may be cm^2 .

- (a) 600 (b) 900 (c) 800 (d) 700

13 In the opposite figure :

A square of perimeter 32 cm. divided into 8 congruent triangles , then the area of the coloured region is cm^2 .

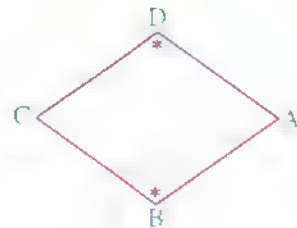
- (a) 4 (b) 8
(c) 16 (d) 32



14 In the opposite figure :

If $m(\angle A) + m(\angle C) = 140^\circ$
, $m(\angle B) = m(\angle D)$
, then $m(\angle B) = \dots\dots$

- (a) 50° (b) 55°
(c) 110° (d) 220°





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By a group of supervisors

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- Final Revision
- Final Examinations

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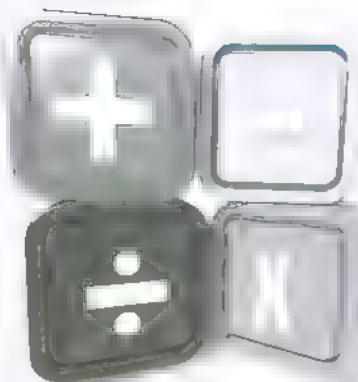
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CONTENTS

First Algebra and Statistics


- 18 Accumulative tests
- Final revision
- Final examinations :
 - School book examinations
(2 models examinations
+ model for the merge students)
 - 15 schools examinations



Second Geometry

- 9 Accumulative tests
- Final revision
- Final examinations :
 - School book examinations
(2 models examinations
+ model for the merge students)
 - 15 schools examinations



 Selected math exams from the multidisciplinary exams of the previous year

First

Algebra and Statistics

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- Final revision 24
- Final examinations : 38
 - School book examinations
(2 models examinations + model for the merge students)
 - 15 schools examinations.



Accumulative Tests

on Algebra and Statistics



Accumulative test

1

on lesson 1 – unit 1

1 Choose the correct answer from the given ones :

1 $\sqrt[3]{2\frac{10}{27}} = \dots\dots\dots$

(a) $\frac{3}{4}$

(b) $\frac{10}{3}$

(c) $\frac{4}{3}$

(d) $\frac{20}{27}$

2 $\sqrt{25} - \sqrt[3]{125} = \dots\dots\dots$

(a) 10

(b) 5

(c) zero

(d) -5

3 $\sqrt{4} = \sqrt[3]{\dots\dots\dots}$

(a) 2

(b) 4

(c) 8

(d) 16

4 $\sqrt[3]{\dots\dots\dots} + \sqrt[3]{27} = \sqrt{64}$

(a) 25

(b) -125

(c) 125

(d) 5

5 If $\sqrt[3]{x} = \frac{1}{4}$, then $x = \dots\dots\dots$

(a) $\frac{1}{2}$

(b) $\frac{1}{16}$

(c) $\frac{1}{64}$

(d) $\frac{1}{12}$

6 If $\sqrt[3]{x^2} = 4$, then $x = \dots\dots\dots$

(a) 8

(b) ± 8

(c) 4

(d) ± 4

7 $\sqrt[3]{x^6} = \sqrt{\dots\dots\dots}$

(a) x

(b) x^2

(c) x^3

(d) x^4

8 The cube whose volume is 1 cm^3 , then the sum of all its edge lengths = $\dots\dots\dots$ cm.

(a) 1

(b) 6

(c) 8

(d) 12

2 Find the S.S. of each of the following equations in \mathbb{Q} :

1 $x^3 + 1 = \text{zero}$

2 $8x^3 + 7 = 8$

1 Choose the correct answer from the given ones :

1 $\sqrt{6} \in \dots\dots\dots$

(a) \mathbb{N}

(b) \mathbb{Q}

(c) \mathbb{Q}

(d) \mathbb{Z}

2 The irrational number located between 2 and 3 is

(a) $\sqrt{7}$

(b) $\sqrt{10}$

(c) 2.5

(d) $\sqrt{3}$

3 If the volume of a cube is 125 cm^3 , then the area of one of its faces is

(a) 25 cm^2

(b) 50 cm^2

(c) 100 cm^2

(d) 125 cm^2

4 The nearest integer to $\sqrt[3]{-28}$ is

(a) -4

(b) -30

(c) -3

(d) 3

5 If $x = \sqrt{2}$, $y = 2$, then which of the following does not represent a rational number ?

(a) $x^2 + y$

(b) $x + y^2$

(c) $\sqrt{x^2 y}$

(d) $\sqrt{2} x y$

6 If $x < \sqrt{7} < x + 1$, then $x = \dots\dots\dots$

(a) 4

(b) 3

(c) 2

(d) 5

7 $\sqrt[3]{-8} + \sqrt{4} = \dots\dots\dots$

(a) 4

(b) 2

(c) zero

(d) 13

8 The S.S. of the equation : $x^2 - 16 = \text{zero}$ is \mathbb{Q} is

(a) \mathbb{Q}

(b) $\{-4\}$

(c) $\{-4, 4\}$

(d) $\{4\}$

2 [a] Prove that : $\sqrt{5}$ is included between 2.2 and 2.3

[b] Without using the calculator, prove that :

$\sqrt[3]{15}$ is included between 2.4 and 2.5

Choose the correct answer from the given ones :

1 The set of real numbers $\mathbb{R} = \dots\dots\dots$

- (a) $\mathbb{R}_+ \cup \mathbb{R}_-$ (b) $\mathbb{R}^* - \mathbb{R}_+$ (c) $\mathbb{Q} \cup \mathbb{Q}$ (d) $\mathbb{Q} \cap \mathbb{Q}$

2 $\mathbb{Q} \cap \mathbb{Q} = \dots\dots\dots$

- (a) \mathbb{Q} (b) \mathbb{Q} (c) \mathbb{R} (d) \emptyset

3 $\mathbb{R}_+ \cup \mathbb{R}_- = \dots\dots\dots$

- (a) \mathbb{R}_+ (b) \mathbb{R}_- (c) \mathbb{R}^* (d) \mathbb{R}

4 The irrational number located between 4 and 5 is $\dots\dots\dots$

- (a) $\sqrt{8}$ (b) $4\sqrt{2}$ (c) $3\sqrt{2}$ (d) $\sqrt{10}$

5 $\sqrt[3]{9} \dots\dots\dots \sqrt{4}$

- (a) $>$ (b) $<$ (c) $=$ (d) \leq

6 Which of the following rational numbers is located between $\frac{1}{5}$ and $\frac{2}{5}$?

- (a) $\frac{2}{10}$ (b) $\frac{1}{10}$ (c) 0.3 (d) -0.3

7 The S.S. of : $x^2 + 25 = \text{zero}$ in \mathbb{Q} is $\dots\dots\dots$

- (a) \emptyset (b) $\{-5, 5\}$ (c) $\{5\}$ (d) $\{-5\}$

8 The S.S. of the equation : $x^3 + 8 = 0$ in \mathbb{R} is $\dots\dots\dots$

- (a) \emptyset (b) $\{2\}$ (c) $\{-8\}$ (d) $\{-2\}$

9 $\sqrt[3]{64} - \sqrt{64} = \dots\dots\dots$

- (a) 4 (b) 8 (c) zero (d) -4

10 If $\frac{1}{a}$, $\frac{a}{\sqrt{5}}$ are two real numbers included between zero and 1, then a can equal $\dots\dots\dots$

- (a) -2 (b) 1 (c) $\sqrt{5}$ (d) 2

1 Choose the correct answer from the given ones :

1 The interval that represents : $X = \{x : x \in \mathbb{R}, -1 < x \leq 4\}$ is

- (a) $[-4, 1]$ (b) $] -4, 1[$ (c) $[-1, 4[$ (d) $] -1, 4]$

2 {The multiplicative identity element, 3} $[0, 3]$

- (a) \in (b) \notin (c) \subset (d) $\not\subset$

3 $\mathbb{R} = \dots\dots\dots$

- (a) $\mathbb{R}_+ \cup \mathbb{R}$ (b) $] -\infty, \infty[$ (c) $] -\infty, 0]$ (d) $[0, \infty[$

4 If $\sqrt[3]{4 - \sqrt[3]{x}} = 5$, then $x = \dots\dots\dots$

- (a) 125 (b) 27 (c) -27 (d) 3

5 The square whose area is 10 cm^2 , then its side length is cm.

- (a) 5 (b) -5 (c) $\sqrt{10}$ (d) $\sqrt[3]{10}$

6 $] -5, 3] -] -5, 3[= \dots\dots\dots$

- (a) $\{3\}$ (b) $\{-5\}$ (c) $\{-5, 3\}$ (d) $\{-3\}$

7 If x is a negative number, then which of the following numbers is positive ?

- (a) x^3 (b) $2x$ (c) x^2 (d) $\frac{x}{2}$

8 $] -\infty, 1[\cup] 1, \infty[= \dots\dots\dots$

- (a) \mathbb{R} (b) $\{1\}$ (c) \emptyset (d) $\mathbb{R} - \{1\}$

2 If $X = [2, 5]$, $Y = [0, 3]$

1 Write X using the description method.

2 Represent X, Y on the number line.

3 Find : $X - Y$ as an interval by using the number line. Is $\sqrt{29} \in X - Y$?

3 If $X = [-1, 4]$, $Y = [3, \infty[$

Find using the number line each of : $X \cup Y, X \cap Y, Y - X$

1 Choose the correct answer from the given ones :

1 $\frac{6}{\sqrt{3}} = \dots\dots\dots$

(a) $\sqrt{2}$

(b) 2

(c) $2\sqrt{3}$

(d) $6\sqrt{3}$

2 $(2^3\sqrt{2})^3 = \dots\dots\dots$

(a) 4

(b) 8

(c) 16

(d) 40

3 The multiplicative inverse of $\frac{\sqrt{2}}{2}$ is $\dots\dots\dots$

(a) $\sqrt{2}$

(b) $2\sqrt{2}$

(c) 2

(d) $-\frac{\sqrt{2}}{2}$

4 $[-1, 5] - \{-1, 5\} = \dots\dots\dots$

(a) $\{-1, 5\}$

(b) $]-1, 5[$

(c) $]-1, 5]$

(d) $[-1, 5[$

5 $\mathbb{Q} \cup \mathbb{Q} = \dots\dots\dots$

(a) zero

(b) \mathbb{Z}

(c) \mathbb{R}

(d) \emptyset

6 The cube whose volume is 8 cm^3 , its total area = $\dots\dots\dots \text{ cm}^2$

(a) 16

(b) 24

(c) 64

(d) 8

7 The rectangle whose dimensions are $(\sqrt{7} - 1) \text{ cm.}$, $(\sqrt{7} + 1) \text{ cm.}$, its area is $\dots\dots\dots \text{ cm}^2$

(a) 8

(b) 7

(c) 6

(d) $2\sqrt{7}$

8 If $x = \sqrt{2} + 3$, $y = \sqrt{2} - 3$, then $x^2 - y^2 = \dots\dots\dots$

(a) $2\sqrt{3}$

(b) $12\sqrt{2}$

(c) $6\sqrt{5}$

(d) $3\sqrt{6}$

2 If $y = \sqrt{2 + \sqrt{3}}$, find the value of : $y^4 - 2y^2 + 1$

3 If $a = 5 - \sqrt{3}$, $b = 5 + \sqrt{3}$, find in the simplest form "Showing steps"

1 $a \cdot b$

2 $a^2 + b^2$

1 Choose the correct answer from the given ones :

1] The multiplicative inverse of the number $\sqrt{32}$ is

- (a) $4\sqrt{2}$ (b) $\frac{\sqrt{2}}{8}$ (c) $2\sqrt{5}$ (d) $\frac{\sqrt{3}}{2}$

2 $\sqrt{5}, \sqrt{20}, \sqrt{45}, \sqrt{80}, \dots$ "In the same pattern"

- (a) $\sqrt{75}$ (b) $\sqrt{90}$ (c) $\sqrt{112}$ (d) $\sqrt{125}$

3 $\sqrt{75} - \sqrt{27} - \sqrt{12} = \dots$

- (a) $\sqrt{3}$ (b) zero (c) $-\sqrt{3}$ (d) $-2\sqrt{3}$

4 The additive inverse of the number $-\sqrt{5}$ is

- (a) $\sqrt{5}$ (b) 5 (c) -5 (d) $\frac{-1}{\sqrt{5}}$

5 $]1, 3] \cap [-3, -1] = \dots$

- (a) \emptyset (b) $\{-3\}$ (c) $\{-1\}$ (d) $\{3\}$

6 The S.S. of the equation : $x^2 + 9 = 0$ in \mathbb{R} is

- (a) $\{3\}$ (b) \emptyset (c) $\{-3\}$ (d) $\{-3, 3\}$

7 $\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{2}} = \dots$

- (a) $2\sqrt{2}$ (b) 1 (c) $\sqrt{2}$ (d) $\frac{\sqrt{2}}{2}$

8 If $x = 2\sqrt{2} - \sqrt{7}$, $y = 2\sqrt{2} + \sqrt{7}$, then $xy - 1 = \dots$

- (a) 5 (b) zero (c) -4 (d) 7

2 If $A =]-\infty, 3[$, $B = [-2, 5]$, find using the number line each of :

1 $A \cap B$

2 $B - A$

3 Simplify each of the following to the simplest form :

1 $\sqrt{18} + \sqrt{54} - 3\sqrt{2} - \frac{1}{2}\sqrt{24}$

2 $\sqrt{128} - \frac{14}{\sqrt{2}} + 6\sqrt{\frac{1}{2}} - (\sqrt{2})^5$

1 Choose the correct answer from the given ones :

1 The conjugate number of the number : $\sqrt{3} + \sqrt{2}$ is

- (a) $\sqrt{3} + \sqrt{2}$ (b) $\sqrt{3} - \sqrt{2}$ (c) $\sqrt{3}$ (d) $-\sqrt{2}$

2 The multiplicative inverse of the number : $1 - \sqrt{2}$ is

- (a) $\sqrt{2} - 1$ (b) $1 - \sqrt{2}$ (c) $-\sqrt{2} - 1$ (d) $1 + \sqrt{2}$

3 If $x = 2 + \sqrt{5}$, $y = 2 - \sqrt{5}$, then $(x - y)^2 =$

- (a) $2\sqrt{8}$ (b) 20 (c) $4\sqrt{5}$ (d) -1

4 $]-3, 2[\cap \mathbb{Z}_+ =$

- (a) $\{1\}$ (b) $\{1, 2\}$ (c) $\{2, -3\}$ (d) \emptyset

5 $\sqrt[3]{16} - \sqrt[3]{-64} =$

- (a) zero (b) 12 (c) 8 (d) -8

6 The irrational number included between 3 and 6 is

- (a) $\sqrt{5}$ (b) $\sqrt{10}$ (c) $\sqrt{25}$ (d) $\sqrt[3]{27}$

7 The multiplicative inverse of the number : $\frac{\sqrt{5}}{5}$ is

- (a) $5\sqrt{5}$ (b) $-\sqrt{5}$ (c) $\sqrt{5}$ (d) $2\sqrt{5}$

8 If $x = \sqrt{7} + \sqrt{3}$ and y is the conjugate number of x , then $xy =$

- (a) 10 (b) 4 (c) 40 (d) 58

2 [a] If $xy = 1$, $y = 2 + \sqrt{3}$, find the value of : $x^2 + \sqrt{48}$ in its simplest form.

[b] Without using the calculator , simplify the following to the simplest form :

$$2\sqrt{5}(\sqrt{5} - 2) + \sqrt{20} - 10\sqrt{\frac{1}{5}}$$

3 If $x = \sqrt{5} + 2$, $y =$ the multiplicative inverse of x , prove that x and y are conjugate numbers , then find the value of : $\left(\frac{x-y}{x+y}\right)^2$

1 Choose the correct answer from the given ones :

1 $\sqrt[3]{16} - \sqrt[3]{2} = \dots$

(a) $\sqrt[3]{14}$

(b) $\sqrt[3]{2}$

(c) $3\sqrt[3]{2}$

(d) 8

2 $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = \dots$

(a) 0

(b) 1

(c) -1

(d) 2

3 $\sqrt[3]{2} + \sqrt[3]{2} = \dots$

(a) $\sqrt[3]{16}$

(b) $\sqrt[3]{8}$

(c) $\sqrt[3]{4}$

(d) $\sqrt[3]{2}$

4 The S.S. of the equation : $x^3 = 27$ in \mathbb{R} is

(a) \emptyset

(b) $\{3\}$

(c) $\{-3\}$

(d) $\{0\}$

5 The set of non-positive real numbers are

(a) $[0, \infty[$

(b) $]0, \infty[$

(c) $] -\infty, 0]$

(d) $] -\infty, 0[$

6 $x = \sqrt[3]{3} + 1$, $y = \sqrt[3]{3} - 1$, then $x + y = \dots$

(a) $3\sqrt[3]{6}$

(b) $2\sqrt[3]{3}$

(c) 3

(d) 0

7 $\sqrt{12} + \sqrt{3} = \dots$

(a) 3

(b) $\sqrt{15}$

(c) $3\sqrt{3}$

(d) $3\sqrt{2}$

8 $\frac{5}{4}\sqrt[3]{5} + \frac{3}{4}\sqrt[3]{5} = \dots$

(a) 5

(b) $\sqrt[3]{20}$

(c) $\sqrt[3]{5}$

(d) $\sqrt[3]{40}$

2 Simplify each of the following to the simplest form :

1 $\sqrt[3]{54} + 4\sqrt[3]{\frac{1}{4}} - \sqrt[3]{-2}$

2 $\sqrt[3]{32} + 4\sqrt[3]{\frac{1}{2}} - (2\sqrt[3]{-2})^2 + (\sqrt{2})^{\text{zero}} - \left(\frac{2}{\sqrt{2}}\right)^2$

3 [a] If $X = [-2, 3]$, $Y =]-\infty, 1]$

, find using the number line each of :

1 $X \cap Y$

2 $X - Y$

[b] If $x = \frac{6}{\sqrt{2}}$, $y = \frac{1}{\sqrt{2}-1}$, find the value of : $\left(y - \frac{1}{3}x\right)^2$

1 Choose the correct answer from the given ones :

- 1 If the volume of a sphere = $\frac{4}{3} \pi \text{ cm}^3$, then its radius = cm.
 (a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\left(\frac{4}{3}\right)^{\text{zero}}$ (d) π
- 2 The volume of a cube is 512 cm^3 , then the perimeter of one face = cm.
 (a) 8 (b) 64 (c) 32 (d) 16
- 3 If the dimensions of a cuboid are $\sqrt{2} \text{ cm}$, $\sqrt{3} \text{ cm}$, and $\sqrt{6} \text{ cm}$, then its volume = cm^3 .
 (a) 6 (b) 36 (c) $6\sqrt{6}$ (d) $18\sqrt{2}$
- 4 $\sqrt[3]{2} + \sqrt[3]{2} = \dots\dots\dots$
 (a) $\sqrt[3]{16}$ (b) $\sqrt[3]{8}$ (c) $\sqrt[3]{4}$ (d) $\sqrt[3]{2}$
- 5 If $a = \sqrt{7} + 4$, $b = \sqrt{7} - 4$, then $a b = \dots\dots\dots$
 (a) 3 (b) 16 (c) 9 (d) 9
- 6 $\mathbb{R}_+ \cap [-1, 3] = \dots\dots\dots$
 (a) $[0, 3]$ (b) $]0, 3]$ (c) $[0, 3[$ (d) $]0, 3[$
- 7 A right circular cylinder whose base area is 20 cm^2 and its volume is 80 cm^3 , then its height = cm.
 (a) 3 (b) 4 (c) 5 (d) 100
- 8 A sphere and a cylinder are equal in volume and there radii are equal in length, then the height of the cylinder = the radius of the sphere.
 (a) 3 (b) 4 (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

2 [a] Find the height of a right circular cylinder whose height is equal to its base radius length and its volume is $64 \pi \text{ cm}^3$

[b] If $x = \frac{4}{\sqrt{7}-\sqrt{3}}$, $y = \sqrt{7}-\sqrt{3}$

Prove that : x, y are two conjugate numbers, **then find the value of :** xy

3 [a] The volume of a sphere is $36 \pi \text{ cm}^3$. Calculate its surface area in terms of π

[b] Simplify to the simplest form : $\sqrt{125} - \sqrt[3]{250} + \frac{1}{2} \sqrt[3]{16} + \sqrt{20}$

1 Choose the correct answer from the given ones :

1 The S.S. of the equation : $x + 5 = |-5|$ in \mathbb{N} is

- (a) \emptyset (b) $\{0\}$ (c) $\{10\}$ (d) $\{-10\}$

2 The S.S. of the inequality : $-2x \geq 6$ in \mathbb{R} is the interval

- (a) $]-\infty, -3]$ (b) $[3, \infty[$ (c) $]3, \infty[$ (d) $]-\infty, -3[$

3 The S.S. of the equation : $\sqrt[3]{3}x - 1 = 2$ in \mathbb{R} is ...

- (a) $\{2\sqrt[3]{3}\}$ (b) $\{\sqrt[3]{3}\}$ (c) $\{2\}$ (d) $\{2\sqrt[3]{2}\}$

4 If three quarters of the volume of a sphere is $8\pi \text{ cm}^3$, then its radius length is cm.

- (a) 64 (b) 8 (c) 4 (d) 2

5 The S.S. of the equation : $(x^2 + 9)(x^3 + 1) = \text{zero}$ in \mathbb{R} is

- (a) \emptyset (b) 1 (c) $\{-1\}$ (d) $\{3, 1\}$

6 The irrational number included between 2 and 3 is

- (a) $2\frac{1}{2}$ (b) $\sqrt[3]{10}$ (c) $\sqrt[3]{7}$ (d) $\sqrt[3]{3}$

7 The S.S. of the inequality : $x + 3 < 3$ in \mathbb{R} is

- (a) $[0, \infty[$ (b) \mathbb{R} (c) $]-\infty, 0]$ (d) \mathbb{R}_+

8 The S.S. of the inequality : $-2 < 3x + 7 \leq 10$ in \mathbb{R} is

- (a) $]-3, 1]$ (b) $]1, 3]$ (c) $[-3, 1]$ (d) $[-3, 1[$

2 [a] The volume of a sphere is $\frac{99000}{7} \text{ cm}^3$, calculate its radius length. ($\pi = \frac{22}{7}$)

[b] Find the S.S. of the inequality : $-3 \leq 2x + 1 < 7$ in \mathbb{R} in the form of an interval, then represent the solution on the number line.

3 [a] If $X = [-1, 4[$, $Y = [2, 6]$ using the number line find each of the following :

- 1 $X \cup Y$ 2 $X \cap Y$

[b] Find in \mathbb{R} the S.S. of the inequality : $x - 1 < 3 - x \leq x + 5$ in the form of an interval and represent it on the number line.

1 Choose the correct answer from the given ones :

- 1 If $(2, -5)$ satisfies the relation : $3x - y + k = 0$, then $k = \dots\dots\dots$
 - (a) 1
 - (b) 11
 - (c) -1
 - (d) -11
- 2 The relation : $2x + y = 6$ is represented by a straight line intersects the y-axis at the point $\dots\dots\dots$
 - (a) $(0, -6)$
 - (b) $(0, 6)$
 - (c) $(6, 0)$
 - (d) $(3, 0)$
- 3 The relation : $2x = 3y$ is represented by a straight line passing through the point $\dots\dots\dots$
 - (a) $(2, 3)$
 - (b) $(0, \frac{3}{2})$
 - (c) $(0, 0)$
 - (d) $(\frac{2}{3}, 0)$
- 4 The S.S. of the equation : $x + 9 = |-5|$ in \mathbb{R} is $\dots\dots\dots$
 - (a) $\{0\}$
 - (b) \emptyset
 - (c) $\{-4\}$
 - (d) $\{4\}$
- 5 The volume of a sphere is $\frac{32}{3} \pi \text{ cm}^3$, then its radius length = $\dots\dots\dots$ cm.
 - (a) 2
 - (b) 4
 - (c) 8
 - (d) 32
- 6 The simplest form of the expression : $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$ is $\dots\dots\dots$
 - (a) $\sqrt{3}$
 - (b) 1
 - (c) $\sqrt{2}$
 - (d) $2\sqrt{3}$
- 7 The point $(3k, 2k)$ lies on the straight line : $x - 3y = 9$, then $k = \dots\dots\dots$
 - (a) -3
 - (b) 1
 - (c) 0
 - (d) 2
- 8 The x-axis is the graphical representation of the relation $y = \dots\dots\dots$
 - (a) 1
 - (b) -1
 - (c) zero
 - (d) x

2 [a] Find four ordered pairs satisfy the relation : $y + 2x = 5$

[b] Without using the calculator , simplify the following to the simplest form :

"Showing steps"

$$\sqrt{12} + \sqrt[3]{54} + 3\sqrt{\frac{1}{3}} - 6\sqrt[3]{\frac{1}{4}}$$

3 [a] Find in \mathbb{R} the S.S. of the inequality :

$-2x + 5 \leq x - 4$ and represent it on the number line.

[b] Graph the relation : $x - 4y = 4$ and if the straight line representing the relation intersects the x-axis at the point A and the y-axis at the point B , find the area of the triangle OAB where O is the origin point.

1 Choose the correct answer from the given ones :

1 The slope of the straight line which passes through the two points (2 , 3) and (3 , 4) is

- (a) $\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) - 1

2 Which of the following ordered pairs satisfies the relation : $y = -x - 3$?

- (a) (1 , -4) (b) (-2 , -5) (c) (1 , -2) (d) (3 , 0)

3 $\sqrt[3]{16} - \sqrt[3]{64} =$

- (a) zero (b) 12 (c) 8 (d) - 8

4 The slope of the straight line perpendicular on y-axis is

- (a) positive. (b) negative. (c) zero. (d) undefined.

5 The slope of the vertical straight line is

- (a) zero (b) 1 (c) - 1 (d) undefined.

6 The lateral area of the cube whose volume is 216 cm^3 equals cm^2

- (a) 36 (b) 6 (c) 144 (d) 216

7 If the straight line which passes through the two points (2 , 3) and (5 , y) is parallel to X-axis , then $y =$

- (a) 3 (b) - 3 (c) zero (d) $\frac{1}{3}$

8 If the slope of the straight line representing the relation : $x + m y = 5$ is undefined , then $m =$

- (a) 1 (b) - 1 (c) 5 (d) zero

2 [a] Represent graphically , then find the slope of the straight line that represents the relation : $x + y = 7$

[b] Find the S.S. in \mathbb{R} for the inequality :

$2x + 3 \leq 5x + 3 \leq 2x + 9$, then represent it on the number line.

3 [a] Prove that the points A , B and C are collinear where A (2 , -3) , B (4 , -5) and C (0 , -1)

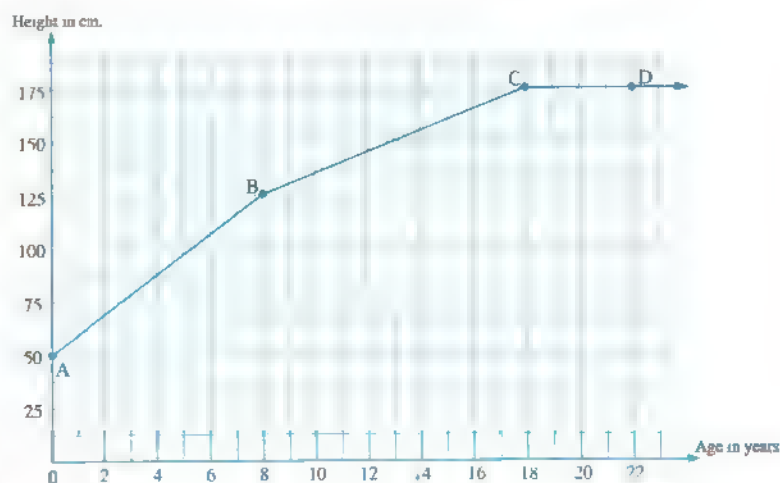
[b] Simplify to the simplest form : $5\sqrt{8} + 4\sqrt[3]{\frac{1}{4}} - 2\sqrt{50} - \sqrt[3]{16}$

1 Choose the correct answer from the given ones :

- 1 If 100 gm. of food have 300 calories , then the number of calories that exist in 30 gm. of the same food equals calories.
 (a) 90 (b) 100 (c) 900 (d) 9000
- 2 The slope of any straight line parallel to X-axis is
 (a) positive. (b) negative. (c) zero. (d) undefined.
- 3 The slope of the straight line which passes through the two points : (2 , k) and (4 , 7) equals 3 , then the value of k =
 (a) 5 (b) 4 (c) 3 (d) 1
- 4 The straight line which represents the relation : $4x = 3y$ is passing through the point
 (a) (4 , 3) (b) (3 , 4) (c) (4 , 0) (d) (0 , 3)
- 5 $\sqrt{9}$ $]-3, \infty[$
 (a) \subset (b) $\not\subset$ (c) \in (d) \notin

2 The opposite figure shows the relation between the height of a person (in cm.) and his age (in years) :

- 1 Find the slope of each of \overrightarrow{AB} , \overrightarrow{BC} and \overrightarrow{CD}
- 2 Calculate the difference between the height of this person when he was 8 years old and his height when he was 30 years old.



- 3 [a] A right circular cylinder , its diameter length is 14 cm. and its height is 10 cm. , find the lateral area and the volume of the cylinder. $(\pi = \frac{22}{7})$
 [b] Represent graphically the relation : $y - 2x = 1$, then find the points of intersection of the straight line with the two axes.

4 If \mathbb{R}_+ is the set of the positive real numbers and $Z = [-2, 3]$, find :

- 1 $\mathbb{R}_+ \cap Z$
- 2 $\mathbb{R}_+ \cup Z$
- 3 \mathbb{R}_+

Accumulative test

14

till lesson 1 – unit 3

1 Choose the correct answer from the given ones :

1 If $(5, a)$ satisfies the relation : $x + y = 3$, then $a = \dots\dots\dots$

- (a) 2 (b) zero (c) -2 (d) 8

2 The irrational number included between 3 and 4 is $\dots\dots\dots$

- (a) 1.5 (b) $\sqrt{5}$ (c) $\sqrt{11}$ (d) 3.5

3 The slope of the straight line passing through $(3, 2)$ and $(4, 2)$ is ..

- (a) undefind. (b) $\frac{4}{7}$ (c) zero (d) $\frac{1}{7}$

4 $\sqrt{50} - \sqrt{8} = \dots\dots\dots$

- (a) $\sqrt{42}$ (b) $3\sqrt{2}$ (c) $2\sqrt{3}$ (d) $\sqrt{58}$

2 The following table shows the marks obtained by 30 students in an examination :

5	9	11	4	9	9	16	7	8	12	2	10	7	12	5
8	15	13	13	9	7	14	19	3	11	14	3	12	13	7

Form the frequency table to these data.

3 [a] Find the S.S. in \mathbb{R} of the following inequality as an interval : $1 < 3x - 2 < 13$

[b] Find three ordered pairs satisfying the relation :

$y = x + 2$, then represent it graphically.


4 [a] Simplify to the simplest form : $4\sqrt{\frac{1}{2}} + \sqrt{32} - \sqrt{72}$

[b] If $x = \sqrt{5} + 2$, $xy = 1$

Find : y , then prove that x and y are two conjugate numbers.

Accumulative test 15 till lesson 2 – unit 3

1 Choose the correct answer from the given ones :

- 1 The slope of the straight line passes through (1, 5) and (5, -3) is
 (a) 4 (b) -2 (c) zero (d) undefined.
- 2 If $x = 2 + \sqrt{5}$, $y = 2 - \sqrt{5}$, then $(x - y)^2 = \dots\dots\dots$
 (a) $2\sqrt{8}$ (b) 20 (c) $4\sqrt{5}$ (d) -1
- 3 The figure  represents the interval
 (a) $[3, 7[$ (b) $[3, 7]$ (c) $]3, 7[$ (d) $]3, 7]$
- 4 If the point (3, a) lies on the straight line : $y + 2x = 5$, then $a = \dots\dots\dots$
 (a) 1 (b) -1 (c) 11 (d) zero

2 The following table is the frequency distribution of wages of 100 workers weekly :

Sets	50 -	60 -	70 -	80 -	90 -	Total
Frequency	5	15	30	40	10	100

- 1 Find the number of workers whose wages are less than 70 pounds weekly.
- 2 Graph the ascending cumulative frequency curve.

3 Find the S.S. in \mathbb{R} of each of the following :

- 1 $2\sqrt{2}x - 1 = 3$
- 2 $-5 \leq 2x - 3 < 7$ and represent the S.S. on the number line.

4 [a] Find the slope of \overleftrightarrow{AB} , where : A (-1, 3) and B (2, 5), is the point C (8, 1) $\in \overleftrightarrow{AB}$?

[b] If $X = [2, \infty[$ and $Y =]-2, 3[$, find using the number line :

- 1 $X \cap Y$ 2 $X \cup Y$ 3 $X - Y$

1 Choose the correct answer from the given ones :

1 The arithmetic mean of a frequency distribution equals

(a) $\frac{\text{sum of } (X \times f)}{\text{sum of } f}$

(b) $\frac{\text{sum of } (X + f)}{\text{sum of } f}$

(c) $\frac{\text{sum of } f \times \text{sum of } X}{\text{sum of } f}$

(d) $\text{sum of } (f + X) \times \frac{2}{\text{sum of } f}$

2 If the lower limit of a set is 15 and its centre is 20 , then its upper limit is

(a) 5

(b) 15

(c) 35

(d) 25

3 The arithmetic mean of the values : 18 , 23 , $2k - 1$, 29 , k is 18 , then k =

(a) 1

(b) 7

(c) 29

(d) 90

4 The slope of any straight line parallel to X-axis is

(a) zero

(b) undefined.

(c) 1

(d) - 1

5 $\sqrt{a} + \sqrt{18} = 4\sqrt{2}$ if a =

(a) $\sqrt{2}$

(b) zero

(c) 2

(d) 3

6 The conjugate of the number : $\sqrt{2} - \sqrt{3}$ is

(a) $\sqrt{2} + \sqrt{3}$

(b) $\sqrt{3} - 2$

(c) $\sqrt{2} - 3$

(d) $-\sqrt{2} + \sqrt{3}$

7 If the arithmetic mean of the lengths of a triangle equals 12 cm.
 , then its perimeter = cm.

(a) 4

(b) 36

(c) 24

(d) 48

8 The mean of the values : $\sqrt{5}$ and $\sqrt{45}$ is

(a) $\sqrt{5}$

(b) $2\sqrt{5}$

(c) $3\sqrt{5}$

(d) $4\sqrt{5}$

2 The following table shows the frequency distribution of extra wages weekly for 100 workers in a factory :

Extra wages in pounds	20 -	30 -	40 -	50 -	X -	70 -
Number of workers	10	k	22	26	20	8

1 Calculate the value of each of X and k

2 Find the arithmetic mean of this distribution.

3 [a] A right circular cylinder of volume is 924 cm^3 and its height 6 cm.

Calculate the diameter length of its base ($\pi = \frac{22}{7}$)

[b] If $X = \frac{4}{\sqrt{7} - \sqrt{3}}$, $y = \frac{4}{\sqrt{7} + \sqrt{3}}$, put X and y in the simplest form

, then find the value of : $X^2 y^2$

Accumulative test 17 till lesson 4 – unit 3

1 Choose the correct answer from the given ones :

- 1 The median of the values : 8 , 4 , 5 , 3 and 7 is
 (a) 8 (b) 5 (c) 3 (d) 7
- 2 The median of the values : 34 , 23 , 25 , 40 , 22 and 4 is
 (a) 22 (b) 23 (c) 24 (d) 25
- 3 The order of the median of the values : 5 , 7 , 6 , 4 and 8 is
 (a) third. (b) fourth. (c) fifth. (d) sixth.
- 4 If the order of the median of a set of values is the fourth , then the number of these values equals
 (a) 3 (b) 5 (c) 7 (d) 9
- 5 The arithmetic mean of five numbers is 7 , then the sum of these numbers equals
 (a) 12 (b) 35 (c) 21 (d) 18
- 6 (3 , 2) does not satisfy the relation
 (a) $y + x = 5$ (b) $3y - x = 3$ (c) $y + x = 7$ (d) $2y - x = 1$
- 7 The volume of the cuboid whose dimensions , are $\sqrt{2}$ cm. , $\sqrt{5}$ cm. and $\sqrt{10}$ cm. is cm^3 .
 (a) 20 (b) 100 (c) 50 (d) 10
- 8 The intersection point of the ascending and descending cumulative curves is (30 , 50) , then the sum of frequencies is
 (a) 30 (b) 50 (c) 100 (d) 60

2 [a] Graph the relation : $y = 2 - x$

[b] The following table shows a frequency distributions :

Sets	20 –	30 –	40 –	50 –	60 –	70 –	Total
Frequency	10	k	22	25	20	8	100

Find : 1 The value of k

2 The median using the descending cumulative frequency.

3 [a] Find the S.S. of the inequality : $-3 \leq 2 - 5x \leq 12$ where $x \in \mathbb{R}$

[b] If $X =]3 , 7]$, $Y = [5 , \infty[$ by using the number line find :

1 $X \cap Y$

2 $X \cup Y$

3 $X - Y$

Accumulative test 18 till lesson 5 – unit 3

1 Choose the correct answer from the given ones :

- 1 The most common value or the most repeated value of a set of values is ...
 (a) the arithmetic mean. (b) the median.
 (c) the mode. (d) the range.
- 2 The mode of the values : 3 , 4 , 5 , 4 , 3 , 4 , 7 is
 (a) 3 (b) 4 (c) 5 (d) 7
- 3 The mode of the values : 11 , 8 , 3 $X + 2$, 11 , 5 is 11 , then $X =$
 (a) 2 (b) 1 (c) 4 (d) 3
- 4 The arithmetic mean of the values : k , $-k$, $3k$ equals
 (a) $3k$ (b) $2k$ (c) $-k$ (d) k
- 5 If $(k, 2k)$ satisfies the relation : $3X - y = 1$, then $k =$
 (a) 1 (b) -1 (c) $\frac{1}{2}$ (d) 5
- 6 $\sqrt{4} - \sqrt[3]{8} =$
 (a) 4 (b) -2 (c) zero (d) -4
- 7 If the volume of a sphere is $36\pi \text{ cm}^3$, then its radius length =
 (a) 3 (b) $\sqrt{3}$ (c) $\sqrt[3]{3}$ (d) 6
- 8 The mode of the values : 8 , $\sqrt{8}$, $\sqrt[3]{8}$, $2\sqrt{2}$ is
 (a) 8 (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$

2 [a] Find the S.S. of the equation : $\sqrt{7}X + 1 = 8$ in \mathbb{R}

[b] Reduce to the simplest form : $(\sqrt{5} - \sqrt{2})^2 + \sqrt{40}$

3 [a] Find the value of y such that the straight line passing through the two points $(3, 4)$ and $(2, y)$ is parallel to the X -axis.

[b] The following table shows the frequency distribution with equal range sets for the weekly wages of 100 workers in a factory :

Sets of wages in L.E.	70–	80–	90–	100–	X –	120–	130–
Number of workers	10	13	$k - 4$	20	16	14	11

Find : 1 The value of each of X and k

2 The mode of wages in L.E. by using the histogram.

Final Revision

of Algebra and Statistics



First Real numbers

Remember that

- $\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}^c$
- $\mathbb{R} - \mathbb{Q} = \mathbb{Q}^c$
- $\mathbb{R}_+ \cap \mathbb{R}_- = \emptyset$
- $\pi \in \mathbb{Q}^c$
- $\mathbb{Q} \cap \mathbb{Q}^c = \emptyset$
- $\mathbb{R} - \mathbb{Q}^c = \mathbb{Q}$
- $\mathbb{R} = \mathbb{R}_+ \cup \{0\} \cup \mathbb{R}_-$
- $\mathbb{R}^* = \mathbb{R} - \{0\}$

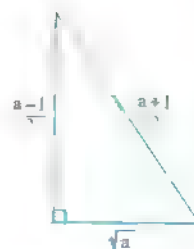
Remember The representing of the irrational number on the number line

Each irrational number can be represented by a point on the number line.

and to draw a line segment with length $= \sqrt{a}$ length unit where $a > 1$

Draw a right-angled triangle in which :

- The length of one side of the right-angle $= \frac{a-1}{2}$ length unit.
- The length of the hypotenuse $= \frac{a+1}{2}$ length unit.

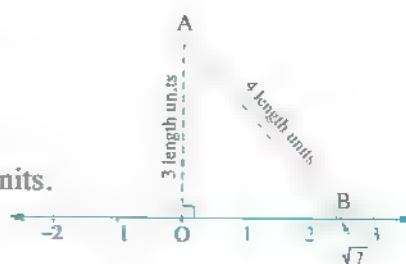


and we can apply this to represent the irrational number $\sqrt{7}$ on the number line as the following :

- From the point which represents the number zero on the number line , we draw a perpendicular line segment as \overline{OA} where $OA = \frac{7-1}{2} = 3$ length units.
- Using the compasses with a distance $= \frac{7+1}{2} = 4$ length units.

and centre at A , draw an arc to cut the number line on the right side of the point O at the point B

, then B is the point which represents $\sqrt{7}$ as in the figure.



- Notice that : To represent the number $(-\sqrt{7})$, we draw the arc which cuts the number line on its left side , not on its right side.
- Notice that : To represent the number $(1 + \sqrt{7})$, we follow the same previous steps but we draw the perpendicular line segment \overline{OA} from the point which represents the number 1 , not the number 0


Remember The operations on intervals

Intervals	Intersection	Union	Difference	Complement
$X = [-1, 5]$ $Y =]-3, 2[$	$X \cap Y = [-1, 2]$	$X \cup Y =]-3, 5[$	$X - Y = [2, 5]$ $Y - X =]-3, -1[$	$\hat{X} = \mathbb{R} - [-1, 5]$ $=]-\infty, -1[\cup]5, \infty[$
$X =]-\infty, 1[$ $Y = [-2, 1]$	$X \cap Y = [-2, 1]$	$X \cup Y =]-\infty, 1]$	$X - Y =]-\infty, -2[\cup \{1\}$ $Y - X = \emptyset$	$\hat{X} =]1, \infty[$
$X = [-1, 5]$ $Y =]-1, 5[$	$X \cap Y =]-1, 5[$	$X \cup Y = [-1, 5]$	$X - Y = \{-1, 5\}$ $Y - X = \emptyset$	$\hat{Y} = \mathbb{R} -]-1, 5[$ $=]-\infty, -1] \cup [5, \infty[$
$X =]-3, 4[$ $Y = \{-3, 4\}$	$X \cap Y = \{4\}$	$X \cup Y = [-3, 4]$	$X - Y =]-3, 4[$ $Y - X = \{-3\}$	$\hat{Y} = \mathbb{R} - \{-3, 4\}$


Remember The operations on the square roots and the cube roots

The square roots

$$\textcircled{1} \sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\text{For Example : } \sqrt{3} \times \sqrt{12} = \sqrt{3 \times 12} = \sqrt{36} = 6$$

$$\textcircled{2} \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \text{ (where } b \neq 0 \text{)}$$

$$\text{For Example : } \frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$$

$$\textcircled{3} \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b} \text{ (where } b \neq 0 \text{)}$$

$$\text{For Example : } \frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$$

The cube roots

$$\textcircled{1} \sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{ab}$$

$$\text{For Example : } \sqrt[3]{3} \times \sqrt[3]{9} = \sqrt[3]{3 \times 9} = \sqrt[3]{27} = 3$$

$$\textcircled{2} \frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\frac{a}{b}} \text{ (where } b \neq 0 \text{)}$$

$$\text{For Example : } \frac{\sqrt[3]{32}}{\sqrt[3]{4}} = \sqrt[3]{\frac{32}{4}} = \sqrt[3]{8} = 2$$

Example Simplify to the simplest form :

$$\textcircled{1} \sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}}$$

$$\textcircled{2} \sqrt{18} - \frac{\sqrt{12}}{\sqrt{6}}$$

$$\textcircled{3} 5\sqrt{2} (2\sqrt{2} + \sqrt{12})$$

$$\textcircled{4} \sqrt[3]{54} + 6\sqrt[3]{16} - 6\sqrt[3]{\frac{1}{4}}$$

$$\textcircled{5} \sqrt[3]{72} + \sqrt[3]{\frac{1}{3}} + \sqrt[3]{-9}$$

Solution

$$\textcircled{1} \sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}} = \sqrt{2 \times 16} - \sqrt{2 \times 36} + 3 \times 2\sqrt{\frac{1}{2}}$$

$$= 4\sqrt{2} - 6\sqrt{2} + 3\sqrt{\frac{1}{2}} \times 4 = 4\sqrt{2} - 6\sqrt{2} + 3\sqrt{2} = \sqrt{2}$$

$$\textcircled{2} \sqrt{18} - \frac{\sqrt{12}}{\sqrt{6}} = \sqrt{2 \times 9} - \sqrt{2} = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$$

$$\textcircled{3} 5\sqrt{2} (2\sqrt{2} + \sqrt{12}) = 5\sqrt{2} \times 2\sqrt{2} + 5\sqrt{2} \times \sqrt{12} = 10\sqrt{4} + 5\sqrt{24} = 10 \times 2 + 5\sqrt{4 \times 6}$$

$$= 20 + 5 \times 2\sqrt{6} = 20 + 10\sqrt{6}$$

$$\textcircled{4} \sqrt[3]{54} + 6\sqrt[3]{16} - 6\sqrt[3]{\frac{1}{4}} = \sqrt[3]{2 \times 27} + 6\sqrt[3]{8 \times 2} - 3 \times 2\sqrt[3]{\frac{1}{4}}$$

$$= 3\sqrt[3]{2} + 6 \times 2\sqrt[3]{2} - 3 \times 3\sqrt[3]{8 \times \frac{1}{4}} = 3\sqrt[3]{2} + 12\sqrt[3]{2} - 3\sqrt[3]{2} = 12\sqrt[3]{2}$$

$$\textcircled{5} \sqrt[3]{72} + \sqrt[3]{\frac{1}{3}} + \sqrt[3]{-9} = \sqrt[3]{8 \times 9} + \sqrt[3]{\frac{1}{3} \times \frac{9}{9}} - \sqrt[3]{9}$$

$$= 2\sqrt[3]{9} + \sqrt[3]{\frac{9}{27}} - \sqrt[3]{9} = 2\sqrt[3]{9} + \frac{1}{3}\sqrt[3]{9} - \sqrt[3]{9} = \frac{4}{3}\sqrt[3]{9}$$



Remember: The two conjugate numbers

If a and b are two positive rational numbers :

then each of the two numbers $(\sqrt{a} + \sqrt{b})$ and $(\sqrt{a} - \sqrt{b})$

is conjugate to the other one and we find that :

- Their sum $= (\sqrt{a} + \sqrt{b}) + (\sqrt{a} - \sqrt{b}) = 2\sqrt{a}$ = twice the first term
- Their product $= (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$
= The square of the first term – the square of the second term

For example : The number $(\sqrt{3} - \sqrt{2})$ its conjugate is $(\sqrt{3} + \sqrt{2})$, then we find that :

- Their sum $= 2\sqrt{3}$
- Their product $= 3 - 2 = 1$

! Remark

If we have a real number whose denominator is written in the form $(\sqrt{a} + \sqrt{b})$ or $(\sqrt{a} - \sqrt{b})$, we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator.

For example :

For writing the number $\frac{12}{\sqrt{6} - \sqrt{2}}$ in the simplest form , we multiply the two terms of the number by the conjugate of the denominator which is $(\sqrt{6} + \sqrt{2})$

$$\therefore \frac{12}{\sqrt{6} - \sqrt{2}} \times \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}} = \frac{12(\sqrt{6} + \sqrt{2})}{6 - 2} = 3(\sqrt{6} + \sqrt{2}) = 3\sqrt{6} + 3\sqrt{2}$$

Important remarks from multiplying by inspection

• We know that : $(X - y)(X + y) = X^2 - y^2$

• And we know also :

$$(X + y)^2 = X^2 + 2Xy + y^2$$

Then

$$\bullet X^2 + Xy + y^2 = (X + y)^2 - Xy$$

$$\bullet X^2 + y^2 = (X + y)^2 - 2Xy$$

$$(X - y)^2 = X^2 - 2Xy + y^2$$





Then

$$\bullet X^2 - Xy + y^2 = (X - y)^2 + Xy$$

or

$$X^2 + y^2 = (X - y)^2 + 2Xy$$

Summary of rules of areas and volumes of some solids

The solid		The lateral area	The total area	The volume
The cube		$4l^2$	$6l^2$	l^3
The cuboid		$2(x+y) \times z$	$2(xy + yz + zx)$	xyz
The cylinder		$2\pi r h$	$2\pi r h + 2\pi r^2$ $= 2\pi r(h + r)$	$\pi r^2 h$
The sphere			$4\pi r^2$	$\frac{4}{3}\pi r^3$

Remember that : The circumference of the circle $= 2\pi r$, the area of the circle $= \pi r^2$

Remember Solving an equation of the first degree in one unknown in \mathbb{R}

- Solving the equation of the first degree in one unknown in \mathbb{R} means finding the real number which satisfies this equation.

And the following example shows how to solve an equation of the first degree in one unknown.

Example

Find in \mathbb{R} the solution set of each of the following equations, then represent the solution on the number line :

① $\sqrt{5}x - 1 = 4$

② $x - \sqrt{3} = 2$

Solution

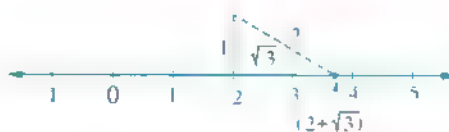
① $\because \sqrt{5}x - 1 = 4 \quad \therefore \sqrt{5}x = 4 + 1 = 5$

$$\therefore x = \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$$

$$\therefore \text{The S.S.} = \{\sqrt{5}\}$$

② $\because x - \sqrt{3} = 2 \quad \therefore x = 2 + \sqrt{3}$

$$\therefore \text{The S.S.} = \{2 + \sqrt{3}\}$$




Remember Solving an inequality of the first degree in one unknown in \mathbb{R}

- Solving the inequality means finding all values of the unknown which satisfy this inequality.
- The solution set of the inequality in \mathbb{R} will be written as an interval.

And the following example shows how to solve an inequality of the first degree in one unknown in \mathbb{R}

Example

Find in \mathbb{R} the solution set of each of the following inequalities, then represent the solution on the number line :

1 $2x + 6 < 2$

2 $5 - 4x \leq -3$

3 $3 < 3 - 5x < 13$

4 $x - 2 \geq 3x - 5$

Solution

1 $\therefore 2x + 6 < 2$

$\therefore 2x < 2 - 6$

$\therefore 2x < -4$

$\therefore x < \frac{-4}{2}$

$\therefore x < -2$

\therefore The S.S. = $] -\infty, -2[$



2 $\therefore 5 - 4x \leq -3$

$\therefore -4x \leq -8$

$\therefore x \geq \frac{-8}{-4}$

(Notice the change in the direction of the symbol of the inequality because we divided by a negative number)

$\therefore x \geq 2$

\therefore The S.S. = $[2, \infty[$



3 $\therefore 3 < 3 - 5x < 13$

(adding -3 to all sides)

$\therefore 0 < -5x < 10$ (dividing all sides by -5)

$\therefore 0 > x > -2$

(Notice the change in the direction of the symbol of the inequality because we divided by a negative number)

\therefore The S.S. = $] -2, 0[$



4 $\therefore x - 2 \geq 3x - 5$

$\therefore x - 3x \geq -5 + 2$

$\therefore -2x \geq -3$

$\therefore x \leq \frac{3}{2}$

\therefore The S.S. = $] -\infty, \frac{3}{2}]$



Second Relation between two variables

Remember The linear relation

It is a relation of the first degree between two variables X and y , it is in the form :
 $aX + by = c$, where a , b and c are real numbers, a and $b \neq 0$ together.

And there is an infinite number of ordered pairs which satisfy this relation and it is enough to get three ordered pairs satisfying the relation at the graphical representation.

Example 1

Find three ordered pairs satisfying the relation : $3X - 2y = 6$

Solution

$$\therefore 3X - 2y = 6$$

$$\therefore -2y = 6 - 3X$$

$$\therefore y = \frac{3X - 6}{2}$$

$$\bullet \text{ Putting } X = 0$$

$$\therefore y = -3$$

$$\therefore (0, -3) \text{ satisfies the relation.}$$

$$\bullet \text{ Putting } X = 1$$

$$\therefore y = -\frac{3}{2}$$

$$\therefore \left(1, -\frac{3}{2}\right) \text{ satisfies the relation.}$$

$$\bullet \text{ Putting } X = 2$$

$$\therefore y = 0$$

$$\therefore (2, 0) \text{ satisfies the relation.}$$

Example 2

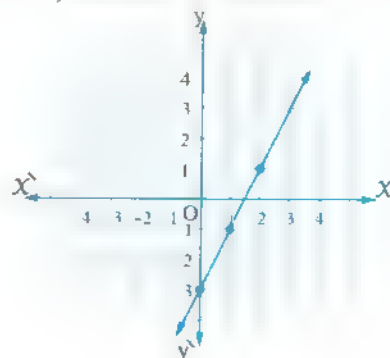
Represent graphically the relation : $2X - y = 3$

Solution

$$\therefore 2X - y = 3$$

$$\therefore y = 2X - 3$$

X	0	1	2
y	-3	-1	1



Remember The slope of the straight line

The slope of the straight line = $\frac{\text{the change in } y\text{-coordinates}}{\text{the change in } X\text{-coordinates}} = \frac{\text{the vertical change}}{\text{the horizontal change}}$

$$\text{i.e. } S = \frac{y_2 - y_1}{X_2 - X_1}, \text{ where } X_1 \neq X_2$$

For example : The slope of the straight line passing through the two points $(2, 3)$, $(-5, 2)$ is :

$$S = \frac{2 - 3}{-5 - 2} = \frac{-1}{-7} = \frac{1}{7}$$

Notice that :

- The slope of the straight line parallel to X -axis = 0
- The slope of the straight line parallel to y -axis is undefined.

Third Statistics

Remember The tables and cumulative frequency curves

The following frequency table shows the weekly wages in pounds of 50 workers in a factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (frequency)	5	12	22	7	4	50

1 Forming the ascending cumulative frequency table and graphing the curve

The upper boundaries of sets	Frequency	Sets of wages	54 –	58 –	62 –	66 –	70 –
		Number of workers (frequency)	5	12	22	7	4
Less than 54	zero	Less than 54 = 0					
Less than 58	5	Less than 58 = 5 + 0 = 5					
Less than 62	17	Less than 62 = 5 + 12 = 17					
Less than 66	39	Less than 66 = 5 + 12 + 22 = 39					
Less than 70	46	Less than 70 = 5 + 12 + 22 + 7 = 46					
Less than 74	50	Less than 74 = 5 + 12 + 22 + 7 + 4 = 50					

"The ascending cumulative frequency table"

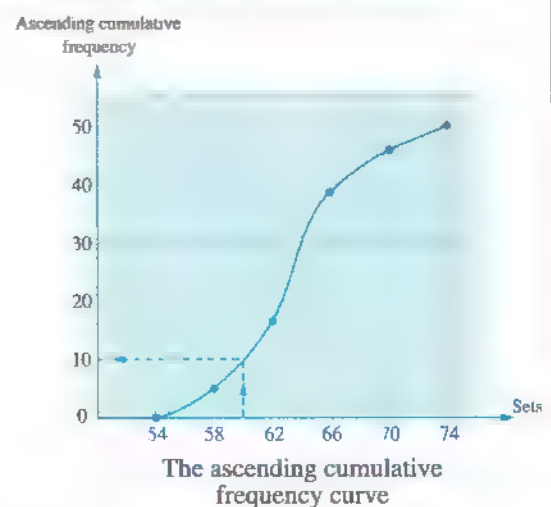
Notice that :

The ascending cumulative frequency begins with zero and ends at the total frequency.

- From the opposite graph , we can find the number of individuals which is less than a certain value.

For Example :

The number of workers whose wages are less than 60 pounds is 10 workers.



2 Forming the descending cumulative frequency table and graphing the curve

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (frequency)	5	12	22	7	4

$$54 \text{ and more} = 5 + 12 + 22 + 7 + 4 = 50$$

$$58 \text{ and more} = 12 + 22 + 7 + 4 = 45$$

$$62 \text{ and more} = 22 + 7 + 4 = 33$$

$$66 \text{ and more} = 7 + 4 = 11$$

$$70 \text{ and more} = 4$$

$$74 \text{ and more} = 0$$

The lower boundaries of sets	Frequency
54 and more	50
58 and more	45
62 and more	33
66 and more	11
70 and more	4
74 and more	zero

"The descending cumulative frequency table"

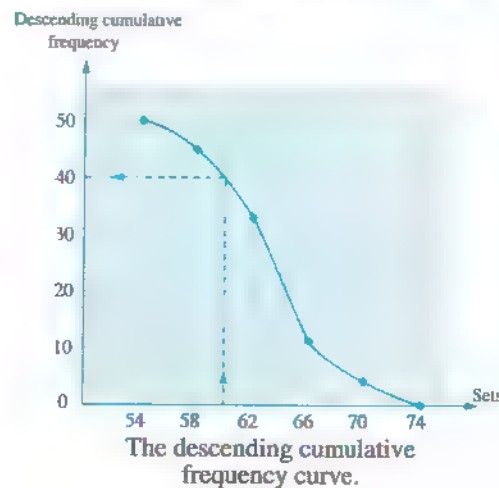
Notice that :

The descending cumulative frequency begins with the total frequency and ends with zero.

- From the opposite graph , we can find the number of individuals which is more than or equal to a certain value.

For Example :

The number of workers whose wages are 60 pounds or more is 40 workers.



Remember The measures of the central tendency

- ① The mean. ② The median. ③ The mode.

The mean

[a] The mean of a set of values (simple frequency distribution)

The mean of a set of values = $\frac{\text{The total of values}}{\text{Number of values}}$

For example : The mean of the numbers : 5 , 3 , 7 , 9 = $\frac{5+3+7+9}{4} = 6$

[b] The mean of a frequency distribution with sets

Example

The following table shows the distribution of the marks of 50 pupils in mathematics :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	8	12	14	9	7	50

Find the mean of these marks.

Solution

- ① Determine the centres of sets according to the rule :

$$\text{The centre of a set} = \frac{\text{the lower limit} + \text{the upper limit}}{2}$$

\therefore The centre of the first set = $\frac{10+20}{2} = 15$... and so on.

Since the lengths of the subsets are equal and each of them = 10 therefore we consider the upper limit of the last set = 60

, then its centre = $\frac{50+60}{2} = 55$

- ② Form the following table :

Set	Centre of the set « X »	Frequency « f »	X × f
10 –	15	8	120
20 –	25	12	300
30 –	35	14	490
40 –	45	9	405
50 –	55	7	385
Total		50	1700

$$\text{The mean} = \frac{\text{The sum of } (X \times f)}{\text{The sum of } f} = \frac{1700}{50} = 34 \text{ marks.}$$

2 The median

[a] The median of a set of values

The median is the middle value in a set of values after arranging it ascendingly or descendingly, such that the number of values which are less than it is equal to the number of values which are greater than it.

We arrange the values ascendingly or descendingly

If the values number is odd, then

The median is the value lying in the middle exactly.

If the values number is even, then

The median

$$= \frac{\text{The sum of the two values lying in the middle}}{2}$$

For example :

If the values are

42, 23, 17, 30 and 20

We arrange them ascendingly as follows

17, 20, (23), 30, 42

The median = 23

For example :

If the values are

27, 13, 23, 24, 13, 21

We arrange them ascendingly as follows

13, 13, [21, 23], 24, 27

$$\text{The median} = \frac{21 + 23}{2} = 22$$

[b] Finding the median of a frequency distribution with sets graphically

For finding the median of a frequency distribution with sets graphically, do the following steps :

- 1 Form the ascending or the descending cumulative frequency table, then draw the cumulative frequency curve of it.
- 2 Find the order of the median = $\frac{\text{The total of frequency}}{2}$
- 3 Determine the point which represents the order of the median on the vertical axis, from this point, draw a horizontal straight line to cut the curve at a point, then from this point, draw a perpendicular to the horizontal axis to intersect it at a point which represents the median.

The following example shows how to find the median using the two curves (the ascending or the descending cumulative frequency curve).

Example

The following table shows the frequency distribution of marks of 50 students in math exam :

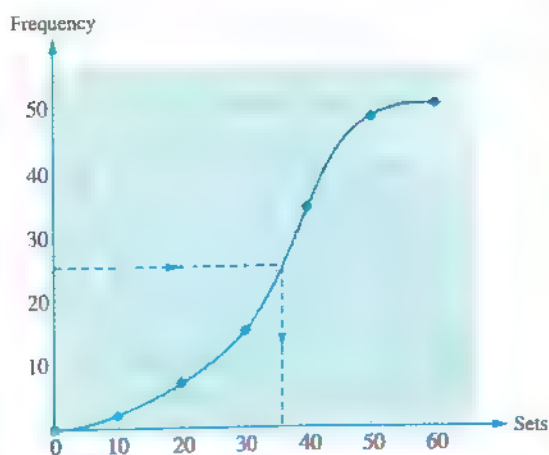
Sets of marks	0 -	10 -	20 -	30 -	40 -	50 -	Total
Number of students	2	5	8	19	14	2	50

Find the median mark of the student.

Solution

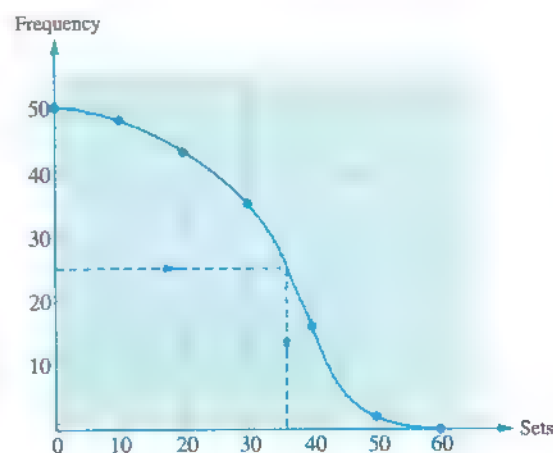
Using the ascending cumulative frequency curve :

The upper boundaries of sets	Frequency
Less than 0	0
Less than 10	2
Less than 20	7
Less than 30	15
Less than 40	34
Less than 50	48
Less than 60	50



Using the descending cumulative frequency curve :

The lower boundaries of sets	Frequency
0 and more	50
10 and more	48
20 and more	43
30 and more	35
40 and more	16
50 and more	2
60 and more	0



$$\therefore \text{The order of the median} = \frac{50}{2} = 25$$

\therefore From the two previous graphs, the median = 36 approximately

The mode

[a] The mode of a set of values

The mode of a set of values is the most common value in the set, or in other words, it is the value which is repeated more than any other values.

For example : The mode of the set of the values : 7, 3, 4, 1, 7, 9, 7, 4 is 7

[b] The mode of a frequency distribution with sets

Example

The following is the frequency distribution of marks of 100 pupils in one of the exams :

Sets of marks	10 –	20 –	30 –	40 –	50 –	Total
Number of pupils	16	24	30	20	10	100

Find the mode mark for these pupils.

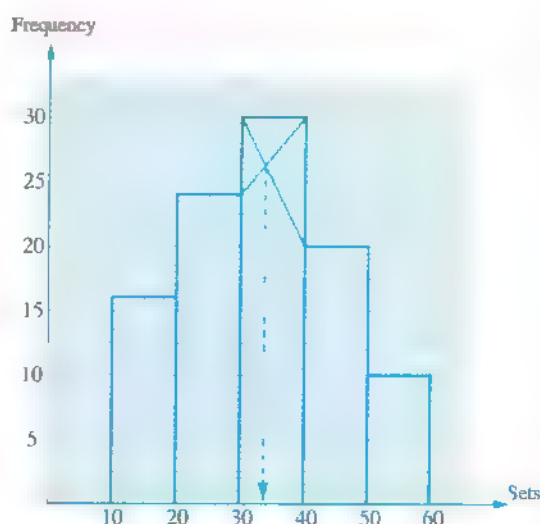
Solution

You can find the mode of that distribution graphically using the histogram as follows :

- 1 Draw two orthogonal axes : one of them is horizontal and the other is vertical to represent the frequency of each set.
- 2 Divide the horizontal axis into a number of equal parts with a suitable drawing scale to represent the sets.
- 3 Divide the vertical axis into a number of equal parts with a suitable drawing scale to represent the greatest frequency in the sets.
- 4 Draw a rectangle whose base is set (10 –) and its height equals the frequency (16)
- 5 Draw a second rectangle adjacent to the first one whose base is set (20 –) and its height equals the frequency (24)
- 6 Repeat drawing the remained adjacent rectangles till the last set (50 –)
- 7 Determine the set which has the greatest frequency then draw two lines as shown in the histogram to intersect at a point.

From this point, draw a vertical line to intersect the horizontal axis at a point which represents the value of the mode.

i.e. The mode mark is 34 approximately.



Final Examinations

on Algebra and Statistics



Model

Answer the following questions :

1 Complete the following :

- 1 The S.S. of the equation : $(x^2 + 3)(x^3 + 1) = 0$ is , $x \in \mathbb{R}$
- 2 If the lower boundary of a set is 10 and the upper boundary is X and its centre is 15 , then $X =$
- 3 $[-2, 2] \cup \{-2, 0\} =$
- 4 The cube whose volume is 8 cm^3 , then the sum of all its edge lengths is cm.
- 5 The multiplicative inverse of the number $(\sqrt{3} + \sqrt{2})$ is in the simplest form.

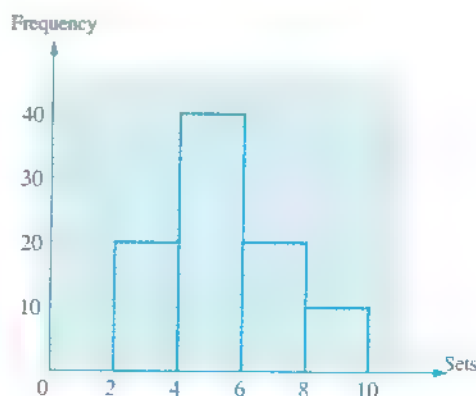
2 Choose the correct answer from the given ones :

- 1) If the radius length of a sphere is 6 cm. , then its volume is
 (a) $6 \pi \text{ cm}^3$ (b) $36 \pi \text{ cm}^3$ (c) $72 \pi \text{ cm}^3$ (d) $288 \pi \text{ cm}^3$
- 2) If the point $(a, 1)$ satisfies the relation $x + y = 5$, then $a =$
 (a) 1 (b) -4 (c) 4 (d) 5
- 3 $(2^3 \sqrt{2})^3 =$
 (a) 4 (b) 8 (c) 16 (d) 40
- 4) The median of the values : 34 , 23 , 25 , 40 , 22 , 4 is
 (a) 22 (b) 23 (c) 24 (d) 25
- 5 If the arithmetic mean of the values : 27 , 8 , 16 , 24 , 6 , k is 14 , then $k =$
 (a) 3 (b) 6 (c) 27 (d) 84

6) In the opposite figure :

The value of the mode =

- (a) 4 (b) 5
- (c) 6 (d) 40



3 [a] Find the value of : $\sqrt{18} + \sqrt[3]{54} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{16}$

[b] If $x = \frac{3}{\sqrt{5} - \sqrt{2}}$ and $y = \sqrt{5} - \sqrt{2}$

, prove that : x and y are two conjugate numbers.

4 [a] The area of a square is 1089 cm^2 . Find the length of its diagonal.

[b] Find the S.S. of the inequality : $\frac{3x+1}{6} < x+1 < \frac{x+4}{2}$ in \mathbb{R}

, then represent it on the number line.

5 [a] The radius length of the base of a right circular cylinder is $4\sqrt{2} \text{ cm}$. and its height is 9 cm . Find its volume in terms of π and if its volume equals the volume of a sphere , find the radius length of the sphere.

[b] Find the arithmetic mean of the following frequency distribution :

The sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

Model 2

Answer the following questions :

1 Complete the following :

1 The additive inverse of the number : $-\sqrt{3} - \sqrt{5}$ is

2 $(\sqrt{8} + \sqrt{2})(\sqrt{8} - \sqrt{2}) = \dots$

3 The conjugate of the number $\frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{2}}$ is

4 If the volume of a sphere is $\frac{9}{2} \pi \text{ cm}^3$, then its diameter length is cm.

5 $[3, 4] - \{3, 5\} = \dots$

2 Choose the correct answer from the given ones :

1 If the volume of a cube is 27 cm^3 , then the area of one of its faces is

(a) 3 cm^2

(b) 9 cm^2

(c) 36 cm^2

(d) 54 cm^2

2 If the mode of the values $4, 11, 8, 2x$ is 4 , then $x = \dots$

(a) 2

(b) 4

(c) 6

(d) 8

3 If the arithmetic mean of the values 18 , 23 , 29 , $2k - 1$, k is 18 , then $k =$..

- (a) 1 (b) 7 (c) 29 (d) 90

4 If the lower limit of a set is 4 and the upper limit is 8 , then its centre is ...

- (a) 2 (b) 4 (c) 6 (d) 8

5 A right circular cylinder the radius length of its base is r cm. and its height equals its diameter length , then its volume = cm^3 .

- (a) πr^3 (b) πr^2 (c) $2\pi r^3$ (d) $2r^3$

6 The solution set of the equation : $x(x^2 - 1) = 0$, $x \in \mathbb{R}$ is

- (a) $\{0\}$ (b) $\{1\}$ (c) $\{-1\}$ (d) $\{0, -1, 1\}$

3 [a] Reduce to the simplest form : $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}$

[b] Prove that : $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = 0$

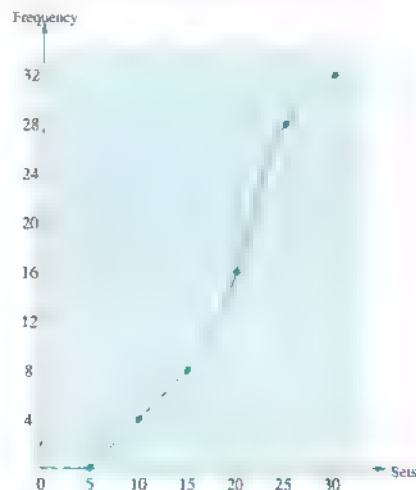
4 [a] Find the S.S. of the inequality : $-2 < 3x + 7 \leq 10$ in \mathbb{R} , then represent the interval of solution on the number line.

[b] If $x = \sqrt{2} + \sqrt{3}$, find the value of : $x^4 - 2x^2 + 1$

5 [a] The opposite graph represents the marks of 32 pupils in an exam.

Complete :

The median mark =



[b] Find the arithmetic mean of the following frequency distribution :

The sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	4	5	6	3	2	20

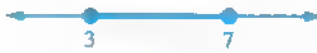
Model for the merge student!*Answer the following questions :***1 Complete each of the following :**

- 1 The conjugate of the number $\sqrt{3} + \sqrt{2}$ is
- 2 $\sqrt{18} + \sqrt{54} - 3\sqrt{2} = \dots\dots\dots$
- 3 The mode for the numbers : 3 , 5 , 3 , 4 , 3 is
- 4 The median of the values : 2 , 3 , 5 , 7 , 9 is
- 5 The solution set of the equation : $x^2 + 9 = 0$ in \mathbb{R} is

2 Choose the correct answer from those given :

- 1 The arithmetic mean for the values : 9 , 6 , 5 , 14 , 1 is
 (a) 7 (b) 3 (c) 5 (d) 9
- 2 The simplest form of the expression : $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$ is
 (a) $\sqrt{3}$ (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{3}$
- 3 The additive inverse of the number $-\sqrt{5}$ is
 (a) $\sqrt{5}$ (b) 5 (c) $\sqrt{2}$ (d) -5
- 4 $[3, 5] - \{3, 5\} = \dots\dots\dots$
 (a) $]3, 5[$ (b) $[3, 5[$ (c) \emptyset (d) $]3, 5]$
- 5 A cube is of volume 64 cm^3 , then its edge length is cm.
 (a) 4 (b) 8 (c) 16 (d) 64

3 Match from the column (A) to the suitable one from the column (B) :

(A)	(B)
1 The S.S. of the equation : $x^2 - 25 = 0$ in \mathbb{R} is	$[0, 2]$
2 $[-3, 2] \cap [0, 2] = \dots\dots\dots$	7
3 If the order of the median is fourth , then the number of values is	$\{5, -5\}$
4 $\sqrt{3}$ is an number.	
5 The S.S. of the inequality : $3 \leq x \leq 7$ on the number line is	irrational

4 Put (✓) for the correct statements and (✗) for the incorrect ones :

1 The arithmetic mean of a set of values = sum of values \div its number. ()

2 If $x = \sqrt{13} - \sqrt{7}$, $y = \sqrt{13} + \sqrt{7}$, then x , y are two conjugate numbers. ()

3 The irrational number $\sqrt{7}$ lies between 2 and 3 ()

4 $\sqrt{75} - 2\sqrt{27} = 7\sqrt{3}$ ()

5 The simplest form of the number $\frac{1}{\sqrt{5}}$ is $\frac{\sqrt{5}}{5}$ ()

5 [a] Complete : If the lower limit of a set is 4 and the upper limit is 8

, then its centre = $\frac{4 + 8}{2} = \dots$

[b] Complete the following table to obtain the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45	Total
Frequency	7	10	12	13	8	50

Sets	The centre of the set « x »	Frequency « f »	$x \times f$
5 –	10	7	$10 \times 7 = 70$
15 –	20	10	$20 \times 10 =$
25 –	$\dots \times 12 = \dots$
35 –	$\dots \times 13 = \dots$
45 –	$\dots \times 8 = \dots$
Total		50

The arithmetic mean = $\frac{\sum (x \times f)}{\sum (f)} = \frac{\dots}{\dots} = \dots$

Answer the following questions :

1 Choose the correct answer :

- 1 The multiplicative inverse of $\sqrt[3]{3}$ is
 (a) $\sqrt[3]{3}$ (b) $-\sqrt[3]{3}$ (c) $\frac{\sqrt[3]{3}}{3}$ (d) $\frac{3}{\sqrt[3]{3}}$
- 2 The S.S. of the equation : $x^2 + 9 = 0$ in \mathbb{R} is
 (a) \emptyset (b) $\{3, -3\}$ (c) $\{3\}$ (d) $\{-3\}$
- 3 If $(k, 3)$ satisfies the relation : $y = 2x + 5$, then $k =$
 (a) 1 (b) -1 (c) 2 (d) 3
- 4 The volume of a cube is 27 cm^3 , then its lateral area = cm^2
 (a) 12 (b) 54 (c) 36 (d) 27
- 5 If $2x + 1 = 7$, then $3x =$
 (a) 6 (b) 9 (c) 12 (d) -12
- 6 The mean of the values : 3 , 2 , 4 , 7 is
 (a) 2 (b) 3 (c) 7 (d) 4

2 Complete :

- 1 $3a^2b \times \dots = 12a^4b^2$
- 2 If the mode of the values : 6 , 9 , $x - 2$, 10 is 6 , then $x =$
- 3 $[2, 7] - \{7\} =$
- 4 The slope of the straight line parallel to x -axis is
- 5 The median of : 24 , 20 , 11 , 36 , 40 is

- 3 [a]** If $x = \sqrt{3} + \sqrt{2}$, $y = \frac{1}{\sqrt{3} + \sqrt{2}}$, find the value of : $\frac{x+y}{xy}$

- [b]** If the slope of the straight line passing through the two points A (4 , k) , B (3 , 2) is 5 , find the value of k

4 [a] Find in \mathbb{R} the S.S. of the inequality :

$-1 \leq 2x + 3 < 5$ and represent the S.S. on the number line.

- [b] Simplify :** $\sqrt{50} + 2\sqrt{18} - \sqrt{32} - 8\sqrt{\frac{1}{2}}$

- 5 a) If the volume of a sphere is $\frac{500}{3} \pi \text{ cm}^3$, find the length of its diameter.

b) Find the mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20



Answer the following questions :

1 Choose the correct answer :

1 $(\sqrt{5} + \sqrt{3})^2 (\sqrt{5} - \sqrt{3})^2 = \dots\dots\dots$

- (a) 2 (b) 3 (c) 4 (d) 8

2 The lower limit of a set is 4 and the upper limit is 8 , then its centre is

- (a) 8 (b) 6 (c) 4 (d) 2

3 $5 \in \dots\dots\dots$

- (a) $\{55\}$ (b) $]1, 5[$ (c) $]-\infty, 4]$ (d) $]-1, \infty[$

4 The mode of the values : 4 , 11 , 8 , 2 X is 8 , then X =

- (a) 2 (b) 4 (c) 9 (d) 11

5 If the volume of a cube is 27 cm^3 , then the perimeter of one of its faces is cm.

- (a) 12 (b) 9 (c) 15 (d) 40

6 If $(-1, 5)$ satisfies the equation : $3x + ky = 7$, then k =

- (a) 2 (b) 0.8 (c) 3 (d) 5

2 Complete :

1 If the volume of a sphere is $\frac{9}{2} \pi \text{ cm}^3$, then its radius length is

2 $(2x - 3)(3x + 5) = 6x^2 + \dots\dots\dots$

3 $[3, 4] - \{3, 5\} = \dots\dots\dots$

4 If A (1, -2) , B (5, -4) , then the slope of \overrightarrow{AB} is

5 The mean of the values : 7 , 11 , 21 , 10 and 16 is

3 a) Simplify to the simplest form :

1 $6\sqrt[3]{16} + \sqrt[3]{54} - 6\sqrt[3]{\frac{1}{4}}$

2 $5\sqrt{2}(2\sqrt{2} + \sqrt{12})$

b) If $x = \frac{4}{\sqrt{7} - \sqrt{3}}$, $y = \sqrt{7} - \sqrt{3}$

, prove that : x and y are conjugate numbers , then find the value of : $(x + y)^2$

Algebra and Statistics

- 4 [a]** Find the total area of a right circular cylinder of volume $72\pi \text{ cm}^3$ and height 8 cm.
(in terms of π)

[b] Find in \mathbb{R} the S.S. of :

- 1** $5 - 3x > 11$, then represent the solution set on the number line.
2 $8x^3 + 7 = 8$

- 5 [a]** Graph the relation : $y = 3x + 1$ and if $(2, a)$ satisfies the relation , find the value of a

[b] Find the arithmetic mean of the following frequency distribution :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	4	6	8	7	5	30



Answer the following questions :

1 Choose the correct answer :

- 1** The slope of the straight line passing through $(4, 1)$, $(6, -3)$ is
(a) -1 (b) 0 (c) 2 (d) -2
- 2** The solution set of : $2x^3 + 54 = 0$ in \mathbb{R} is
(a) $\{3\}$ (b) $\{-3\}$ (c) $\{-3, 3\}$ (d) \emptyset
- 3** If $(6k, 4k)$ satisfies the relation : $x + y = 50$, then $k =$
(a) 0 (b) 10 (c) 15 (d) 5
- 4** If the order of the median of some values is tenth , then the number of these values is
(a) 19 (b) 20 (c) 21 (d) 22
- 5** If $2x = 14$, then $6x =$
(a) 12 (b) 28 (c) 36 (d) 42
- 6** $]-1, 3] \cup \{0, -1\} =$
(a) $]0, 3]$ (b) $]-1, 3[$ (c) $[-1, 3]$ (d) $[0, 3]$

2 Complete each of the following :

- 1** The volume of the sphere whose radius length equals 14 cm. is ($\pi \approx \frac{22}{7}$)
2 If the mode of the values : 16 , 18 , $x - 3$, 14 is 16 , then $x =$

- 3 The median of the values : 29 , 24 , 30 , 23 , 18 , 28 is
- 4 If the slope of a straight line equals zero , then the line is parallel to
- 5 If the lower limit of a set is 28 and the upper limit of it is 32 , then the centre of the set equals

3 [a] If $X =]-\infty, 4]$ and $Y =]2, \infty[$, find using the number line :

- 1 $X \cap Y$ 2 $X \cup Y$ 3 \bar{X}

[b] A right circular cylinder whose volume is 704 cm^3 and its diameter length is 8 cm. , then find its height. $(\pi \approx \frac{22}{7})$

4 [a] Find the solution set in \mathbb{R} of the inequality :
 $-4 \leq 5x + 1 < 11$ and represent it on the number line.

[b] Simplify : $\sqrt[3]{54} + \sqrt[3]{50} + \sqrt[3]{16} + \sqrt[3]{8}$

5 [a] Graph the relation : $y = 2x + 2$

[b] Find the arithmetic mean of the following data :

Sets	20 -	22 -	24 -	26 -	Total
Frequency	16	12	14	8	50



Answer the following questions :

1 Choose the correct answer :

1 $2\sqrt{x} \times 3\sqrt{x} = \dots\dots\dots$ (where $x > 0$)

- (a) $6x^2$ (b) $6x$ (c) $5x^2$ (d) $5x$

2 If $(m, 2)$ satisfies the relation : $x + 2y = 7$, then $m = \dots\dots\dots$

- (a) -4 (b) -3 (c) 3 (d) 4

3 $(\sqrt{5} - 2) + (\sqrt{5} + 2) = \dots\dots\dots$

- (a) 1 (b) 2 (c) 4 (d) $2\sqrt{5}$

4 The volume of a cube is 27 cm^3 , then the area of one of its faces is cm^2

- (a) 3 (b) 6 (c) 9 (d) 12

5 If $a = \frac{2}{\sqrt{3}-1}$, $b = \sqrt{3}-1$, then $2ab = \dots\dots\dots$

- (a) 1 (b) 2 (c) 3 (d) 4

6 The arithmetic mean of the values : 7 , 4 , 9 , 10 , 11 , 16 , 13 is

- (a) 13 (b) 11 (c) 10 (d) 9

2 Complete the following :

- [1] Let A (1 , 3) , B (2 , 5) , then the slope of \overline{AB} equals
- [2] The S.S. of the equation : $(X + 3)(X - 1) = 0$ in \mathbb{R} is
- [3] The median of the values : 6 , 7 , 9 , 10 , 8 , 5 , 4 is
- [4] The mode of the values : 5 , 6 , 7 , 6 , 9 , 5 , 7 , 5 , 9 , 4 , 6 , 9 , 5 is ..
- [5] $[1 , 5] - \{1 , 5\} =$

3 [a] If $X = [2 , 8]$, $Y =]-3 , 4[$, find each of the following using the number line :

- [1] $X \cap Y$ [2] $X \cup Y$

- [b] Find the S.S. of the inequality : $5X + 1 \geq 21$ in \mathbb{R} and represent the solution set on the number line.

4 [a] Find the value of : $\sqrt{20} + \sqrt{45} - \sqrt{80}$ (showing the steps of your answer)

- [b] Find the volume of a right circular cylinder of height 10 cm. and its radius length is 7 cm.

5 [a] Represent graphically the relation : $y = 3 - X$

- [b] Find the arithmetic mean of the following frequency distribution :

The set	0 -	10 -	20 -	30	40 -	Total
Frequency	4	5	6	3	2	20



Answer the following questions :

1 Choose the correct answer :

- [1] The S.S. of the equation : $X^2 + 5 = 0$ in \mathbb{R} is
 (a) 5 (b) $\{\sqrt{5}, -\sqrt{5}\}$ (c) $\{\sqrt{5}\}$ (d) \emptyset
- [2] If the point (a , 1) satisfies the relation : $X + y = 5$, then a =
 (a) -4 (b) 1 (c) 4 (d) 5
- [3] If four times a number is 48 , then third of this number is
 (a) 12 (b) 6 (c) 4 (d) 8
- [4] $[-1 , 5] -]-1 , 5[=$
 (a) \emptyset (b) $\{-1 , 5\}$ (c) $[-1 , 5]$ (d) $]-1 , 5[$

5 The irrational number between 3 and 4 is

- (a) $\sqrt{17}$ (b) $\sqrt{6}$ (c) $\sqrt[3]{29}$ (d) 3.6

6 A cube the sum of its edge lengths is 48 cm. , then its volume is cm^3

- (a) 64 (b) 6 (c) 4 (d) 46

2 Complete :

1 If the lower limit of a set is 4 and its centre is 6 , then its upper limit is

2 If $\frac{1}{x} = \sqrt{5} - 2$, then $x = \dots\dots\dots$ (in its simplest form)

3 A sphere its diameter length is 6 cm. , then its volume is cm^3

4 If A (-1, 4) , B (x, 2) and the slope of $\overrightarrow{AB} = -2$, then $x = \dots\dots\dots$

5 The S.S. of : $\sqrt{5}x \leq 5$ is in \mathbb{R}

3 [a] A right circular cylinder , its radius length equals its height and its volume is $216\pi \text{ cm}^3$
Find the height of the right cylinder.

[b] Find the S.S. in \mathbb{R} :

1 $5 > 2x - 3 > -1$ (represent it on the number line)

2 $(2x - 1)^3 = 125$

4 [a] If $X =]-\infty, 1]$ and $Y = [-2, 4[$, find :

1 $X \cap Y$

2 $Y - X$

3 \bar{X}

[b] Simplify : $5\sqrt{8} + 2\sqrt[3]{2} - 2\sqrt{50} - \sqrt[3]{16}$

[c] If $x = \sqrt{7} + \sqrt{4}$, $y = \frac{3}{x}$

1 Prove that : x and y are two conjugate numbers.

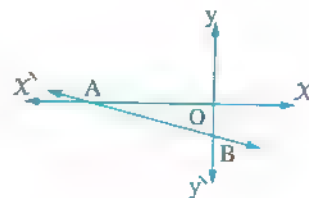
2 Find : $x^2 + 2xy + y^2$

5 [a] If the relation : $x + 4y = -4$ is represented in the opposite figure where A is the intersection point with x -axis and B is the intersection point with y -axis , then find :

1 The coordinates of A and B

2 The area of $\triangle ABO$ where O is the origin point.

3 The slope of \overrightarrow{AB}



[b] From the following frequency distribution :

Sets	5	15	25 -	35 -	45 -	Total
Frequency	7	10	12	13	k	50

1 Find k

2 Find the arithmetic mean.



Answer the following questions :

1 Choose the correct answer :

1 $\sqrt{4} \dots\dots\dots] - 2, \infty[$

- (a) \in (b) \notin (c) \subset (d) $\not\subset$

2 $\sqrt{\frac{x}{y}} = \dots\dots\dots$ (where $y > 0$)

- (a) $\frac{1}{y} \sqrt{x}$ (b) $\frac{1}{x} \sqrt{y}$ (c) $\frac{1}{y} \sqrt{xy}$ (d) $\frac{x}{y}$

3 The order of the median of the values : 4 , 5 , 6 , 7 and 8 is the

- (a) third. (b) fourth. (c) fifth. (d) sixth.

4 If $x = (-2)^4$, $y = -2^4$, then

- (a) $x = y$ (b) $x > y$ (c) $x < y$ (d) $x \leq y$

5 If $(2k, k)$ satisfies the relation : $y + 2x = 5$, then $k = \dots\dots\dots$

- (a) 5 (b) 4 (c) 2 (d) 1

6 If the mean of the values : 9 , 5 , 6 , x , 14 is 7 , then $x = \dots\dots\dots$

- (a) 3 (b) 2 (c) 1 (d) 5

2 Complete :

1 The additive inverse of the number $-5 + \sqrt{3}$ is

2 If the mode of the values : 4 , 11 , 8 , $2x$ is 4 , then $x = \dots\dots\dots$

3 The cube whose volume is 8 cm^3 , then the sum of all edge lengths is cm.

4 If the lower limit of a set is 4 and the upper limit is 8 , then its centre is

5 The straight line which represents the relation : $2x + 7y = 14$ intersects x -axis at the point (..... ,)

3 [a] If $x = \sqrt{7} - \sqrt{6}$, $y = \frac{1}{x}$, prove that : $(x + y)^2 = 28$

[b] If $A(3, 4)$, $B(5, a)$ and the slope of $\overrightarrow{AB} = 3$, find the value of a

[c] Find the lateral area of a right circular cylinder of volume $72\pi \text{ cm}^3$ and height 8 cm.

4 [a] Graph the relation : $y = 2 - x$

[b] Simplify : 1 $\sqrt{32} - 6\sqrt{\frac{1}{2}}$ 2 $\sqrt[3]{128} + \sqrt[3]{16}$

[c] If $X =] - \infty, 2[$ and $Y = [-1, 5]$, find using the number line :

- 1 $X \cap Y$ 2 $X \cup Y$ 3 \bar{X}

5 [a] Complete : The S.S. of the equation : $x^2 + 1 = 0$ in \mathbb{R} is

[b] Find in \mathbb{R} the S.S. of the inequality :

$5 - 3x > 11$, then represent the S.S. on the number line.

[c] Find the mean of the following data :

Sets	5	15 -	25 -	35	45	Total
Frequency	4	5	6	3	2	20



Answer the following questions :

1 Choose the correct answer :

[1] The mode for the values : 3 , 5 , 3 , 4 , 3 is

- (a) 3 (b) 4 (c) 5 (d) 12

[2] Let A (3 , 5) and B (5 , -1) , then the slope of \overleftrightarrow{AB} =

- (a) $-\frac{1}{3}$ (b) -3 (c) 3 (d) $\frac{1}{3}$

[3] If the point (a , 1) satisfies the relation : $x + y = 5$, then a =

- (a) 1 (b) -4 (c) 4 (d) 5

[4] The solution set of the equation : $x^2 + 9 = 0$ in \mathbb{R} is

- (a) \emptyset (b) $\{-3\}$ (c) $\{3\}$ (d) $\{3, -3\}$

[5] $4.274 \approx$ (to the nearest $\frac{1}{10}$)

- (a) 4 (b) 4.2 (c) 4.3 (d) 4.27

[6] The lower limit of a set is 4 and the upper limit is 8 , then its centre is

- (a) 2 (b) 4 (c) 6 (d) 8

2 Complete the following :

[1] The surface area of a sphere of diameter length 14 cm. equals

[2] $(\sqrt{8} + \sqrt{2})(\sqrt{8} - \sqrt{2}) =$

[3] The conjugate of the number $\frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{2}}$ is

[4] A cube whose volume is 8 cm^3 , then the sum of lengths of all its edges equals

[5] The S.S. of the equation : $x(x^3 - 1) = 0$ in \mathbb{R} is

3 [a] Find in the simplest form : $6\sqrt{\frac{1}{2}} + \frac{1}{3}\sqrt[3]{54} - \sqrt{18} - \sqrt[3]{2}$

[b] If $x = \sqrt{5} + \sqrt{2}$ and $y = \sqrt{5} - \sqrt{2}$, find the value of : $\frac{x+y}{xy-1}$

4 [a] Find the S.S. in \mathbb{R} of the inequality : $2x + 1 \leq 7$, then represent it on the number line.

[b] Find the volume of the sphere whose diameter length is 4.2 cm. ($\pi = \frac{22}{7}$)

5 [a] If the slope of \overleftrightarrow{AB} is 3 where $A = (3, 4)$, $B = (4, y)$, find the value of y

[b] Find the arithmetic mean of the following distribution :

Sets	4 -	8 -	12 -	16	20 -	Total
Frequency	2	4	8	6	4	24



Answer the following questions :

1 Choose the correct answer :

1 The solution set of the equation : $x + 5 = 5$ in \mathbb{N} is

(a) $\{0\}$ (b) $\{10\}$ (c) $\{-10\}$ (d) \emptyset

2 The rational number that lies between 0.2 , 0.3 is

(a) 0.21 (b) 0.11 (c) 0.31 (d) 0.33

3 $\sqrt[3]{x^6} = \sqrt{\dots\dots\dots}$

(a) x^3 (b) x^2 (c) x (d) x^4

4 If $(2, -5)$ satisfies the relation : $3x - y + c = 0$, then $c = \dots\dots\dots$

(a) 1 (b) -1 (c) 11 (d) -11

5 If the arithmetic mean of the set of values : 18 , 23 , 29 , $2k - 1$, k is 18 , then $k = \dots\dots\dots$

(a) 1 (b) 7 (c) 29 (d) 19

6 The median of the values : 34 , 23 , 25 , 40 , 22 , 4 is

(a) 22 (b) 23 (c) 24 (d) 25

2 Complete :

1 $0.3 = \dots\dots\dots$ (in the form of $\frac{a}{b}$)

2 $\sqrt[3]{343} = \dots\dots\dots$

3 The slope of any line parallel to X-axis is

- 4] The mode is the common value in the set.
- 5] If the order of the median of some values is the fourth , then the number of the values is

3 [a] Find the solution set of : $5x - 3 < 2x + 9$ in \mathbb{R}

[b] Find the value of : $\sqrt{18} + \sqrt{54} - 3\sqrt{2} - \frac{1}{2}\sqrt{24}$

4 [a] The radius length of the base of a right circular cylinder is 4 cm. and its height is 9 cm.
Find the volume in terms of π

[b] If A (2 , - 1) , B (10 , 3) and C (2 , 3) , find the slope of each of \overrightarrow{AB} and \overrightarrow{BC}

5 [a] Find : $[-1, 4] - [-3, 2]$ by using the number line.

[b] The following table shows the frequency distribution for the score of 50 students in an examination :

Sets	2 -	6 -	10 -	14	18 -	22 -	26	Total
Frequency	3	5	9	10	12	7	4	50

Find the mean of the students score.

9 El-Monofia Governorate



Answer the following questions :

1 Choose the correct answer :

1] The degree of the algebraic term $2x^3y^2$ is the

- (a) second. (b) third. (c) fourth. (d) fifth.

2] If the radius length of a sphere is 6 cm. , then its volume is cm^3

- (a) 6π (b) 36π (c) 72π (d) 288π

3] If x is a negative number , then the number is positive.

- (a) x^2 (b) x^3 (c) $2x$ (d) $\frac{1}{2}x$

4] $\sqrt{8} - 2\sqrt{2} = \dots\dots\dots$

- (a) 4 (b) 8 (c) zero (d) 2

5] If $|x| = 7$, then $x = \dots\dots\dots$

- (a) 7 (b) -7 (c) ± 7 (d) 8

6] The arithmetic mean for five values is 13 , then the sum of these values is

- (a) 70 (b) 56 (c) 65 (d) 13

2 Complete :

- 1 The slope of the straight line parallel to X-axis is
- 2 If the mode of the values : 18 , 11 , 4 , 2 X is 18 , then X =
- 3 If (k , 2) represents the relation : $X + 2y = 5$, then k =
- 4 If the order of the median of some values is the seventh , then the number of these values is
- 5 The median of : a + 2 , a , a - 2 , a - 1 , a + 1 is

3 [a] Simplify : $\sqrt[3]{75} - 6\sqrt{\frac{1}{3}} - 3\sqrt{12}$

[b] If $A = [-2, 3]$, $B = [1, \infty[$, find using the number line :

1 $A \cap B$

2 $A \cup B$

[c] The diameter length of a cylinder is 7 cm. and its height is 10 cm. Find the lateral area of the cylinder.

4 [a] Represent the relation : $2X + y = 4$, then find the slope of the straight line representing this relation.

[b] If $X = \frac{1}{\sqrt{7} + \sqrt{6}}$, $y = \sqrt{7} + \sqrt{6}$, prove that : X and y are two conjugate numbers , then find : $(X + y)^2$ in the simplest form.

5 [a] Find the S.S. in \mathbb{R} for the inequality :

$\sqrt[3]{-8} \leq X + 1 \leq \sqrt{9}$, then represent it on the number line.

[b] From the following frequency distribution :

The set	10 -	20 -	30 -	40 -	50 -	Total
Frequency	10	20	25	k	15	100

Find : 1 The value of k

2 The arithmetic mean.



Answer the following questions :

1 Choose the correct answer :

1 The S.S. in \mathbb{R} for the equation : $X^3 + 27 = 0$ is

(a) $\{-3\}$

(b) $\{2\}$

(c) $\{3\}$

(d) \emptyset

- 2 If the mode of the values : 3 , 6 , $X + 1$, 6 , 3 , 1 is 6 , then $X = \dots\dots\dots$
 (a) 1 (b) 2 (c) 5 (d) 0
- 3 The cube whose volume is 64 cm^3 , the length of one of its edges is $\dots\dots\dots$ cm.
 (a) 8 (b) 3 (c) 16 (d) 4
- 4 If $X < \sqrt{51} < X + 1$, $X \in \mathbb{Z}$, then $X = \dots\dots\dots$
 (a) 8 (b) 7 (c) 6 (d) 5
- 5 $\sqrt{7} + \sqrt{7} = \dots\dots\dots$
 (a) $\sqrt{28}$ (b) 7 (c) 14 (d) $\sqrt{14}$
- 6 If the point (a , 1) satisfies the relation $X + y = 5$, then a = $\dots\dots\dots$
 (a) 1 (b) 2 (c) 5 (d) 4

2 Complete :

- 1 $\sqrt[3]{\dots\dots\dots} = -\sqrt{4}$
- 2 If the order of the median of some values is seventh , then the number of these values is $\dots\dots\dots$
- 3 If the lower limit of a set is 8 and the upper limit of the same set is 10 , then the centre of this set is $\dots\dots\dots$
- 4 $[-3 , 6] \cap [3 , 9] = \dots\dots\dots$
- 5 The slope of X -axis is $\dots\dots\dots$

3 [a] Reduce to the simplest form : $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}$

[b] Prove that : $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = 0$

[c] Find in \mathbb{R} the solution set of the inequality : $-3 < 4X - 7 < 5$

- 4 [a] A right circular cylinder whose height is 10 cm. and its volume is $90\pi \text{ cm}^3$
 Find the length of the radius of its base.

- [b] If $X = [-3 , 4]$, $Y =]1 , \infty[$, find each of the following using the number line :

1 $X \cap Y$

2 $X \cup Y$

3 $X - Y$

5 [a] Simplify : $\sqrt{50} + \sqrt{18} - \sqrt{32}$

- [b] Find the arithmetic mean of the following frequency distribution :

Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	4	5	6	3	2	20



Answer the following questions :

1 Choose the correct answer from those given :

1 $[3, 5] -]3, 5[= \dots\dots\dots$

- (a) \emptyset (b) $[3, 5]$ (c) $]3, 5[$ (d) $\{3, 5\}$

2 If the point $(a, 1)$ satisfies the relation : $x + y = 5$, then $a = \dots\dots\dots$

- (a) -4 (b) 1 (c) 4 (d) 5

3 If the lower limit of a set is 4 and the upper limit is 8 , then its centre is $\dots\dots\dots$

- (a) 2 (b) 4 (c) 6 (d) 8

4 If the radius length of a sphere is 6 cm. , then its volume is $\dots\dots\dots \text{cm}^3$

- (a) 6π (b) 36π (c) 72π (d) 288π

5 $\sqrt{100 - 36} = 10 - \dots\dots\dots$

- (a) -6 (b) 2 (c) 4 (d) 6

6 The intersection point of the ascending and descending cumulative curves determines the $\dots\dots\dots$ on the sets axis.

- (a) order of the median (b) median
(c) mean (d) mode

2 Complete each of the following :

1 $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \dots\dots\dots$ (in the same pattern)

2 The slope of any straight line parallel to x -axis is $\dots\dots\dots$

3 If $n \in \mathbb{Z}_+$, $n < \sqrt{26} < n + 1$, then $n = \dots\dots\dots$

4 The arithmetic mean of the set of values : $3 - x, 5 + x, 4$ equals $\dots\dots\dots$

5 If the mode of the values : $4, 11, 8, 2, x$ is 4 , then $x = \dots\dots\dots$

3 [a] Find the slope of \overleftrightarrow{AB} where $A(-1, 3)$ and $B(2, 5)$, is the point $C(8, 1) \in \overleftrightarrow{AB}$?

[b] If $x = \sqrt{7} + \sqrt{5}$, $xy = 2$, find the value of : $\frac{x+y}{xy}$

4 [a] Find the S.S. of the inequality : $-2 \leq 3x + 7 < 10$ in \mathbb{R} , then represent the interval of solution on the number line.

[b] Find the height of a right circular cylinder whose height is equal to its base radius length and its volume is $72\pi \text{ cm}^3$.

5 [a] Simplify to the simplest form : $\sqrt[3]{18} + \sqrt[3]{54} - 3\sqrt[3]{2} = \frac{1}{2}\sqrt[3]{16}$

[b] Find the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

12 Ismailia Governorate



Ministry of Education
Ismailia Governorate

Answer the following questions :

1 Choose the correct answer :

1 The slope of y-axis is

- (a) 0 (b) $\frac{1}{2}$ (c) undefined. (d) - 1

2 The mean of 8 , 19 , 11 , 12 , 10 is

- (a) 12 (b) 15 (c) 20 (d) 11

3 The multiplicative inverse of $\frac{\sqrt{6}}{2}$ is

- (a) $-\frac{\sqrt{6}}{2}$ (b) $\frac{\sqrt{6}}{3}$ (c) $\frac{\sqrt{6}}{2}$ (d) $2\sqrt{6}$

4 If the age of Ali now is X years , then his age after 12 years is years.

- (a) $X + 12$ (b) $X - 12$ (c) $X + 15$ (d) $12 X$

5 $\sqrt[3]{125} = \sqrt{\dots}$

- (a) 5 (b) 100 (c) 10 (d) 25

6 If the mode of : 7 , 10 , k + 3 , 9 is 7 , then k =

- (a) 3 (b) 10 (c) 4 (d) 9

2 Complete :

1 $4a^5 \times 5a^2 = \dots$

2 The median of : 15 , 7 , 16 , 9 , 4 , 20 is

3 $\{2, 7\} - \{2, 7\} = \dots$

4 If (3 , k) satisfies the relation : $2x + y = 10$, then k =

5 $\{1, 2, 3\} \cap \{2, 4, 5\} = \dots$

3 [a] The area of a sphere is 616 cm^2 . Find its diameter length $\left(\pi = \frac{22}{7}\right)$

[b] Graph the relation : $y = 2x$

[c] Find the slope of \overrightarrow{AB} where A (- 1 , 5) , B (2 , 6)

4 [a] Simplify : $\sqrt{72} + 2\sqrt{32} - 3\sqrt{2}$

[b] Find the S.S. in \mathbb{R} and represent it on the number line of : $1 < 3 - 2x \leq 11$

5 [a] If $A = [-2, 3]$, $B =]0, 5[$, using the number line find :

1 $A \cup B$

2 $A \cap B$

3 $A - B$

[b] From the following frequency distribution :

Sets	10 -	20 -	30 -	40 -	50 -	Total
Frequency	7	10	8	6	9	40

Find the mean.

13

Kafr El-Sheikh Governorate



Math Supervision

Answer the following questions :

1 Choose the correct answer :

1 The S.S. of the equation : $x(x^2 + 4) = 0$ in \mathbb{R} is

(a) $\{4\}$

(b) $\{0\}$

(c) $\{4, 0\}$

(d) $\{4, -4\}$

2 The slope of the straight line which is perpendicular to X-axis is

(a) 1

(b) zero

(c) -1

(d) undefined.

3 If the arithmetic mean of the numbers : 5, 4, $x-3$, 6, 4 is 4, then $x =$

(a) 5

(b) 4

(c) 6

(d) 3

4 If the mode of the numbers : 5, 2, 4, $x-2$ is 5, then $x =$

(a) 4

(b) 6

(c) 7

(d) 5

5 If $-2x < 6$, then x

(a) < 6

(b) > -3

(c) > 6

(d) > -6

6 $\mathbb{Z} \cap \mathbb{N} =$

(a) $\{0\}$

(b) \mathbb{Z}_+

(c) \mathbb{N}

(d) \mathbb{Q}

2 Complete the following :

1 The multiplicative inverse of the number $\sqrt{10} - 3$ is

2 $[3, 5] -]3, 5[=$

3 The median of the numbers : 41, 19, 15, 30, 20 is

4 $\sqrt{18} - \sqrt{2} =$

5 If the slope of the straight line passing through (2, k), (3, -1) is 2, then $k =$

3 [a] Find the lateral area of the right circular cylinder of volume $150\pi \text{ cm}^3$ and height 6 cm.

[b] Find in the simplest form : $3\sqrt{2} + \sqrt{8} - \sqrt{18}$

4 [a] Find in \mathbb{R} the S.S. of the inequality : $x < 2x - 1 < x + 3$

[b] If $x = \sqrt{7} - \sqrt{5}$, $y = \frac{2}{x}$, find : $\frac{x+y}{xy}$ in the simplest form.

5 [a] If $(-1, 5)$ satisfies the relation : $3x + ky = 7$, then find k

[b] The following table shows the frequency of marks of 50 students :

Sets	2	6	10	14	ℓ	22	26	Total
Frequency	3	6	8	10	11	k	4	50

Find : 1 The value of each of ℓ and k

2 The arithmetic mean for the marks of students.

11 Souhag Governorate



Ministry of Education, Government of Lebanon

Answer the following questions :

1 Choose the correct answer :

1) The simplest form of $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$ is

(a) $\sqrt{3}$ (b) 1 (c) $\sqrt{2}$ (d) $\sqrt[3]{3}$

2 The volume of a cube is 64 cm^3 , then its edge length is ... cm.

(a) 4 (b) 8 (c) 16 (d) 64

3 The mean of the values : 34, 23, 25, 40, 22, 12 is

(a) 22 (b) 23 (c) 24 (d) 26

4 If the point $(k, 1)$ satisfies the relation : $x + y = 5$, then $k =$

(a) 1 (b) -4 (c) 4 (d) 5

5 $(2\sqrt[3]{2})^3 =$

(a) 4 (b) 8 (c) 16 (d) 40

6 If the mode of the values : 4, 11, 8, 2x is 4, then $x =$

(a) 2 (b) 4 (c) 6 (d) 8

2 Complete :

1 The S.S. of : $x^2 + 9 = 0$ in \mathbb{R} is

2 $\sqrt{8} + \sqrt{18} - 3\sqrt{2} =$

Algebra and Statistics

- 3 The mode of : 3 , 5 , 3 , 4 , 3 is
- 4 $]-2, 2[\cup \{-2, 2\} = \dots\dots\dots$
- 5 If the volume of a sphere $= \frac{9}{2} \pi \text{ cm}^3$, then its diameter length equals cm.
- 3 [a] Find in the simplest form : $\sqrt{18} + \sqrt{32} - 3\sqrt{2} - \frac{1}{2}\sqrt{8}$
- [b] If $x = \sqrt{5} - \sqrt{2}$, $y = \frac{3}{\sqrt{5} - \sqrt{2}}$, prove that : x and y are two conjugate numbers.
- 4 [a] Represent graphically the linear relation : $y = 2 - x$
- [b] Find the solution set of the inequality :
 $-2 < 3x + 7 \leq 10$ in \mathbb{R} , then represent the S.S. on the number line.
- 5 [a] A right circular cylinder of radius length 4 cm. and its height is 9 cm.
 Find its volume in terms of π
- [b] Find the arithmetic mean of the following frequency distribution :

Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	7	10	12	13	8	50

15

Aswan Governorate



Aswan Educational Directorate

Secondary Education

Answer the following questions :

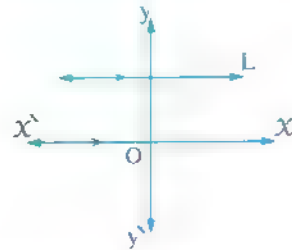
- 1 Choose the correct answer :
- 1 The multiplicative inverse of $\frac{\sqrt{3}}{5}$ is
- (a) $-\frac{\sqrt{3}}{5}$ (b) $\frac{5}{3}$ (c) $\frac{3}{5}$ (d) $\frac{5\sqrt{3}}{3}$
- 2 If $x = \sqrt{6} - \sqrt{2}$, $y = \frac{4}{x}$, then $y = \dots\dots\dots$
- (a) 4 (b) $\sqrt{6} + \sqrt{2}$ (c) 10 (d) $\sqrt{8}$
- 3 If the ordered pair $(2k, k)$ satisfies the relation : $y + 2x = 5$, then $k = \dots\dots\dots$
- (a) 1 (b) 2 (c) 3 (d) 4
- 4 If the lower boundary of a set is 4 and the upper boundary is 8, then its centre is
- (a) 2 (b) 4 (c) 6 (d) 8
- 5 $[1, 5] - \{1, 5\} = \dots\dots\dots$
- (a) $[2, 4]$ (b) $]1, 5[$ (c) $]0, \infty[$ (d) $]1, 5]$

6 In the opposite figure :

The slope of the straight line

L is

- (a) positive. (b) negative.
 (c) zero. (d) undefined.

**2 Complete each of the following :**

1 $\sqrt[3]{64} = \sqrt{\quad}$

2 In the relation : $y = 3x + 4$, if $y = 1$, then $x = \dots\dots\dots$

3 If the mode of the values : 12 , 7 , $x + 1$, 7 , 12 is 7 , then $x = \dots\dots\dots$

4 $[-2, 5[\cap \mathbb{R}_+ = \dots\dots\dots$

5 The median of the set of values : 34 , 23 , 25 , 40 , 22 , 4 is

3 [a] Find in the simplest form the value of : $\sqrt[3]{128} + \sqrt[3]{16} + 2\sqrt[3]{-54}$

[b] If $x = \sqrt{3} + 1$ and $y = \frac{2}{\sqrt{3} + 1}$

[1 Prove that : x and y are conjugate.

2 Find the value of : $\frac{x+y}{xy}$ in the simplest form.

4 [a] If $X =]-1, 4]$ and $Y = [3, \infty[$, using the number line find each of the following :

[1] $X \cup Y$

[2] $X - Y$

[3] $X \cap Y$

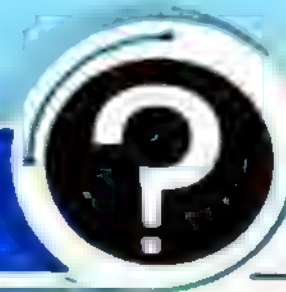
[b] Find the S.S. in \mathbb{R} of : $-2 \leq 3x + 7 \leq 10$ and represent it on the number line.

5 [a] Represent graphically the relation $y = 2 - x$ and if $(-4, b)$ satisfies the relation , find the value of b

[b] Find the arithmetic mean of the following frequency distribution :

Sets of marks	5	15 –	25 –	35 –	45 –	Total
Number of pupils	7	10	12	13	8	50

Some Schools Examinations



on Algebra and Statistics

1

Cairo Governorate

Near City Educ. Administration
St. Fatime Language School

Answer the following questions :

1 Choose the correct answer :

1 $[0, 5] \cup [3, 8[= \dots\dots\dots$

(a) $]3, 5]$

(b) $[3, 5]$

(c) $[0, 8]$

(d) $[0, 8[$

2 $\sqrt{12} - \sqrt{3} = \dots\dots\dots$

(a) 3

(b) $\sqrt{3}$

(c) $2\sqrt{3}$

(d) $3\sqrt{3}$

3 The S.S. in \mathbb{R} of the equation $x(x^2 - 1) = 0$ is $\dots\dots\dots$

(a) $\{0\}$

(b) $\{1\}$

(c) $\{-1\}$

(d) $\{0, -1, 1\}$

4 The arithmetic mean of the values 27, 8, 16, 24, 6, k is 14, then k = $\dots\dots\dots$

(a) 3

(b) 6

(c) 27

(d) 84

5 The additive inverse of the number $-\sqrt{5}$ is $\dots\dots\dots$

(a) $\sqrt{5}$

(b) 5

(c) $\sqrt{2}$

(d) -5

6 The radius length of a sphere is 6 cm., then its volume is $\dots\dots\dots$

(a) $6\pi \text{ cm}^3$

(b) $36\pi \text{ cm}^3$

(c) $72\pi \text{ cm}^3$

(d) $288\pi \text{ cm}^3$

2 Complete :

1 $[1, 5] \cap]-2, 3] = \dots\dots\dots$

2 The mode of the set of the values 3, 4, 7, 4, 2 is $\dots\dots\dots$

3 The volume of the cuboid whose dimensions are $\sqrt{2}, \sqrt{3}, \sqrt{6}$ cm. is $\dots\dots\dots \text{ cm}^3$

4 The S.S. in \mathbb{R} of $3 < 2x - 1 < 5$ as an interval is $\dots\dots\dots$

5 The slope of any line parallel to x-axis is $\dots\dots\dots$

3 [a] If $a = \sqrt{3} + \sqrt{2}$, $b = \sqrt{3} - \sqrt{2}$, find the value of : $a^2 - ab + b^2$

[b] Find the S.S. for each of the following inequalities in \mathbb{R} , in the form of an interval, then represent the S.S. on the number line :

1 $5x - 3 < 2x + 9$

2 $1 \leq 3 - 2x < 5$

4 [a] If $M = [2, \infty[$, $J =]-2, 3[$, find each of the following using the number line :

1 $M \cap J$

2 $M - J$

[b] Simplify : $\frac{\sqrt{3}}{\sqrt{5} - \sqrt{3}} + \frac{\sqrt{5}}{\sqrt{5} + \sqrt{3}}$

Algebra and Statistics

5 [a] Reduce to the simplest form : $2\sqrt{18} + \sqrt{50} + \frac{1}{3}\sqrt{162}$

[b] Find the arithmetic mean of the following frequency distribution :

The Set	5 -	15 -	25 -	35 -	45 -	Total
Frequency	4	5	6	3	2	20

2

Cairo Governorate

El-Maadi Zone
Directing Mathematics

Answer the following questions :

1 Choose the correct answer :

1 The multiplicative inverse of $\frac{\sqrt{3}}{12}$ is

- (a) $4\sqrt{3}$ (b) 2 (c) $2\sqrt{3}$ (d) $6\sqrt{3}$

2 The conjugate of the number $2 - \sqrt{3}$ is

- (a) $\sqrt{3} - 2$ (b) $2 - \sqrt{3}$ (c) $\sqrt{2} - 3$ (d) $2 + \sqrt{3}$

3 The volume of the cuboid whose dimensions are $\sqrt{8}$, $\sqrt{3}$, $\sqrt{6}$ is

- (a) 144 (b) 12 (c) $\sqrt{120}$ (d) 20

4 The median for the values 7, 8, 9, 6 and 5 is

- (a) 7 (b) 8 (c) 9 (d) 10

5 $4^3 + 4^3 + 4^3 + 4^3 = \dots\dots\dots$

- (a) 4^{20} (b) 4^4 (c) 4^{12} (d) 16^3

6 If $(2k, k)$ satisfies the relation $2x + y = 15$, then $k = \dots\dots\dots$

- (a) 1 (b) 2 (c) 3 (d) 4

2 Complete :

1 $[2, 7] -]2, 7[= \dots\dots\dots$

2 If the mode of the values 8, 11, 4, $2x$ is 4, then $x = \dots\dots\dots$

3 $\mathbb{R} \cap \mathbb{R}_- = \dots\dots\dots$

4 The slope of the straight line passing through the two points A (5, 3), B (2, 1) is

5 The solution set in \mathbb{R} for $x^2 + 4 = 16$ is

3 [a] Put in the simplest form : $2\sqrt{8} + \sqrt{50} - \sqrt{32}$

[b] Find the solution set in \mathbb{R} for : $3x - 4 \leq 5$ and represent it on the number line.

4 [a] If $x = \frac{2}{\sqrt{7}-\sqrt{5}}$, $y = \sqrt{7}-\sqrt{5}$, find : $(x+y)^2$

[b] Represent graphically the relation : $y = 3x - 2$

5 [a] If the volume of a sphere equals $\frac{500}{3} \pi \text{ cm}^3$, find the length of its radius.

[b] The following table shows the frequency of marks of 50 students :

Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	7	10	12	13	8	50

Find the mean of the marks of the students.

3

Cairo Governorate

El-Khelifa and El-Mokattam Zone
El-Helmia Exper. Lang. School



Answer the following questions :

1 Choose the correct answer :

- 1 The S.S. in \mathbb{R} for the equation : $x^3 + 8 = 0$ is
 (a) $\{4\}$ (b) $\{2\}$ (c) \emptyset (d) $\{-2\}$
- 2 If the mode of the values 3 , 5 , $x + 1$, 5 , 3 , 1 is 5 , then $x =$
 (a) 5 (b) 4 (c) 3 (d) 6
- 3 The cube whose volume is 8 cm^3 , the area of one of its faces is cm^2
 (a) 4 (b) 8 (c) 16 (d) 64
- 4 If $x < \sqrt{15} < x + 1$, $x \in \mathbb{Z}$, then $x =$
 (a) 3 (b) 4 (c) 5 (d) \emptyset
- 5 $\sqrt{3} + \sqrt{3} =$
 (a) -3 (b) $\sqrt{12}$ (c) 12 (d) 3
- 6 Which of the following ordered pairs satisfies the relation $2x + y = 5$?
 (a) $(-1, 3)$ (b) $(1, 3)$ (c) $(3, 1)$ (d) $(2, 2)$

2 Complete :

- 1 $\sqrt[3]{\dots} = -\sqrt{9}$
- 2 If $(-1, 5)$ satisfies the relation $3x + ky = 7$, then $k =$
- 3 If the order of the median of some values is fifth, then the number of these values is
- 4 $[-2, 5] \cap [3, 7] =$
- 5 If the lower limit of a set is 4 and the upper limit of the same set is 10, then the centre of this set is

Algebra and Statistics

- 3 [a] The volume of a sphere is $562.5 \pi \text{ cm}^3$, find its surface area.
 [b] If $x = \frac{4}{\sqrt{7} + \sqrt{3}}$, $y = \sqrt{7} + \sqrt{3}$, then find the numerical value of: $x^2 - 2xy + y^2$

- 4 [a] Find in \mathbb{R} the S.S. of: $-1 < 3x + 5 \leq 14$ and represent it on the number line.
 [b] Graph the relation: $2x + y = 1$
 [c] If $A =]-\infty, 3[$, $B = [-1, 5]$
 , find the following using the number line: 1 $A \cap B$ 2 $A - B$

- 5 [a] Find the slope of \overline{AB} where $A(-1, 3)$, $B(2, 5)$
 Is the point $C(8, 1) \in \overline{AB}$?
 [b] The following table shows the marks of 50 students in an examination:

Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	7	10	12	13	8	50

Find the arithmetic mean of this frequency distribution.

4

Giza Governorate

El-Haram Directorate
Al Maarefa Exp. Language School

Answer the following questions:

- 1 Complete the following:
 1 $\sqrt[3]{4} = \sqrt[3]{\dots}$
 2 $] -3, 4[\cup \{-3\} = \dots$
 3 The mode of the values 7, 3, 8, 2, 3, 4, 3, 7 is
 4 If $(3k, 2k)$ satisfies the relation $2x - y + 2 = 12$, then $k = \dots$
 5 The slope of the straight line which passes through $A(2, -5)$, $B(3, -2)$ is

- 2 Choose the correct answer:

- 1 The multiplicative inverse of $\frac{\sqrt{2}}{4}$ is
 (a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d) 2
 2 $[2, 5] -]2, 5[= \dots$
 (a) $\{2, 5\}$ (b) $[2, 5[$ (c) $]2, 5]$ (d) \emptyset
 3 The mean of the values 4, 7, 3, 9, 2 is
 (a) 2 (b) 3 (c) 5 (d) 7
 4 The S.S. of the equation $x^2 + 36 = 0$ in \mathbb{R} is
 (a) $\{6\}$ (b) $\{-6\}$ (c) $\{6, -6\}$ (d) \emptyset

5 If $5x = 35$, then $2x + 1 = \dots\dots\dots$

- (a) 9 (b) 15 (c) 8 (d) 7

6 The order of the median of 5, 2, 3, 9, 7, 1, 6 is $\dots\dots\dots$

- (a) 9 (b) 5 (c) 4 (d) 2

3 [a] If $X = [-2, 4]$, $Y =]1, 6]$

, find by using the number line : 1 \bar{X} 2 $X \cap Y$ 3 $X - Y$

[b] Find in \mathbb{R} the S.S. of the inequality : $2x + 1 < 7$

4 [a] Find in the simplest form : $2\sqrt{18} + \sqrt{50} - \sqrt{162}$

[b] If $x = 3 + \sqrt{5}$, $y = \frac{4}{3 + \sqrt{5}}$

, prove that : x, y are conjugate numbers and find the value of : $x^2 - 2xy + y^2$

5 [a] A lead cuboid in which its dimensions are 77 cm., 24 cm. and 21 cm. It was melted to form a sphere. Find the radius length of that sphere ($\pi = \frac{22}{7}$)

[b] Find the median by using the ascending cumulative frequency curve :

Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	4	5	6	3	2	20

5

Giza Governorate

Abo El-Nomros Educational Zone
Royal House Language Schools



Answer the following questions :

1 Choose the correct answer :

1 $(\sqrt{8} + \sqrt{2})^2 = \dots\dots\dots$

- (a) $\sqrt{10}$ (b) 10 (c) 18 (d) $\sqrt{18}$

2 The slope of any line // X-axis is $\dots\dots\dots$

- (a) 1 (b) undefined (c) -1 (d) zero

3 The multiplicative inverse of $(-2\frac{1}{3})$ is $\dots\dots\dots$

- (a) $\frac{1}{3}$ (b) $-\frac{7}{3}$ (c) $\frac{3}{7}$ (d) $-\frac{3}{7}$

4 The median of the values 34, 23, 25, 40, 22 is $\dots\dots\dots$

- (a) 22 (b) 23 (c) 24 (d) 25

5 $2a^2b \times \dots\dots\dots = 12a^3b$

- (a) $6ab$ (b) $6a$ (c) $6b$ (d) $6ab^2$

Algebra and Statistics

6 The mode of the values 8 , 5 , $x + 3$, 5 , 8 is 8 , then $x = \dots\dots\dots$

- (a) 5 (b) 8 (c) 3 (d) - 5

2 Complete :

- 1 The point (3 , $\dots\dots\dots$) satisfies $2x + y = 10$
 2 The mean of x , $2x$, $3x$ is $\dots\dots\dots$
 3 If $2x = y$, then $x : y = \dots\dots\dots$; $\dots\dots\dots$
 4 If the centre of a set is 4 and the upper limit of this set is 8 , then the lower limit of this set is $\dots\dots\dots$
 5 $[2 , 3] - \{2 , 3\} = \dots\dots\dots$

3 [a] If $x = \sqrt{7} - \sqrt{6}$, $y = \frac{1}{x}$, find the value of : $(x + y)^2$ (Show the steps).

[b] Find in \mathbb{R} the S.S. of : $-15 \leq 2x - 3 \leq 5$

[c] Simplify : $\sqrt[3]{54} + 8\sqrt[3]{\frac{1}{4}} + 5\sqrt[3]{16}$

4 [a] If $X =]-\infty , 5]$ and $Y =]1 , 9[$, find by using the number line :

- 1 $X \cap Y$ 2 $X \cup Y$ 3 $X - Y$

[b] Find the slope of the straight line passing through the two points (2 , 4) , (4 , 5)

5 [a] Find the S.S. in \mathbb{R} : $125x^3 - 7 = 20$

[b] Find the mode of the following distribution :

The Set	2 -	6 -	10 -	14 -	18 -	22 -	26 -	Total
Frequency	3	5	8	10	7	5	2	40

6

Alexandria Governorate

East Educational Zone
Maths Supervision

Answer the following questions :

1 Choose the correct answer from the given ones :

- 1 The arithmetic mean for the values : 9 , 6 , 5 , 14 , 1 is $\dots\dots\dots$
 (a) 7 (b) 3 (c) 5 (d) 9
 2 The additive inverse of the number $-\sqrt{5}$ is $\dots\dots\dots$
 (a) $\sqrt{5}$ (b) 5 (c) $\sqrt{2}$ (d) - 5

Final Examinations

- 3 If the lower limit of a set is 4 and the upper limit is 8 , then its centre is
 (a) 2 (b) 4 (c) 6 (d) 8
- 4 The simplest form of the expression : $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$ is
 (a) $\sqrt{3}$ (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{3}$
- 5 If the radius length of a sphere is 6 cm. , then its volume is $\pi \text{ cm}^3$
 (a) 6 (b) 36 (c) 72 (d) 288
- 6 $(2^3\sqrt{2})^3 = \dots\dots\dots$
 (a) 4 (b) 8 (c) 16 (d) 40

2 Complete the following :

- 1 If $3^x = 1$, then $x = \dots\dots\dots$
- 2 The median of the values 2 , 9 , 3 , 7 , 5 is
 3 $]-2, 2] \cup \{-2, 0\} = \dots\dots\dots$
- 4 The mode for the numbers : 3 , 5 , 3 , 4 , 3 is
 5 A cube whose volume is 8 cm^3 , then the sum of lengths of all its edges is

- 3 [a] Find the value of : $\sqrt{18} + \sqrt[3]{54} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{16}$ (with steps).

[b] Represent graphically the relation : $y = 2 - x$

- 4 [a] Find the S.S. of the inequality : $-2 < 3x + 7 \leq 10$ in \mathbb{R} , then represent the interval of solution on the number line.

[b] Reduce to the simplest form : $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}$ (with steps).

- 5 [a] If $(\sqrt{3})^x = (2\sqrt{2}-\sqrt{5})(2\sqrt{2}+\sqrt{5})$, then what is the value of x ?

[b] Find the arithmetic mean of the following frequency distribution :

The Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	7	10	12	13	8	50

7 Alexandria Governorate

El-Montazah Educational Zone
Math's Supervision



Answer the following questions :

1 Choose the correct answer :

1 $\frac{3}{4} = \dots\dots\dots \%$

- (a) 70 (b) 50 (c) 75 (d) 25

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Algebra and Statistics

- 2 [2, 7] -]2, 7[=
- (a)]2, 7] (b) [2, 7[(c) {2, 7} (d) [2, ∞[
- 3 The median of the values 3, 7, 2, 9, 5, 11 is
- (a) 9 (b) 6 (c) 8 (d) 11
- 4 The remainder of subtracting $-5x$ from $3x$ equals
- (a) $-2x$ (b) $8x$ (c) $2x$ (d) $8x^2$
- 5 If $(a, 4)$ satisfies the relation $x - y = -1$, then the value of a is
- (a) $\sqrt{3}$ (b) 5 (c) 27 (d) 3
- 6 If the lower limit of a set is 4 and its centre is 9, then its upper limit is
- (a) 36 (b) 5 (c) 13 (d) 14

2 Complete :

- 1 $\sqrt[3]{5} + \dots = \text{zero}$
- 2 $\mathbb{R}^+ \cup \mathbb{R}^- = \dots$
- 3 $\sqrt{a} + \sqrt{b}$ its conjugate is and their sum is
- 4 The mode of the set of values 4, 5, $k+1$, 3 is 3, then $k = \dots$
- 5 The slope of the straight line parallel to x -axis equals

3 [a] Simplify :

1 $\sqrt{32} - \sqrt{50} + 4\sqrt{\frac{1}{2}}$

2 $\sqrt[3]{16} - \frac{1}{3}\sqrt[3]{54}$

- [b] If $x = \sqrt{7} + \sqrt{5}$, $y = \frac{2}{x}$, find the value of $\frac{x+y}{xy}$ in the simplest form.

4 [a] Find in \mathbb{R} the S.S. of the following inequality : $-1 \leq 3 - 2x < 5$,

then represent the interval of solution on the number line.

- [b] Find the height of a right circular cylinder whose height is equal to its base radius length and its volume is $72\pi \text{ cm}^3$

[c] Graph the relation : $x + 2y = 3$

5 [a] Find the slope of \overline{AB} , where $A(-1, 3)$ and $B(2, 5)$. Is the point $C(8, 1) \in \overline{AB}$?

[b] Find the mean of the following frequency data :

Sets	8 -	12 -	16 -	20 -	24 -	Total
Frequency	4	10	16	12	8	50

8

El-Kalyoubia Governorate

Directorate of Education
Inspection of Mathematics

Answer the following questions :

1 Choose the correct answer :

- 1 Let A (3 , 5) and B (5 , - 1) , then the slope of \overrightarrow{AB} =
 (a) $-\frac{1}{3}$ (b) - 3 (c) 3 (d) $\frac{1}{3}$
- 2 If the point (a , 1) satisfies the relation $x + y = 5$, then a =
 (a) 1 (b) - 4 (c) 4 (d) 5
- 3 The median of the values 34 , 23 , 25 , 40 , 22 , 4 is
 (a) 22 (b) 23 (c) 24 (d) 25
- 4 If the mode of the set of values 4 , 11 , 8 , 2 x is 4 , then x =
 (a) 2 (b) 4 (c) 6 (d) 8
- 5 The arithmetic mean for the values 9 , 6 , 5 , 14 , 1 is
 (a) 7 (b) 3 (c) 5 (d) 9
- 6 The mode for the values 3 , 5 , 3 , 4 , 3 is
 (a) 3 (b) 4 (c) 5 (d) 12

2 Complete :

- 1 25% = (in the form of $\frac{a}{b}$ in the simplest form)
- 2 The sum of the two square roots of the number $2\frac{1}{4}$ is
- 3 $|-0.75| = \dots\dots\dots$
- 4 $\sqrt[3]{-125} = \dots\dots\dots$
- 5 The multiplicative inverse for $(\sqrt{3} + \sqrt{2})$ in its simplest form is

3 [a] Find the value of x if : $x^3 - 1000 = 0$ [b] Find the circumference of the circle whose area is $3\pi \text{ cm}^2$ 4 [a] Find : $[2, \infty[\cap]-2, 3[$ (by using the number line)[b] Simplify the following to the simplest form : $(\sqrt{2} + 5)(3 + \sqrt{2})$ 5 [a] Graph the straight line that represents the relation : $x + 2y = 3$

[b] Find the arithmetic mean of the following frequency distribution :

The Set	5 -	15 -	25 -	35 -	45 -	Total
Frequency	4	5	6	3	2	20

Algebra and Statistics

9

El-Gharbia Governorate

Central Mathematics Supervision
Official Languages Schools

Answer the following questions :

1 Choose the correct answer :

- 1 If the radius length of a sphere is 6 cm. , then its volume is
(a) $6 \pi \text{ cm}^3$ (b) $36 \pi \text{ cm}^3$ (c) $72 \pi \text{ cm}^3$ (d) $288 \pi \text{ cm}^3$
- 2 If the point (a , 1) satisfies the relation $x + y = 5$, then a =
(a) 1 (b) -4 (c) 4 (d) 5
- 3 The median of the values 34 , 23 , 25 , 40 , 22 , 4 is
(a) 22 (b) 23 (c) 24 (d) 25
- 4 The solution set of the equation $x(x^2 - 1) = 0$, $x \in \mathbb{R}$ is
(a) {1} (b) {0} (c) {-1} (d) {0 , 1 , -1}
- 5 If the arithmetic mean of the values 18 , 21 , 29 , $2k + 1$, k is 18 , then k =
(a) 1 (b) 7 (c) 29 (d) 90
- 6 $\sqrt{3 \frac{3}{8}} = \frac{3}{2} \sqrt{\frac{\dots}{\dots}}$
(a) $\frac{3}{8}$ (b) $\frac{3}{2}$ (c) $\frac{27}{8}$ (d) $\frac{729}{64}$

2 Complete the following :

- 1 If the lower boundary of a set is 10 and the upper boundary is x and its centre is 15 , then $x = \dots\dots\dots$
- 2 The multiplicative inverse of the number $(\sqrt{3} + \sqrt{2})$ is (in the simplest form).
- 3 $[3 , 4] - \{3 , 5\} = \dots\dots\dots$
- 4 $\sqrt{64} - \sqrt[3]{64} = \dots\dots\dots$
- 5 The slope of the straight line passing through (2 , 3) and (5 , -1) is

3 [a] If $x = \sqrt{7} + \sqrt{5}$, $y = \frac{2}{\sqrt{7} + \sqrt{5}}$

- 1 Prove that : x and y are two conjugate numbers.
- 2 Find : xy , $(x + y)^2$

[b] Find in the simplest form : $\sqrt{12} + \sqrt[3]{54} - \sqrt{3} - \sqrt[3]{16}$ 4 [a] Graph the relation : $2x + 3y = 6$, if the straight line representing this relation intersects the x -axis at A and the y -axis at B , find the area of the triangle OAB where O is the origin point.[b] Find the solution set in \mathbb{R} : $8x^3 + 7 = 8$

5 [a] Find the solution set for the inequality : $2x - 1 \geq 5$ in \mathbb{R}

[b] Find the arithmetic mean of the following frequency distribution :

The Set	5 -	15 -	25 -	35 -	45 -	Total
Frequency	4	5	6	3	2	20

10 El-Dakahlia Governorate

Talkha Educational Directorate
A.M.D.I. School



Answer the following questions :

1 Choose the correct answer from the given ones :

1 If $x = 3 + \sqrt{3}$ and $y = 3 - \sqrt{3}$, then $x - y = \dots\dots\dots$

- (a) $6\sqrt{3}$ (b) -6 (c) $\sqrt{6}$ (d) $2\sqrt{3}$

2 If the order of the median of a set of values is the fifth, then the number of these values is $\dots\dots\dots$

- (a) 6 (b) 10 (c) 11 (d) 9

3 The result of $(1 + \sqrt{5})(1 - \sqrt{5}) = \dots\dots\dots$

- (a) 2 (b) -4 (c) $-2\sqrt{5}$ (d) $2\sqrt{5}$

4 If A (3, -2), B (0, 4), then the slope of $\overline{AB} = \dots\dots\dots$

- (a) -2 (b) 2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

5 The mean of the values 2, 8, 6, 4 is $\dots\dots\dots$

- (a) 3 (b) 4 (c) 5 (d) 6

6 The multiplicative inverse of $\frac{\sqrt{3}}{6}$ is $\dots\dots\dots$

- (a) $-\frac{\sqrt{3}}{6}$ (b) $6\sqrt{3}$ (c) $2\sqrt{3}$ (d) $-2\sqrt{3}$

2 Complete the following :

1 $[-3, 7] - \{-3, 7\} = \dots\dots\dots$

2 The S.S. of the equation $x^2 + 9 = 0$ in \mathbb{R} is $\dots\dots\dots$

3 If the mode of 14, 8, $x + 5$, 8 and 14 is 8, then $x = \dots\dots\dots$

4 The slope of the straight line perpendicular to y-axis is $\dots\dots\dots$

5 If the volume of a sphere is $\frac{9}{2} \pi \text{ cm}^3$, then its radius length is $\dots\dots\dots$

3 [a] Find in the simplest form : $\sqrt{18} + \sqrt[3]{54} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{16}$

[b] If $X = [-3, 4]$, $Y =]1, \infty[$, find each of the following using the number line :

1 $X \cap Y$

2 $X - Y$

Algebra and Statistics

4 [a] Find in \mathbb{R} the S.S. of the inequality : $-7 \leq -3x + 1 < 13$ and represent it on the number line.

[b] If $x = \sqrt{6} + \sqrt{5}$, $y = \frac{1}{\sqrt{6} + \sqrt{5}}$:

1 Prove that : x , y are two conjugate numbers.

2 Find : the numerical value of $(x - y)^2$

5 [a] Graph the relation $y + 3x = 6$ and find the slope of the straight line.

[b] Find the arithmetic mean of the following frequency distribution :

Sets	10 -	20 -	30 -	40 -	50 -	Total
Frequency	5	15	20	25	10	75

11

Ismailia Governorate

Directorate of Education
Meth's Supervision

Answer the following questions :

1 Choose the correct answer :

1 A (2, 5), B (3, 7), then the slope of $\overrightarrow{AB} = \dots\dots\dots$

(a) $\frac{1}{2}$ (b) 2 (c) -2 (d) 5

2 $]3, 5[\cup \{3, 5\} = \dots\dots\dots$

(a) $]3, 5[$ (b) $\{3, 5\}$ (c) $[3, 5]$ (d) $[3, 5[$

3 The median of 4, 11, 8, 16, 9, 14 is $\dots\dots\dots$

(a) 10 (b) 8 (c) 16 (d) 9

4 $\mathbb{Q} \cup \mathbb{Q} = \dots\dots\dots$

(a) \emptyset (b) \mathbb{R} (c) \mathbb{Z} (d) \mathbb{N}

5 The slope of X-axis is $\dots\dots\dots$

(a) negative. (b) positive. (c) undefined. (d) zero.

6 $\mathbb{Z}^+ \cap \mathbb{Z}^- = \dots\dots\dots$

(a) zero (b) \emptyset (c) \mathbb{Z} (d) \mathbb{N}

2 Complete :

1 The mean of 12, 13, 10, 11, 14 is $\dots\dots\dots$

2 The multiplicative inverse of $\sqrt{3} - \sqrt{2}$ is $\dots\dots\dots$

3 The mode of 5, 11, 6, 2, 11, 7 is $\dots\dots\dots$

4 If $\frac{x}{y} = 1$, then $x - y = \dots\dots\dots$

5 $\sqrt{5^2 - 4^2} = \dots\dots\dots$

3 [a] Find the S.S. in \mathbb{R} of : $8 \leq 3x + 2 \leq 17$ and represent it on the number line.

[b] Simplify : $\sqrt{72} + 3\sqrt{18} - 2\sqrt{\frac{1}{2}}$

4 [a] The volume of a cylinder is 1540 cm^3 , if its height is 10 cm. , find its diameter length. ($\pi = \frac{22}{7}$)

[b] Graph the relation : $y = -3$

5 [a] If $X = [-1, \infty[$, $Y =]-4, 3]$, using the number line find :

1 $X \cap Y$

2 $X \cup Y$

3 X^c

[b] Find the mean of the following frequency distribution :

Sets	10 -	20 -	30 -	40 -	50 -	Total
Frequency	8	12	14	9	7	50

12

Damietta Governorate

Damietta Inspection of mathematics
Official Language Schoole



Answer the following questions :

1 Choose the correct answer from those given :

1 $\sqrt{25} - \sqrt[3]{-125} = \dots\dots\dots$

(a) zero

(b) 10

(c) 5

(d) ± 5

2 The multiplicative inverse of $\frac{\sqrt{2}}{6}$ is $\dots\dots\dots$

(a) $\sqrt{2}$

(b) $2\sqrt{2}$

(c) $3\sqrt{6}$

(d) $3\sqrt{2}$

3 If the lower limit of a set is 4 and the upper limit is 8 , then its centre is $\dots\dots\dots$

(a) 8

(b) 6

(c) 4

(d) 2

4 The solution set of the equation $x^2 + 9 = 0$ in \mathbb{R} is $\dots\dots\dots$

(a) $\{3\}$

(b) $\{-3\}$

(c) \emptyset

(d) $\{-3, 3\}$

5 The arithmetic mean of the values $6 - k$, 12 , 18 and $k + 4$ is $\dots\dots\dots$

(a) 9

(b) 10

(c) 15

(d) 40

6 If the volume of a cube is 27 cm^3 , then the perimeter of one of its faces is $\dots\dots\dots \text{ cm}$.

(a) 12

(b) 9

(c) 36

(d) 3

2 Complete each of the following :

1 The slope of the straight line passing through the points (1 , -1) and (-3 , 7) is $\dots\dots\dots$

2 If the ordered pair (k , 2k) satisfies the relation $x + y = 15$, then $k = \dots\dots\dots$

3 The point of intersection of the ascending and descending cumulative frequency curves determines $\dots\dots\dots$ on the set-axis.

Algebra and Statistics

4 If three times of a number is 60 , then $\frac{1}{5}$ of this number equals

5 If the mode of the values 5 , 9 , 5 , $x + 3$, 9 is 9 , then $x =$

3 [a] If $x = \sqrt{5} + \sqrt{2}$, $y = \frac{3}{x}$, then find the value of : $\frac{x+y}{xy}$ in its simplest form.

[b] Find in \mathbb{R} the solution set of the inequality : $-3 \leq 4x - 7 \leq 5$

[c] A right circular cylinder whose height is 8 cm. and its volume is $72\pi \text{ cm}^3$
Find the length of the radius of its base.

4 [a] Find in its simplest form : $\sqrt{50} + \sqrt[3]{54} - 10\sqrt{\frac{1}{2}} - \sqrt[3]{16}$

[b] If $X = [-1, 5[$ and $Y = [2, \infty[$, find using the number line :

1 $X \cup Y$

2 $X \cap Y$

3 $X - Y$

5 [a] Find three ordered pairs satisfying the relation $2x + y = 7$, then represent it graphically.

[b] Find the arithmetic mean of the following frequency distribution :

Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	4	5	6	3	2	20

13 Kafr El-Sheikh Governorate

Directorate of Education
Math's Supervision



Answer the following questions :

1 Choose the correct answer :

1 $(\sqrt{5} + \sqrt{3})^2 (\sqrt{5} - \sqrt{3})^2 =$

(a) 2

(b) 3

(c) 4

(d) 8

2 If the lower limit of a set is 4 and the upper limit is 8 , then its centre is

(a) 8

(b) 6

(c) 4

(d) 2

3 $2 \in$

(a) $]-1, \infty[$

(b) $]2, 5[$

(c) $]-\infty, 1[$

(d) $\{22\}$

4 If $(-1, 5)$ satisfies the relation $3x + ky = 7$, then $k =$

(a) 7

(b) 4

(c) 3

(d) 2

5 If the slope of the straight line $ax + by + 1 = 0$ is undefined , then =

(a) $a = b$

(b) $a = \text{zero}$

(c) $b = \text{zero}$

(d) $a = -b$

6 The intersection point of the ascending and descending cumulative frequency curves determines the on the sets axis.

(a) mode

(b) median

(c) mean

(d) centre

2 Complete :

- 1 The slope of the straight line passing through the two points (2, 6) and (-1, 3) equals
- 2 If the mode of the values 4, 11, 8, 2, x is 4, then $x =$
- 3 If the mean of the values 9, 6, 5, 14 is k , then $k =$
- 4 If the volume of a sphere $= 36\pi \text{ cm}^3$, then its diameter length = cm.
- 5 The degree of the algebraic term $3x^2y^2$ is

- 3 [a] Find the volume of the right circular cylinder whose diameter length of its base is 10 cm. and its height is 7 cm. ($\pi = \frac{22}{7}$)

[b] If $X =]-\infty, 5]$, $Y =]1, 7]$

, find by using the number line : 1 $X \cap Y$ 2 $X \cup Y$ 3 $Y - X$

[c] Find the S.S. of the equation : $8x^3 + 7 = 8$ in \mathbb{R}

- 4 [a] Represent graphically the relation $y = x + 2$ and if $(-4, a)$ satisfies the relation, find the value of a

[b] Simplify : $\sqrt{18} + \sqrt{50} - 2\sqrt{8}$

[c] Find in \mathbb{R} the S.S. of the inequality : $-8 < 3x + 1 \leq 4$

- 5 [a] If $x = \sqrt{3} + \sqrt{2}$, $y = \frac{1}{\sqrt{3} + \sqrt{2}}$, then find the value of : $\frac{x+y}{xy}$

[b] From the following frequency table with equal sets :

The Set	10 -	20 -	30 -	40 -	50 -	60 - 70	Total
Frequency	12	15	25	27	$k + 4$	4	100

1 Find the value of k

2 Calculate the median.

14

Souhag Governorate

Maths Supervision



Answer the following questions :

- 1 Choose the correct answer from those given :

- 1 If the mode of the values 5, 8, $6 + x$, 9 is 9, then $x =$
 (a) 5 (b) 6 (c) 3 (d) 8
- 2 The volume of a cube is 27 cm^3 , then the area of one of its faces is
 (a) 3 cm^2 (b) 9 cm^2 (c) 36 cm^2 (d) 54 cm^2

Algebra and Statistics

- 3 The slope of any line parallel to x -axis equals
- (a) 1 (b) undefined (c) -1 (d) zero
- 4 The multiplicative inverse of $\frac{2\sqrt{3}}{6}$ is
- (a) $\sqrt{2}$ (b) 6 (c) $\sqrt{3}$ (d) zero
- 5 $\mathbb{Q} \cup \mathbb{Q} = \dots\dots\dots$
- (a) \emptyset (b) 0 (c) \mathbb{R} (d) \mathbb{Z}
- 6 If $(-1, 5)$ satisfies the relation $3x + ky = 7$, then $k = \dots\dots\dots$
- (a) 5 (b) 6 (c) 2 (d) 7

2 Complete the following :

- 1 $[1, 5] - \{1, 5\} = \dots\dots\dots$
- 2 The S.S. of the equation : $x(x^2 - 1) = 0$ in \mathbb{R} is
- 3 $(2x^2y) \times (\dots\dots\dots) = 12x^3y$
- 4 The arithmetic mean of the values 8, 6, 3, 7, 1 is
- 5 $\sqrt[3]{64} + \sqrt{16} = \dots\dots\dots$

3 [a] Use the following table to find the relation between x, y :

x	-1	0	1	2
y	-1	1	3	5

- [b] Find the S.S. of the inequality : $-2 < 3x + 7 \leq 10$ in \mathbb{R} , then represent the interval of the S.S. on the number line.

4 [a] If $x = \sqrt{3} + \sqrt{2}$, $y = \frac{1}{\sqrt{3} + \sqrt{2}}$, then find the value of : $\frac{x+y}{xy}$

- [b] If $X =]-2, 1]$, $Y = [0, 3[$, use the number line to find :

- 1 $X \cap Y$ 2 $X \cup Y$ 3 $X - Y$

5 [a] Simplify : 1 $\sqrt{50} + \sqrt{18} - \sqrt{32}$ 2 $\sqrt[3]{54} + 8\sqrt[3]{\frac{1}{4}} + 5\sqrt[3]{16}$

- [b] Find the arithmetic mean of the following frequency distribution :

Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	4	5	6	3	2	20

15

Luxor Governorate

Luxor Directorate
El-Salem Private Language School

Answer the following questions :

1 Choose the correct answer :

- 1 The smallest prime number is
(a) 0 (b) 1 (c) 2 (d) 3
- 2 If the mode of the set of values 4 , 11 , 8 , 2 x is 4 , then x =
(a) 2 (b) 4 (c) 6 (d) 8
- 3 If (2 , 5) satisfies the relation $3x + y = c$, then c =
(a) 1 (b) -1 (c) 11 (d) -11
- 4 The solution set of the equation $x^2 + 9 = 0$ in \mathbb{R} is
(a) \emptyset (b) $\{-3\}$ (c) $\{3\}$ (d) $\{3, -3\}$
- 5 The lower limit of a set is 4 and the upper limit is 8 , then its centre is
(a) 2 (b) 4 (c) 6 (d) 8
- 6 $4.274 \approx$ (to the nearest $\frac{1}{10}$)
(a) 4 (b) 4.2 (c) 4.3 (d) 4.27

2 Complete :

- 1 $[2, 7] - \{2, 7\} =$
- 2 The coefficient of the algebraic term $5a^3b^2$ is
- 3 The mean of 3 , 5 , 7 , 4 , 1 is
- 4 The slope of any line parallel to y-axis is
- 5 The median of the values 3 , 7 , 6 , 9 , 2 is

3 [a] Simplify to the simplest form : $\sqrt{27} - \sqrt{12} + \sqrt{300}$ [b] If $a = \sqrt{5} + \sqrt{3}$, $b = \sqrt{5} - \sqrt{3}$, find : $a^2 + 2ab + b^2$ 4 [a] Find the S.S. in \mathbb{R} of the inequality : $2x + 1 \leq 7$, then represent it on the number line.[b] Find the volume of the sphere whose diameter length is 4.2 cm. ($\pi = \frac{22}{7}$)5 [a] Let A (2 , -1) , B (10 , 3) and C (2 , 3). Find the slope of each of \overline{AB} and \overline{BC}

[b] Find the arithmetic mean of the following distribution :

Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	4	5	6	3	2	20

Final
Examinations of

Algebra and
Statistics
2019



Some Schools Examinations on Algebra and Statistics

1

Cairo Governorate

Al-Nozha Administration
Al Farouk Islamic Language School

Answer the following questions :

1 Choose the correct answer from the given ones :

(1) The irrational number lies between 3 and 4 is

(a) 3.5

(b) $3\frac{1}{8}$ (c) $\sqrt{13}$ (d) $\sqrt{20}$ (2) $]-2, 1] \cap \{-2, 0, 1\} = \dots\dots\dots$ (a) $\{-2, 0, 1\}$ (b) $\{1\}$ (c) $\{0, 1\}$ (d) $[-2, 1]$ (3) If $x = \sqrt{3} + 2$ and $y = \sqrt{3} - 2$, then $(xy, x + y) = \dots\dots\dots$ (a) $(5, 2\sqrt{3})$ (b) $(5, 9)$ (c) $(1, 2\sqrt{3})$ (d) $(-1, 2\sqrt{3})$ (4) The line represented the relation : $3x + 8y = 24$ intersects the y-axis at the point(a) $(0, 8)$ (b) $(8, 0)$ (c) $(0, 3)$ (d) $(3, 0)$ (5) If the arithmetic mean of the set of the values $m, m + 5, m + 4, m + 3$ is 9, then $m = \dots\dots\dots$

(a) 2

(b) 6

(c) 9

(d) 10

2 Complete each of the following :

(1) The slope of a straight line which passes through $(-3, 1)$ and $(-2, 5)$ is(2) If the mode of the set of the values $17, 8, k + 5, 8, 17$ is 8, then $k = \dots\dots\dots$ (3) The multiplicative inverse of $\frac{\sqrt{13} - \sqrt{10}}{3}$ is (In the simplest form)(4) The radius length of a sphere whose volume is $\frac{9}{2} \pi \text{ cm}^3$ is cm.

(5) If the order of the median of the set of values is fifth, then the number of these values equals

3 [a] If $A =]-1, 3]$ and $B = [0, 5[$, then find :(1) $A \cap B$ (2) $B - A$ (3) $\mathbb{R}_+ \cap B$ [b] Simplify : $2\sqrt{27} + \frac{1}{3}\sqrt[3]{54} - \sqrt{75} + \sqrt[3]{16}$ 4 [a] Find in \mathbb{R} the S.S. of each of the following :(1) $\frac{(2x-1)^3}{3} = 9$ (2) $-1 < 3 - 2x \leq 5$ [b] If $x = 2\sqrt{3} - \sqrt{2}$ and $y = \sqrt{12} + \sqrt{2}$ Find the value of : $\frac{x+y}{xy+2}$

5 [a] If $(a, 3)$ and $(3, b)$ satisfies the relation $2x - y = 1$

(1) Find the value of a and b

(2) Find the slope of the straight line which represented the relation : $2x - y = 1$

[b] From the following frequency table :

Sets	10 -	20 -	30 -	40 -	50 -	60 -	Total
Frequency	10	17	20	32	$k + 2$	4	100

(1) Find the value of k

(2) Graph the frequency histogram , then find the mode.

2 Cairo Governorate

Western Cairo Educational Zone
Mathematics Inspection



Answer the following questions :

1 Choose the correct answer :

(1) If the volume of a cube is 64 cm^3 , then its edge length is

(a) 32 cm.

(b) 16 cm.

(c) 8 cm.

(d) 4 cm.

(2) The figure  represents the solution of the inequality in \mathbb{R}

(a) $x > -3$

(b) $x \geq -3$

(c) $x < -3$

(d) $x \leq -3$

(3) $\sqrt{3}(\sqrt{11} + \sqrt{3}) = \dots\dots\dots$

(a) $3\sqrt{11} + 2$

(b) $\sqrt{33} + 3$

(c) $11\sqrt{3} + 2$

(d) $2\sqrt{11} + 3$

(4) $(3, 2)$ does not satisfy the relation

(a) $y + x = 5$

(b) $3y - x = 3$

(c) $y + x = 7$

(d) $x - y = 1$

(5) The arithmetic mean of the values : 5 , 12 , 17 , 6 is

(a) 10

(b) 12

(c) 4

(d) 17

2 Complete each of the following :

(1) $\sqrt[3]{-64} + \sqrt{16} = \dots\dots\dots$

(2) If the mode of the set of the values : 15 , 9 , $x + 1$, 9 and 15 is 9 , then $x = \dots\dots\dots$

(3) The multiplicative inverse of the number $\frac{3}{\sqrt{3}}$ is $\frac{\dots}{\sqrt{3}}$

(4) If the volume of a sphere = $\frac{9}{16} \pi \text{ cm}^3$, then its radius length = cm.

(5) If the order of the median of the set of values is fourth , then the number of these values is

Algebra and Statistics

3 [a] If $x = \sqrt{3} - 2$ and $y = \sqrt{3} + 2$, find the value of : $\left(\frac{x-y}{x+y}\right)^2$

[b] Simplify the following to the simplest form : $\sqrt{98} - \sqrt{128} - \sqrt{18} + 4\sqrt{2}$

4 [a] If $X =]-\infty, 2[$ and $Y = [-1, 5]$, find using the number line :

(1) $X \cap Y$

(2) $X - Y$

[b] Find the slope of the straight line passing through the two points : A (1, 3) and B (2, 3)

5 [a] Find the solution set for the following equation in \mathbb{R} , then represent the solution on the number line : $-8 \leq 3x + 1 \leq 4$

[b] Find the mean of the following frequency distribution :

Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	3	10	12	10	5	40

3 Cairo Governorate

New Cairo Educational Zone
Akhnaton Egyptian College



Answer the following questions :

1 Complete the following :

(1) The S.S. of the equation : $x^3 - 27 = 0$ in \mathbb{R} is

(2) $[1, 5] - \{1, 5\} = \dots\dots\dots$

(3) The slope of the straight line which passes through the two points (2, -2) and (4, 2) is

(4) A cube whose volume is 8 cm^3 , the length of its edge = cm.

(5) The arithmetic mean of 10, 6, 5, 14, 15 is

2 Choose the correct answer :

(1) If $x = \sqrt{3} + 2$ and $y = \sqrt{3} - 2$, then $xy = \dots\dots\dots$

(a) 1

(b) -1

(c) -4

(d) 3

(2) $]-1, 3[\cap [-3, -1] = \dots\dots\dots$

(a) \emptyset

(b) $\{-3\}$

(c) $\{-1\}$

(d) $\{3\}$

(3) If the lower limit of a set is 6 and the upper limit is 10, then its centre is

(a) 4

(b) 6

(c) 10

(d) 8

(4) The multiplicative inverse of $\frac{\sqrt{5}}{10}$ is

- (a) $\sqrt{10}$ (b) $\sqrt{5}$ (c) $2\sqrt{5}$ (d) $-2\sqrt{5}$

(5) The S.S. of $x + 2 \geq 1$ in \mathbb{R} is

- (a) $[-1, \infty[$ (b) $] -1, \infty[$ (c) $[1, 2]$ (d) $[1, 2[$

[3] [a] Simplify : $\sqrt[3]{16} - \frac{1}{3}\sqrt[3]{54} + \sqrt[3]{-2}$

[b] Find the S.S. of : $-2 < 3x + 7 \leq 10$ in \mathbb{R} , then represent the interval of the solution set on the number line.

[4] [a] If $x = \sqrt{5} + \sqrt{2}$ and $y = \sqrt{5} - \sqrt{2}$, then find the value of : $\frac{x+y}{x-y-1}$

[b] If $X = [-2, 1]$ and $Y = [0, \infty[$ Find :

- (1) $X \cap Y$ (2) $X \cup Y$ (3) $Y - X$

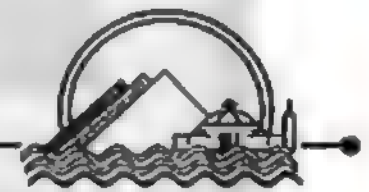
[5] [a] Find the arithmetic mean of the following frequency distribution :

Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	4	5	6	3	2	20

[b] Represent graphically the relation : $2y - x = 2$

4 Giza Governorate

Al-Agoza Directorate
Supervision of math.



Answer the following questions :

[1] Complete :

- The S.S. of the equation $x^2 + 9 = 0$ in \mathbb{R} is
- $\sqrt[3]{16} = \sqrt[3]{\dots\dots\dots}$
- The multiplicative inverse of the number $2\sqrt{3}$ is
- $\{8, 9, 10\} \cap]8, 10[= \dots\dots\dots$
- The length of the edge of a cube of volume $15 \frac{5}{8} \text{ cm}^3$ is

[2] Choose the correct answer :

- The mean of the set of numbers : 5, 12, 17, 6 is
(a) 40 (b) 20 (c) 5 (d) 10
- The S.S. of the equation : $x^2 - 1 = 8$ in \mathbb{R} is
(a) \emptyset (b) $\{3\}$ (c) $\{-3\}$ (d) $\{-3, 3\}$

Algebra and Statistics

(3) The conjugate of $\frac{1}{\sqrt{3}-\sqrt{2}}$ is

(a) $\sqrt{3}-\sqrt{2}$

(b) $3-\sqrt{2}$

(c) $3+\sqrt{2}$

(d) $\sqrt{3}+\sqrt{2}$

(4) The value of b that makes $(-2, 3)$ satisfies the relation : $3x + by = 3$ is

(a) 3

(b) 2

(c) 1

(d) -3

(5) If the mode of the values : 5 , $x+3$, 9 , 4 is 9 , then $x =$

(a) 5

(b) 4

(c) 6

(d) 3

[3] [a] Represent graphically the relation : $y = 2x - 3$

[b] If $X =]-\infty, 2]$ and $Y = [-1, 8]$, using the number line , find :

(1) $X \cup Y$

(2) $X - Y$

(3) $X \cap Y$

[4] [a] Simplify :

(1) $\sqrt{50} + \sqrt{18} - \sqrt{32}$

(2) $\sqrt[3]{54} + 8\sqrt[3]{\frac{1}{4}} + 5\sqrt[3]{16}$

[b] Find the slope of the straight line passing through the two points : A (5 , -3) and B (6 , 2)

[5] [a] Write two ordered pairs satisfying the relation : $y = x + 1$

[b] Find the arithmetic mean of the following frequency distributive :

Sets	10 -	20 -	30 -	40 -	50 -	Total
Frequency	10	20	25	30	15	100

5

Giza Governorate

El-Haram Educational Zone
Pyramide Language School



Answer the following questions :

[1] Complete the following :

(1) $\sqrt[3]{64} = \sqrt{\dots}$

(2) If $a = \sqrt{5} - 2$, $b = \sqrt{5} + 2$, then $a^2 b^2 =$

(3) The S.S. of the equation $x^2 + 5 = 0$ in \mathbb{R} is

(4) $[-1, 5] \cap [3, 7] =$

(5) If $a^2 + b^2 = 25$ and $ab = 5$, then $\frac{a}{b} + \frac{b}{a} =$

2 Choose the correct answer :

(1) $(\sqrt{2} + \sqrt{8})^2 = \dots\dots\dots$

(a) 18

(b) $\sqrt{10}$

(c) 4

(d) 10

(2) The sum of the real numbers of the interval $[-150, 150]$ is $\dots\dots\dots$

(a) 300

(b) -300

(c) zero

(d) 150

(3) The volume of a cuboid whose dimensions $\sqrt{2}$ cm. , $\sqrt{3}$ cm. , $\sqrt{6}$ cm. is $\dots\dots\dots$

(a) 6 cm^3

(b) 36 cm^3

(c) $6\sqrt{6} \text{ cm}^3$

(d) $18\sqrt{2} \text{ cm}^3$

(4) $\sqrt{(10)^2 - (6)^2} = \dots\dots\dots$

(a) 4

(b) 8

(c) ± 4

(d) ± 8

(5) $\sqrt[3]{3\sqrt{3}} = \dots\dots\dots$

(a) 3

(b) $\frac{1}{2}$

(c) $\sqrt[3]{3}$

(d) $\sqrt{3}$

3 [a] Simplify the following :

(1) $6\sqrt{\frac{5}{2}} + 20\sqrt{\frac{2}{5}}$

(2) $4\sqrt[3]{\frac{1}{2}} + 3\sqrt[3]{32} - \sqrt[3]{4}$

[b] Find the S.S. in $\mathbb{R} : (x-1)^2 = 4$

4 [a] If (3 , 2) satisfies the relation $x + 2y = m$, then find the value of m

[b] Find the slope of the straight line passes through the two points (3 , 5) and (4 , 7)

[c] Represent graphically : $y = x + 2$

5 [a] Find the median of : 28 , 25 , 24 , 26 , 27

[b] Find the arithmetic mean of the following frequency distribution :

Sets	10 -	20 -	30 -	40 -	50 -	Sum
Frequency	4	6	8	7	5	30

6 Alexandria Governorate

Middle Educational Zone
Math's Supervision



Answer the following questions :

1 Complete each of the following :

(1) If $3^x = 1$, then $x = \dots\dots\dots$

(2) The S.S. of the equation : $x(x^3 - 1) = 0$ in \mathbb{R} is $\dots\dots\dots$

Algebra and Statistics

(3) $]5, 7[\cup \{5, 7\} = \dots\dots\dots$

(4) If the arithmetic mean of the values : 9 , 6 , 5 , 14 , k is 7 , then k =

(5) If the slope of the straight line : $kx + 2y = 5$ is zero , then k =**2** Choose the correct answer from the given ones :

(1) $(2\sqrt[3]{2})^3 = \dots\dots\dots$

(a) 4

(b) 8

(c) 16

(d) 40

(2) If the volume of a cube is 27 cm^3 , then the area of its face is cm^2

(a) 3

(b) 9

(c) 36

(d) 54

(3) If the order of the median of a set of values is the fourth , then the number of values is

(a) 3

(b) 5

(c) 7

(d) 9

(4) If the mode of the set of values : 5 , 9 , 5 , $x - 2$, 9 is 9 , then $x = \dots\dots\dots$

(a) 5

(b) 57

(c) 9

(d) 11

(5) If $(-1, 5)$ satisfies the relation : $3x + ky = 7$, then k =

(a) 2

(b) -2

(c) 1

(d) 10

3 [a] Find the value of : $\sqrt{18} + \sqrt[3]{54} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{16}$

[b] If $x = \sqrt{5} + \sqrt{2}$ and $y = \sqrt{5} - \sqrt{2}$, find the value of : $\frac{x+y}{x-y-1}$ **4** [a] Write in the form of an interval the S.S. of the inequality : $x + 4 \geq 2x - 3 > x + 1$ [b] Represent graphically the relation : $y = 2 - x$ **5** [a] The volume of a sphere is $\frac{99000}{7} \text{ cm}^3$. Calculate its radius length. $(\pi = \frac{22}{7})$

[b] Find the arithmetic mean of the following frequency distribution :

Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	7	10	12	13	8	50

7

Alexandria Governorate

El-Montazah Educational Zone
Math's Supervision

Answer the following questions :

1 Complete each of the following :(1) The multiplicative inverse for $-\frac{\sqrt{2}}{6}$ is(2) If $5x - 3y = 0$, then $x : y = \dots\dots\dots$:

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- (3) The slope of any line parallel to X-axis =
- (4) $\sqrt{5} + \sqrt{2}$ its conjugate is and their product is
- (5) If $(-1, 5)$ satisfies the relation $3x + ky = 7$, then $k = \dots\dots\dots$

2 Choose the correct answer :

- (1) If $|a| = 5$, then $a = \dots\dots\dots$
- (a) 5 (b) -5 (c) ± 5 (d) $\sqrt{5}$
- (2) The order of the median of the set of values : 4, 5, 6, 7, 8 is
- (a) third. (b) fourth. (c) fifth. (d) sixth.
- (3) The S.S. of the inequality $-2x \geq 6$ in \mathbb{R} is
- (a) $]-\infty, -3[$ (b) $]-\infty, -3]$ (c) $[-3, \infty[$ (d) $[-3, \infty[$
- (4) $\{8, 9, 10\} -]8, 10[= \dots\dots\dots$
- (a) \emptyset (b) $\{9\}$ (c) \mathbb{N} (d) $\{8, 10\}$
- (5) The mode of the set of values : 5, 9, 5, $x-2$, 9 is 9, then $x = \dots\dots\dots$
- (a) 5 (b) 57 (c) 9 (d) 11

3 [a] Find in the simplest form : $2\sqrt{18} + \sqrt{50} + \frac{1}{3}\sqrt{162}$

[b] If $a - b = 2\sqrt{7}$, then find the value of : $a(a - b)^2 - b(a - b)^2$

[c] Find the slope of line \overline{AB} , where A $(-1, 3)$ and B $(2, 5)$ Is the point C $(8, 1) \in \overline{AB}$?

4 [a] Find the S.S. of the inequality : $-1 < 2x - 3 \leq 5$ in \mathbb{R} and represent the interval of solution on the number line.

[b] Find the lateral area for right circular cylinder of volume 924 cm^3

, and its height 6 cm.

$$\left(\pi = \frac{22}{7}\right)$$

5 [a] If $(\sqrt{3})^x = (2\sqrt{2} - \sqrt{5})(2\sqrt{2} + \sqrt{5})$, then what is the value of x ?

[b] By using the following distribution :

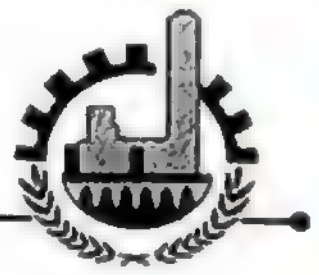
Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	3	10	$k - 2$	10	5	40

- (1) Find the value of k
- (2) Find the arithmetic mean.

Algebra and Statistics

8 El-Kalyoubia Governorate

Mathematics Inspection



Answer the following questions :

1 Choose the correct answer :

(1) $\mathbb{Q} \cap \mathbb{Q} = \dots\dots\dots$

- (a) \mathbb{R} (b) \mathbb{R}_+ (c) \mathbb{R}_- (d) \emptyset

(2) The S.S. of the equation : $x^3 + 27 = 0$ in \mathbb{R} is

- (a) $\{3\}$ (b) $\{-3\}$ (c) \emptyset (d) $\{3\sqrt{3}, -3\sqrt{3}\}$

(3) $\{x : x \in \mathbb{R}, x < 1\} = \dots\dots\dots$

- (a) $\{0, -1, -2\}$ (b) $]-\infty, 1]$ (c) $]-\infty, 1[$ (d) $]1, \infty[$

(4) The mode of values : 3, 5, 3, 6, 5, 3, 7 is

- (a) 3 (b) 5 (c) 7 (d) 6

(5) The arithmetic mean of the values : 6, 19, 32, 25, 8 is

- (a) 90 (b) 32 (c) 18 (d) 6

2 Complete the following :

(1) If $3^x = 1$, then $x = \dots\dots\dots$ (2) The conjugate of the number $\frac{4}{\sqrt{7}-\sqrt{3}}$ is(3) The total area of a cube of edge length 4 cm. is cm^2 (4) If the point (6, a) lies on the straight line whose equation is $x + y = 3$, then $a = \dots\dots\dots$

(5) The median of the set of the values : 2, 9, 3, 7, 5 is

3 [a] If $x = \sqrt{5} + \sqrt{2}$ and $y = \sqrt{5} - \sqrt{2}$ Find the value of : $\frac{x+y}{x-y+1}$ [b] If $X = [-1, 2]$ and $Y = [1, \infty[$ Find :

- (1) $X \cap Y$ (2) $X \cup Y$

4 [a] Find the S.S. of the inequality : $7 \geq 2x + 1 > 3$ [b] The radius length of the base of a right cylinder is $4\sqrt{2}$ cm. and its height is 9 cm. Find its volume in terms of π

5 [a] Find the slope of \overline{AB} where A (2 , -1) and B (-1 , 3) , then draw \overline{AB} on 2-dimensions coordinate.

[b] Find the arithmetic mean of the following frequency distribution :

The sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	3	4	7	4	2	20

9 El-Sharkia Governorate

Directorate of Education
Dept. of Governmental L. Schools



Answer the following questions :

1 Complete each of the following :

- (1) $[2, 7[\cup \{2, 7\} = \dots\dots\dots$
- (2) If the volume of a cube is 64 cm^3 , then its lateral area = $\dots\dots\dots \text{ cm}^2$
- (3) If (k , 4) satisfies the relation $x + 2y = 15$, then k = $\dots\dots\dots$
- (4) If $a = \sqrt{5} + 1$ and $b = \sqrt{5} - 1$, then $a - b = \dots\dots\dots$
- (5) The mean of the numbers 3 , 4 , 6 , 7 is $\dots\dots\dots$

2 Choose the correct answer :

- (1) The additive inverse of $\sqrt{5} - \sqrt{3}$ is $\dots\dots\dots$
 - (a) $\sqrt{5} - \sqrt{3}$
 - (b) $\sqrt{3} + \sqrt{5}$
 - (c) $-\sqrt{5} - \sqrt{3}$
 - (d) $\sqrt{3} - \sqrt{5}$
- (2) The S.S. of the equation $x^2 + 16 = 0$ in \mathbb{R} is $\dots\dots\dots$
 - (a) $\{4\}$
 - (b) \emptyset
 - (c) $\{4, -4\}$
 - (d) $\{-4\}$
- (3) $(\sqrt{5} + \sqrt{3})^2 (\sqrt{5} - \sqrt{3})^2 = \dots\dots\dots$
 - (a) 4
 - (b) 2
 - (c) 8
 - (d) 3
- (4) The slope of any line parallel to X-axis equals $\dots\dots\dots$
 - (a) 1
 - (b) undefined
 - (c) -1
 - (d) zero
- (5) If $5x = 35$, then $2x + 1 = \dots\dots\dots$
 - (a) 7
 - (b) 15
 - (c) 8
 - (d) 71

3 [a] Find the value of : $\sqrt{50} - \sqrt{8} + 2\sqrt{\frac{1}{2}} - \sqrt{18}$

[b] If $x = \frac{4}{3 + \sqrt{5}}$ and $y = 3 + \sqrt{5}$ Prove that : x and y are conjugate numbers , then find the value of : $(x + y)^2$

Algebra and Statistics

4 [a] If $A =] - 2, 6]$ and $B = [4, \infty[$, use the number line to find :

(1) $A \cup B$

(2) $A \cap B$

[b] If the volume of a sphere is $36 \pi \text{ cm}^3$. Find the length of its radius, then calculate its total area ($\pi = 3.14$)

5 [a] Graph the linear relation : $y = 2x - 1$

[b] Solve in \mathbb{R} the inequality : $x + 2 \leq 3x + 2 < x + 16$

[c] Find the mean of the following data :

Sets	20 -	30 -	40 -	50 -	60 -	70 -	Total
Frequency	10	15	22	25	20	8	100

10 El-Dakahlia Governorate

Math's Supervision (E.L.S)



Answer the following questions :

1 Complete the following :

(1) $[-5, 9] - \{-5, 9\} = \dots\dots\dots$

(2) The S.S. of the equation : $x^3 + 8 = 0$ in \mathbb{R} is $\dots\dots\dots$

(3) If the mode of 14, 9, $x + 5$, 9 and 14 is 9, then $x = \dots\dots\dots$

(4) The slope of the straight line parallel to x -axis is $\dots\dots\dots$

(5) If the volume of the sphere is $\frac{1}{6} \pi \text{ cm}^3$, then its radius length = $\dots\dots\dots$

2 Choose the correct answer :

(1) If $x = 5 + \sqrt{3}$ and $y = 5 - \sqrt{3}$, then $x - y = \dots\dots\dots$

(a) 10

(b) -10

(c) $\sqrt{6}$

(d) $2\sqrt{3}$

(2) If the order of the median of the set of values is the fourth, then the number of values is $\dots\dots\dots$

(a) 8

(b) 10

(c) 7

(d) 9

(3) $(1 + \sqrt{7})(1 - \sqrt{7}) = \dots\dots\dots$

(a) 2

(b) -4

(c) $-2\sqrt{7}$

(d) -6

(4) If A (2, -2) and B (1, 4), then the slope of $\overrightarrow{AB} = \dots\dots\dots$

(a) -2

(b) 2

(c) -6

(d) $-\frac{1}{2}$

(5) The mean of the values 3, 7, 8, 2 is $\dots\dots\dots$

(a) 2

(b) 4

(c) 5

(d) 6

3 [a] Simplify to the simplest form : $2\sqrt{18} + \sqrt[3]{54} - 12\sqrt{\frac{1}{2}} - 5\sqrt[3]{16}$

[b] If $X = [-2, 5]$ and $Y =]2, \infty[$

Find : (1) $X \cap Y$

(2) $Y - X$

4 [a] Find in \mathbb{R} the S.S. of the inequality : $-9 \leq -3x + 2 < 17$

[b] If $x = \sqrt{7} + \sqrt{6}$ and $y = \frac{1}{\sqrt{7} + \sqrt{6}}$

(1) Prove that : x and y are conjugate. (2) Find : the numerical value of $x^2 - y^2$

5 [a] Graph : $y + 2x = 4$ Does the point $(-1, 6)$ belong to the straight line ?

[b] Using the following distribution , find the arithmetic mean :

Sets	10 -	20 -	30 -	40 -	50 -
Frequency	6	14	21	24	10

11 Ismailia Governorate

Directorate of Education
El-Manar Language School



Answer the following questions :

1 Complete the following :

(1) $[-1, 5] -]-1, 5[= \dots\dots\dots$

(2) If $(k, 5)$ satisfies the relation : $2y + 2x = 8$, then $k = \dots\dots\dots$

(3) The S.S. of the equation $x^3 + 125 = 0$ in \mathbb{R} is $\dots\dots\dots$

(4) The additive inverse of $\sqrt{7} + \sqrt{3}$ is $\dots\dots\dots$

(5) If the dimensions of a rectangle is $(\sqrt{11} + 2)$ cm. and $(\sqrt{11} - 2)$ cm.
 , then its area = $\dots\dots\dots$ cm²

2 Choose the correct answer :

(1) If the mode of the values 8 , 7 , 8 , 5 , $x - 5$, 5 is 8 , then $x = \dots\dots\dots$

(a) 8

(b) 10

(c) 5

(d) 13

(2) The slope of the straight line passing through the two points $(-2, 2)$ and $(-8, 5)$
 is $\dots\dots\dots$

(a) $\frac{-7}{10}$

(b) $\frac{10}{7}$

(c) $\frac{-6}{12}$

(d) -2

Algebra and Statistics

(3) If the volume of a cube is 27 cm^3 , then the sum of edges of this cube is cm.

- (a) 36 (b) 3 (c) 12 (d) 27

(4) The median of the values 31, 13, 9, 60, 1, 45, 4 is

- (a) 60 (b) 13 (c) 31 (d) 163

(5) $]-\infty, 0] = \dots\dots\dots$

- (a) \mathbb{R}_+ (b) \mathbb{R}_-
(c) set of non positive real numbers. (d) set of non negative real numbers.

3 [a] Find the simplest form of : $\sqrt[3]{54} - \frac{1}{2}\sqrt[3]{16} + \sqrt[3]{-2}$

[b] If $x = \sqrt{5} + \sqrt{3}$ and $y = \frac{2}{\sqrt{5} + \sqrt{3}}$, find the value of : $\frac{x+y}{xy}$

4 [a] Find the S.S. in \mathbb{R} of the inequality :

$-2 < 3x + 7 \leq 10$ and represent it on the number line.

[b] If $X =]-\infty, 5]$ and $Y =]1, 9[$ Find using the number line :

- (1) $X \cap Y$ (2) $X \cup Y$ (3) $X - Y$ (4) \bar{X}

5 [a] If the volume of a sphere is $288\pi \text{ cm}^3$ find its area.

[b] The following table shows the frequency distribution of marks of 40 students in an algebra exam :

Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	7	9	12	x	4	40

(1) Find the value of x

(2) Find the arithmetic mean.

12 Port Said Governorate

Educational Directorate
Math inspection



Answer the following questions :

1 Choose the correct answer :

(1) The multiplicative inverse to the number $\frac{3}{\sqrt{2}}$ is

- (a) $\frac{\sqrt{2}}{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $2\sqrt{3}$

(2) The solution set of the equation : $x^3 = 8$ in \mathbb{R} is

- (a) \emptyset (b) $\{2\}$ (c) $\{-2\}$ (d) $\{0\}$

(3) $\mathbb{Q} \cup \mathbb{Q} = \dots\dots\dots$

(a) \emptyset

(b) 0

(c) \mathbb{R}

(d) \mathbb{Z}

(4) The conjugate of the number $\sqrt{2} - \sqrt{3}$ is

(a) $\sqrt{2} + \sqrt{3}$

(b) $\sqrt{3} - 2$

(c) $2 - \sqrt{3}$

(d) $-\sqrt{2} + \sqrt{3}$

(5) The arithmetic mean of the values 2 , 5 , 8 is

(a) 5

(b) 4

(c) 3

(d) 2

2] Complete each of the following :

(1) The mode of the values 5 , 5 , 6 , 4 , 5 is

(2) The slope of the straight line which parallel to the x -axis =

(3) $[2 , 8[\cup \{8\} = \dots\dots\dots$

(4) $\sqrt[3]{\dots\dots\dots} = \sqrt{4}$

(5) A cube of side length 3 cm. , then its volume = cm^3 **3] [a] Find the solution set in \mathbb{R} to the following inequality in the form of an interval :**

$x - 2 > 3$

[b] If $x = \sqrt{3} + \sqrt{2}$ and $y = \sqrt{3} - \sqrt{2}$ Find the value of : $x \times y$ **4] [a] Without using calculator , simplify : $\sqrt{2} + \sqrt{8} - \sqrt{18}$** **[b] Find the slope of the straight line which passes through the two points (2 , 3) and (1 , 2)****5] [a] Write three ordered pairs satisfy the relation : $x + y = 5$** **[b] Find the arithmetic mean for the following frequency distribution :**

Sets	2 -	4 -	6 -	Total
Frequency	2	4	2	8

13 Kafr El-Sheikh Governorate

General Maths Supervision

**Answer the following questions :****1] Choose the correct answer :**

(1) The mean of the values : 21 , 19 , 27 , 3 , 5 is

(a) 90

(b) 32

(c) 18

(d) 15

Algebra and Statistics

(2) If $x = \sqrt{7} - \sqrt{5}$ and $y = \sqrt{7} + \sqrt{5}$, then $(xy)^3 = \dots\dots\dots$

- (a) 4 (b) 6 (c) 8 (d) 9

(3) $[1, 3] - \{1, 3\} = \dots\dots\dots$

- (a) $]1, 3[$ (b) $] - 1, - 3[$ (c) $[1, 3[$ (d) $] - 1, 3[$

(4) $\mathbb{R} = \dots\dots\dots$

- (a) $[0, \infty]$ (b) $] - \infty, \infty[$ (c) $[0, \infty[$ (d) $] - \infty, 0]$

(5) If A (2, 7) and B (5, - 2), then the slope of $\overline{AB} = \dots\dots\dots$

- (a) - 2 (b) 2 (c) - 3 (d) 3

2 Complete :

(1) The volume of a sphere whose diameter length is 6 cm. = $\dots\dots\dots \pi \text{ cm}^3$

(2) The S.S. for the equation $x^3 + 8 = 0$ in \mathbb{R} is $\dots\dots\dots$

(3) If $(k, 2k)$ satisfies $x + y = 15$, then $k = \dots\dots\dots$

(4) The slope of any line parallel to the x -axis = $\dots\dots\dots$

(5) If the area of one face of a cube = 9 cm^2 , then its volume = $\dots\dots\dots \text{ cm}^3$

3 [a] Simplify : $\sqrt{18} + \sqrt[3]{54} - 3\sqrt{2} - \sqrt[3]{16}$

[b] Find in \mathbb{R} the S.S. of the following inequality : $- 1 \leq 5x + 4 \leq 14$
 , then represent the S.S. on the number line.

4 [a] If $x = \sqrt{6} + \sqrt{5}$ and $y = \sqrt{6} - \sqrt{5}$ Find : $(x + y)^2$

[b] If $X =] - 3, 2]$ and $Y =] - 1, 5]$, then find :

- (1) $X \cap Y$ (2) $X \cup Y$

5 [a] Represent the relation $x + y = 3$ on the coordinate plane.

[b] Find the mean for the following frequency distribution :

Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	4	5	6	3	2	20

14

Beni Suef Governorate

Directorate Of Official Language School
Education administration

Answer the following questions :

1 Choose the correct answer :

(1) The irrational number lies between -2 and -1 is(a) -3 (b) $-1\frac{1}{2}$ (c) $-\sqrt{3}$ (d) $\sqrt{2}$ (2) $\sqrt[3]{x^6} = \sqrt{\dots}$ (a) x^3 (b) x^2 (c) x (d) x^4 (3) $|-5| - |5| = \dots$ (a) -10 (b) -5 (c) 0 (d) 10 (4) $(3, 2)$ does not satisfy the relation(a) $y + x = 5$ (b) $3y - x = 3$ (c) $y + x = 7$ (d) $x - y = 1$ (5) If the volume of a right circular cylinder is $90\pi \text{ cm}^3$ and its height is 10 cm , then the radius length of its base equals cm.(a) 3 (b) 4.5 (c) 5 (d) 9

2 Complete :

(1) If $(a, 3)$ satisfies the relation $2x - y = 7$, then $a = \dots$ (2) $\left(-\frac{5}{7}\right) \times \left(-\frac{7}{5}\right) = \dots$ (3) If the arithmetic mean of the values $9, 6, 5, 14, x$ is 7 , then $x = \dots$

(4) The point of intersection of the ascending and descending cumulative frequency curves determines on the set-axis.

(5) If the sum of five numbers equals 30 , then the arithmetic mean of these numbers is3 [a] Simplify to the simplest form : $\sqrt[3]{-16} + \frac{14}{\sqrt{7}} - \sqrt{28} + \sqrt[3]{54}$ [b] If $x = \frac{4}{3 + \sqrt{5}}$ and $y = 3 + \sqrt{5}$, Find the value of : $x^2 + y^2$ 4 [a] If $X = [-1, 4]$, $Y = [3, \infty[$ and $Z = \{3, 4\}$

, find each of the following using the number line :

(1) $X - Y$ (2) $Y \cap Z$ [b] Find the solution set of the inequality $3 - 2x \leq 7$ in \mathbb{R} in the form of an interval, then represent the solution on the number line.

Algebra and Statistics

5 [a] Let A (2 , - 1) , B (10 , 3) and C (2 , 3) , find the slope of each of : \overrightarrow{AB} and \overrightarrow{AC}

[b] The following table shows the frequency distribution of the weekly bonus of 100 workers in a factory :

Bonus in L.E.	20 -	30 -	40 -	50 -	m -	70 -
Number of workers	10	k	22	26	20	8

- ① Find the value of each of k and m
- ② Graph the frequency histogram , then find the mode value of the weekly bonus.

Assiut Governorate

Badr Language School



Answer the following questions :

1 Choose the correct answer from those given :

- ① If the volume of a cube is 27 cm^3 , then the area of one of its faces is
 (a) 3 cm^2 (b) 9 cm^2 (c) 36 cm^2 (d) 54 cm^2
- ② The S.S. of the equation : $x^2 + 3 = 0$ in \mathbb{R} is =
 (a) \emptyset (b) $\{-\sqrt{3}\}$ (c) $\{\sqrt{3}\}$ (d) $\{-\sqrt{3}, \sqrt{3}\}$
- ③ If $x = \sqrt{3} + 2$ and $y = \sqrt{3} - 2$, then $(x y , x + y) = \dots\dots\dots$
 (a) $(1 , 2\sqrt{3})$ (b) $(-1 , 2\sqrt{3})$ (c) $(5 , 2\sqrt{3})$ (d) $(5 , 9)$
- ④ If the median of the set of the values : $k + 1 , k + 2 , k + 5 , k + 4 , k + 3$ where k is a positive number is 13 , then $k = \dots\dots\dots$
 (a) 2 (b) 5 (c) 10 (d) 13
- ⑤ If the mode of the set of values : 4 , 11 , 8 , $2x$ is 4 , then $x = \dots\dots\dots$
 (a) 2 (b) 4 (c) 6 (d) 8

2 Complete :

- ① If $(-1 , 5)$ satisfies the relation $3x + ky = 7$, then $k = \dots\dots\dots$
- ② $[2 , 6] - \{2 , 6\} = \dots\dots\dots$
- ③ If the arithmetic mean of the values 9 , 6 , 5 , 14 , k is 7 , then $k = \dots\dots\dots$
- ④ The slope of the straight line passing through the two points (2 , 6) and $(-1 , 3)$ is
- ⑤ The multiplicative inverse of the number $\sqrt{3} - \sqrt{2}$ is (in the simplest form)

3 [a] If $x = \sqrt{5} + \sqrt{2}$ and $y = \sqrt{5} - \sqrt{2}$, find the value of : $\frac{x+y}{xy-1}$

[b] Find the S.S. of the inequality : $-5 \leq 2x - 3 < 5$ in \mathbb{R} , then represent it on the number line.

4 [a] Prove that : $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = 0$

[b] Represent graphically the relation : $y = 2 - x$

5 [a] If $X =]-\infty, 2[$ and $Y = [-1, 5]$ find as an intervals using the number line :

① $X \cup Y$

② $X \cap Y$

③ $X - Y$

[b] Find the arithmetic mean of the following frequency distribution :

Sets	5 -	15 -	25 -	35 -	45 -	Total
Frequency	7	10	12	13	8	50

Second

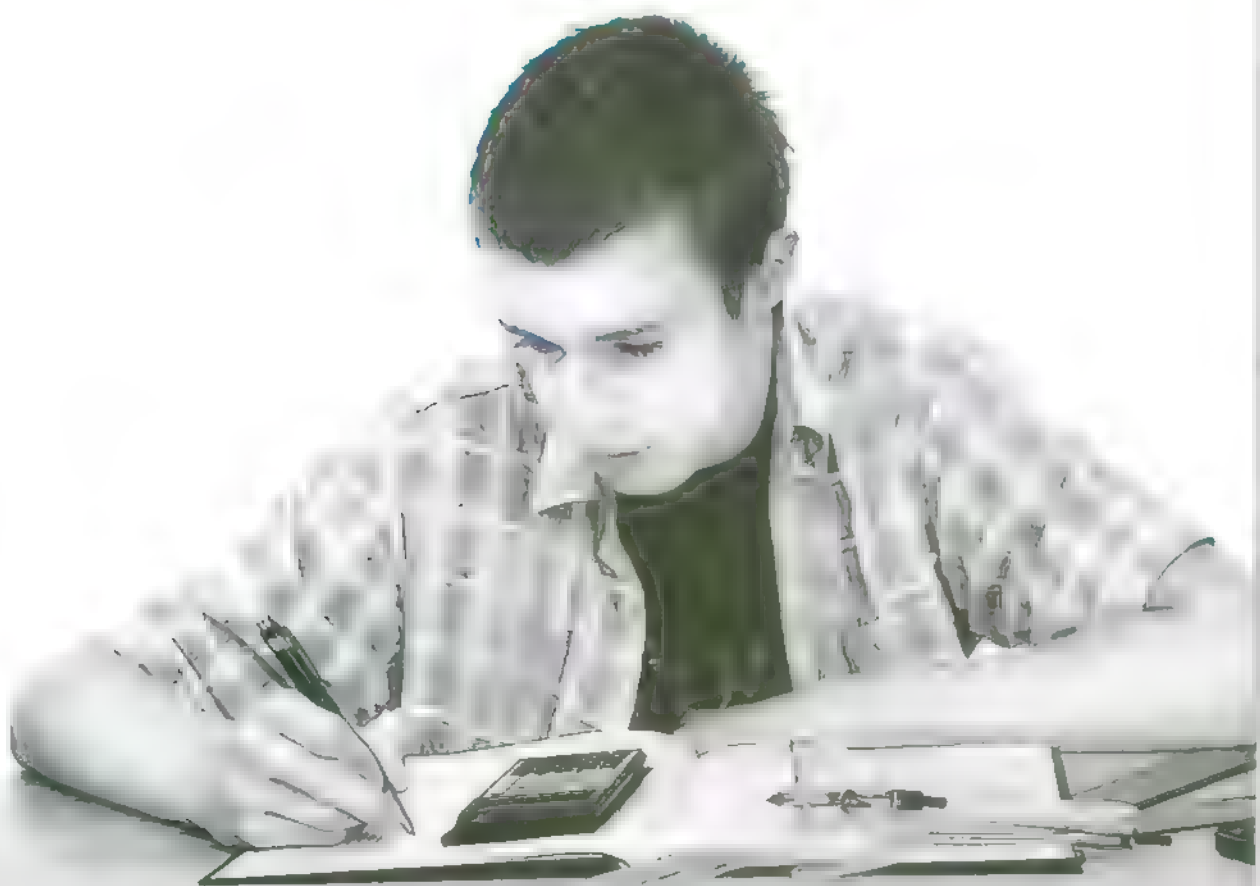
Geometry

- **9 Accumulative tests** **63**
- **Final revision** **81**
- **Final examinations :** **90**
 - School book examinations
(2 models examinations + model for the merge students)
 - 15 schools examinations.



Accumulative Tests

on Geometry •



1 Choose the correct answer from the given ones :

1 The number of medians of the triangle is

- (a) 1 (b) 2 (c) 3 (d) 4

2 The points of concurrence of the medians of the triangle divides each median in the ratio from the vertex.

- (a) 2 : 1 (b) 1 : 2 (c) 3 : 1 (d) 3 : 2

3 If M is the point of intersection of medians of $\triangle ABC$, \overline{AD} is a median, then $AD = \dots\dots\dots$

- (a) 2 AM (b) $\frac{2}{3}$ MD (c) $\frac{3}{2}$ AM (d) 4 MD

4 If \overline{AD} is a median in $\triangle ABC$, M is the point of intersection of the medians, $AM = 12$ cm., then $AD = \dots\dots\dots$ cm.

- (a) 8 (b) 4 (c) 18 (d) 9

5 The point of intersection of medians of the triangle divides each of them in the ratio 4 : from the base.

- (a) 2 (b) 8 (c) 1 (d) 4

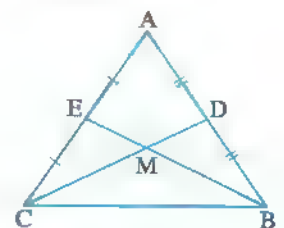
6 In $\triangle XYZ$, \overline{XD} is a median, M is the point of intersection of the medians, then $XM \dots\dots\dots MD$

- (a) > (b) < (c) = (d) \leq

7 In the opposite figure :

$BM = 6$ cm., then $ME = \dots\dots\dots$ cm.

- (a) 3 (b) 6
(c) 7 (d) 9



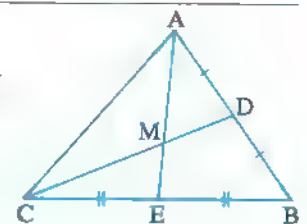
8 In $\triangle ABC$, \overline{AD} is a median, M is the point of intersection of its medians, then $(AM)^2 = \dots\dots\dots (AD)^2$

- (a) 2 (b) $\frac{3}{2}$ (c) $\frac{4}{9}$ (d) $\frac{1}{2}$

2 [a] In the opposite figure :

ABC is a triangle in which D, E are the midpoints of \overline{AB} , \overline{BC} respectively, $\overline{AE} \cap \overline{CD} = \{M\}$, if $AM = 4$ cm., $CD = 9$ cm.

Find : The length of each of \overline{AE} , \overline{MC}

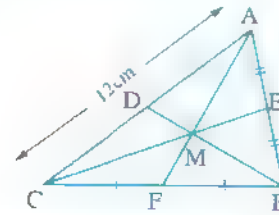


[b] In the opposite figure :

E is the midpoint of \overline{AB}

, F is the midpoint of \overline{BC} , $AC = 12$ cm.

Find with proof : The length of \overline{AD}

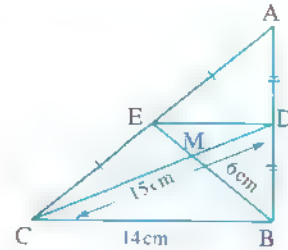


3 In the opposite figure :

M is the point of intersection of the medians of $\triangle ABC$

, $BM = 6$ cm. , $BC = 14$ cm. , $DC = 15$ cm.

Find : The perimeter of $\triangle MDE$



1 Choose the correct answer from the given ones :

- 1 The length of the side opposite to the angle of measure 30° in the right-angled triangle = the hypotenuse.

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{5}$

- 2 If ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , then the length of the median drawn from B equals cm.

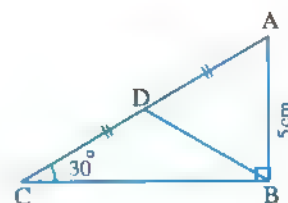
(a) 10 (b) 5 (c) 3 (d) 4

- 3 In the opposite figure :

ABC is a right-angled triangle at B

, D is the midpoint of \overline{AC} , $m(\angle ACB) = 30^\circ$

, AB = 5 cm. , then BD = cm.



(a) 5 (b) 10 (c) 2.5 (d) 15

- 4 The point of intersection of medians of the triangle divides each median in the ratio from the base.

(a) 2 : 1 (b) 3 : 6 (c) 3 : 2 (d) 1 : 3

- 5 If \overline{BD} is a median in $\triangle ABC$, $BD = \frac{1}{2} AC$, then

(a) $m(\angle ABC) = 90^\circ$ (b) $m(\angle BAC) = 90^\circ$
(c) $m(\angle ABC) = 30^\circ$ (d) $m(\angle ACB) = 90^\circ$

- 6 If M is the point of intersection of the medians of $\triangle ABC$, D is the midpoint of \overline{BC} , then MD : AD =

(a) 1 : 2 (b) 2 : 3 (c) 1 : 3 (d) 3 : 2

- 7 If M is the point of intersection of the medians of $\triangle ABC$, \overline{AD} is a median of length 9 cm. , then AM = cm.

(a) 6 (b) 3 (c) 4 (d) 2

- 8 A rectangle , its diagonals intersect at M , the length of its diagonal is 6 cm. , then the length of the median \overline{AM} is

(a) 1 cm. (b) 2 cm. (c) 3 cm. (d) 4 cm.

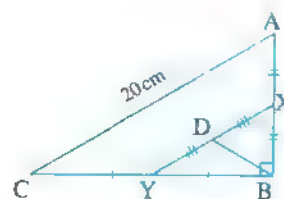
2 [a] In the opposite figure :

$m(\angle ABC) = 90^\circ$, X is the midpoint of \overline{AB}

, Y is the midpoint of \overline{BC}

, D is the midpoint of \overline{XY} , $AC = 20$ cm.

Find : The length of \overline{BD}

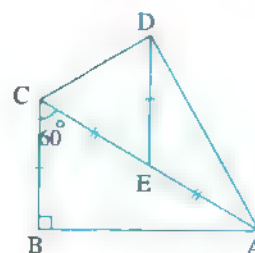

[b] In the opposite figure :

ABC is a right-angled triangle at B

, $m(\angle ACB) = 60^\circ$

, E is the midpoint of \overline{AC} , $DE = BC$

Prove that : $m(\angle ADC) = 90^\circ$


3 In the opposite figure :

ABC is a right-angled triangle at B

, $m(\angle C) = 30^\circ$, D is the midpoint of \overline{BC}

, E is the midpoint \overline{AC} , $\overline{AD} \cap \overline{BE} = \{M\}$

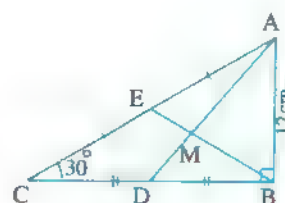
, if $AB = 12$ cm. , $AD = 15$ cm.

Find with proof :

1 The length of \overline{AE}

2 The length of \overline{ME}

3 The perimeter of $\triangle AME$



1 Choose the correct answer from the given ones :

- 1 The measure of the exterior angle of the equilateral triangle equals
 (a) 60° (b) 90° (c) 120° (d) 180°

- 2 The two base angles of the isosceles triangle are
 (a) complementary. (b) congruent.
 (c) supplementary. (d) different.

3 In the opposite figure :

ABC is an equilateral triangle
 $\overline{DE} \parallel \overline{CA}$, then $m(\angle D) = \dots\dots\dots$

- (a) 100° (b) 60°
 (c) 120° (d) 150°



- 4 The point of intersection of the medians of the triangle divides each median in the ratio from the base.

- (a) 1 : 2 (b) 2 : 1 (c) 3 : 1 (d) 1 : 3

- 5 ABC is a right-angled triangle at B, $AC = 20$ cm. , D is the midpoint of \overline{AC}
 , then $BD = \dots\dots\dots$ cm.

- (a) 10 (b) 8 (c) 6 (d) 5

- 6 XYZ is an isosceles triangle in which $m(\angle Y) = 100^\circ$, then $m(\angle Z) = \dots\dots\dots$

- (a) 100° (b) 80° (c) 60° (d) 40°

- 7 If $\triangle XYZ$ is right-angled at Y , $m(\angle X) = 60^\circ$, $XZ = 10$ cm.
 , then $XY = \dots\dots\dots$ cm.

- (a) 10 (b) 6 (c) 8 (d) 5

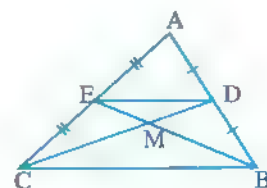
- 8 In $\triangle ABC$, if $AB = AC$, $m(\angle A) = 2 m(\angle B)$, then $m(\angle C) = \dots\dots\dots$

- (a) 30° (b) 45° (c) 60° (d) 90°

2 [a] In the opposite figure :

\overline{BE} , \overline{CD} are two medians in $\triangle ABC$ intersect at point M
 , the perimeter of $\triangle MDE = 12$ cm.

Find : The perimeter of $\triangle MBC$



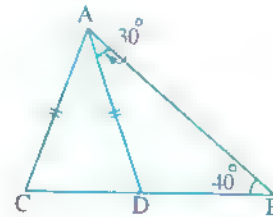
[b] In the opposite figure :

$$AD = AC, B \in \overline{CD}$$

$$, m(\angle B) = 40^\circ$$

$$, m(\angle BAD) = 30^\circ$$

Prove that : $AB = CB$



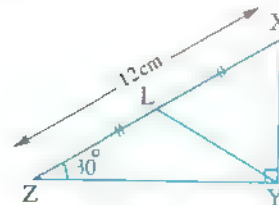
3 [a] In the opposite figure :

XYZ is a right-angled triangle at Y

, L is the midpoint of \overline{XZ} , $m(\angle Z) = 30^\circ$

$$, XZ = 12 \text{ cm.}$$

Find : The perimeter of $\triangle XLY$

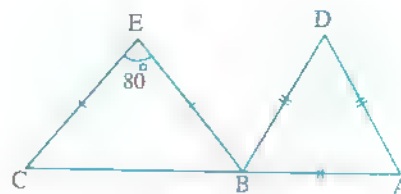


[b] In the opposite figure :

$B \in \overline{AC}$, $\triangle ABD$ is equilateral

$$, EB = EC, m(\angle E) = 80^\circ$$

Find : $m(\angle DBE)$



1 Choose the correct answer from the given ones :

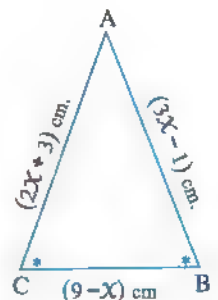
- 1 If the measures of two angles in a triangle are 42° , 69° , then the triangle is ...
 - (a) isosceles.
 - (b) scalene.
 - (c) equilateral.
 - (d) otherwise.
- 2 If the length of the median drawn from the vertex of the right angle in the right-angled triangle equals the hypotenuse.
 - (a) half
 - (b) double
 - (c) quarter
 - (d) third
- 3 A right-angled triangle , the measure of one of its angles is 45° , then it is
 - (a) isosceles triangle.
 - (b) scalene triangle.
 - (c) equilateral triangle.
 - (d) otherwise.

4 In the opposite figure :

ABC is a triangle in which

$m(\angle B) = m(\angle C)$, then $x =$

- (a) $\frac{2}{5}$
- (b) $\frac{4}{5}$
- (c) 2
- (d) 4



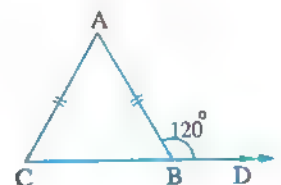
5 If ABC is a triangle , $AB = BC$, then $\angle C$ is

- (a) acute.
- (b) right.
- (c) obtuse.
- (d) straight.

2 [a] In the opposite figure :

$AB = AC$, $m(\angle ABD) = 120^\circ$

Prove that : $\triangle ABC$ is equilateral.



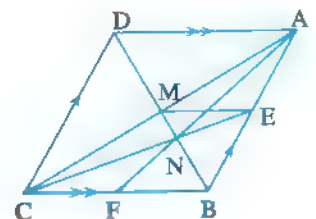
[b] In the opposite figure :

ABCD is a parallelogram its diagonals

intersect at M , if $N \in \overline{BM}$

where $BN = 2 NM$, $\overrightarrow{CN} \cap \overrightarrow{AB} = \{E\}$

Prove that : $EM = \frac{1}{2} BC$

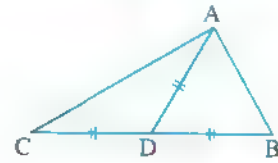


3 [a] In the opposite figure :

ABC is a triangle

, $AD = BD = CD$

Find : $m(\angle BAC)$

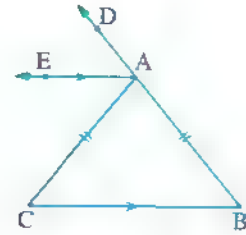


[b] In the opposite figure :

$D \in \overrightarrow{BA}$, $AB = AC$

, $\overrightarrow{AE} \parallel \overrightarrow{BC}$

Prove that : \overrightarrow{AE} bisects $\angle DAC$

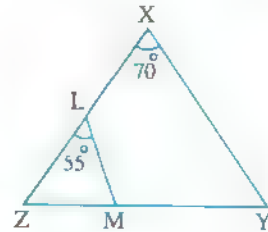


4 In the opposite figure :

$XZ = XY$, $m(\angle MLZ) = 55^\circ$

, $m(\angle X) = 70^\circ$

Prove that : $ML = MZ$



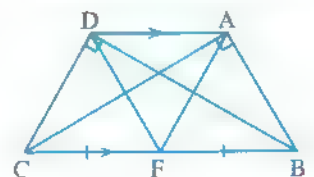
1 Choose the correct answer from the given ones :

- 1 The median of the isosceles triangle from the vertex angle bisects it and to the base.
 (a) axis of symmetry (b) parallel (c) congruent (d) perpendicular
- 2 An isosceles triangle , the measure of one of its angles is 60° , then the number of its axes of symmetry is
 (a) 4 (b) 3 (c) 2 (d) 1
- 3 The length of the hypotenuse in the right-angled triangle equals ... the length of the side opposite to the angle of the measure 30°
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 2 (d) 3
- 4 The number of medians of the isosceles triangle is
 (a) zero (b) 1 (c) 2 (d) 3
- 5 If ABC is a triangle , $AB = AC$, $m(\angle B) = 50^\circ$, then $m(\angle A) =$
 (a) 80° (b) 110° (c) 40° (d) 50°
- 6 The triangle which has no axes of symmetry is
 (a) the isosceles triangle. (b) the scalene triangle.
 (c) the equilateral triangle. (d) the right-angled triangle.
- 7 If \overleftrightarrow{AB} is the axis of symmetry of \overline{FD} , then $\frac{AD}{AF} =$
 (a) zero (b) 1 (c) $\frac{1}{2}$ (d) 2
- 8 ABC is an equilateral triangle , X is the point of intersection of its axes of symmetry , \overleftrightarrow{AX} cuts \overline{BC} at D , if $DX = 5$ cm. , then $AX =$
 (a) 10 cm. (b) 15 cm. (c) 2.5 cm. (d) 7.5 cm.

2 [a] In the opposite figure :

\overline{AF} , \overline{DF} are two medians

Prove that : $AF = FD$



Accumulative tests

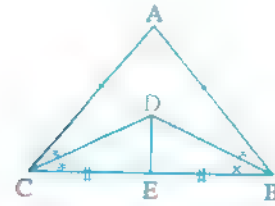
[b] In the opposite figure :

$AB = AC$, \overrightarrow{BD} bisects $\angle ABC$

, \overrightarrow{CD} bisects $\angle ACB$

, E is the midpoint of \overline{BC}

Prove that : $\overline{DE} \perp \overline{BC}$



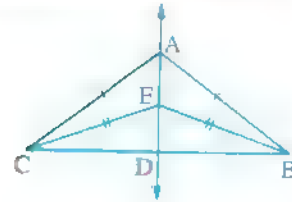
3 [a] In the opposite figure :

ABC is a triangle in which $AB = AC = 10$ cm.

, $BE = EC$, $BC = 16$ cm.

, $\overline{AE} \cap \overline{BC} = \{D\}$

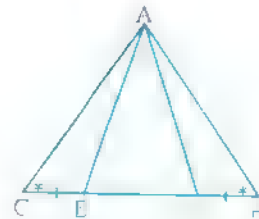
Find : The length of \overline{AD}



[b] In the opposite figure :

$BD = EC$, $m(\angle B) = m(\angle C)$

Prove that : $m(\angle ADE) = m(\angle AED)$



1 Choose the correct answer from the given ones :

1 If $x - z > y - z$, then x y

- (a) = (b) > (c) < (d) \leq

2 If $C \in$ the axis of symmetry of \overline{AB} , then $AC - BC =$

- (a) zero (b) 1 (c) 3 (d) 2

3 If the measure of the vertex angle of an isosceles triangle is 80° , then the measure of one of its base angles is

- (a) 45° (b) 40° (c) 50° (d) 100°

4 If $\triangle ABC$ is right-angled at B , $AB = \frac{1}{2} AC$, then $m(\angle A) =$

- (a) 45° (b) 30° (c) 90° (d) 60°

5 In the opposite figure :

$C \in \overleftrightarrow{AB}$, $D \in \overleftrightarrow{AB}$

, $m(\angle ACE) < m(\angle BDF)$

, then $m(\angle ECD)$ $m(\angle FDC)$

- (a) > (b) <
(c) = (d) \leq

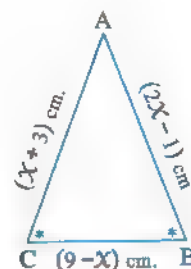


2 [a] In the opposite figure :

ABC is a triangle ,

$m(\angle B) = m(\angle C)$

Find : The perimeter of $\triangle ABC$

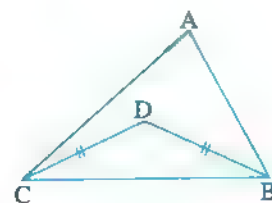


[b] In the opposite figure :

$m(\angle ABC) > m(\angle ACB)$

, $BD = CD$

Prove that : $m(\angle ABD) > m(\angle ACD)$

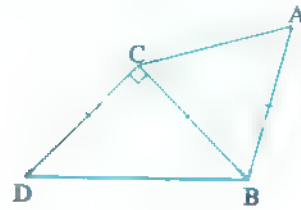


3 [a] In the opposite figure :

$$AB = BC = AC = CD$$

$$, m(\angle BCD) = 90^\circ$$

Find : $m(\angle ABD)$



[b] In the opposite figure :

ABC is a triangle, D, E are the midpoint of \overline{AB} , \overline{AC} respectively

, $\overline{CD} \cap \overline{BE} = \{M\}$, if $CD = 15$ cm.

, $EM = 4$ cm. , $DE = 6$ cm.

Find : The perimeter of $\triangle MBC$



4 [a] In the opposite figure :

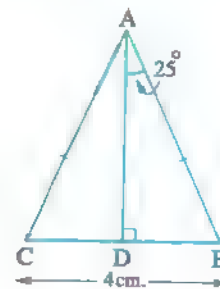
ABC is a triangle, $AB = AC$, $\overline{AD} \perp \overline{BC}$

, $m(\angle BAD) = 25^\circ$, $BC = 4$ cm.

Find :

1 $m(\angle DAC)$

2 The length of \overline{DC}

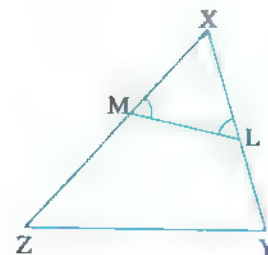


[b] In the opposite figure :

$$XZ > XY$$

$$, m(\angle XLM) = m(\angle XML)$$

Prove that : $ZM > YL$



1 Choose the correct answer from the given ones :

1 In $\triangle ABC$, if $AB > AC$, then $m(\angle C)$ $m(\angle B)$

(a) $<$

(b) $>$

(c) $=$

(d) \leq

2 In $\triangle XYZ$, \overline{XY} is the shortest side, then the angle of the smallest measure is

(a) X

(b) Y

(c) Z

(d) otherwise.

3 If M is the point of intersection of medians of $\triangle ABC$, \overline{AD} is a median, then $AD : MD =$

(a) 2 : 3

(b) 3 : 2

(c) 3 : 1

(d) 1 : 3

4 A triangle has 3 axes of symmetry, then the measure of the exterior angle at one of its vertices equals

(a) 90°

(b) 80°

(c) 120°

(d) 60°

5 In $\triangle ABC$, $AB = 7$ cm., $BC = 5$ cm., $AC = 6$ cm., then $m(\angle B)$ $m(\angle C)$

(a) $>$

(b) $<$

(c) $=$

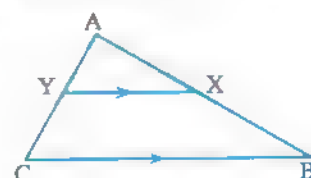
(d) \equiv

2 [a] In the opposite figure :

ABC is a triangle in which $AB > AC$

, $\overline{XY} \parallel \overline{BC}$

Prove that : $m(\angle AYX) > m(\angle AXY)$



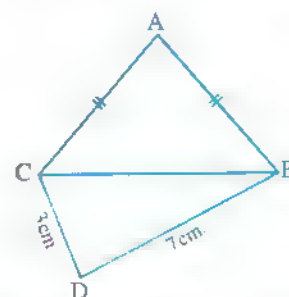
[b] In the opposite figure :

$AB = AC$

, $BD = 7$ cm.

, $DC = 3$ cm.

Prove that : $m(\angle ACD) > m(\angle ABD)$



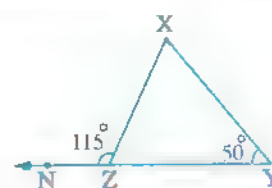
3 [a] In the opposite figure :

XYZ is a triangle in which

$m(\angle Y) = 50^\circ$, $N \in \overrightarrow{YZ}$

, $m(\angle XZN) = 115^\circ$

Prove that : $\triangle XYZ$ is isosceles

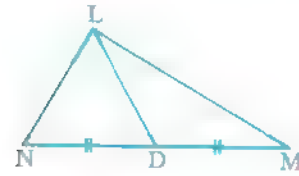


[b] In the opposite figure :

If the perimeter of $\triangle LMD >$ the perimeter of $\triangle LDN$

, D is the midpoint of \overline{MN}

Prove that : $m(\angle N) > m(\angle M)$

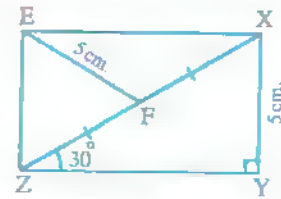


4 [a] In the opposite figure :

$m(\angle Y) = 90^\circ$, $m(\angle XZY) = 30^\circ$

, $XY = EF = 5$ cm. , F is the midpoint of \overline{XZ}

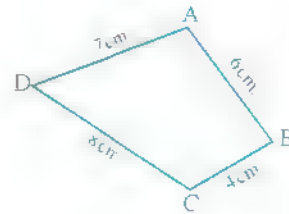
Prove that : $m(\angle XEZ) = 90^\circ$



[b] In the opposite figure :

From the data on the figure.

Prove that : $m(\angle ABC) > m(\angle ADC)$



1 Choose the correct answer from the given ones :

1 In $\triangle ABC$, if $m(\angle B) > m(\angle C)$, then

- (a) $AB > AC$ (b) $BC > AC$ (c) $AC > AB$ (d) $AB > BC$

2 In $\triangle XYZ$, $m(\angle Y) = 130^\circ$, then the longest side is

- (a) \overline{XZ} (b) \overline{XY} (c) \overline{YZ} (d) its median.

3 In $\triangle XYZ$, $m(\angle Z) = 70^\circ$, $m(\angle Y) = 60^\circ$, then YZ XZ

- (a) $<$ (b) $>$ (c) $=$ (d) twice

4 In $\triangle ABC$, if $AB > AC$, then $m(\angle B)$ $m(\angle C)$

- (a) $>$ (b) $<$ (c) $=$ (d) \geq

5 If the measures of two angles in a triangle are 48° , 84° , then its type is

- (a) isosceles. (b) equilateral. (c) scalene. (d) right-angled.

6 If A lies on the axis of symmetry of \overline{BC} , then \overline{AB} \overline{AC}

- (a) $=$ (b) $//$ (c) \perp (d) \equiv

7 If $\triangle ABC$ is an obtuse-angled triangle at C, then BC AB

- (a) $>$ (b) $<$ (c) $=$ (d) \geq

8 The longest side in $\triangle XYZ$ where $m(\angle Y) = m(\angle X) + m(\angle Z)$ is

- (a) \overline{XY} (b) \overline{XZ} (c) \overline{YZ} (d) otherwise.

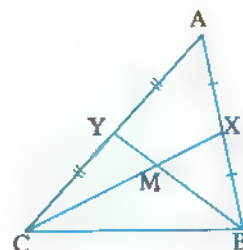
2 [a] In $\triangle ABC$, $m(\angle A) = (5x + 2)^\circ$, $m(\angle B) = (6x - 10)^\circ$, $m(\angle C) = (x + 20)^\circ$

Order the lengths of the sides of the triangle ascendingly

[b] In the opposite figure :

X, Y are the midpoints
of \overline{AB} , \overline{AC} respectively
, $XM > YM$

Prove that : $m(\angle MBC) > m(\angle MCB)$

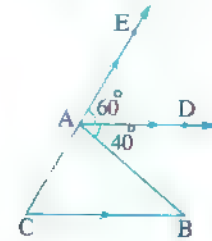


3 [a] In the opposite figure :

$\overrightarrow{AD} \parallel \overrightarrow{BC}$, $m(\angle EAD) = 60^\circ$

, $m(\angle BAD) = 40^\circ$

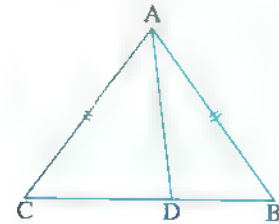
Prove that : $AB > AC$



[b] In the opposite figure :

$AB = AC$, $D \in \overline{BC}$

Prove that : $AB > AD$



1 Choose the correct answer from the given ones :

- 1 The sum of the lengths of any two sides in a triangle is the length of the third side.
 (a) greater than (b) equal to (c) smaller than (d) twice
- 2 Which of the following numbers can be the lengths of sides of a triangle ?
 (a) 5 , 3 , 2 (b) 6 , 3 , 2 (c) 6 , 3 , 3 (d) 3 , 3 , 3
- 3 If z , 12 , $2z$ are the lengths of sides of a triangle , then the greatest value of $z =$
 (a) 12 (b) 11 (c) 4 (d) 3
- 4 The measure of the exterior angle of the equilateral triangle equals
 (a) 60° (b) 80° (c) 120° (d) 180°
- 5 If ABC is a right-angled triangle at B , then
 (a) $AC < AB$ (b) $AC < BC$ (c) $AB < AC$ (d) $BC > AB$
- 6 ABC is a right-angled triangle at B , if $AC = 20$ cm. , then the length of the median drawn from B equals cm.
 (a) 10 (b) 8 (c) 6 (d) 5
- 7 A triangle has one axis of symmetry and the lengths of two sides in it are 3 cm. , 8 cm. , then its perimeter = cm.
 (a) 14 (b) 19 (c) 11 (d) 24
- 8 In $\triangle XYZ$, $XY = 8$ cm. , $YZ = 5$ cm. , then its perimeter \in
 (a) $]3 , 13[$ (b) $[16 , 26]$ (c) $]16 , 26[$ (d) $\{16 , 26\}$

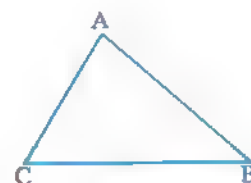
2 [a] In $\triangle ABC$, $m(\angle A) = 50^\circ$, $m(\angle B) = 70^\circ$

Order the lengths of the sides of the triangle descendingly.

[b] In the opposite figure :

ABC is a triangle.

Prove that : $AB < \frac{1}{2}$ the perimeter of $\triangle ABC$

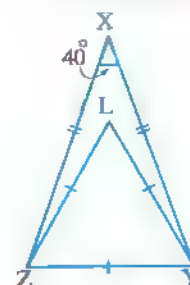


3 In the opposite figure :

LYZ is an equilateral triangle

, $XY = XZ$, $m(\angle X) = 40^\circ$

Find : $m(\angle XZL)$



Final Revision

of Geometry



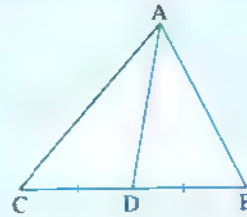
Revision for the important theorems , corollaries and rules

of

Geometry

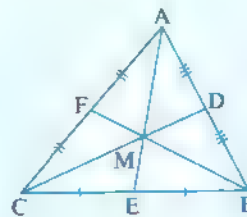
Medians of triangle

The median of the triangle is the line segment drawn from any vertex of the triangle to the midpoint of the opposite side of this vertex.



If D is the midpoint of \overline{BC} , then \overline{AD} is a median in $\triangle ABC$

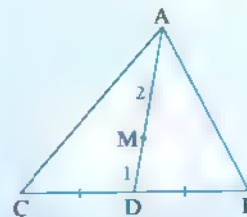
The medians of a triangle are concurrent.



If \overline{CD} , \overline{BF} and \overline{AE} are the medians of $\triangle ABC$ where $\overline{CD} \cap \overline{BF} \cap \overline{AE} = \{M\}$, then M is the intersection point of the medians of $\triangle ABC$

The point of concurrence of the medians of the triangle divides each median in the ratio of :

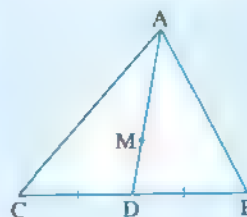
- 1 : 2 from the base.
- 2 : 1 from the vertex.



If M is the intersection point of the medians of $\triangle ABC$, then :

- $DM = \frac{1}{2} AM$
- $AM = 2 DM$
- $DM = \frac{1}{3} AD$
- $AM = \frac{2}{3} AD$

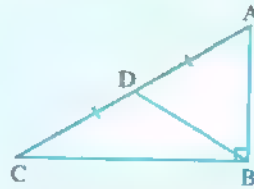
The point which divides the median of a triangle by the ratio 1 : 2 from the base is the point of the intersection of the medians of the triangle.



If $DM : MA = 1 : 2$, then M is the intersection point of the medians of $\triangle ABC$

Right-angled triangle

The length of the median from the vertex of the right angle equals half the length of the hypotenuse.



If $\triangle ABC$ is right-angled at B, \overline{BD} is a median in it, then

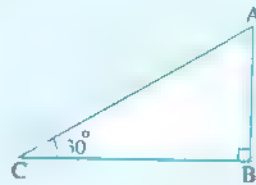
$$BD = \frac{1}{2} AC$$

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right.



If \overline{BD} is a median of $\triangle ABC$, $BD = \frac{1}{2} AC$, then $m(\angle ABC) = 90^\circ$

The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.



If $\triangle ABC$ is right-angled at B in which :

$$m(\angle C) = 30^\circ$$

$$\text{, then } AB = \frac{1}{2} AC$$

In the right-angled triangle, the hypotenuse is the longest side of the triangle.

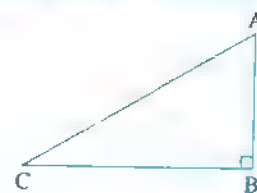


If $\triangle ABC$ is right-angled at B, then

$$AC > AB, AC > BC$$

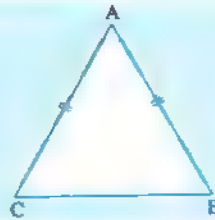
If $\triangle ABC$ is right-angled at B, then :

- $(AC)^2 = (AB)^2 + (BC)^2$
- $(AB)^2 = (AC)^2 - (BC)^2$
- $(BC)^2 = (AC)^2 - (AB)^2$



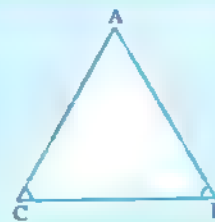
The isosceles triangle

The base angles of the isosceles triangle are congruent.



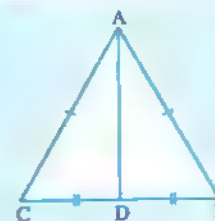
If $\triangle ABC$ in which :
 $AB = AC$, then
 $m(\angle B) = m(\angle C)$

If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.



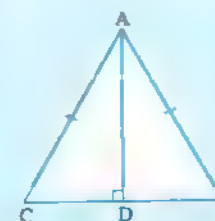
If $\triangle ABC$ in which :
 $m(\angle B) = m(\angle C)$
 , then $AB = AC$

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.



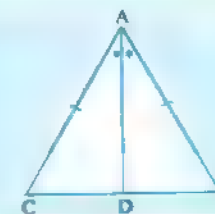
If $\triangle ABC$ in which :
 $AB = AC$, \overline{AD} is a median
 , then \overline{AD} bisects $\angle BAC$
 $\overline{AD} \perp \overline{BC}$

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.



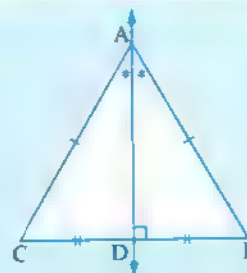
If $\triangle ABC$ in which :
 $AB = AC$, $\overline{AD} \perp \overline{BC}$
 , then D is the midpoint of \overline{BC} ,
 \overline{AD} bisects $\angle BAC$

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.



If $\triangle ABC$ in which :
 $AB = AC$, \overline{AD} bisects $\angle BAC$, then D is the midpoint of \overline{BC} , $\overline{AD} \perp \overline{BC}$

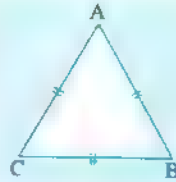
The number of axes of symmetry of the isosceles triangle equals 1



If $\triangle ABC$ in which :
 $AB = AC$, $\overline{AD} \perp \overline{BC}$ and intersect it at D
 , then \overleftrightarrow{AD} is the axis of symmetry of the triangle ABC

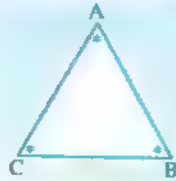
The equilateral triangle

If the triangle is an equilateral, then it is equiangular where each angle measure is 60°



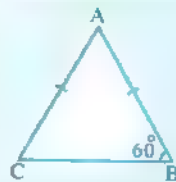
If $\triangle ABC$ in which :
 $AB = BC = CA$, then
 $m(\angle A) = m(\angle B) = m(\angle C) = 60^\circ$

If the angles of a triangle are congruent, then the triangle is equilateral.



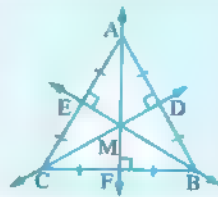
If $\triangle ABC$ in which :
 $m(\angle A) = m(\angle B) = m(\angle C)$,
 then $AB = BC = CA$

The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle.



If $\triangle ABC$ in which :
 $AB = AC$, $m(\angle B) = 60^\circ$,
 then $\triangle ABC$ is an equilateral triangle.

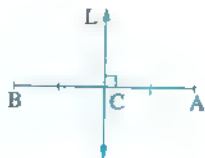
The equilateral triangle has three axes of symmetry.



If $\triangle ABC$ is an equilateral triangle,
 $\overline{AF} \perp \overline{BC}$, $\overline{CD} \perp \overline{AB}$, $\overline{BE} \perp \overline{AC}$,
 then \overline{AF} , \overline{CD} and \overline{BE} are the axes of symmetry of the triangle ABC

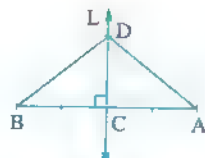
The axis of symmetry

The axis of symmetry of a line segment is the straight line perpendicular to it from its middle.



If the straight line $L \perp \overline{AB}$,
 $C \in \overline{AB}$ where $CA = CB$,
 $C \in$ the straight line L ,
 then L is the axis of \overline{AB}

Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).



If the straight line L is the axis of \overline{AB} , $D \in$ the straight line L , then $DA = DB$

If a point is at equal distances from the two terminals of a line segment, then this point lies on the axis of this line segment.

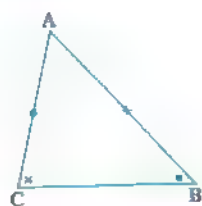


If $CA = CB$, then
 C lies on the axis of \overline{AB}

Inequality relations in the triangle

Comparing the measures of angles in a triangle

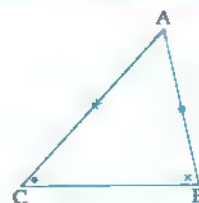
If two sides have unequal lengths, the longer is opposite to the angle of the greater measure



If $AB > AC$, then $m(\angle C) > m(\angle B)$

Comparing the lengths of sides in a triangle

If two angles are unequal in measure, then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.

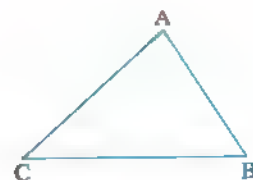


If $m(\angle B) > m(\angle C)$, then $AC > AB$

Triangle inequality

In any triangle, the sum of the lengths of any two sides is greater than the length of the third side.

- $AB + BC > AC$
- , $BC + CA > AB$
- , $CA + AB > BC$



Notice that :

- The length of any side of a triangle is greater than the difference between the lengths of the two other sides and less than their sum.

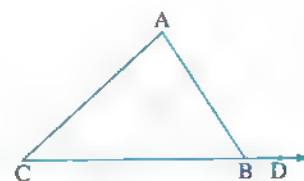
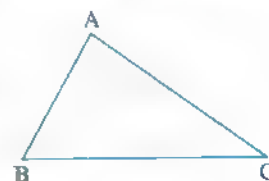
In $\triangle ABC$:

$$AC - AB < BC < AC + AB$$

- The measure of any exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

In $\triangle ABC$:

- $m(\angle ABD) > m(\angle A)$
- , $m(\angle ABD) > m(\angle C)$



Proofs of the important theorems

Theorem

In the right-angled triangle, the length of the median from the vertex of the right angle equals half the length of the hypotenuse.

Given

ABC is a triangle in which $m(\angle ABC) = 90^\circ$,

\overline{BD} is a median in the triangle ABC

R.T.P.

$$BD = \frac{1}{2} AC$$

Construction

Draw \overline{BD} and take the point $E \in \overline{BD}$ such that $BD = DE$

Proof

In the figure $ABCE$: $\because \overline{AC}$ and \overline{BE} bisect each other

\therefore The figure $ABCE$ is a parallelogram.

$$\because m(\angle ABC) = 90^\circ$$

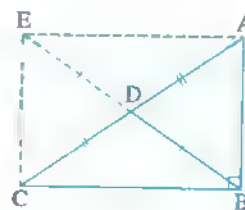
\therefore The figure $ABCE$ is a rectangle.

$$\therefore BE = AC$$

$$\therefore BD = \frac{1}{2} BE$$

$$\therefore BD = \frac{1}{2} AC$$

(Q.E.D.)



Theorem

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right.

Given

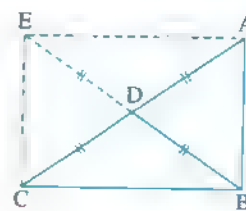
In $\triangle ABC$, \overline{BD} is a median and $DA = DB = DC$

R.T.P.

$$m(\angle ABC) = 90^\circ$$

Construction

Draw \overline{BD} , then take the point $E \in \overline{BD}$ such that $BD = DE$



Proof

$$\therefore BD = \frac{1}{2} BE = \frac{1}{2} AC$$

$$\therefore BE = AC$$

\therefore In the figure $ABCE$:

\overline{AC} and \overline{BE} are equal in length and bisect each other.

\therefore The figure $ABCE$ is a rectangle.

$$\therefore m(\angle ABC) = 90^\circ$$

(Q.E.D.)

Theorem

The base angles of the isosceles triangle are congruent.

Given

ABC is a triangle in which $\overline{AB} \equiv \overline{AC}$

R.T.P.

$\angle B \equiv \angle C$

Construction

Draw $\overline{AD} \perp \overline{BC}$ where $\overline{AD} \cap \overline{BC} = \{D\}$

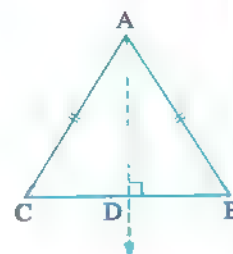
Proof

$\therefore \Delta ADB, ADC$ in which :

$$\begin{cases} m(\angle ADB) = m(\angle ADC) = 90^\circ & (\text{const.}) \\ \overline{AB} \equiv \overline{AC} & (\text{given}) \\ \overline{AD} \text{ is a common side} \end{cases}$$

$\therefore \Delta ADB \equiv \Delta ADC$, then we deduce that $\angle B \equiv \angle C$

(Q.E.D.)



Theorem

If two angles of a triangle are congruent, then the two sides opposite to these two angles are congruent and the triangle is isosceles.

Given

ABC is a triangle in which $\angle B \equiv \angle C$

R.T.P.

$\overline{AB} \equiv \overline{AC}$

Construction

bisect $\angle BAC$ by \overline{AD} to intersect \overline{BC} at D

Proof

$\therefore \angle B \equiv \angle C$

$\therefore m(\angle B) = m(\angle C)$

$\therefore \overline{AD}$ bisects $\angle BAC$

$\therefore m(\angle BAD) = m(\angle CAD)$

\therefore The sum of measures of the interior angles of the triangle $= 180^\circ$

$\therefore m(\angle ADB) = m(\angle ADC)$

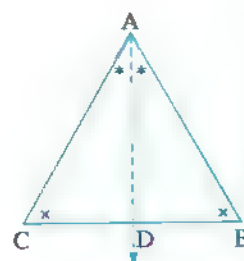
\therefore In ΔABD and ΔACD :

$$\begin{cases} \overline{AD} \text{ is a common side} \\ m(\angle BAD) = m(\angle CAD) & (\text{const.}) \\ m(\angle ADB) = m(\angle ADC) & (\text{by proof}) \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACD$, then we deduce that

$\overline{AB} \equiv \overline{AC}$, then ΔABC is an isosceles triangle.

(Q.E.D.)



Theorem

In a triangle, if two sides have unequal lengths, the longer is opposite to the angle of the greater measure.

Given

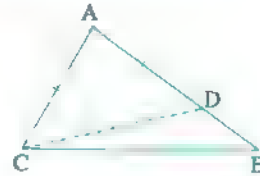
ABC is a triangle in which $AB > AC$

R.T.P.

$m(\angle ACB) > m(\angle ABC)$

Construction

Take $D \in \overline{AB}$ such that $AD = AC$

**Proof**

In $\triangle ACD : \because AD = AC \therefore m(\angle ADC) = m(\angle ACD)$ (1)

$\therefore \angle ADC$ is an exterior angle of $\triangle DBC$

$\therefore m(\angle ADC) > m(\angle B)$ (2)

From (1) and (2) : $\therefore m(\angle ACD) > m(\angle B)$

$\therefore m(\angle ACB) > m(\angle ACD)$

$\therefore m(\angle ACB) > m(\angle ABC)$ (Q.E.D.)

Theorem

In a triangle, if two angles are unequal in measure, then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.

Given

ABC is a triangle in which $m(\angle C) > m(\angle B)$

R.T.P.

$AB > AC$

Proof

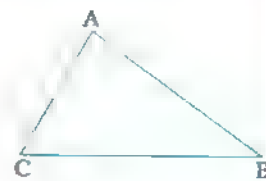
$\therefore \overline{AB}$ and \overline{AC} are two line segments.

\therefore One of the following cases should be verified.

① $AB > AC$

② $AB = AC$

③ $AB < AC$



Unless $AB > AC$, then either $AB = AC$ or $AB < AC$

• If : $AB = AC$, then $m(\angle C) = m(\angle B)$ and this contradicts the given where $m(\angle C) > m(\angle B)$

• If : $AB < AC$, then $m(\angle C) < m(\angle B)$ according to the preceding theorem.

Again this contradicts the given, where $m(\angle C) > m(\angle B)$

\therefore It should be that $AB > AC$

(Q.E.D.)

Final Examinations

on Geometry



Model 1

Answer the following questions :

1 Complete the following :

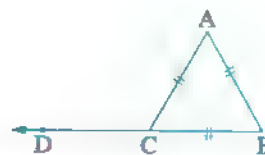
- 1] The longest side in the right-angled triangle is
- 2] If the lengths of two sides in a triangle are 2 cm. and 7 cm. , then
..... < the length of the third side <
- 3] If the measures of two angles in a triangle are different , then the greater in measure of them is opposite to
- 4] If the length of the median drawn from a vertex of a triangle equals half the opposite side to this vertex in length , then
- 5] If the measure of an angle in the isosceles triangle equals 60° , then the triangle is

2 Choose the correct answer from those given :

1] In the opposite figure :

$\triangle ABC$ is equilateral , then $m(\angle ACD) = \dots\dots\dots$

- (a) 45° (b) 60°
(c) 120° (d) 135°



- 2] In $\triangle ABC$ which is right-angled at B , if $AC = 20$ cm. , then the length of the median of the triangle drawn from B equals

- (a) 10 cm. (b) 8 cm. (c) 6 cm. (d) 5 cm.

- 3] XYZ is a triangle in which : $m(\angle Z) = 70^\circ$ and $m(\angle Y) = 60^\circ$, then $YZ \dots\dots\dots XY$

- (a) > (b) < (c) = (d) twice

- 4] The lengths which can be lengths of sides of a triangle are

- (a) 0 , 3 , 5 (b) 3 , 3 , 5 (c) 3 , 3 , 6 (d) 3 , 3 , 7

- 5] The triangle in which the measures of two angles of it are 42° and 69° is

- (a) an isosceles triangle. (b) an equilateral triangle.
(c) a scalene triangle. (d) a right-angled triangle.

6] In the opposite figure :

$m(\angle C) = 2 m(\angle A)$

, $BC = 6$ cm.

, then $AC = \dots\dots\dots$ cm.

- (a) 3 (b) 6
(c) 9 (d) 12



3 [a] Complete : ABC is a triangle in which $AB > AC$, then $m(\angle C) \dots\dots\dots m(\angle B)$

[b] In the opposite figure :

$m(\angle A) = 50^\circ$, $AB = AC$

and $\triangle DBC$ is equilateral

Find : $m(\angle ABD)$

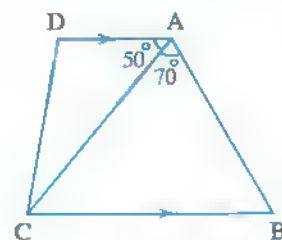
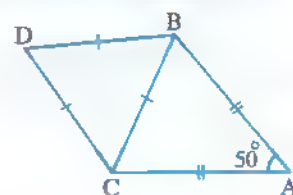
[c] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$

, $m(\angle BAC) = 70^\circ$

and $m(\angle DAC) = 50^\circ$

Prove that : $BC > AC$



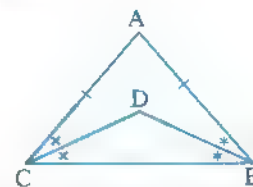
4 [a] Prove that : The two base angles of the isosceles triangle are congruent.

[b] In the opposite figure :

$AB = AC$, \overline{BD} bisects $\angle B$

and \overline{CD} bisects $\angle C$

Prove that : $\triangle DBC$ is isosceles.



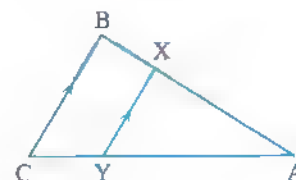
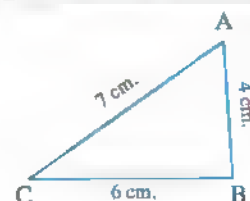
5 [a] In the opposite figure :

Arrange the angles
of $\triangle ABC$ descendingly
due to their measures.

[b] In the opposite figure :

$AB > BC$, $\overline{XY} \parallel \overline{BC}$

Prove that : $AX > XY$



Model 2

Answer the following questions :

1 Choose the correct answer from those given :

[1] The triangle which has three axes of symmetry is

(a) scalene. (b) isosceles. (c) right-angled. (d) equilateral.

2 The sum of lengths of two sides in a triangle is the length of the third side.

(a) greater than (b) smaller than (c) equal to (d) twice

3 If the lengths of two sides in an isosceles triangle are 8 cm. and 4 cm. , then the length of the third side is cm.

(a) 4 (b) 8 (c) 3 (d) 12

4. In $\triangle ABC$, if $m(\angle B) = 130^\circ$, then the longest side of it is

- (a) \overline{BC} (b) \overline{AC} (c) \overline{AB} (d) its median.

5. $\triangle XYZ$ is an isosceles triangle in which : $m(\angle X) = 100^\circ$, then $m(\angle Y) = \dots$

- (a) 100° (b) 80° (c) 60° (d) 40°

6. In the opposite figure :

$x + y = \dots$

- (a) 100° (b) 140°
(c) 180° (d) 280°



2. Complete the following :

1. If the measure of an angle in a right-angled triangle is 45° , then the triangle is ..
2. The length of any side in a triangle ... the sum of lengths of the two other sides.
3. If $\overline{AB} \equiv \overline{XY}$, then $AB = \dots$
4. In $\triangle ABC$, if $m(\angle A) = 30^\circ$ and $m(\angle B) = 90^\circ$, then $BC = \dots AC$
5. The axis of symmetry of a line segment is the straight line which ... at its midpoint.

3. [a] In $\triangle ABC$, $AB = 7$ cm., $BC = 5$ cm. and $AC = 6$ cm.

Arrange its angles ascendingly due to their measures.

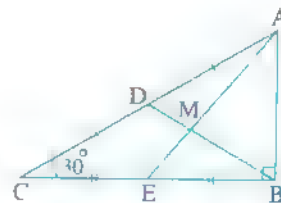
[b] In the opposite figure :

$\triangle ABC$ is right-angled at B

, $m(\angle C) = 30^\circ$, D is the midpoint of \overline{AC}

, E is the midpoint of \overline{BC} , $AC = 9$ cm.

Find : The length of each of \overline{BD} , \overline{BM} and \overline{AB}



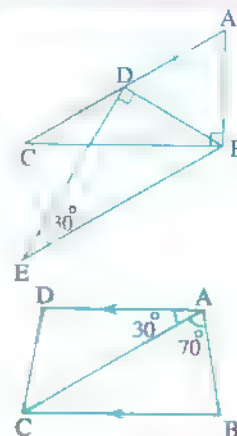
4. [a] In the opposite figure :

$m(\angle ABC) = m(\angle BDE) = 90^\circ$

, $m(\angle E) = 30^\circ$

, D is the midpoint of \overline{AC}

Prove that : $AC = BE$



[b] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle BAC) = 70^\circ$

, $m(\angle DAC) = 30^\circ$

Prove that : $AC > BC$

5. [a] Complete :

If the measures of two angles of a triangle are different, then the greater in measure is opposite to ..

[b] In the opposite figure :

$\overline{AB} \parallel \overline{XY}$ and \overline{AB} bisects $\angle YAZ$

Prove that : $XZ > YZ$



Model for the merge student

Answer the following questions :

1 Complete each of the following :

- 1 The point of concurrence of the medians of the triangle divides each median in the ratio from the base.
- 2 In the right-angled triangle , the length of the median drawn from the vertex of the right angle equals
- 3 The base angles of the isosceles triangle are
- 4 In $\triangle ABC$, if $m(\angle B) = 70^\circ$, $m(\angle C) = 50^\circ$, then AC AB
- 5 The median of the isosceles triangle from the vertex angle ,

2 Choose the correct answer from those given :

- 1 If ABC is an equilateral triangle , then $m(\angle B) =$
 (a) 30° (b) 60° (c) 70° (d) 90°
- 2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals the length of the hypotenuse.
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 2
- 3 If the measure of the vertex angle of an isosceles triangle is 80° , then the measure of one of the base angles equals
 (a) 60° (b) 40° (c) 30° (d) 50°
- 4 The number of axes of symmetry of the isosceles triangle is
 (a) 1 (b) 2 (c) 3 (d) zero
- 5 In $\triangle ABC$, if $m(\angle A) = 50^\circ$, $m(\angle B) = 60^\circ$, then the longest side is
 (a) \overline{AB} (b) \overline{BC} (c) \overline{AC}

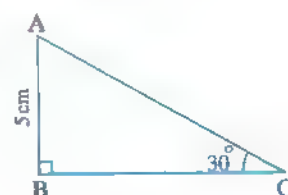
3 In the opposite figure :

$\triangle ABC$ is a right-angled triangle at B , $m(\angle C) = 30^\circ$, $AB = 5$ cm.

Find : The length of \overline{AC}

$$\therefore m(\angle B) = \dots\dots\dots^\circ , m(\angle C) = \dots\dots\dots^\circ$$

$$\therefore AB = \frac{1}{2} \times \dots\dots\dots \therefore AC = \dots\dots\dots \text{ cm.}$$



- 4 [a] In $\triangle ABC$, $m(\angle A) = 40^\circ$, $m(\angle B) = 75^\circ$, $m(\angle C) = 65^\circ$

Arrange the lengths of the sides of the triangle descendingly.

The order is : , ,

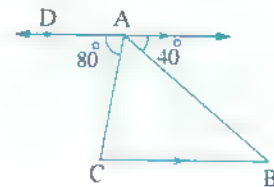
- [b] In the opposite figure :

$\overrightarrow{AD} \parallel \overrightarrow{BC}$

Complete :

1 $m(\angle B) = \dots\dots\dots^\circ$

2 The side is the longest side of $\triangle ABC$



- 5 In the opposite figure :

$AB = AC = CD = AD = 10$ cm.

$m(\angle BAC) = 70^\circ$

Put (✓) or (✗) :

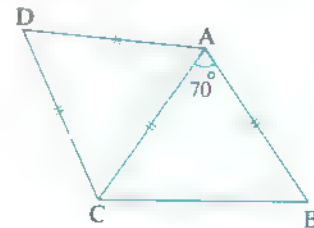
1 $m(\angle B) = 55^\circ$

2 $m(\angle D) = 70^\circ$

3 $m(\angle DCB) = 120^\circ$

4 $AB + AD = 20$ cm.

5 $AB + BC = BC + CD$



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Answer the following questions :

1 Choose the correct answer :

1] In the opposite figure :

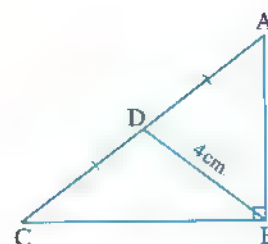
$AC = \dots\dots\dots$ cm.

(a) 4

(b) 6

(c) 8

(d) 2



2] If ΔABC is right-angled at A and $AB = AC$, then $m(\angle B) = \dots\dots\dots$

(a) 30°

(b) 45°

(c) 60°

(d) 90°

3] In ΔABC , if $AB = 6$ cm. , $AC = 7$ cm. , then $BC \in \dots\dots\dots$

(a) $]6, 13]$

(b) $[6, 7]$

(c) $]1, 13[$

(d) $[1, 7[$

4. In ΔXYZ , if $XY < XZ$, then $\dots\dots\dots$

(a) $m(\angle Y) \leq m(\angle Z)$

(b) $m(\angle Y) > m(\angle Z)$

(c) $m(\angle Y) = m(\angle Z)$

(d) $m(\angle Z) > m(\angle Y)$

5 If ΔABC is right-angled at B , $m(\angle A) = 55^\circ$, then the number of axes of symmetry of ΔABC equals $\dots\dots\dots$

(a) 1

(b) 2

(c) 3

(d) zero

6 The triangle in which the measures of two angles of it are 42° and 69° is $\dots\dots\dots$ triangle.

(a) an isosceles

(b) an equilateral

(c) a scalene

(d) a right-angled

2 Complete the following :

1 Any point on the axis of symmetry of a line segment is $\dots\dots\dots$ from its terminals.

2] The longest side in the right-angled triangle is $\dots\dots\dots$

3 The point of intersection of the medians of the triangle divides each of them by the ratio $\dots\dots\dots$; $\dots\dots\dots$ from the vertex.

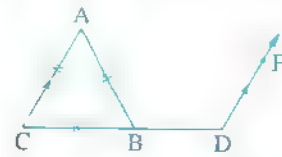
4 The measure of any exterior angle of an equilateral triangle equals $\dots\dots\dots^\circ$

5] The sum of the lengths of any two sides in a triangle is $\dots\dots\dots$ the length of the third side.

3 [a] In the opposite figure :

$\triangle ABC$ is an equilateral triangle , $\overrightarrow{DF} \parallel \overrightarrow{AC}$

Find by proof : $m(\angle D)$

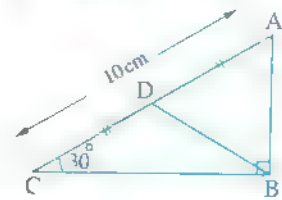


[b] In the opposite figure :

$m(\angle ABC) = 90^\circ$, $m(\angle C) = 30^\circ$

, $AC = 10$ cm. , $AD = DC$

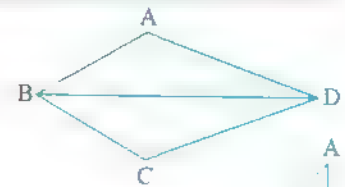
Find : The perimeter of $\triangle ABD$



4 [a] In the opposite figure :

$AB < AD$, $BC < CD$

Prove that : $m(\angle ABC) > m(\angle ADC)$

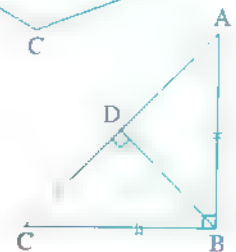


[b] In the opposite figure :

$m(\angle ABC) = 90^\circ$, $\overline{BD} \perp \overline{AC}$

, $AB = BC$

Prove that : $\triangle DCB$ is an isosceles triangle.



5 [a] XYZ is a triangle in which $m(\angle X) = 60^\circ$, $m(\angle Y) = 50^\circ$

Order the lengths of the sides of the triangle descendingly.

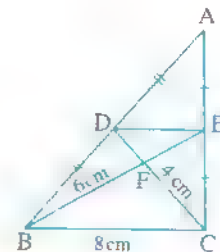
[b] In the opposite figure :

ABC is a triangle in which D , E are the midpoints of \overline{AB} , \overline{AC}

, $FC = 4$ cm. , $FB = 6$ cm.

, $BC = 8$ cm.

Find : The perimeter of $\triangle DFE$



Answer the following questions :

1 Choose the correct answer from those given :

- 1 A triangle has one line of symmetry , the lengths of two sides are 4 cm. and 8 cm. , then the length of the third side is cm.

(a) 3 (b) 4 (c) 8 (d) 6

- 2 The point of intersection of the medians of the triangle divides each median in the ratio of from the base.

(a) 2 : 1 (b) 2 : 3 (c) 1 : 2 (d) 1 : 3

Geometry

- 3 If $m(\angle A) = 50^\circ$, then the measure of its reflex angle is
- (a) 40° (b) 130° (c) 310° (d) 180°
- 4 If the length of the side of an equilateral triangle is 10 cm., then the length of its height is cm.
- (a) 10 (b) 5 (c) $5\sqrt{3}$ (d) 6
- 5 In $\triangle ABC$, if $AB = 6$ cm., $AC = 7$ cm., then the length of $\overline{BC} \in$
- (a) $[6, 7]$ (b) $]1, 7[$ (c) $[1, 13]$ (d) $]1, 13[$
- 6 In the opposite figure :
- $x + y =$
- (a) 180° (b) 360°
(c) 240° (d) 280°

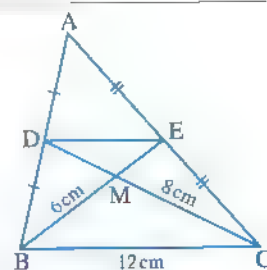


2 Complete :

- 1 If the measures of two angles in a triangle are different, then the greater angle in measure of them is
- 2 In the triangle ABC, if $m(\angle A) = 50^\circ$, $m(\angle B) = 60^\circ$, then the longest side is
- 3 The median drawn from the vertex angle of an isosceles triangle and
- 4 In $\triangle ABC$, if $m(\angle A) = 30^\circ$, $m(\angle B) = 90^\circ$, then $AC =$ BC
- 5 The perpendicular bisector of a line segment is called

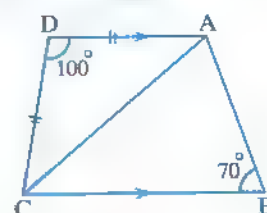
3 [a] In the opposite figure :

In $\triangle ABC$: \overline{BE} , \overline{CD} are two medians, $MB = 6$ cm.,
 $, BC = 12$ cm., $MC = 8$ cm.
 Find : The perimeter of $\triangle MDE$



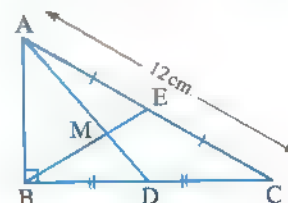
[b] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $AD = DC$
 $, m(\angle D) = 100^\circ$, $m(\angle B) = 70^\circ$
 Prove that : 1 $AC > AB$ 2 $\triangle ABC$ is isosceles.



4 [a] In the opposite figure :

$\triangle ABC$ is right-angled at B
 $, E$ and D are the midpoints of \overline{AC} , \overline{BC} respectively
 $, AC = 12$ cm.
 Find : The length of each of \overline{BE} , \overline{ME}



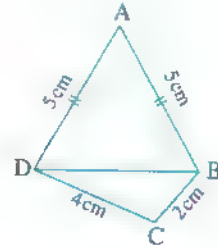
[b] In the opposite figure :

ABCD is a quadrilateral

, $AB = AD = 5 \text{ cm}$.

, $BC = 2 \text{ cm}$, $DC = 4 \text{ cm}$.

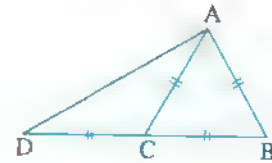
Prove that : $m(\angle ABC) > m(\angle ADC)$



5 [a] In the opposite figure :

$AB = BC = AC = CD$

Prove that : $m(\angle BAD) = 90^\circ$

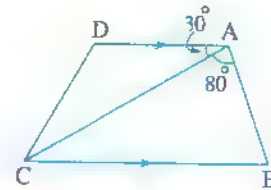


[b] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle BAC) = 80^\circ$

, $m(\angle DAC) = 30^\circ$

Prove that : $BC > AB$



3

Cairo Governorate



New Cairo, Education 17

17

Answer the following questions :

1 Choose the correct answer :

1 In $\triangle ABC$, if $AB = AC$, $m(\angle B) = 40^\circ$, then $m(\angle A) = \dots\dots\dots$

- (a) 70° (b) 55° (c) 100° (d) 40°

2 The point of concurrence of the medians of the triangle divides each median at the ratio from the vertex.

- (a) 1 : 2 (b) 2 : 1 (c) 2 : 3 (d) 1 : 3

3 In $\triangle ABC$, if $AB = 7 \text{ cm}$, $BC = 10 \text{ cm}$, then the length of \overline{AC} must satisfy which of the following inequalities ?

- (a) $3 \leq AC \leq 17$ (b) $3 < AC < 17$ (c) $10 < AC < 20$ (d) $14 < AC < 20$

4 If $\triangle ABD$ is obtuse-angled at B and C is the midpoint of \overline{BD} , then the longest side in $\triangle ABD$ is

- (a) \overline{AB} (b) \overline{AC} (c) \overline{AD} (d) \overline{BD}

5 In $\triangle ABC$, if $m(\angle A) = 64^\circ$, $m(\angle B) = 35^\circ$, then the longest side of the triangle is

- (a) \overline{AB} (b) \overline{AC} (c) \overline{BC} (d) otherwise.

Geometry

6. ABCD is a rectangle, M is the point of intersection of its diagonals, if the length of the diagonal is 6 cm., then the length of the median \overline{AM} is cm.

(a) 3 (b) 6 (c) 9 (d) 12

2 Complete each of the following :

1. The length of the side which is opposite to the angle of measure 30° in the right-angled triangle equals the length of the hypotenuse.
2. In the right-angled triangle, the longest side is the
3. The straight line drawn from the vertex of the isosceles triangle, perpendicular to the base this vertex.
4. The measure of the exterior angle of the equilateral triangle equals $^\circ$
5. The number of axes of symmetry of the isosceles triangle is

3 [a] In the opposite figure :

ABC is a triangle, $AB = AC$, $m(\angle B) = (X + 5)^\circ$

, $m(\angle C) = (2X - 15)^\circ$

Find : $m(\angle A)$ (show all of your work)



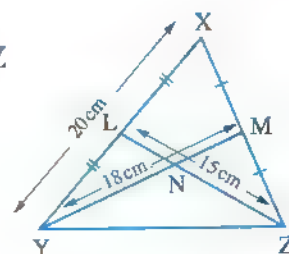
[b] In the opposite figure :

N is the point of concurrence of the medians of the triangle XYZ

, $LZ = 15$ cm., $YM = 18$ cm.

, $XY = 20$ cm.

Find : The perimeter of the triangle NLY



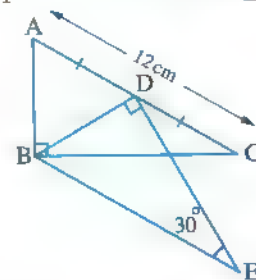
[c] In the opposite figure :

$m(\angle ABC) = m(\angle BDE) = 90^\circ$

, D is the midpoint of \overline{AC}

, $m(\angle E) = 30^\circ$, $AC = 12$ cm.

Find with proof : The length of \overline{BE}



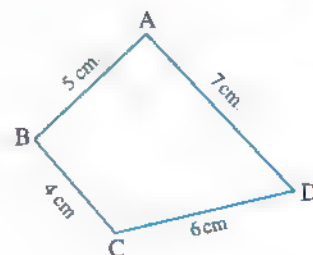
4 [a] In the opposite figure :

ABCD is a quadrilateral in which :

$AB = 5$ cm., $BC = 4$ cm., $CD = 6$ cm.

, $AD = 7$ cm.

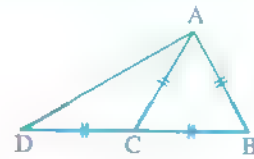
Prove that : $m(\angle ABC) > m(\angle ADC)$



[b] In the opposite figure :

$$AB = AC = CB = CD$$

Prove that : $\overline{AB} \perp \overline{AD}$



[c] XYZ is a triangle in which : $XY = 10$ cm. , $YZ = 6$ cm. and $XZ = 8$ cm.

Arrange the measures of the angles of the triangle.

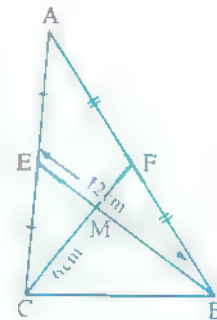
[d] In the opposite figure :

ABC is a triangle in which : F , E are the midpoints of \overline{AB} and \overline{AC} respectively

$$EB = 12 \text{ cm.}$$

$$MC = 6 \text{ cm.}$$

Find with proof : The length of each of \overline{EM} and \overline{MF}

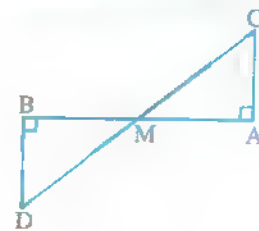


5 [a] In the opposite figure :

$$\overline{DC} \cap \overline{AB} = \{M\}$$

$$m(\angle A) = m(\angle B) = 90^\circ$$

Prove that : $DC > AB$



[b] ABC is a triangle in which : $m(\angle A) = (6x)^\circ$

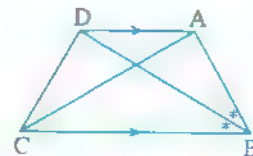
$$m(\angle B) = (4x - 9)^\circ, m(\angle C) = 3(x - 2)^\circ$$

Arrange the lengths of the sides of the triangle.

[c] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, \overline{BD} bisects $\angle ABC$

Prove that : $\triangle BAD$ is an isosceles triangle.



4

Giza Governorate



Answer the following questions :

1 Choose the correct answer :

1 If the measures of two angles of a triangle are 40° , 100° , then the triangle is triangle.

(a) an isosceles (b) an equilateral (c) a scalene (d) a right-angled

2 The angle whose measure is more than 90° and less than 180° is angle.

(a) an acute (b) an obtuse (c) a straight (d) a reflex

Geometry

- 3 If the lengths of two sides in an isosceles triangle are 7 cm. and 3 cm. , then the length of the third side is cm.
 (a) 3 (b) 10 (c) 7 (d) 4
- 4 In $\triangle ABC$, if $m(\angle B) = 120^\circ$, then the longest side in it is
 (a) \overline{BC} (b) \overline{AC} (c) \overline{AB} (d) its median.
- 5 If $\triangle ABC$ is right-angled at B , $AB = 3$ cm. , $BC = 4$ cm. , then the length of the median from B is cm.
 (a) 5 (b) 4 (c) 2.5 (d) 6
- 6 In $\triangle ABC$, if $m(\angle A) = 30^\circ$, $m(\angle B) = 90^\circ$ and $AC = 10$ cm. , then $BC =$
 (a) 20 cm. (b) 15 cm. (c) 10 cm. (d) 5 cm.

2 Complete each of the following :

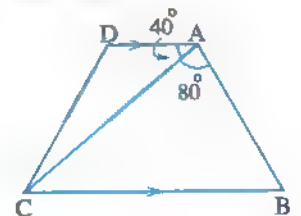
- 1 The angle of measure 70° complements an angle of measure $^\circ$
- 2 In $\triangle ABC$, if $AB = 3$ cm. , $BC = 5$ cm. , then $AC \in].....,[$
- 3 If $\overline{AB} \equiv \overline{CD}$ and $AB = 6$ cm. , then $AB + CD =$ cm.
- 4 The bisector of the vertex angle of an isosceles triangle and
- 5 The point of intersection of the medians of the triangle divides each median in the ratio : from the vertex.

3 [a] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle BAC) = 80^\circ$

and $m(\angle DAC) = 40^\circ$

Prove that : $BC > AC$



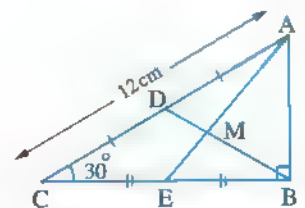
[b] In the opposite figure :

$\triangle ABC$ is right-angled at B , $m(\angle C) = 30^\circ$

, D is the midpoint of \overline{AC}

, E is the midpoint of \overline{BC} , $AC = 12$ cm.

Find : The length of each of \overline{BD} , \overline{BM} and \overline{AB}

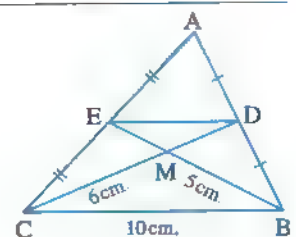


4 [a] In the opposite figure :

D and E are the midpoints of \overline{AB} and \overline{AC} respectively

, $BC = 10$ cm. , $MB = 5$ cm. and $MC = 6$ cm.

Find : The perimeter of $\triangle MDE$

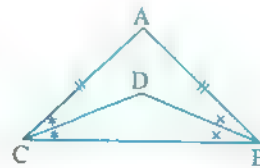


[b] In the opposite figure :

$AB = AC$, \overline{BD} bisects $\angle ABC$

and \overline{CD} bisects $\angle ACB$

Prove that : $\triangle DBC$ is an isosceles triangle.

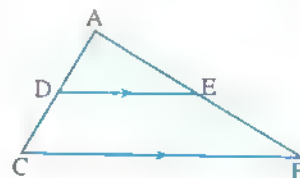


5 [a] In the opposite figure :

ABC is a triangle in which :

$AB > AC$ and $\overline{DE} \parallel \overline{BC}$

Prove that : $m(\angle ADE) > m(\angle AED)$

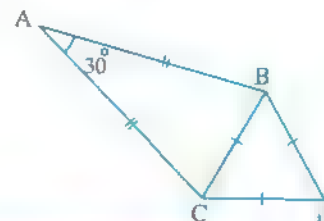


[b] In the opposite figure :

$m(\angle A) = 30^\circ$, $AB = AC$

and $\triangle DBC$ is equilateral.

Find : $m(\angle ABD)$



5

Glenn Governorate



Bozdog El Dakrouy Directorate

103

Answer the following questions :

1 Choose the correct answer :

1 The lengths 9 cm. , 4 cm. and may be the side lengths of an isosceles triangle.

- (a) 9 cm. (b) 13 cm. (c) 5 cm. (d) 4 cm.

2 In $\triangle ABC$, if $m(\angle B) = 130^\circ$, then the longest side of it is

- (a) \overline{BC} (b) \overline{AC} (c) \overline{AB} (d) its median.

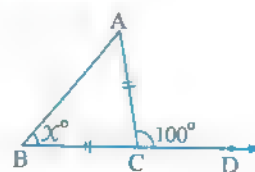
3 In the opposite figure :

$CA = CB$, $m(\angle B) = x^\circ$

, $m(\angle ACD) = 100^\circ$ where $C \in \overline{BD}$

, then $x = \dots\dots\dots$

- (a) 50° (b) 100° (c) 150° (d) 200°



4 The measure of the exterior angle of an equilateral triangle equals

- (a) 30° (b) 60° (c) 90° (d) 120°

5 In $\triangle ABC$, if $AB = 6$ cm. and $AC = 7$ cm. , then $BC \in \dots\dots\dots$

- (a) $[6, 13]$ (b) $[6, 7]$ (c) $]1, 13[$ (d) $[1, 7[$

8 In the opposite figure :

$AD = DC$, $m(\angle C) = 30^\circ$

, $m(\angle ABC) = 90^\circ$, $AB = 5$ cm.

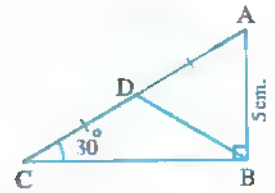
, then the perimeter of $\triangle ABD = \dots\dots\dots$ cm.

(a) 5

(b) 15

(c) 20

(d) 25



2 Complete the following :

1 ABC is a triangle in which $AB = AC$ and $m(\angle A) = 60^\circ$, if its perimeter = 18 cm.

, then $BC = \dots\dots\dots$ cm.

2 The number of the axes of symmetry of the equilateral triangle equals $\dots\dots\dots$

3 The longest side of the right-angled triangle is the $\dots\dots\dots$

4 If the angles of a triangle are congruent , then the triangle is $\dots\dots\dots$

5 In $\triangle ABC$, if $AB > BC$, then $m(\angle A) \dots\dots\dots m(\angle C)$

3 [a] In the opposite figure :

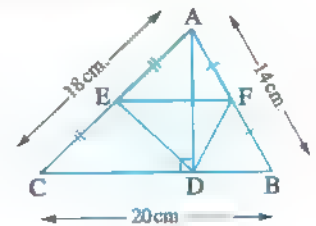
ABC is a triangle in which $AB = 14$ cm.

, $AC = 18$ cm. , $BC = 20$ cm.

, E is the midpoint of \overline{AC}

, F is the midpoint of \overline{AB} and $\overline{AD} \perp \overline{BC}$

Find : The perimeter of $\triangle DEF$

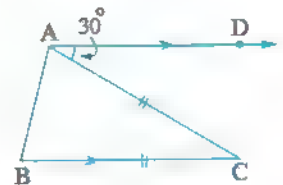


[b] In the opposite figure :

ABC is a triangle in which $AC = BC$

, $\overline{AD} \parallel \overline{BC}$, $m(\angle DAC) = 30^\circ$

Find with proof : The measures of the angles of $\triangle ABC$

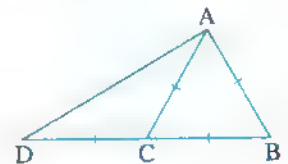


4 [a] In the opposite figure :

$AB = BC = AC = DC$

Prove that :

$m(\angle BAD) = 90^\circ$



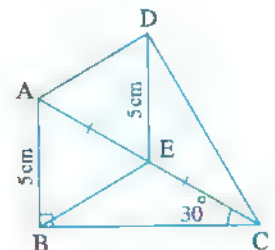
[b] In the opposite figure :

$m(\angle ABC) = 90^\circ$, E is the midpoint of \overline{AC}

, $m(\angle ACB) = 30^\circ$

, $AB = DE = 5$ cm.

Prove that : $m(\angle ADC) = 90^\circ$



- 5 [a] In $\triangle ABC$, $m(\angle A) = 40^\circ$, $m(\angle B) = 75^\circ$, $m(\angle C) = 65^\circ$
 , arrange the lengths of the sides of this triangle descendingly.

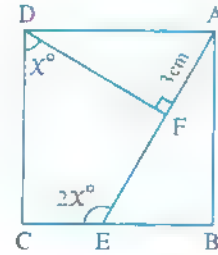
[b] In the opposite figure :

ABCD is a square , $E \in \overline{BC}$

where $m(\angle FDC) = x^\circ$ and $m(\angle FEC) = 2x^\circ$

, $\overline{DF} \perp \overline{AE}$, $AF = 3$ cm.

Calculate : The area of the square ABCD



6 Alexandria Governorate



El Montazah Educational Zone
 Modern Language School

Answer the following questions :

1 Complete :

- 1 If $\triangle ABC$ is a right-angled triangle at B , $m(\angle A) = 30^\circ$, $AC = 10$ cm.
 , then $CB = \dots\dots\dots$ cm.
- 2 In $\triangle ABC$, $m(\angle A) = m(\angle B) = m(\angle C)$, then the measure of the exterior angle
 equals $\dots\dots\dots^\circ$
- 3 In $\triangle ABC$, $AB = AC$, $m(\angle B) = x + 30^\circ$, $m(\angle C) = 2x + 5^\circ$, then $x = \dots\dots\dots^\circ$
- 4 In a triangle , if two angles are unequal in measure , then the greater angle in measure
 is opposite to $\dots\dots\dots$
- 5 In any triangle , the sum of the lengths of any two sides $\dots\dots\dots$ the length of
 the third side.

2 Choose the correct answer :

- 1 If \overline{AD} is a median of $\triangle ABC$ and M is the point of concurrence of the medians , then
 $AM = \dots\dots\dots AD$
 (a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) 2
- 2 The measure of one of the base angles of an isosceles triangle is 65° , then the measure
 of its vertex angle equals $\dots\dots\dots^\circ$
 (a) 65 (b) 50 (c) 130 (d) 55
- 3 In the triangle ABC , if $m(\angle A) = 50^\circ$, $m(\angle B) = 60^\circ$, then the longest side
 is $\dots\dots\dots$
 (a) \overline{AB} (b) \overline{BC} (c) \overline{AC} (d) 110 cm.
- 4 The numbers which can not be side lengths of a triangle are $\dots\dots\dots$
 (a) 3 , 3 , 3 (b) 3 , 3 , 4 (c) 3 , 3 , 5 (d) 3 , 3 , 6

Geometry

- 5 The number of the axes of symmetry of the scalene triangle is

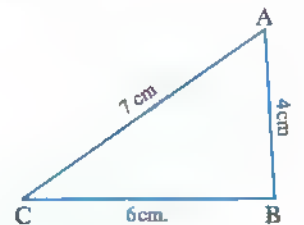
(a) 1 (b) 2 (c) 3 (d) 0

- 6 If $\triangle XYZ$ is right-angled at Y , then XZ YZ

(a) < (b) \leq (c) > (d) =

- 3 [a] In the opposite figure :

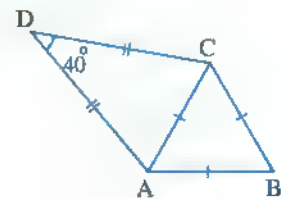
Arrange the angles of $\triangle ABC$ descendingly due to their measures.



- [b] In the opposite figure :

$m(\angle D) = 40^\circ$, $DA = DC$
and $\triangle ABC$ is an equilateral triangle.

Find : $m(\angle DCB)$



- 4 [a] In the opposite figure :

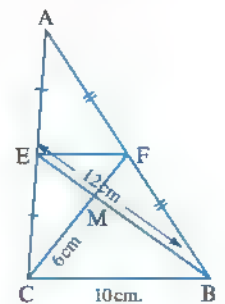
ABC is a triangle

, F and E are the midpoints of \overline{AB} and \overline{AC} respectively

If $BE = 12$ cm, $CM = 6$ cm.

, $BC = 10$ cm.

, then find : The perimeter of $\triangle MEF$

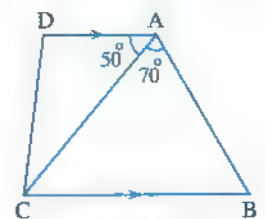


- [b] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle CAB) = 70^\circ$

, $m(\angle DAC) = 50^\circ$

Prove that : $BC > AC$



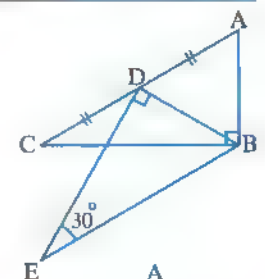
- 5 [a] In the opposite figure :

$m(\angle ABC) = m(\angle BDE) = 90^\circ$

, $m(\angle E) = 30^\circ$

, D is the midpoint of \overline{AC}

Prove that : $AC = BE$

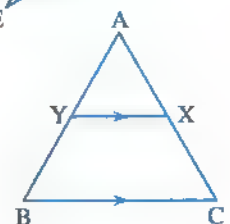


- [b] In the opposite figure :

ABC is a triangle in which :

$AB = AC$, $\overline{XY} \parallel \overline{CB}$

Prove that : $\triangle AXY$ is an isosceles triangle.





Answer the following questions :

1 Choose the correct answer :

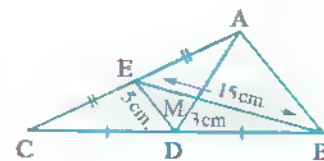
- 1 An isosceles triangle has two sides of lengths 6 cm. and 12 cm. , then the length of the third side equals cm.
 (a) 6 (b) 9 (c) 12 (d) 18
- 2 In $\triangle XYZ$, if $m(\angle Y) = 115^\circ$, then the longest side is
 (a) \overline{XY} (b) \overline{YZ}
 (c) \overline{ZX} (d) the median of the triangle.
- 3 The lengths 5 cm. , 4 cm. and cm. are lengths of sides of a triangle.
 (a) 8 (b) 9 (c) 12 (d) 10
- 4 The triangle having two angles of measures 74° and 53° is triangle.
 (a) an isosceles (b) an equilateral (c) a scalene (d) a right-angled
- 5 The intersection point of the medians of a triangle divides each median by the ratio 1 : from the base.
 (a) 1 (b) 2 (c) 3 (d) 4
- 6 If two sides of a triangle have unequal lengths , then the smaller side is opposite to the angle of the measure from that is opposite to the other side.
 (a) greater (b) smaller (c) equal (d) otherwise

2 Complete each of the following :

- 1 The length of the median of the right-angled triangle drawn from the vertex of the right angle equals the length of the hypotenuse.
- 2 The number of the axes of symmetry of an isosceles triangle is
- 3 The measure of the exterior angle of the equilateral triangle equals $^\circ$
- 4 The two angles of the base of an isosceles triangle are
- 5 The sum of the measures of the accumulative angles at a point equals $^\circ$

3 [a] In the opposite figure :

If E is the midpoint of \overline{AC} and D is the midpoint of \overline{BC}
 , $ED = 5$ cm. , $MD = 3$ cm. and $BE = 15$ cm.
 , find : The perimeter of $\triangle AMB$



- [b] ABC is a triangle in which : $m(\angle B) = 40^\circ$, $m(\angle C) = 80^\circ$
 Arrange its side lengths ascendingly.

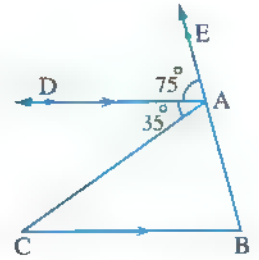
4 [a] In the opposite figure :

$$\overrightarrow{AD} \parallel \overrightarrow{BC}$$

$$, m(\angle EAD) = 75^\circ$$

$$\text{and } m(\angle DAC) = 35^\circ$$

Prove that : $AC > AB$



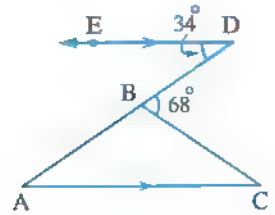
[b] In the opposite figure :

$$\overrightarrow{DE} \parallel \overrightarrow{AC}$$

$$, m(\angle EDA) = 34^\circ$$

$$\text{and } m(\angle DBC) = 68^\circ$$

Prove that : $\triangle ABC$ is an isosceles triangle.

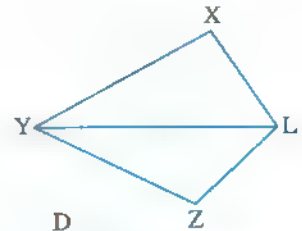


5 [a] In the opposite figure :

$$\text{If } XY > XL$$

$$, YZ > ZL$$

, prove that : $m(\angle XLZ) > m(\angle XYZ)$



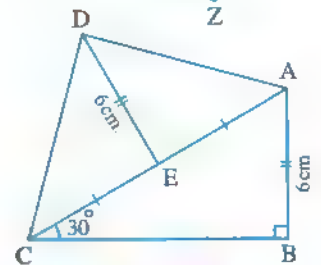
[b] In the opposite figure :

$$m(\angle B) = 90^\circ, m(\angle ACB) = 30^\circ$$

, E is the midpoint of \overline{AC} and $AB = DE = 6 \text{ cm}$.

Find : 1 The length of \overline{AC}

2 $m(\angle ADC)$



Answer the following questions :

1 Choose the correct answer :

1 In any isosceles triangle , the type of the base angles is

- (a) acute. (b) right. (c) obtuse. (d) reflex.

2 The medians of the triangle intersect at

- (a) 4 points. (b) 3 points. (c) 2 points. (d) a point.

3 ABC is a triangle in which $m(\angle A) = 100^\circ$, then the greatest side in length in the triangle is

- (a) \overline{AB} (b) \overline{AC} (c) \overline{BC} (d) \overline{BD}

4 The numbers which can be lengths of sides of a triangle are

- (a) 0 , 3 , 5 (b) 3 , 3 , 5 (c) 3 , 3 , 6 (d) 3 , 3 , 7

- 5] The triangle which has three axes of symmetry is
- (a) scalene. (b) isosceles. (c) right-angled. (d) equilateral.
- 6] If $\triangle ABC$ is an equilateral triangle, then $m(\angle B) = \dots\dots\dots$
- (a) 30° (b) 60° (c) 70° (d) 90°

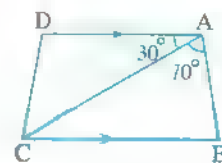
2 Complete :

- 1 In $\triangle ABC$, if the point D is the midpoint of \overline{AB} and the point E is the midpoint of \overline{AC} , then $DE = \dots\dots\dots BC$
- 2 The base angles in the isosceles triangle are in measure.
- 3 In the triangle, the smallest angle in measure is opposite to side in length.
- 4 In the triangle ABC, if $AB = AC$, $m(\angle A) = 70^\circ$, so $m(\angle C) = \dots\dots\dots^\circ$
- 5 The point of concurrence of the medians of the triangle divides each median in the ratio of from the base.

3 [a] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle BAC) = 70^\circ$
 $m(\angle DAC) = 30^\circ$

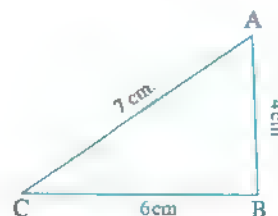
Prove that : $AC > BC$



[b] In the opposite figure :

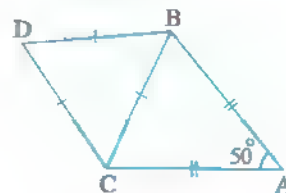
$AB = 4 \text{ cm}$, $BC = 6 \text{ cm}$,
 $AC = 7 \text{ cm}$.

Arrange the measures of the angles of the triangle ABC descendingly.



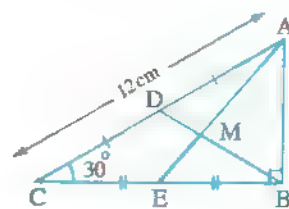
4 [a] In the opposite figure :

$m(\angle A) = 50^\circ$, $AB = AC$
 and $\triangle DBC$ is an equilateral triangle.
 Find : $m(\angle ABD)$



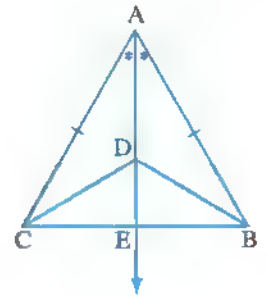
[b] In the opposite figure :

$\triangle ABC$ is right-angled at B, $m(\angle C) = 30^\circ$
 D is the midpoint of \overline{AC}
 E is the midpoint of \overline{BC} , $AC = 12 \text{ cm}$.
 Find : The length of each of \overline{BD} , \overline{BM} and \overline{AB}



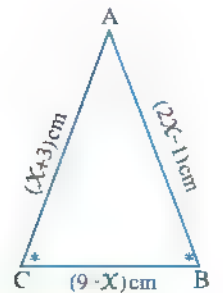
5 [a] In the opposite figure :

ABC is a triangle in which :
 $AB = AC$, \overrightarrow{AE} bisects $\angle BAC$
 $\overrightarrow{AE} \cap \overrightarrow{BC} = \{E\}$, $D \in \overrightarrow{AE}$
Prove that : **1** $BE = \frac{1}{2} BC$
2 $BD = CD$



[b] In the opposite figure :

ABC is a triangle in which :
 $m(\angle B) = m(\angle C)$
 $AB = (2x - 1) \text{ cm.}$
 $AC = (x + 3) \text{ cm.}$, $BC = (9 - x) \text{ cm.}$
Find : The perimeter of the triangle ABC



9

El-Sharkia Governorate



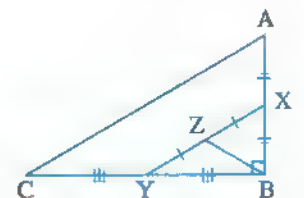
Nehia Educational Zone

Governmental Language School

Answer the following questions :

1 Complete the following :

- 1** The base angles of the isosceles triangle are
- 2** In $\triangle ABC$, if $\overrightarrow{AB} \perp \overrightarrow{BC}$ and $AB = BC$, then $m(\angle A) = \dots\dots\dots^\circ$
- 3** In $\triangle ABC$, if $AB > AC$, then $m(\angle C) \dots\dots\dots m(\angle B)$
- 4** The triangle whose side lengths are $(2x - 1) \text{ cm.}$, $(x + 3) \text{ cm.}$, 7 cm. becomes an equilateral triangle when $x = \dots\dots\dots \text{ cm.}$
- 5 In the opposite figure :**
 $AC = \dots\dots\dots BZ$



2 Choose the correct answer from those given :

- 1** The sum of lengths of any two sides in a triangle is the length of the third side.
 - (a) smaller than
 - (b) greater than
 - (c) equal to
 - (d) twice

- 2 The measure of the exterior angle of the equilateral triangle equals
 (a) 30° (b) 60° (c) 90° (d) 120°
- 3 The length of the hypotenuse of the right-angled triangle equals the length of the median drawn from the vertex of the right angle.
 (a) third (b) quarter (c) half (d) twice
- 4 The lengths of two sides in a triangle are 4 cm. and 9 cm. and it has one axis of symmetry, then the length of the third side is
 (a) 4 cm. (b) 5 cm. (c) 9 cm. (d) 13 cm.
- 5 The quadrilateral ABCD in which \overline{BD} is an axis of symmetry of \overline{AC} may be a
 (a) rhombus. (b) rectangle. (c) parallelogram. (d) trapezium.

6 In the opposite figure :

$x + y = \dots\dots\dots$

- (a) 100° (b) 280°
 (c) 140° (d) 80°

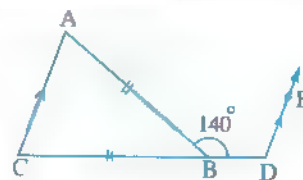


3 [a] In the opposite figure :

$AB = BC$, $m(\angle ABD) = 140^\circ$

and $\overline{AC} \parallel \overline{DE}$

Find : $m(\angle EDC)$



[b] In the opposite figure :

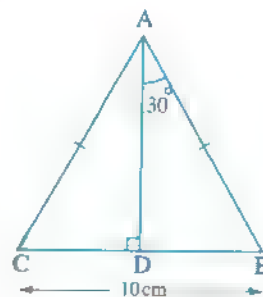
$AB = AC$, $BC = 10$ cm.

, $m(\angle BAD) = 30^\circ$

and $\overline{AD} \perp \overline{BC}$

Find : 1 The length of each of \overline{BD} and \overline{AD}

2 The area of $\triangle ABC$



4 [a] In the opposite figure :

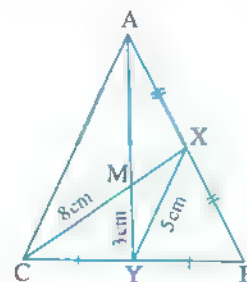
ABC is a triangle, X is the midpoint of \overline{AB}

, Y is the midpoint of \overline{BC} , $XY = 5$ cm.

, $\overline{XC} \cap \overline{AY} = \{M\}$ where $CM = 8$ cm.

, $YM = 3$ cm.

Find : The perimeter of $\triangle MXY$

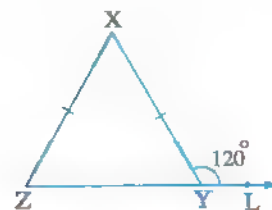


[b] In the opposite figure :

$$XY = XZ, m(\angle XYL) = 120^\circ, L \in \overrightarrow{ZY}$$

Prove that :

$\triangle XYZ$ is an equilateral triangle.

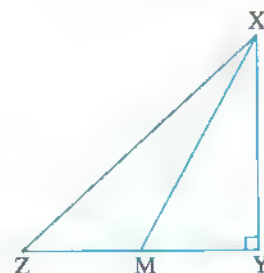


5 [a] In the opposite figure :

XYZ is a right-angled triangle

at Y and $M \in \overline{YZ}$

Prove that : $XZ > XM$



[b] In the opposite figure :

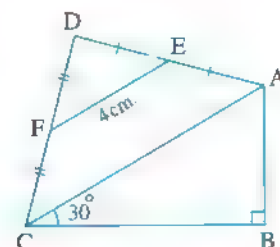
$ABCD$ is a quadrilateral in which :

$m(\angle B) = 90^\circ$, E is the midpoint of \overline{AD}

, F is the midpoint of \overline{CD}

, $m(\angle ACB) = 30^\circ$ and $EF = 4$ cm.

Find by proof : The length of \overline{AB}



10

El-Gharbia Governorate



The Central Maths Commission
Ministry of Education and Scientific Research

Answer the following questions :

1 Choose the correct answer :

1 In $\triangle ABC$, if $m(\angle C) = 65^\circ$, $m(\angle A) = 75^\circ$, then

- (a) $AB > BC$ (b) $AB < AC$ (c) $BC > AB$ (d) $AB = AC$

2 The sum of measures of two angles in the equilateral triangle equals

- (a) 180° (b) 60° (c) 360° (d) 120°

3 The numbers 5, 4, can be lengths of sides of a triangle.

- (a) 8 (b) 9 (c) 10 (d) 12

4 If M is the point of intersection of the medians of $\triangle ABC$ and D is the midpoint of \overline{BC} , then $AD = \dots\dots\dots$

- (a) $2 AM$ (b) $3 MD$ (c) $\frac{2}{3} MD$ (d) AM

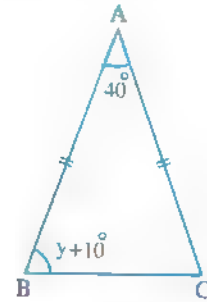
5 If $\triangle ABC$ is right-angled at B , then

- (a) $AC < AB$ (b) $AC > BC$ (c) $AB = AC$ (d) $BC > AC$

6 In the opposite figure :

$y = \dots\dots\dots$

- (a) 30°
- (b) 40°
- (c) 60°
- (d) 70°

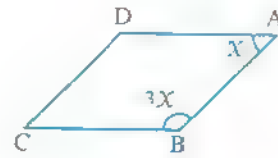


2 Complete the following :

- 1 In $\triangle XYZ$, if $XY = XZ$, $\overline{XL} \perp \overline{YZ}$, then \overline{XL} bisects each of $\dots\dots\dots$ and $\dots\dots\dots$.
- 2 The number of axes of symmetry of the isosceles triangle is $\dots\dots\dots$.
- 3 If ABC is a right-angled triangle at B, $AB = BC$, then $m(\angle C) = \dots\dots\dots^\circ$.
- 4 The longest side of the right-angled triangle is $\dots\dots\dots$.

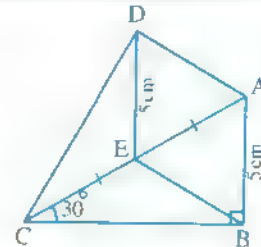
5 In the opposite figure :

ABCD is a parallelogram
 , then $X = \dots\dots\dots^\circ$



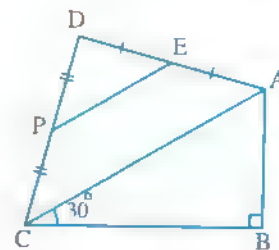
3 [a] In the opposite figure :

ABC is a right-angled triangle at B
 $m(\angle ACB) = 30^\circ$, $AB = 5$ cm.
 and E is the midpoint of \overline{AC}
 If $DE = 5$ cm. , prove that : $m(\angle ADC) = 90^\circ$



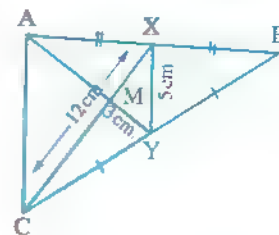
[b] In the opposite figure :

$m(\angle B) = 90^\circ$, $m(\angle ACB) = 30^\circ$
 E is the midpoint of \overline{AD}
 , P is the midpoint of \overline{CD}
 Prove that : $AB = EP$



4 [a] In the opposite figure :

M is the intersection point of the medians
 of $\triangle ABC$, $XY = 5$ cm.
 $CX = 12$ cm. , $MY = 3$ cm.
 Find with proof : The perimeter of $\triangle MAC$



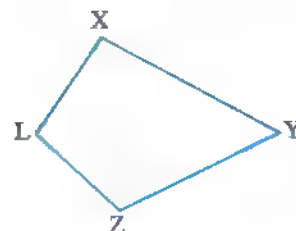
Geometry

[b] In the opposite figure :

$XY > XL$ and $YZ > ZL$

Prove that :

$m(\angle XLZ) > m(\angle XYZ)$



5 [a] In the opposite figure :

ABC is a triangle in which $AB = AC$

, \overrightarrow{AE} bisects $\angle BAC$

Prove that :

1 $BE = \frac{1}{2} BC$

2 $BD = CD$

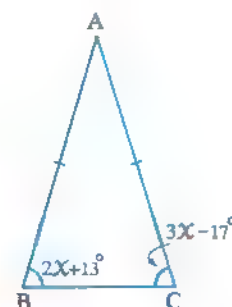
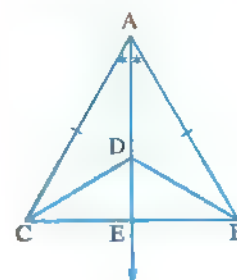
[b] In the opposite figure :

$AB = AC$, $m(\angle B) = 2x + 13^\circ$

, $m(\angle C) = 3x - 17^\circ$

Find :

The measures of the angles of $\triangle ABC$



11

Suez Governorate



Answer the following questions :

1 Choose the correct answer :

1 In $\triangle ABC$, if $AB = 3$ cm., $BC = 5$ cm., then $AC \in$

(a) $]3, 5[$

(b) $[3, 5]$

(c) $]2, 8[$

(d) $[2, 8]$

2 If the lengths of two sides of an isosceles triangle are 5 cm. and 10 cm., then the length of the third side is cm.

(a) 10

(b) 5

(c) 15

(d) 4

3 In $\triangle ABC$, if $m(\angle A) = 100^\circ$, then the longest side of it is

(a) \overline{AB}

(b) \overline{AC}

(c) \overline{BC}

(d) its median.

4 In $\triangle ABC$, if $2m(\angle A) = m(\angle B) + m(\angle C)$, then $m(\angle A) =$ °

(a) 45

(b) 90

(c) 60

(d) 120

5 If $A \in$ the axis of symmetry of \overline{BC} , then \overline{AB} \overline{AC}

(a) \equiv

(b) $=$

(c) $//$

(d) \perp

- 6 The point of intersection of the medians of the triangle divides each of them in the ratio from the vertex.

(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 1 : 2

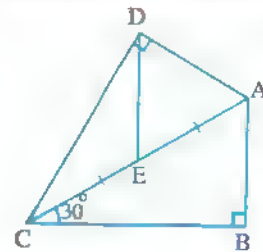
2 Complete :

- 1 The base angles of an isosceles triangle are in measure.
 2 If $\triangle ABC$ has one axis of symmetry and $m(\angle A) = 120^\circ$, then $m(\angle B) = \dots\dots^\circ$
 3 In $\triangle ABC$, if $AB > AC$, then $m(\angle C) > \dots\dots\dots$
 4 The bisector of the vertex angle of an isosceles triangle and
 5 In a triangle, if two angles are unequal in measure, then the greater angle in measure is opposite to

3 [a] In the opposite figure :

$m(\angle B) = 90^\circ$, $m(\angle ADC) = 90^\circ$
 $m(\angle ACB) = 30^\circ$
 \overline{DE} is a median in $\triangle ADC$

Prove that : $AB = DE$



- [b] In $\triangle ABC$, if $AB = 7$ cm., $BC = 5$ cm., $AC = 6$ cm., arrange the measures of the angles of the triangle ABC ascendingly.

4 [a] In the opposite figure :

$AB > BC$, $AD > CD$

Prove that :

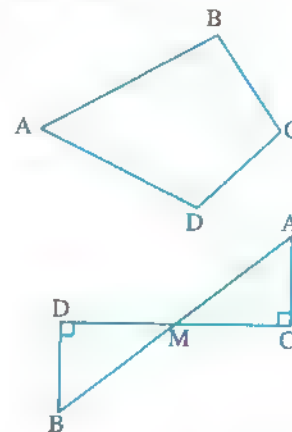
$m(\angle C) > m(\angle A)$

[b] In the opposite figure :

$\overline{AB} \cap \overline{CD} = \{M\}$

$m(\angle C) = m(\angle D) = 90^\circ$

Prove that : $AB > DC$

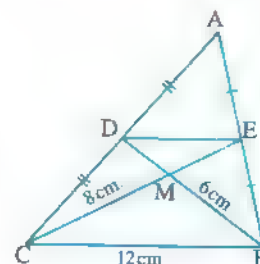


5 [a] In the opposite figure :

If D, E are the midpoints of \overline{AC} , \overline{AB}

$MB = 6$ cm., $MC = 8$ cm., $BC = 12$ cm.

Find : The perimeter of $\triangle MDE$

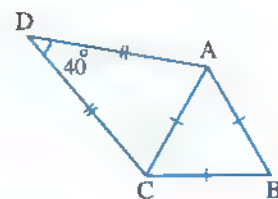


[b] In the opposite figure :

$$AB = BC = AC, DA = DC$$

$$, m(\angle D) = 40^\circ$$

Find : $m(\angle BAD)$



12

Port Said Governorate



Educational Directorate
Math Department

Answer the following questions :

1 Choose the correct answer :

- 1 In $\triangle ABC$, if $AC = 4$ cm. , $BC = 3$ cm. , then $m(\angle B)$ $m(\angle A)$
 (a) $>$ (b) $<$ (c) $=$ (d) \leq
- 2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals the length of the hypotenuse.
 (a) half (b) twice (c) third (d) quarter
- 3 In $\triangle ABC$, if $m(\angle A) = 100^\circ$ and $AB = AC$, then $m(\angle ABC) = \dots$
 (a) 80° (b) 60° (c) 40° (d) 30°
- 4 The point of intersection of the medians of the triangle divides each of them in the ratio from the base.
 (a) $1 : 3$ (b) $3 : 1$ (c) $1 : 2$ (d) $2 : 1$
- 5 If $\triangle ABD$ is obtuse-angled at B and C is the midpoint of \overline{BD} , then the longest side is
 (a) \overline{AB} (b) \overline{AC} (c) \overline{AD} (d) \overline{BD}
- 6 The triangle whose side lengths are 2 cm. , $(x + 3)$ cm. and 5 cm. , becomes an isosceles triangle when $x = \dots$ cm.
 (a) 1 (b) 2 (c) 3 (d) 4

2 Complete :

- 1 The median of an isosceles triangle from the vertex angle bisects and is perpendicular to
- 2 The measure of the exterior angle at any vertex of the equilateral triangle is $^\circ$
- 3 The base angles of the isosceles triangle are
- 4 ABC is a triangle in which $AB = 4$ cm. , $BC = 6$ cm. , then $AC \in] \dots , \dots [$
- 5 The longest side in the right-angled triangle is

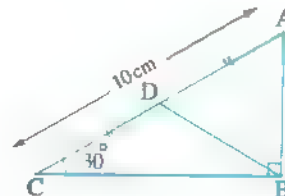
- 3 [a]** In $\triangle ABC$, if $m(\angle A) = (6x)^\circ$, $m(\angle B) = (4x - 9)^\circ$ and $m(\angle C) = 3(x - 2)^\circ$, arrange the side lengths of $\triangle ABC$ ascendingly.

[b] In the opposite figure :

$$m(\angle ABC) = 90^\circ, m(\angle C) = 30^\circ$$

$$AD = DC \text{ and } AC = 10 \text{ cm.}$$

Find : The perimeter of $\triangle ABD$



4 [a] In the opposite figure :

$$\text{If } \overline{AC} \cap \overline{BD} = \{M\}$$

$$\text{, } \overline{AD} \parallel \overline{BC} \text{ and } MB = MC$$

, prove that :

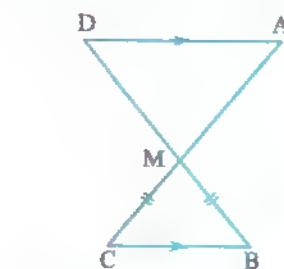
$\triangle MAD$ is isosceles.

[b] In the opposite figure :

$$m(\angle BAC) = 70^\circ, m(\angle B) = 55^\circ$$

$$\text{and } m(\angle ACD) = 90^\circ$$

Prove that : $AD > AB$



5 [a] In the opposite figure :

$$m(\angle D) = 40^\circ, DA = DC$$

and $\triangle ABC$ is equilateral

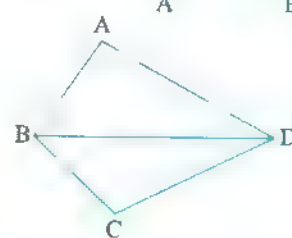
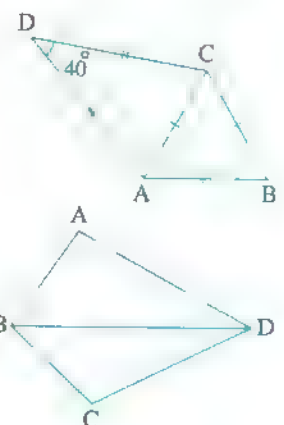
Find : $m(\angle DCB)$

[b] In the opposite figure :

$$AB < AD \text{ and } BC < CD$$

Prove that :

$$m(\angle ABC) > m(\angle ADC)$$



Answer the following questions :


1 Complete each of the following :

- 1 If the measure of one of the base angles of an isosceles triangle equals 50° , then the measure of the vertex angle equals $^\circ$
- 2 The supplementary of the obtuse angle is angle.

Geometry

- 3 The longest side in the right-angled triangle is
- 4 The perpendicular straight line on a line segment from its midpoint is called
- 5 If 4 cm. , 7 cm. are the lengths of two sides in a triangle , then < the length of the third side <

2 Choose the correct answer :

- 1 The point of intersection of the medians of the triangle divides each of them in the ratio of from the base.
 (a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3
- 2 In $\triangle ABC$, if $m(\angle B) = 70^\circ$, $m(\angle C) = 50^\circ$, then AB AC
 (a) $>$ (b) $<$ (c) $=$ (d) \geq
- 3 The number of the quadrilaterals in the figure  is
 (a) 3 (b) 4 (c) 5 (d) 6
- 4 In the right-angled triangle , the length of the median from the vertex of the right angle equals the length of the hypotenuse.
 (a) $\frac{1}{2}$ (b) double (c) $\frac{1}{3}$ (d) $\frac{1}{4}$
- 5 The sum of the measures of the accumulative angles at a point equals $^\circ$
 (a) 90 (b) 180 (c) 360 (d) 308
- 6 The number of lines of symmetry of $\triangle ABC$ in which $AB = AC$, $m(\angle B) = 60^\circ$ is
 (a) 3 (b) 2 (c) 1 (d) zero

3 [a] In the opposite figure :

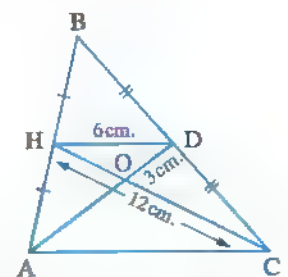
$HD = 6$ cm. , $HC = 12$ cm.

, H is the midpoint of \overline{AB}

and D is the midpoint of \overline{BC}

, $DO = 3$ cm.

Calculate : The perimeter of the triangle AOC

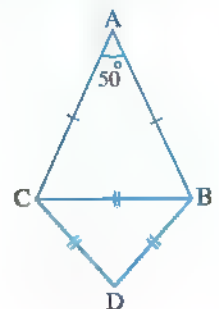


[b] In the opposite figure :

$AB = AC$, $m(\angle A) = 50^\circ$

$\triangle CDB$ is equilateral.

Find with proof : $m(\angle ABD)$



4 [a] In the opposite figure :

$$AB = AC, BD < CD$$

Prove that :

$$m(\angle ABD) > m(\angle ACD)$$

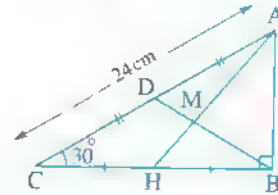
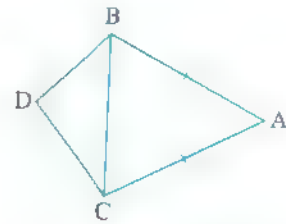
[b] In the opposite figure :

$\triangle ABC$ is right-angled at B

, $\overline{AH}, \overline{BD}$ are two medians

$$, m(\angle C) = 30^\circ, AC = 24 \text{ cm.}$$

Find : The length of each of $\overline{AB}, \overline{BD}, \overline{BM}$



5 [a] In the opposite figure :

\overline{BD} bisects $\angle ABC$

, $\overline{HD} \parallel \overline{BC}$

Prove that :

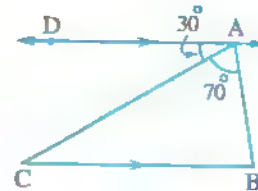
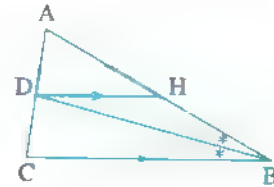
$\triangle HBD$ is an isosceles triangle.

[b] In the opposite figure :

$\overline{AD} \parallel \overline{BC}, m(\angle BAC) = 70^\circ$

$$, m(\angle DAC) = 30^\circ$$

Prove that : $AC > BC$



14 El-Fayoum Governorate



Test 51, Session 7

Mathematics 1101

Answer the following questions :

1 Choose the correct answer from those given :

1 In $\triangle ABC$, if $(AB)^2 = (BC)^2 - (AC)^2$, $m(\angle C) = 42^\circ$, then $m(\angle B) = \dots$

- (a) 40° (b) 90° (c) 48° (d) 110°

2 The scalene triangle has axes of symmetry.

- (a) 3 (b) 2 (c) 1 (d) 0

3 If A lies on the axis of symmetry of $\triangle ABC$, then $AB \dots AC$

- (a) $<$ (b) $>$ (c) $=$ (d) \leq

Geometry

- 4 If \overline{AD} is a median of $\triangle ABC$, M is the point of concurrence of the medians, then $MD = \dots\dots\dots AD$
 (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
- 5 If 10 cm., 5 cm. and X cm. are side lengths of an isosceles triangle, then $X = \dots\dots\dots$ cm.
 (a) 5 (b) 10 (c) 15 (d) 4
- 6 The measure of the exterior angle of the equilateral triangle equals
 (a) 60° (b) 90° (c) 50° (d) 120°

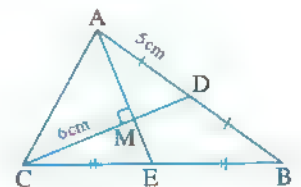
2 Complete the following :

- 1 The total area of a cuboid = 120 cm^2 and its lateral area = 96 cm^2 , then the area of its base equals $\dots\dots\dots \text{ cm}^2$
- 2 The base angles of the isosceles triangle are
- 3 ABC is a right-angled triangle at B , $m(\angle C) = 30^\circ$, $AB = 5 \text{ cm}$, then $AC = \dots\dots\dots \text{ cm}$.
- 4 In $\triangle ABC$, if $m(\angle C) = 30^\circ$, $m(\angle A) = 70^\circ$, then the smallest side in length is
- 5 In any triangle, if the lengths of two sides are not equal, then the greater side in length is opposite to

3 [a] In the opposite figure :

M is the concurrence point of the medians of $\triangle ABC$,
 $\overline{AM} \perp \overline{CD}$, $AD = 5 \text{ cm}$, $MC = 6 \text{ cm}$.

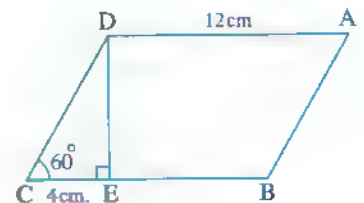
Find with proof : The length of \overline{ME}



[b] In the opposite figure :

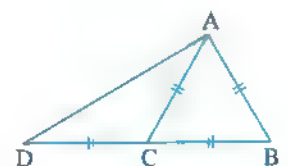
$ABCD$ is a parallelogram
 $m(\angle C) = 60^\circ$, $\overline{DE} \perp \overline{BC}$
 $AD = 12 \text{ cm}$, $CE = 4 \text{ cm}$.

Find with proof : The perimeter of the parallelogram $ABCD$



4 [a] In the opposite figure :

ABC is an equilateral triangle
 $D \in \overline{BC}$, $BC = CD$
 Prove that : $\overline{AB} \perp \overline{AD}$



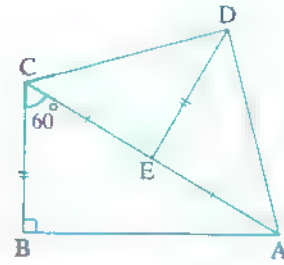
[b] In the opposite figure :

ABC is a right-angled triangle at B

, $m(\angle ACB) = 60^\circ$, E is the midpoint of \overline{AC}

, $DE = BC$

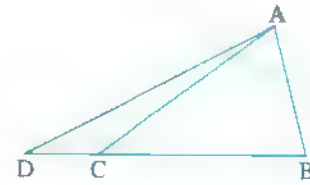
Prove that : $m(\angle ADC) = 90^\circ$



5 [a] In the opposite figure :

$C \in \overline{BD}$, $AC > AB$

Prove that : $m(\angle B) > m(\angle D)$

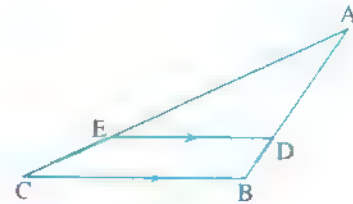


[b] In the opposite figure :

ABC is an obtuse-angled triangle at B

, $\overline{DE} \parallel \overline{BC}$

Prove that : $AE > AD$



Answer the following questions :

1 Complete the following :

- [1] In the right-angled triangle , the is the longest side.
- [2] In $\triangle ABC$, if D is the midpoint of \overline{BC} and $AD = \frac{1}{2} BC$, then $m(\angle A) = \dots^\circ$
- [3] In $\triangle ABC$, if $m(\angle B) = 65^\circ$ and $m(\angle C) = 50^\circ$, then the shortest side in $\triangle ABC$ is
- [4] In $\triangle ABC$, if the point X is the midpoint of \overline{BC} , then \overline{AX} is called
- [5] The measure of the exterior angle of the equilateral triangle is°

2 Choose the correct answer :

- [1] In $\triangle ABC$, if $m(\angle B) > m(\angle C)$, then
 - (a) $AB < AC$
 - (b) $AB = AC$
 - (c) $AB > AC$
 - (d) $\overline{AB} \equiv \overline{AC}$
- [2] The point of concurrence of the medians of the triangle divides each median in the ratio of from the base.
 - (a) 1 : 2
 - (b) 1 : 3
 - (c) 2 : 1
 - (d) 3 : 1

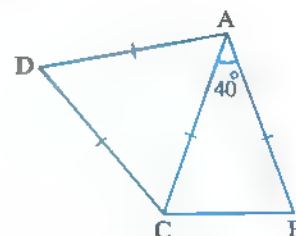
- 3 The lengths of two sides in a triangle are 4 cm. , 9 cm. and it has one axis of symmetry , then the length of the third side is cm.
 (a) 4 (b) 5 (c) 9 (d) 13
- 4 The number of axes of symmetry of the equilateral triangle equals
 (a) 0 (b) 1 (c) 2 (d) 3
- 5 If $\triangle ABC$ is right-angled at B , $AB = 6$ cm. , $BC = 8$ cm. , then the length of the median drawn from B is cm.
 (a) 10 (b) 8 (c) 6 (d) 5
- 6 The lengths which can be lengths of sides of a triangle are
 (a) 0 , 3 , 5 (b) 3 , 3 , 5 (c) 3 , 3 , 6 (d) 3 , 3 , 7

3 [a] In the opposite figure :

$$AB = AC = AD = CD$$

$$, m(\angle BAC) = 40^\circ$$

Find : $m(\angle BCD)$



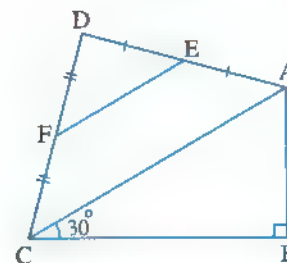
[b] In the opposite figure :

$$m(\angle B) = 90^\circ , m(\angle ACB) = 30^\circ$$

, E is the midpoint of \overline{AD}

, F is the midpoint of \overline{CD}

Prove that : $AB = EF$



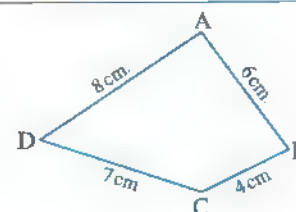
4 [a] In the opposite figure :

ABCD is a quadrilateral in which :

$$AB = 6 \text{ cm. , } BC = 4 \text{ cm.}$$

$$, CD = 7 \text{ cm. , } DA = 8 \text{ cm.}$$

Prove that : $m(\angle BCD) > m(\angle BAD)$



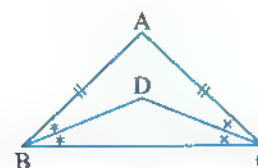
[b] In the opposite figure :

ABC is a triangle in which :

$$AB = AC , \overline{BD} \text{ bisects } \angle ABC$$

$$, \overline{CD} \text{ bisects } \angle ACB$$

Prove that : $\triangle DBC$ is an isosceles triangle.

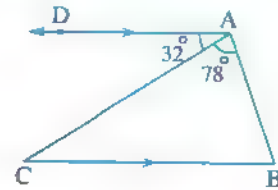


5 [a] In the opposite figure :

$\overrightarrow{AD} \parallel \overrightarrow{BC}$, $m(\angle BAC) = 78^\circ$

, $m(\angle CAD) = 32^\circ$

Prove that : $AC > AB$



[b] In the opposite figure :

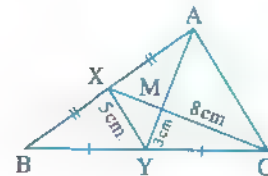
ABC is a triangle , X is the midpoint of \overline{AB}

, Y is the midpoint of \overline{BC}

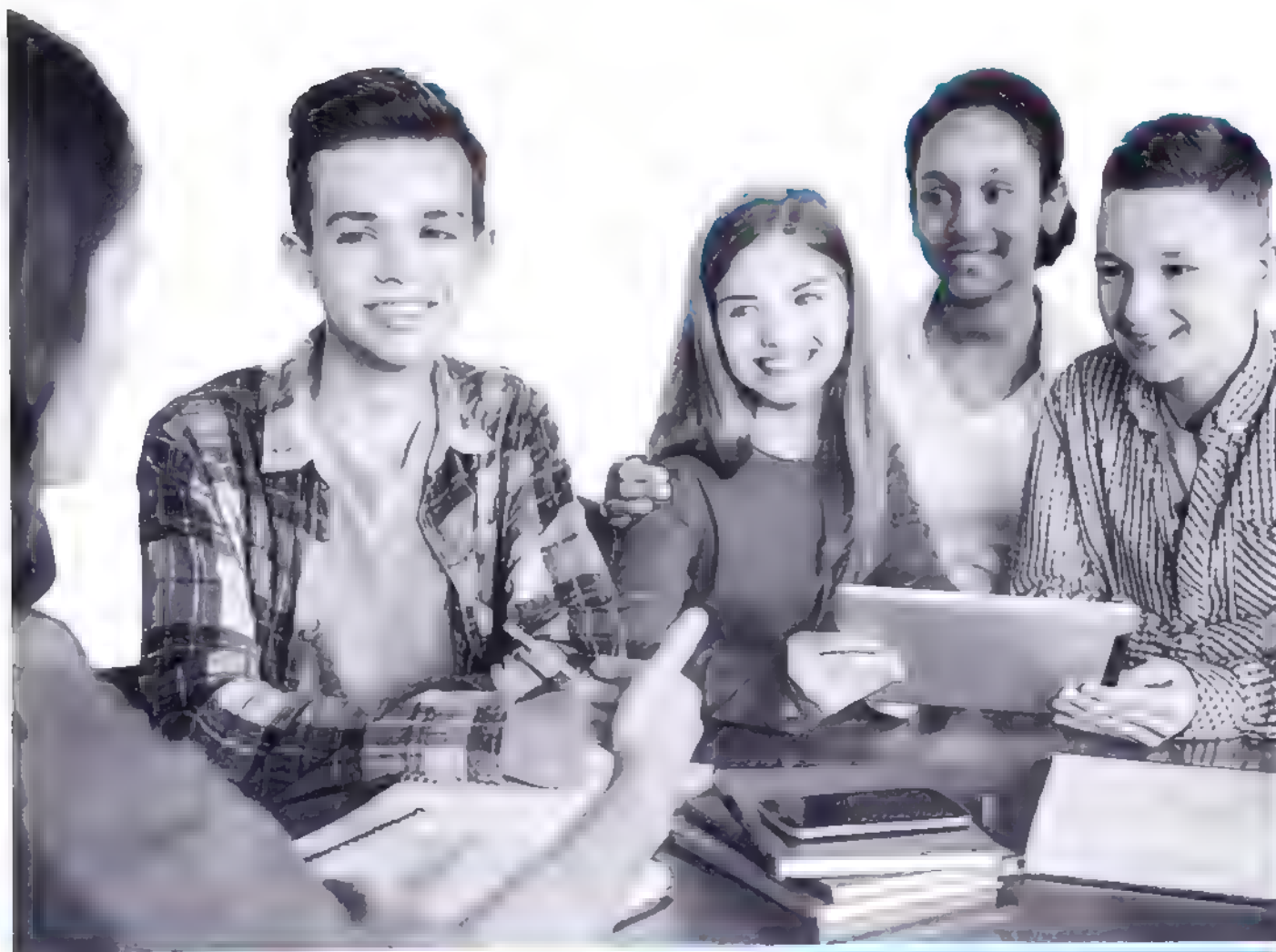
, $\overline{XC} \cap \overline{AY} = \{M\}$, $XY = 5$ cm.

, $CM = 8$ cm. , $YM = 3$ cm.

Find : The perimeter of $\triangle MAC$



Multidisciplinary Exams



Selected math exams from the multidisciplinary exams of the previous year



Cairo Governorate



West Cairo Educational Directorate

Choose the correct answer :

- 1 The slope of the straight line passes through A (3 , 5) and B (5 , - 1) is
 (a) $-\frac{1}{3}$ (b) - 3 (c) 3 (d) $\frac{1}{3}$
- 2 The ordered pair (2 , 5) satisfies the relation
 (a) $y = 3x - 1$ (b) $y = 3x$ (c) $y = 3x + 1$ (d) $y = x - 2$
- 3 The solution set of the equation : $x^3 + 8 = 0$ in \mathbb{R} is
 (a) {4} (b) {2} (c) \emptyset (d) {- 2}
- 4 The median of the values : 34 , 23 , 25 , 40 , 22 , 4 is
 (a) 22 (b) 23 (c) 24 (d) 25
- 5 The two diagonals are equal in the
 (a) rectangle. (b) rhombus. (c) trapezium. (d) triangle.
- 6 In ΔABC , $m(\angle A) = 100^\circ$, and $AB = AC$, then $m(\angle ABC) =$
 (a) 30° (b) 80° (c) 60° (d) 40°
- 7 XYZ is a triangle , $m(\angle Z) = 70^\circ$, $m(\angle Y) = 60^\circ$, then ZY XY
 (a) > (b) < (c) = (d) twice
- 8 If ΔABC is right-angled at B , $AC = 20$ cm. , then the length of the median drawn from B is cm.
 (a) 10 (b) 8 (c) 6 (d) 5



Giza Governorate



El-Dokki Directorate
of Education

Choose the correct answer :

- 1 $(\sqrt{8} + \sqrt{2})(\sqrt{8} - \sqrt{2}) =$
 (a) 8 (b) 6 (c) 5 (d) 4
- 2 The solution set of the equation : $x(x^3 - 1) = 0$ in \mathbb{R} is
 (a) {0} (b) {0 , - 1} (c) {1} (d) {0 , 1}

- 3 The volume of cube = 27 cm^3 , then the area of one face of it is ... cm^2
 (a) 3 (b) 9 (c) 36 (d) 54
- 4 If the mode of the values : 4 , 11 , 8 , 2 X is 4 , then X = ...
 (a) 2 (b) 4 (c) 6 (d) 8
- 5 ΔABC is right-angled at B , if $AC = 20 \text{ cm}$. , then the length of the median from the vertex of B is cm .
 (a) 5 (b) 6 (c) 8 (d) 10
- 6 The number of symmetric axes of isosceles triangle is
 (a) 3 (b) 2 (c) 1 (d) nothing.
- 7 ΔXYZ , $m(\angle Z) = 80^\circ$ and $m(\angle Y) = 70^\circ$, then YZ XY
 (a) $>$ (b) $<$ (c) $=$ (d) twice
- 8 If the measure of one of the two base angles in an isosceles triangle is 40° , then the measure of its vertex =
 (a) 100° (b) 60° (c) 50° (d) 30°



Choose the correct answer :

- 1 If the mode of the set of values : 6 , 10 , 12 , 14 , 3 X is 6 , then X =
 (a) 8 (b) 6 (c) 4 (d) 2
- 2 Let A (1 , 6) and B (3 , 0) , then the slope of \overrightarrow{AB} =
 (a) -3 (b) 3 (c) $-\frac{1}{3}$ (d) $\frac{1}{3}$
- 3 If the volume of the sphere is $\frac{4}{3} \pi \text{ cm}^3$, then its diameter length is ... cm .
 (a) 1 (b) 2 (c) 3 (d) 4
- 4 $[-1, 3] \cap [-3, -1] = \dots\dots\dots$
 (a) $\{-1\}$ (b) $\{-3\}$ (c) \emptyset (d) $\{3\}$
- 5 If the lengths of two sides in an isosceles triangle are 6 cm. and 3 cm. , then the length of the third side is ...
 (a) 3 (b) 4 (c) 6 (d) 12
- 6 XYZ is a triangle in which $m(\angle X) = 70^\circ$ and $m(\angle Y) = 60^\circ$, then YZ XY
 (a) $>$ (b) $<$ (c) $=$ (d) twice

[7] ABC is a triangle in which $AB = 3$ cm. , $BC = 5$ cm. , then $AC \in \dots\dots\dots$

- (a) $[2, 8]$ (b) $]2, 8[$ (c) $]2, 8]$ (d) $[2, 8[$

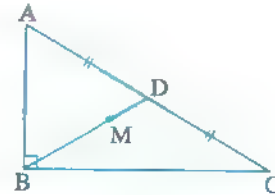
[8] In the opposite figure :

$m(\angle ABC) = 90^\circ$, $AC = 12$ cm.

, D is the midpoint of \overline{AC}

, M is the point of intersection medians

, then $BM = \dots\dots\dots$ cm.



- (a) 6 (b) 2
(c) 3 (d) 4

4 El-Sharkia Governorate



Kafr Sakr Educational Directorate

Choose the correct answer :

[1] The solution set of : $x^2 + 16 = 0$ in \mathbb{R} is $\dots\dots\dots$

- (a) 4 (b) -4 (c) 16 (d) \emptyset

[2] If $(k, 2)$ satisfies the relation $x + 2y = 5$, then $k = \dots\dots\dots$

- (a) 1 (b) 2 (c) 3 (d) 4

[3] The median of the values : 13 , 15 , 10 , 8 , 23 is $\dots\dots\dots$

- (a) 13 (b) 10 (c) 15 (d) 8

[4] The radius length of a sphere is 6 cm. , its volume = $\dots\dots\dots \pi \text{ cm}^3$.

- (a) 288 (b) 72 (c) 36 (d) 6

[5] If the measures of two angles in a triangle are 70° and 40° , then its type according to sides is $\dots\dots\dots$

- (a) equilateral. (b) isosceles. (c) scalene. (d) right.

[6] In $\triangle ABC$, if $m(\angle A) = 30^\circ$, $m(\angle B) = 90^\circ$, $AC = 10$ cm. , then $BC = \dots\dots\dots$ cm.

- (a) 4 (b) 5 (c) 10 (d) 6

[7] The point of intersection of the medians of the triangle divides each median with the ratio $\dots\dots\dots$ from the base.

- (a) 1 : 3 (b) 2 : 1 (c) 1 : 2 (d) 2 : 3

[8] In $\triangle ABC$, if $AB = 6$ cm. , $AC = 7$ cm. , then $BC \in \dots\dots\dots$

- (a) $]1, 13[$ (b) $[6, 7]$ (c) $]6, 13[$ (d) $[7, 13]$



Choose the correct answer :

- 1 The additive inverse of the number : $3 - \sqrt{2}$ is
 (a) $3 + \sqrt{2}$ (b) $-3 - \sqrt{2}$ (c) $-\sqrt{2} + 3$ (d) $\sqrt{2} - 3$
- 2 The S.S. in \mathbb{R} of the equation : $x^2 + 4 = 0$ is
 (a) $\{0\}$ (b) \emptyset (c) $\{2, -2\}$ (d) $\{2\}$
- 3 If the point $(a, 3)$ satisfies the relation : $2x + y = 5$, then $a =$
 (a) 1 (b) 2 (c) 3 (d) 4
- 4 If the order of the median of a set of values is the fourth , then the number of values =
 (a) 4 (b) 5 (c) 6 (d) 7
- 5 The triangle whose measures of two angles are 70° and 40° is triangle.
 (a) an equilateral (b) an isosceles (c) a scalene (d) a right-angled
- 6 The measure of the exterior angle of an equilateral triangle equals
 (a) 120 (b) 90 (c) 60 (d) 30
- 7 If the measure of the vertex angle of an isosceles triangle equals 60° , then the triangle has axes of symmetry.
 (a) zero (b) one (c) two (d) three
- 8 The point of intersection of the medians of the triangle divides each of them in the ratio from the vertex.
 (a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1



Choose the correct answer :

- 1 In $\triangle ABC$, if $m(\angle C) = 120^\circ$, then the longest side in it is
 (a) \overline{BC} (b) \overline{AC} (c) \overline{AB} (d) its median.
- 2 Any triangle has medians.
 (a) 1 (b) 2 (c) 3 (d) 4
- 3 The conjugate of $\sqrt{3} - \sqrt{2}$ is
 (a) $\sqrt{3} + \sqrt{2}$ (b) $\sqrt{6}$ (c) $-\sqrt{3} + \sqrt{2}$ (d) $\sqrt{3} - \sqrt{2}$

- 4 The additive inverse of $\sqrt{3}$ is
- (a) $-\sqrt{3}$ (b) 3 (c) -3 (d) $\sqrt{3}$
- 5 $\sqrt{9} = \sqrt[3]{\dots}$
- (a) 6 (b) 18 (c) 8 (d) 27
- 6 $[3, 7] - \{3\} = \dots$
- (a) $]3, 7[$ (b) $[3, 7[$ (c) $[3, 7]$ (d) $]3, 7]$
- 7 The measure of the exterior angle of the equilateral triangle equals°
- (a) 30 (b) 60 (c) 90 (d) 120
- 8 The point of intersection of the medians of the triangle divides each median in the ratio of from the base.
- (a) 2 : 1 (b) 2 : 3 (c) 1 : 2 (d) 1 : 3



Kafr El-Sheikh Governorate

East Kafr El-Sheikh
Educational Directorate

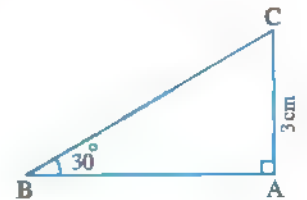
Choose the correct answer :

- 1 The median of the numbers : 8 , 2 , 7 , 5 , 4 is
- (a) 2 (b) 3 (c) 4 (d) 5
- 2 If the length of edge of a cube is 2 cm. , then its volume = cm.³
- (a) 4 (b) 8 (c) 16 (d) 32
- 3 If the point (a , 1) satisfies the relation : $x + y = 4$, then a =
- (a) 1 (b) 2 (c) 3 (d) 4
- 4 The slope of the straight line which parallel to X-axis =
- (a) undefined. (b) 0 (c) 1 (d) 2
- 5 The number of medians in the right-angled triangle is
- (a) 1 (b) 2 (c) 3 (d) 4
- 6 3 cm. , 4 cm. and cm. are lengths of sides of triangle.
- (a) 1 (b) 5 (c) 7 (d) 9
- 7 In $\triangle ABC$: $AB + BC$ AC
- (a) = (b) > (c) < (d) //
- 8 If ABC is a right-angled triangle at B , then the longest side is
- (a) \overline{AB} (b) \overline{BC} (c) \overline{CD} (d) \overline{AC}



Choose the correct answer :

- 1 If $B(1, -2)$, $A(-3, 6)$, then the slop of \overrightarrow{AB} is
 (a) 4 (b) 6 (c) -2 (d) -4
- 2 The arithmetic mean of : 7 , 8 , 9 , 6 , 10 is
 (a) 3 (b) 5 (c) 8 (d) 4
- 3 If $x = 2\sqrt{2} - \sqrt{7}$, $y = 2\sqrt{2} + \sqrt{7}$, then $xy - 1 =$
 (a) 5 (b) 0 (c) -4 (d) 7
- 4 The S.S. of inequality $1 \leq x + 5 \leq 7$ at \mathbb{R} is
 (a) $[-4, 2[$ (b) $[-4, 2]$ (c) $[-4, 2]$ (d) $]-\infty, -2[$
- 5 If M is the concurrent point of the medians in ΔABC , \overline{BD} is median its length = 9 cm.
 , then $BM : MD =$
 (a) 1 : 2 (b) 3 : 1 (c) 2 : 1 (d) 2 : 3
- 6 If ABC is a triangle , $m(\angle B) = 70^\circ$, $m(\angle C) = 50^\circ$, then AB AC
 (a) < (b) > (c) = (d) \geq
- 7 In the opposite figure :
 $m(\angle B) = 30^\circ$, $AC = 3$ cm.
 , then $BC =$ cm.
 (a) 3 (b) 5
 (c) 6 (d) 8
- 8 If ABC is a triangle , $AB = 3$ cm. , $BC = 5$ cm. , then $AC \in$
 (a) $[2, 8]$ (b) $]2, 8]$ (c) $]2, 8[$ (d) $\{2, 8\}$



Choose the correct answer :

- 1 If the volume of a cube is 27 cm^3 , then the area of one of its faces = cm^2
 (a) 3 (b) 9 (c) 36 (d) 8
- 2 If $(-1, 5)$ satisfies the relation : $3x + ky = 7$, then $k =$
 (a) 5 (b) 6 (c) 2 (d) 7
- 3 If the mode of the numbers : 5 , 8 , $6 + x$, 9 is 9 , then $x =$
 (a) 7 (b) 3 (c) 4 (d) 5

4 $2 \in \dots$

- (a) $]2, 5]$ (b) $]2, 5[$ (c) $\{1, 5\}$ (d) $[1, 5[$

5 Which of the following sets can represent the lengths of the sides of a triangle ?

- (a) $\{2, 3, 4\}$ (b) $\{2, 3, 5\}$ (c) $\{2, 3, 6\}$ (d) $\{2, 3, 7\}$

6 The equilateral triangle has of symmetry.

- (a) one axis (b) two axes (c) three axes (d) no axes

7 If the measure of the vertex angle in an isosceles triangle is 80° , then the measure of one of its base angles is

- (a) 80° (b) 40° (c) 50° (d) 90°

8 The point of the intersection of the medians of a triangle divides each median in the ratio from the base.

- (a) $2 : 1$ (b) $1 : 2$ (c) $3 : 1$ (d) $1 : 3$

10

Qena Governorate



Nakkada Educational Directorate

Choose the correct answer :

1 The volume of sphere whose radius length 6 cm. is cm^3

- (a) 36π (b) 288π (c) 9π (d) 12π

2 The S.S. of the inequality : $-2 < 3x + 7 \leq 10$ in \mathbb{R} is

- (a) $[-3, 1[$ (b) $[-3, 1]$ (c) $[1, 3]$ (d) $]-3, 1]$

3 If $x = \sqrt{5} + 2$, $y = \sqrt{5} - 2$, then $\frac{x-y}{xy} = \dots\dots\dots$

- (a) 4 (b) $2\sqrt{5}$ (c) 0 (d) 3

4 If the mode of the values : 4, 11, 8, 2, x is 4, then $x = \dots\dots\dots$

- (a) 2 (b) 4 (c) 6 (d) 8

5 The triangle which has two angles of measures 42° , 69° is

- (a) isosceles. (b) equilateral. (c) scalene. (d) right-angled.

6 In $\triangle ABC$, if $AB = 4$ cm., $BC = 6$ cm., then the length of $\overline{AC} \in] \dots\dots, \dots\dots [$

- (a) 2, 10 (b) 3, 5 (c) 4, 6 (d) 1, 5

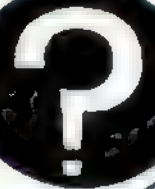
7 ABC is a right-angled triangle at B , $AC = 20$ cm., then the length of the median drawn from B is cm.

- (a) 5 (b) 10 (c) 15 (d) 20

8 The number of the medians in the isosceles triangle

- (a) 0 (b) 1 (c) 2 (d) 3

Some Schools Examinations



on Geometry

1

Cairo Governorate

Centre Cairo Educative Zone
Saint Joseph College Khoronfish

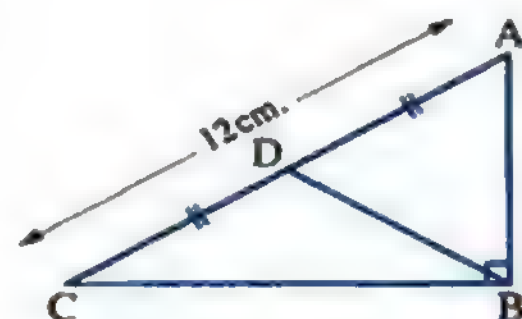
Answer the following questions :

1 Choose the correct answer from the given ones :

- 1 In $\triangle ABC$, if $AB = 6$ cm. and $AC = 7$ cm. , then $BC \in \dots\dots\dots$
 (a) $]6, 13]$ (b) $[6, 7]$ (c) $]1, 13[$ (d) $[1, 7[$
- 2 The point of intersection of the medians of the triangle divides each of them in the ratio of $\dots\dots\dots$ from the vertex.
 (a) $1 : 2$ (b) $1 : 3$ (c) $2 : 1$ (d) $2 : 3$
- 3 The measure of any exterior angle of the equilateral triangle equals $\dots\dots\dots^\circ$
 (a) 60 (b) 100 (c) 120 (d) 150
- 4 In $\triangle ABC$, if \overline{AD} is a median , M is the point of intersection of its medians , then $AM = \dots\dots\dots AD$
 (a) $\frac{1}{2}$ (b) 2 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$
- 5 $\triangle XYZ$ is an isosceles triangle in which $m(\angle X) = 110^\circ$, then $m(\angle Y) = \dots\dots\dots^\circ$
 (a) 110 (b) 35 (c) 60 (d) 45
- 6 In $\triangle ABC$, if $\overline{AB} \perp \overline{BC}$ and $AB = BC$, then $m(\angle A) = \dots\dots\dots^\circ$
 (a) 30 (b) 45 (c) 60 (d) 90

2 Complete the following :

- 1 The number of axes of symmetry of the equilateral triangle equals $\dots\dots\dots$
- 2 The base angles in an isosceles triangle are $\dots\dots\dots$
- 3 The longest side in the right-angled triangle is $\dots\dots\dots$
- 4 The bisector of the vertex angle of the isosceles triangle $\dots\dots\dots$
- 5 In the opposite figure :

AC = 12 cm. , then BD = $\dots\dots\dots$ cm.

3 [a] In $\triangle ABC$, if $m(\angle A) = (6x)^\circ$, $m(\angle B) = (4x - 9)^\circ$

and $m(\angle C) = 3(x - 2)^\circ$

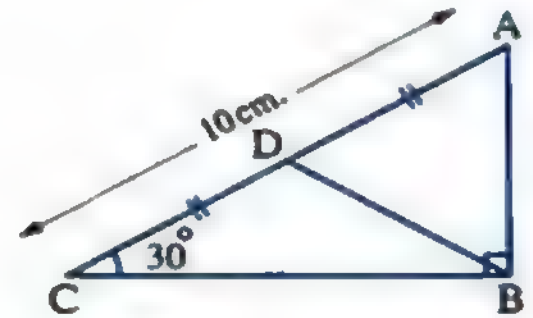
Arrange the side lengths of $\triangle ABC$ ascendingly.

[b] In the opposite figure :

$m(\angle ABC) = 90^\circ$, $m(\angle C) = 30^\circ$

, $AD = DC$ and $AC = 10$ cm.

Find : The perimeter of $\triangle ABD$



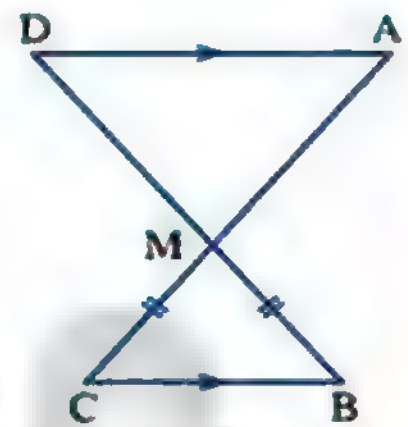
4 [a] In the opposite figure :

If $\overline{AC} \cap \overline{BD} = \{M\}$

, $\overline{AD} \parallel \overline{BC}$ and $MB = MC$

, prove that :

$\triangle MAD$ is isosceles.

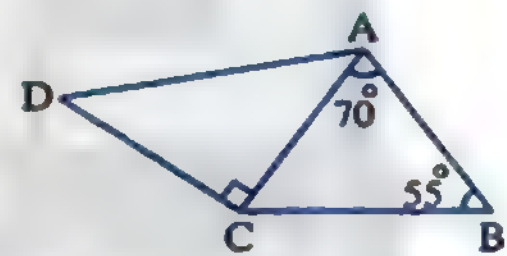


[b] In the opposite figure :

$m(\angle BAC) = 70^\circ$, $m(\angle B) = 55^\circ$

and $m(\angle ACD) = 90^\circ$

Prove that : $AD > AB$



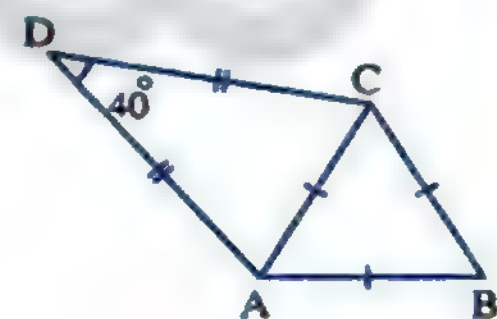
5 [a] In the opposite figure :

$m(\angle D) = 40^\circ$

, $DA = DC$

and $\triangle ABC$ is an equilateral triangle.

Find : $m(\angle DCB)$

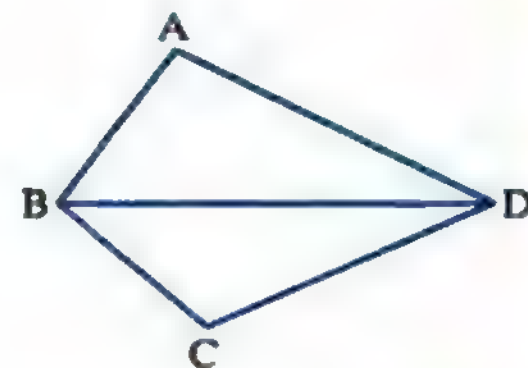


[b] In the opposite figure :

$AB < AD$ and $BC < CD$

Prove that :

$m(\angle ABC) > m(\angle ADC)$



Geometry

2

Cairo Governorate

Hadeik El-Kobba Educational Zone



Answer the following questions :

1 Complete :

- 1 The median of an isosceles triangle from the vertex angle bisects and is perpendicular to
- 2 The measure of the exterior angle at any vertex of the equilateral triangle is°
- 3 The base angles of the isosceles triangle are
- 4 ABC is a triangle in which $AB = 4$ cm. , $BC = 6$ cm. , then $AC \in]..... ,[$
- 5 The longest side in the right-angled triangle is

2 Choose the correct answer :

- 1 In ΔABC , if $AC = 4$ cm. , $BC = 3$ cm. , then $m(\angle B) \dots\dots\dots m(\angle A)$
 (a) $>$ (b) $<$ (c) $=$ (d) \leq
- 2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals the length of the hypotenuse.
 (a) half (b) twice (c) third (d) quarter
- 3 In ΔABC , if $m(\angle A) = 100^\circ$ and $AB = AC$, then $m(\angle ABC) = \dots\dots\dots$
 (a) 80° (b) 60° (c) 40° (d) 30°
- 4 The point of intersection of the medians of the triangle divides each of them in the ratio from the base.
 (a) $1 : 3$ (b) $3 : 1$ (c) $1 : 2$ (d) $2 : 1$
- 5 If ΔABD is obtuse-angled at B and C is the midpoint of \overline{BD} , then the longest side is
 (a) \overline{AB} (b) \overline{AC} (c) \overline{AD} (d) \overline{BD}
- 6 The triangle whose side lengths are 2 cm. , $(x + 3)$ cm. and 5 cm. becomes an isosceles triangle when $x = \dots\dots\dots$ cm.
 (a) 1 (b) 2 (c) 3 (d) 4

Final Examinations

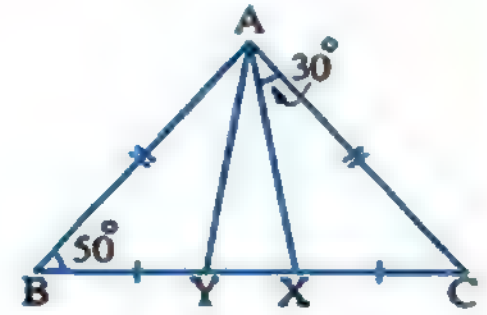
3 [a] In the opposite figure :

ABC is a triangle , $AB = AC$, $XC = YB$

, $m(\angle B) = 50^\circ$, $m(\angle CAX) = 30^\circ$

1 Prove that : $\triangle AXY$ is an isosceles triangle.

2 Find : $m(\angle AYB)$

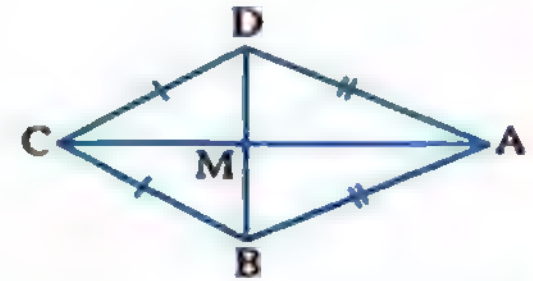


[b] In the opposite figure :

$\overline{BD} \cap \overline{AC} = \{M\}$

, $AB = AD$ and $BC = DC$

Prove that : M is the midpoint of \overline{BD}

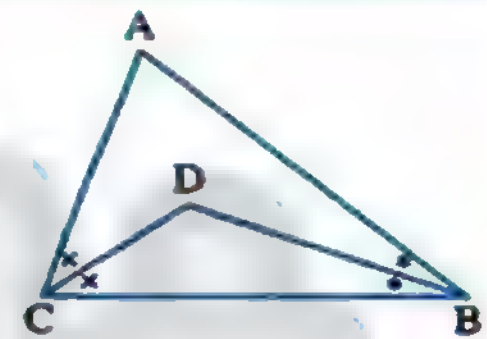


4 [a] In the opposite figure :

ABC is a triangle in which $AB > AC$, \overline{BD} bisects $\angle ABC$

, \overline{CD} bisects $\angle ACB$

Prove that : $BD > CD$

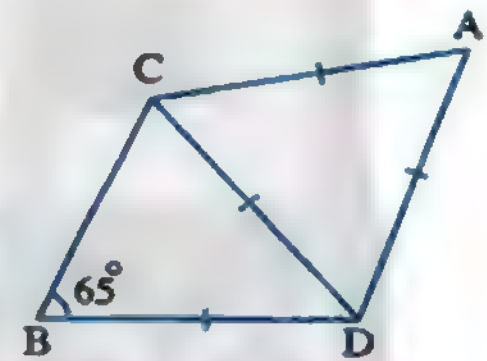


[b] In the opposite figure :

$AD = DC = AC = BD$

, $m(\angle B) = 65^\circ$

Find with proof : $m(\angle BDA)$



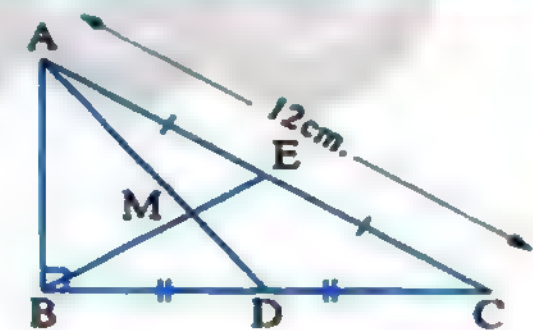
5 [a] In the opposite figure :

$\triangle ABC$ is right-angled at B

, E and D are the midpoints of \overline{AC} and \overline{BC} respectively

, $AC = 12$ cm.

Find the length of each of : \overline{BE} and \overline{ME}



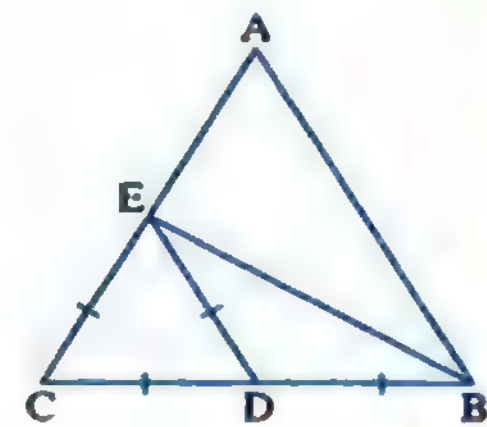
[b] In the opposite figure :

ABC is a triangle , $D \in \overline{BC}$ and $E \in \overline{AC}$

such that $BD = CD = CE = DE$

Prove that : 1 $BC > BE$

2 $AB + BD > AE$



Geometry

3

Cairo Governorate

Rod El-Farag Educational Zone
S.T. Mary's School

Answer the following questions :

1 Choose the correct answer from the given ones :

- 1 In the triangle XYZ , if $m(\angle Z) = 70^\circ$ and $m(\angle Y) = 60^\circ$, then YZ XY
(a) $>$ (b) $=$ (c) $<$ (d) twice
- 2 The measure of the exterior angle of the equilateral triangle equals
(a) 45° (b) 60° (c) 90° (d) 120°
- 3 The intersection point of the medians of a triangle divides each of them from the direction of the base in the ratio
(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 2 : 3
- 4 ABCD is a rectangle , M is the point of intersection of its diagonals , if the length of the diagonal is 6 cm. , then the length of the median \overline{AM} equals cm.
(a) 3 (b) 6 (c) 9 (d) 12
- 5 ABC is an isosceles triangle where $AB = AC$ and $m(\angle A) = 100^\circ$, then $m(\angle B) =$
(a) 60° (b) 50° (c) 40° (d) 30°
- 6 The number of axes of symmetry of the isosceles triangle equals
(a) 0 (b) 1 (c) 2 (d) 3

2 Complete :

- 1 If the measures of two angles of a triangle are different , then the greater in measure is opposite to
- 2 The bisector of the vertex angle of the isosceles triangle ,
- 3 The base angles of the isosceles triangle are
- 4 In any triangle , the sum of the lengths of any two sides the length of the third side.
- 5 $\triangle ABC$ is right-angled at B , $m(\angle A) = 30^\circ$, $AC = 10$ cm. , then $CB =$ cm.

3 [a] ABC is a triangle in which $AB = AC$, \overline{BD} bisects $\angle ABC$, \overline{CD} bisects $\angle ACB$, $\overline{BD} \cap \overline{CD} = \{D\}$ Prove that : $\triangle DBC$ is an isosceles triangle.

Final Examinations

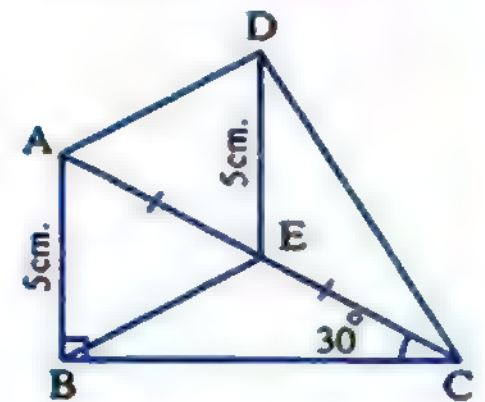
[b] In the opposite figure :

ABC is a right-angled triangle at B

, $m(\angle ACB) = 30^\circ$, $AB = 5$ cm.

, E is the midpoint of \overline{AC} , if $DE = 5$ cm.

, prove that : $m(\angle ADC) = 90^\circ$



4 [a] In the opposite figure :

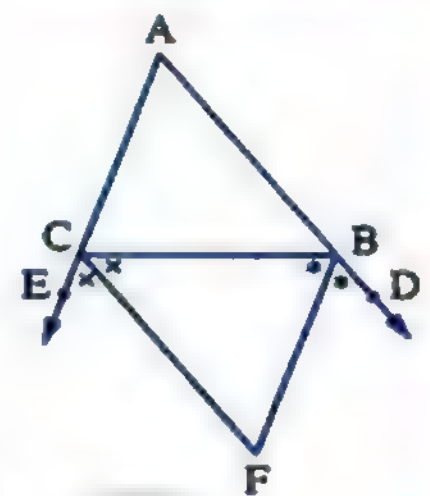
ABC is a triangle in which $AB > AC$, $D \in \overline{AB}$, $E \in \overline{AC}$

, \overline{BF} bisects $\angle DBC$, \overline{CF} bisects $\angle BCE$

, $\overline{BF} \cap \overline{CF} = \{F\}$

Prove that : 1 $m(\angle FBC) > m(\angle BCF)$

2 $CF > BF$

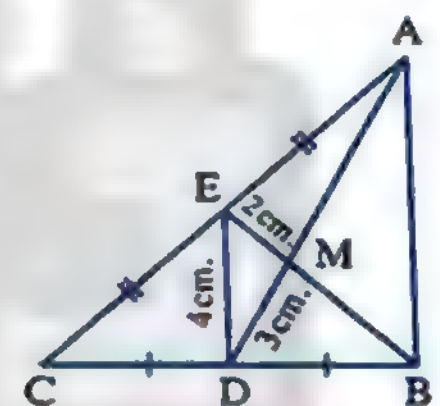


[b] In the opposite figure :

ABC is a triangle in which $ME = 2$ cm. , $MD = 3$ cm.

, $DE = 4$ cm. , D and E are the midpoints of \overline{BC} , \overline{AC} respectively

Find : The perimeter of $\triangle MAB$

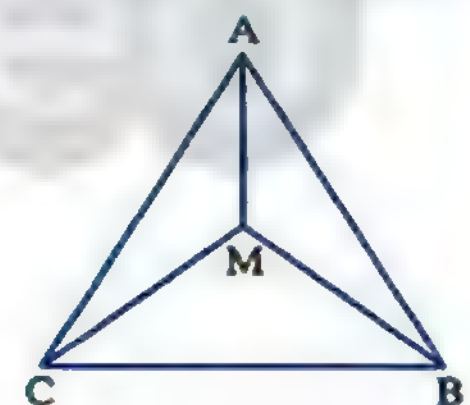


5 [a] In the opposite figure :

ABC is a triangle in which

M is a point inside it.

Prove that : $MA + MB + MC > \frac{1}{2}$ the perimeter of $\triangle ABC$



[b] In the opposite figure :

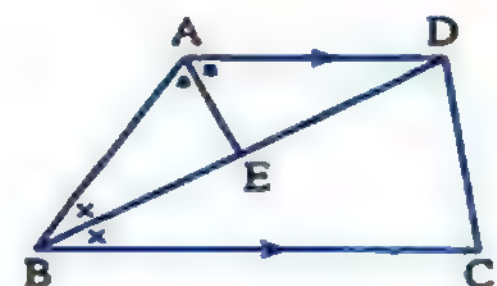
ABCD is a quadrilateral in which $\overline{AD} \parallel \overline{BC}$

, \overline{BD} bisects $\angle ABC$, \overline{AE} bisects $\angle BAD$

Prove that : 1 $AB = AD$

2 $\overline{AE} \perp \overline{BD}$

3 $BE = ED$



Geometry

4

Giza Governorate

Boulaq El Dekroun Directorate of Education
Der El-Hanan Ling. Sch. for Girls

Answer the following questions :

1 Choose the correct answer :

- 1 The number of axes of symmetry of the isosceles triangle equals
(a) 3 (b) 2 (c) 1 (d) 0
- 2 The point of intersection of the medians of the triangle divides each of them in the ratio of from the base.
(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 1 : 2
- 3 ΔXYZ is right-angled at Y , then XZ YZ
(a) $>$ (b) $<$ (c) $=$ (d) \leq
- 4 If 10 cm. , 5 cm. and X cm. are side lengths of an isosceles triangle , then $X =$
(a) 10 (b) 5 (c) 15 (d) 4
- 5 The measure of the exterior angle of an equilateral triangle equals°
(a) 30 (b) 60 (c) 90 (d) 120
- 6 In the opposite figure :
 $x + y =$
(a) 100° (b) 140°
(c) 180° (d) 280°



2 Complete the following :

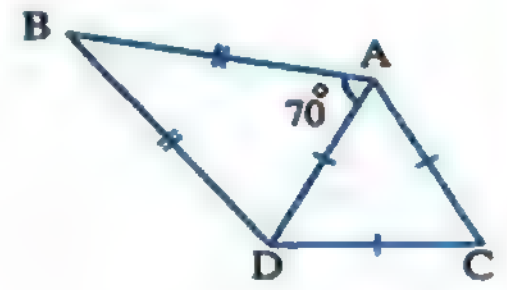
- 1 In ΔABC , if $m(\angle B) = 70^\circ$, $m(\angle C) = 50^\circ$, then AC AB
- 2 In ΔABC , if $m(\angle A) = m(\angle B) + m(\angle C)$, then the longest side is
- 3 The axis of symmetry of a line segment is the straight line which from its midpoint.
- 4 ABC is a triangle in which $AB = 4$ cm. , $CB = 7$ cm.
 , then $AC \in]$, [
- 5 If \overline{AD} is a median in ΔABC , and M is the point of intersection of its medians and $AM = 12$ cm. , then $AD =$

3 [a] In the opposite figure :

$$AB = BD, m(\angle BAD) = 70^\circ$$

, $\triangle ADC$ is an equilateral triangle.

Find : $m(\angle BDC)$

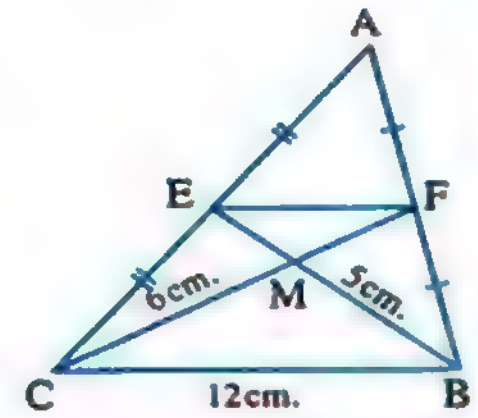


[b] In the opposite figure :

ABC is a triangle , F and E are the midpoints of \overline{AB} and \overline{AC} respectively.

If $BM = 5$ cm. , $CM = 6$ cm. , $BC = 12$ cm.

, then find : The perimeter of $\triangle MEF$



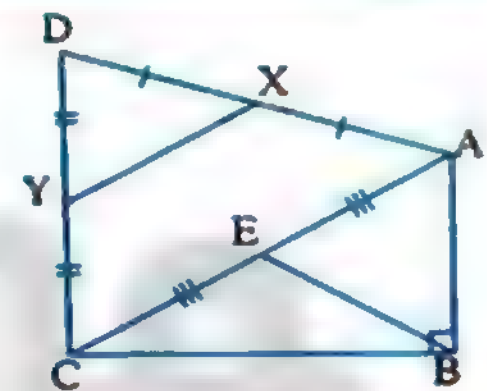
4 [a] In the opposite figure :

$$m(\angle ABC) = 90^\circ$$

, E is the midpoint of \overline{AC}

and X, Y are the midpoints of \overline{DA} and \overline{DC}

Prove that : $XY = BE$



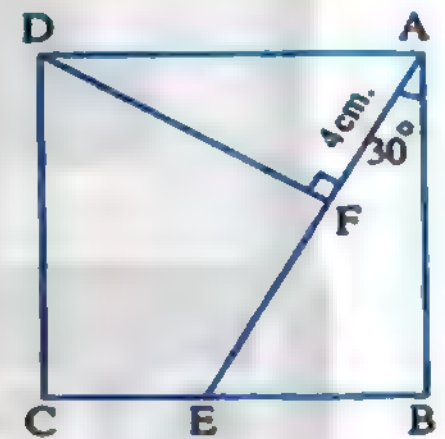
[b] In the opposite figure :

$ABCD$ is a square , $E \in \overline{BC}$

where $m(\angle BAE) = 30^\circ$ and $\overline{DF} \perp \overline{AE}$

, if $AF = 4$ cm.

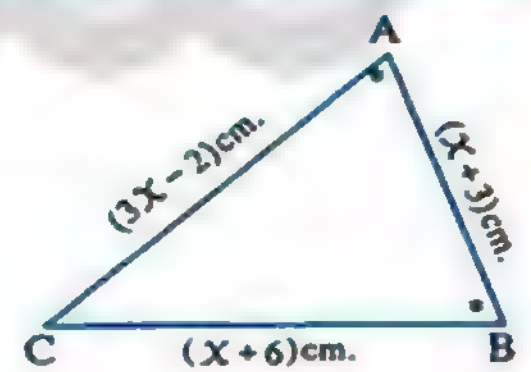
, calculate : The area of the square $ABCD$



5 [a] In the opposite figure :

$$m(\angle A) = m(\angle B)$$

Find : The perimeter of $\triangle ABC$



[b] In the opposite figure :

ABC is a triangle in which :

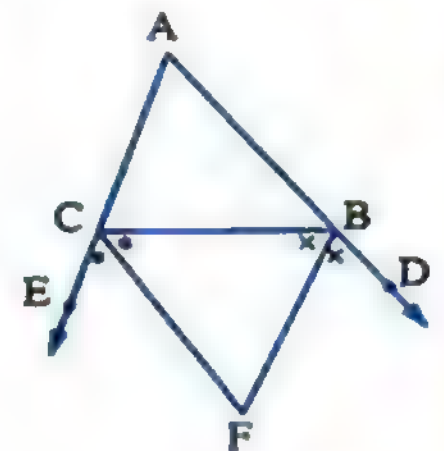
$$AB > AC, D \in \overline{AB}, E \in \overline{AC}$$

, \overline{BF} bisects $\angle DBC$, \overline{CF} bisects $\angle BCE$

$$, \overline{BF} \cap \overline{CF} = \{F\}$$

Prove that : 1 $m(\angle FBC) > m(\angle BCF)$

$$2 \quad CF > BF$$



Geometry

5

Giza Governorate

6th October Directorate
Om El-Moamneen Lang. School

Answer the following questions :

1 Choose the correct answer :

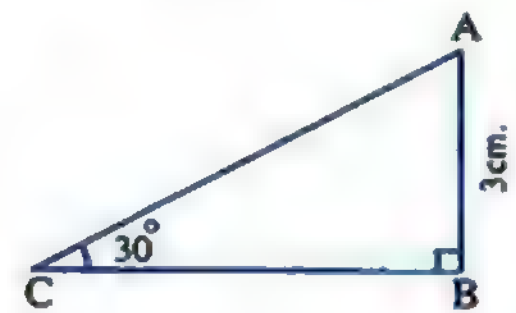
- 1 If ABC is an isosceles triangle , $m(\angle A) = 60^\circ$, $AB = 4$ cm.
 , then its perimeter = cm.
(a) 4 (b) 12 (c) 6 (d) 9
- 2 XYZ is a triangle in which $m(\angle Z) = 70^\circ$, $m(\angle Y) = 60^\circ$, then YZ XY
(a) $>$ (b) $<$ (c) $=$ (d) \geq
- 3 In $\triangle ABC$, if $m(\angle B) = 90^\circ$, then the longest side is
(a) \overline{BC} (b) \overline{AB} (c) \overline{AC} (d) its median.
- 4 A triangle has one axis of symmetry , the lengths of two sides are 4 cm. and 8 cm.
 , then the length of the third side is cm.
(a) 3 (b) 6 (c) 4 (d) 8
- 5 The point of intersection of the medians of the triangle divides each of the medians in
 the ratio from the base.
(a) 2 : 1 (b) 3 : 2 (c) 2 : 4 (d) 3 : 4
- 6 If the length of any side of a triangle $= \frac{1}{3}$ the perimeter of the triangle , then the
 number of axes of symmetry of the triangle equals
(a) 3 (b) 1 (c) 2 (d) zero

2 Complete :

- 1 The bisector of the vertex angle of the isosceles triangle and

- 2 In the opposite figure :

The length of \overline{AC} =



- 3 In $\triangle ABC$, $m(\angle A) = m(\angle B) = m(\angle C)$, then the measure of the exterior angle
 equals
- 4 If the lengths of two sides of a triangle are 4 cm. , 7 cm. , then the length of the third
 side belongs to] , [
- 5 If $\angle X$ and $\angle Y$ are two supplementary angles , $\angle X \equiv \angle Y$, then $m(\angle X) = \dots\dots\dots^\circ$

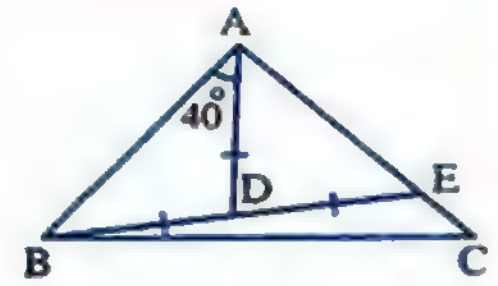
3 [a] In the opposite figure :

$$AD = BD = ED, m(\angle DAB) = 40^\circ$$

Prove that :

1 $AD < AB$

2 $BC > AC$

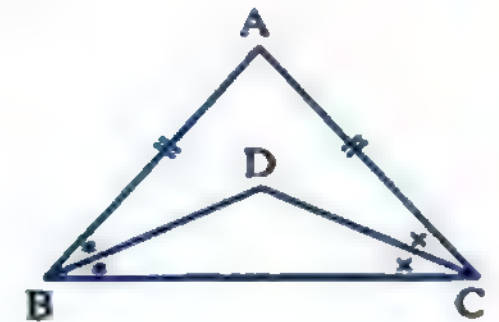


[b] In the opposite figure :

$$AB = AC, \overline{BD} \text{ bisects } \angle ABC$$

$$\text{and } \overline{CD} \text{ bisects } \angle ACB$$

Prove that : $\triangle DBC$ is an isosceles triangle.



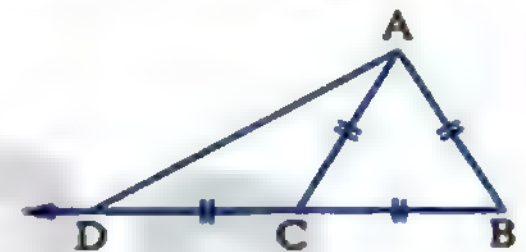
4 [a] ABC is a triangle in which $m(\angle A) = (6x)^\circ$, $m(\angle B) = (4x - 9)^\circ$

, $m(\angle C) = 3(x - 2)^\circ$ Arrange the lengths of the sides of the triangle ascendingly.

[b] In the opposite figure :

$$AB = AC = CB = CD$$

Prove that : $\overline{AB} \perp \overline{AD}$



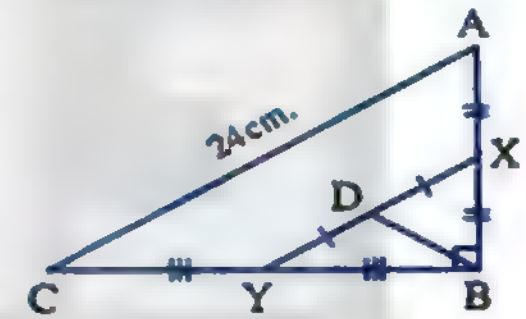
5 [a] In the opposite figure :

$$m(\angle ABC) = 90^\circ, X \text{ is the midpoint of } \overline{AB}$$

$$, Y \text{ is the midpoint of } \overline{BC}$$

$$, D \text{ is the midpoint of } \overline{XY}, AC = 24 \text{ cm.}$$

Find : The length of \overline{BD}



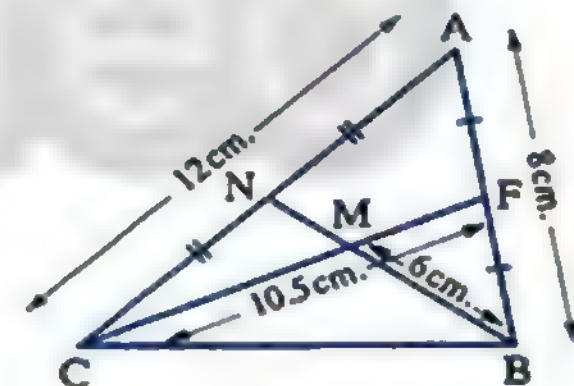
[b] In the opposite figure :

$$F \text{ and } N \text{ are the midpoints of } \overline{AB} \text{ and } \overline{AC} \text{ respectively}$$

$$, AB = 8 \text{ cm.}, AC = 12 \text{ cm.}, BM = 6 \text{ cm.}$$

$$, CF = 10.5 \text{ cm.}$$

Find : The perimeter of the figure AFMN



6

Alexandria Governorate

Middle Educational Zone
Math Supervision

Answer the following questions :

1 Complete each of the following :

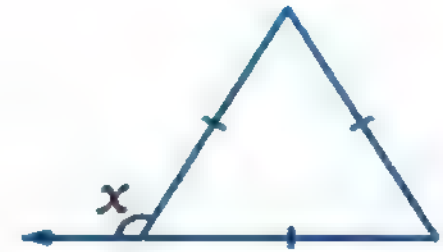
1 If $m(\angle A) = 65^\circ$, then $m(\text{complementary } \angle A) = \dots\dots\dots^\circ$

2 In $\triangle ABC$, $m(\angle A) = 50^\circ$, $m(\angle C) = 80^\circ$, then $CB = \dots\dots\dots$

Geometry

3 In the opposite figure :

$$x = \dots\dots\dots^\circ$$



4 The number of axes of symmetry for the rectangle equals

5 In $\triangle ABC$, $m(\angle B) = 70^\circ$, $m(\angle C) = 45^\circ$, then $BC \dots\dots\dots AC$

6 The medians of the triangle are

2 Choose the correct answer :

1 The sum of lengths of two sides in a triangle is the length of the third side.

- (a) $>$ (b) $<$ (c) $=$ (d) twice

2 The triangle which has no axis of symmetry is

- (a) scalene. (b) isosceles. (c) equilateral. (d) right-angled.

3 The numbers which can not be side lengths of a triangle are

- (a) 3 , 3 , 3 (b) 3 , 3 , 4 (c) 3 , 3 , 5 (d) 3 , 3 , 6

4 \overline{BE} is a median in $\triangle ABC$, M is the point of concurrence of the medians
If $BM = 6$ cm. , then $ME = \dots\dots\dots$ cm.

- (a) 2 (b) 3 (c) 4 (d) 9

5 The angle whose measure is 180° is called angle.

- (a) an acute (b) an obtuse (c) a straight (d) a reflex

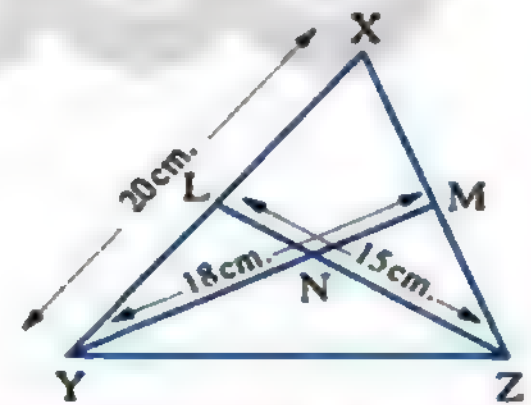
3 [a] $\triangle ABC$ is right-angled at B , if $m(\angle A) = 75^\circ$, arrange the lengths of its sides descendingly.

[b] In the opposite figure :

N is the point of concurrence of
the medians of $\triangle XYZ$

, $LZ = 15$ cm. , $YM = 18$ cm. , $XY = 20$ cm.

Find : The perimeter of $\triangle NLY$



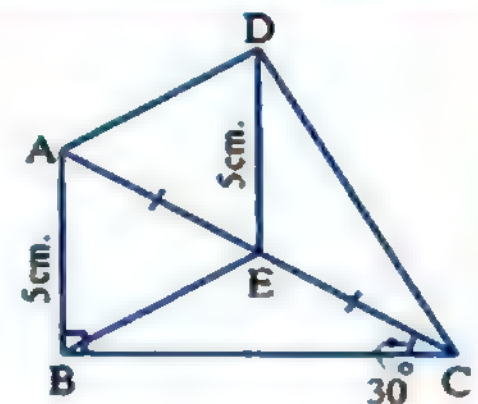
4 [a] In the opposite figure :

$m(\angle ABC) = 90^\circ$, E is the midpoint of \overline{AC}

, $m(\angle ACB) = 30^\circ$

, $AB = DE = 5$ cm.

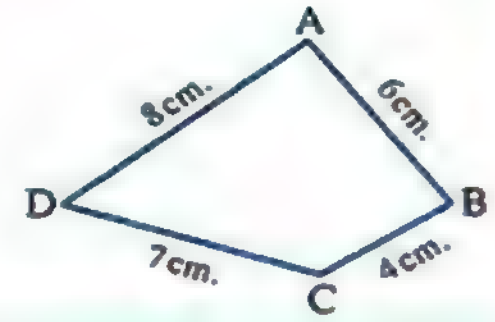
Prove that : $m(\angle ADC) = 90^\circ$



Final Examinations

[b] In the opposite figure :

Prove that : $m(\angle BCD) > m(\angle BAD)$



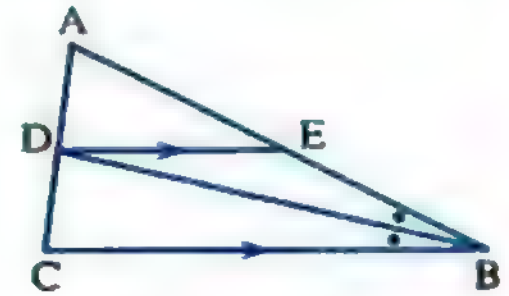
5 [a] In the opposite figure :

\overline{BD} bisects $\angle ABC$

, $\overline{DE} \parallel \overline{BC}$

Prove that :

$\triangle EBD$ is an isosceles triangle.

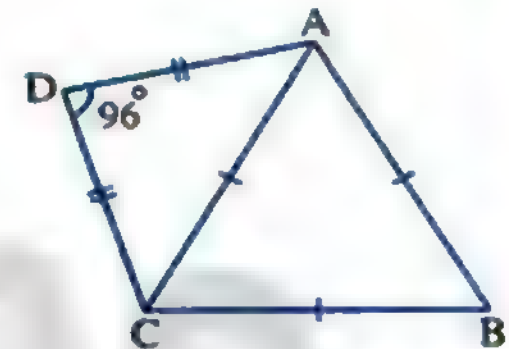


[b] In the opposite figure :

$\triangle ABC$ is equilateral , $DA = DC$

, $m(\angle ADC) = 96^\circ$

Find : $m(\angle DAB)$



7

Alexandria Governorate

Agamy Educational Zone
Inspector of Maths



Answer the following questions :

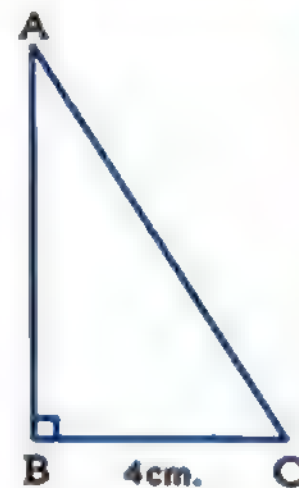
1 Choose the correct answer :

- 1 XYZ is a triangle in which $m(\angle Z) = 70^\circ$, $m(\angle Y) = 60^\circ$, then $YZ \dots\dots\dots XY$
 (a) $>$ (b) $<$ (c) $=$ (d) twice
- 2 The two diagonals are perpendicular in the
 (a) rectangle. (b) rhombus. (c) trapezium. (d) triangle.
- 3 The measure of the exterior angle of the equilateral triangle equals $^\circ$
 (a) 360 (b) 120 (c) 60 (d) 180
- 4 If the lengths of two sides in an isosceles triangle are 3 cm. , 7 cm. , then the length of the third side is cm.
 (a) 3 (b) 7 (c) 10 (d) 4
- 5 The point of concurrence of the medians of the triangle divides each median in the ratio from its base.
 (a) 2 : 1 (b) 1 : 3 (c) 1 : 4 (d) 1 : 2
- 6 If the side length of an equilateral triangle is 10 cm. , then its height equals cm.
 (a) 5 (b) 10 (c) $5\sqrt{3}$ (d) 30

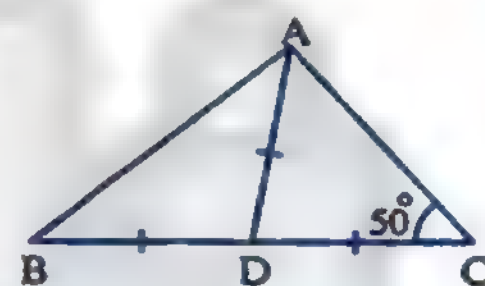
Geometry

2 Complete :

- 1 If the isosceles triangle has an angle of measure 45° , then the triangle is
- angled triangle.
- 2 The sum of lengths of any two sides of a triangle is the length of the third side.
- 3 In the opposite figure :
If $m(\angle C) = 2 m(\angle A)$
, $CB = 4 \text{ cm.}$
, then $AC = \dots\dots\dots \text{ cm.}$

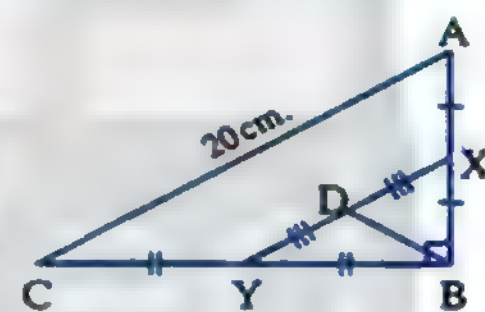


- 4 If the two side lengths in a triangle are 4 cm. , 7 cm. , then the length of the third side $\in]\dots\dots\dots , \dots\dots\dots[$
- 5 In the opposite figure :
 $AD = DC = BD$
, $m(\angle C) = 50^\circ$
, then $m(\angle B) = \dots\dots\dots^\circ$



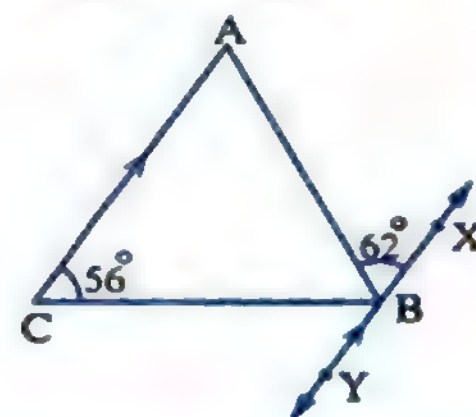
3 [a] In the opposite figure :

$m(\angle ABC) = 90^\circ$, D is the midpoint of \overline{XY}
, X , Y are the midpoints of \overline{AB} , \overline{BC} respectively , $AC = 20 \text{ cm.}$
Find : The length of \overline{BD}



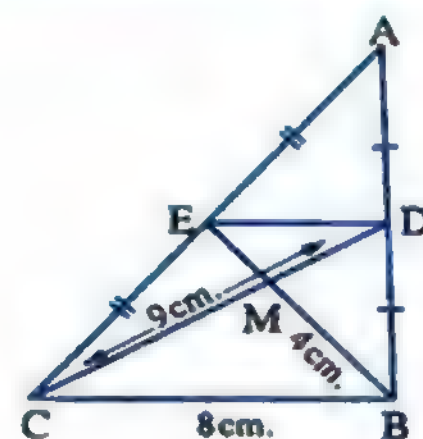
[b] In the opposite figure :

$B \in \overline{XY}$, $\overline{XY} \parallel \overline{AC}$
, $m(\angle ABX) = 62^\circ$
and $m(\angle C) = 56^\circ$
Prove that : $AC = BC$



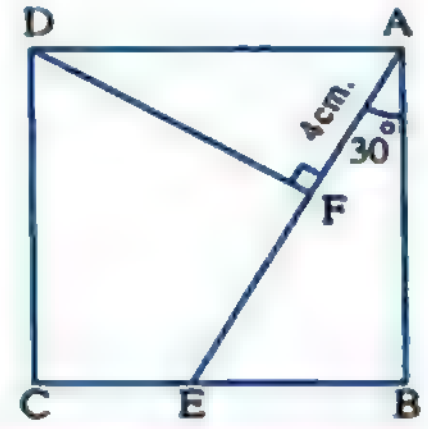
4 [a] In the opposite figure :

D , E are the midpoints of \overline{AB} and \overline{AC} respectively
, $DC = 9 \text{ cm.}$, $MB = 4 \text{ cm.}$ and $BC = 8 \text{ cm.}$
Find : The perimeter of $\triangle DME$



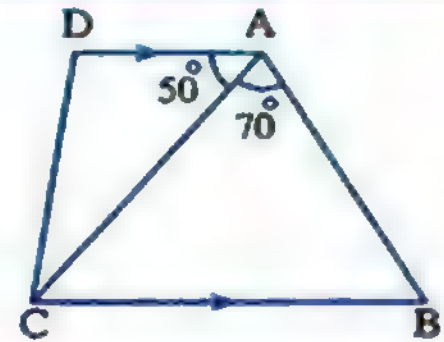
[b] In the opposite figure :

ABCD is a square , $E \in \overline{BC}$
 , where $m(\angle BAE) = 30^\circ$ and $\overline{DF} \perp \overline{AE}$
 , if $AF = 4$ cm.
 , calculate : The area of the square ABCD



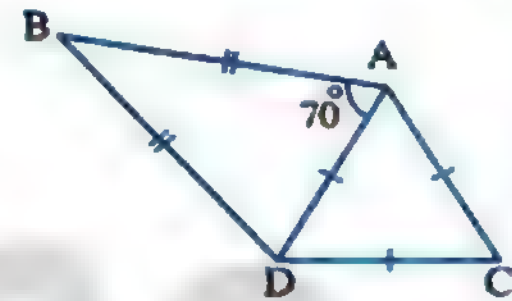
5 [a] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle CAB) = 70^\circ$
 , $m(\angle DAC) = 50^\circ$
 Prove that : $BC > AC$



[b] In the opposite figure :

$AB = BD$, $m(\angle BAD) = 70^\circ$
 , $\triangle ADC$ is equilateral
 Find : $m(\angle BDC)$



8

El-Kalyoubia Governorate

Directorate of Education
 Inspection of Mathematics



Answer the following questions :

1 Choose the correct answer :

- 1 ABC is an equilateral triangle , then $m(\angle A) = \dots\dots\dots^\circ$
 (a) 45 (b) 60 (c) 120 (d) 35
- 2 $\triangle XYZ$ is an isosceles triangle , $m(\angle X) = 100^\circ$, then $m(\angle Y) = \dots\dots\dots^\circ$
 (a) 100 (b) 80 (c) 60 (d) 40
- 3 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals $\dots\dots\dots$ the length of the hypotenuse.
 (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) 2
- 4 The number of axes of symmetry of the isosceles triangle equals $\dots\dots\dots$
 (a) 0 (b) 1 (c) 2 (d) 3
- 5 If the lengths of two sides of an isosceles triangle are 2 cm. , 5 cm. , then the length of the third side equals $\dots\dots\dots$ cm.
 (a) 2 (b) 3 (c) 4 (d) 5
- 6 In the triangle ABC , if $m(\angle A) = 50^\circ$, $m(\angle B) = 60^\circ$, then the longest side is $\dots\dots\dots$
 (a) \overline{AB} (b) \overline{BC} (c) \overline{AC} (d) 110 cm.

Geometry

2 Complete :

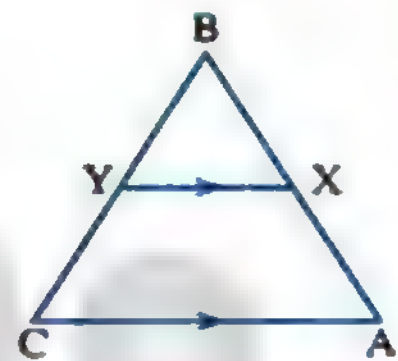
- 1 The medians of a triangle are
- 2 The longest side of the right-angled triangle is the
- 3 If $AB = AC$ in the triangle ABC , then ABC is triangle.
- 4 XYZ is a triangle , $m(\angle Z) = 40^\circ$, $m(\angle Y) = 30^\circ$, then XY XZ
- 5 If the lengths of two sides of a triangle are 6 cm. and 9 cm. , then the length of the third side \in , [

- 3 [a] In $\triangle ABC$, $m(\angle A) = 40^\circ$, $m(\angle B) = 75^\circ$, $m(\angle C) = 65^\circ$
Arrange the lengths of the sides of this triangle descendingly.

[b] In the opposite figure :

$$AB = BC , \overline{XY} \parallel \overline{AC}$$

Prove that : $BX = BY$

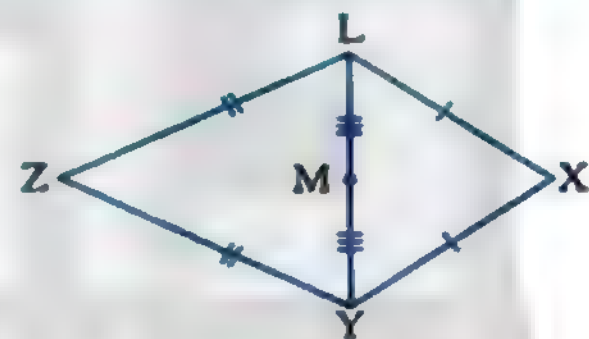


- 4 [a] In the opposite figure :

$$XY = XL , ZY = ZL$$

$$, LM = MY$$

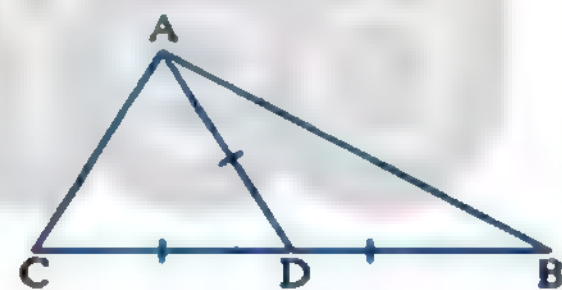
Prove that : X , M , Z are on the same straight line.



[b] In the opposite figure :

$$AB > AC , DB = DC = AD$$

Prove that : $m(\angle BAD) < m(\angle CAD)$



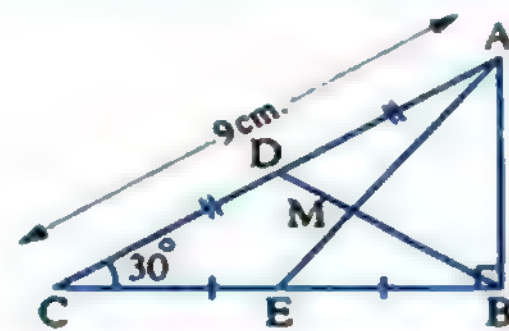
- 5 [a] In the opposite figure :

$\triangle ABC$ is a right-angled triangle at B

, $m(\angle C) = 30^\circ$, D is the midpoint of \overline{AC}

, E is the midpoint of \overline{BC} , $AC = 9$ cm.

Find the length of each of : \overline{BD} , \overline{BM} , \overline{AB} , \overline{MD}



[b] ABC is a triangle such that

$$m(\angle A) = (2x)^\circ , m(\angle C) = (x + 40)^\circ , m(\angle B) = (3x - 10)^\circ$$

Prove that : $AB = AC$

9

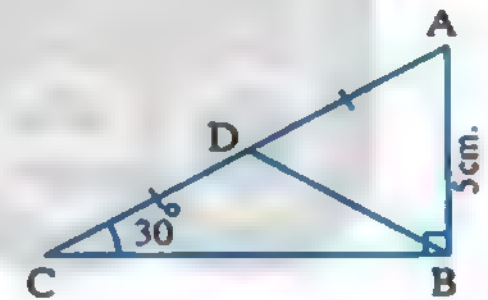
El-Sharkia Governorate

Zagazig English Language School
for Girls

Answer the following questions :

1 Choose the correct answer :

- 1 In $\triangle ABC$, $m(\angle A) = 60^\circ$, $m(\angle C) = 45^\circ$, then
 (a) $AB < AC$ (b) $AB = AC$ (c) $AB > AC$ (d) $AB = BC$
- 2 If M is the point of concurrence of the medians of $\triangle ABC$, \overline{AD} is a median , then $MA = \dots\dots\dots$
 (a) $2 AD$ (b) $\frac{2}{3} AD$ (c) $\frac{3}{2} AD$ (d) $\frac{1}{2} MD$
- 3 In $\triangle ABC$, $AB = 4 \text{ cm.}$, $BC = 6 \text{ cm.}$, then $AC \in \dots\dots\dots$
 (a) $]2 , 4[$ (b) $[2 , 10]$ (c) $]2 , 10[$ (d) $[0 , 10]$
- 4 The number of axes of symmetry of the equilateral triangle equals
 (a) zero (b) 1 (c) 2 (d) 3
- 5 In $\triangle ABC$, $AB = AC$, $m(\angle B) = x + 30^\circ$, $m(\angle C) = 2x + 5^\circ$, then $x = \dots\dots\dots$
 (a) 25° (b) 20° (c) 35° (d) 3°
- 6 In the opposite figure :
 $AD = DC$, $m(\angle C) = 30^\circ$, $m(\angle ABC) = 90^\circ$, $AB = 5 \text{ cm.}$, then the perimeter of $\triangle ABD = \dots\dots\dots \text{ cm.}$
 (a) 5 (b) 15 (c) 20 (d) 25



2 Complete :

- 1 ABCD is a rectangle , $AB = 3 \text{ cm.}$, $BC = 4 \text{ cm.}$, then $BD = \dots\dots\dots \text{ cm.}$
- 2 In $\triangle ABC$, if D is the midpoint of \overline{BC} and $AD = \frac{1}{2} BC$, then $m(\angle CAB) = \dots\dots\dots^\circ$
- 3 The longest side in the right-angled triangle is
- 4 If $\triangle ABC \equiv \triangle XYZ$, then $AC - XZ = \dots\dots\dots$
- 5 The median that is drawn from the vertex angle of an isosceles triangle and

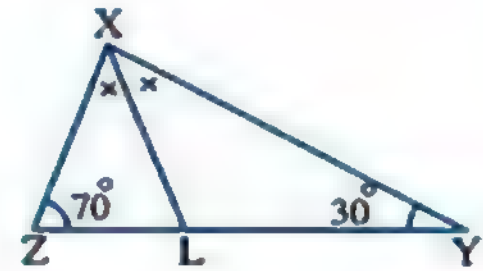
Geometry

3 [a] In the opposite figure :

\overline{XL} bisects $\angle YXZ$, $m(\angle Y) = 30^\circ$
 $m(\angle Z) = 70^\circ$

1 Find : $m(\angle LXZ)$ and $m(\angle XLZ)$

2 Prove that : $\triangle XLZ$ is an isosceles triangle.

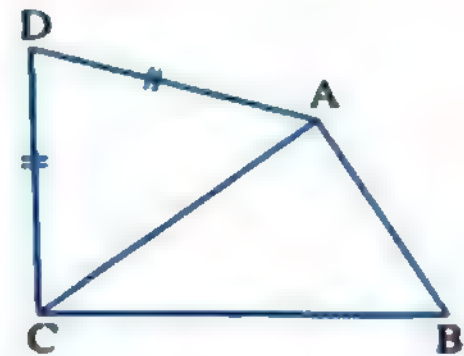


[b] In the opposite figure :

ABCD is a quadrilateral

, $AD = DC$, $BC > AB$

Prove that : $m(\angle BAD) > m(\angle BCD)$



4 [a] In the opposite figure :

X is the midpoint of \overline{AC} , $AB = 8$ cm.

, Y is the midpoint of \overline{BC} , $AM = 5$ cm. , $BX = 6$ cm.

Find : The perimeter of $\triangle XMY$

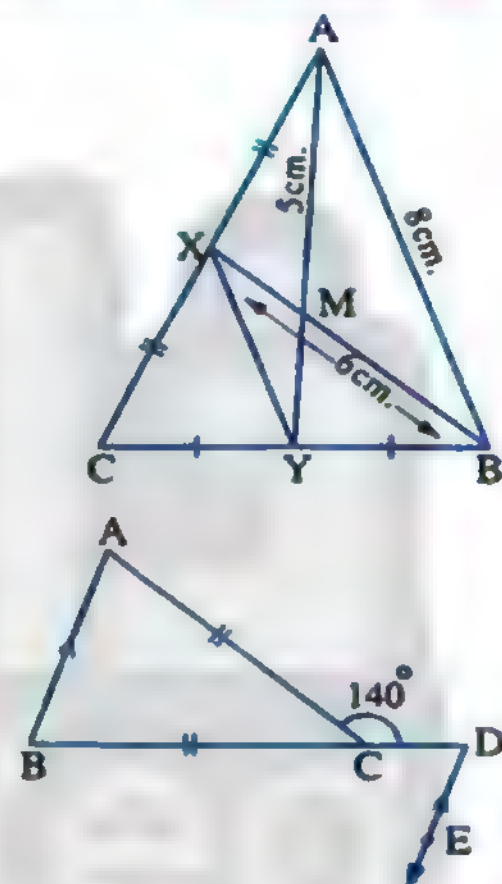
[b] In the opposite figure :

$C \in \overline{BD}$, $CA = CB$

, $\overline{AB} \parallel \overline{DE}$

, $m(\angle ACD) = 140^\circ$

Find : $m(\angle A)$ and $m(\angle BDE)$



5 [a] In the opposite figure :

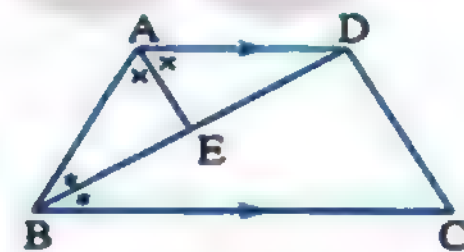
ABCD is a quadrilateral , $\overline{AD} \parallel \overline{BC}$

, \overline{BD} bisects $\angle ABC$

, \overline{AE} bisects $\angle BAD$

Prove that : 1 $AD = AB$

2 $\overline{AE} \perp \overline{BD}$



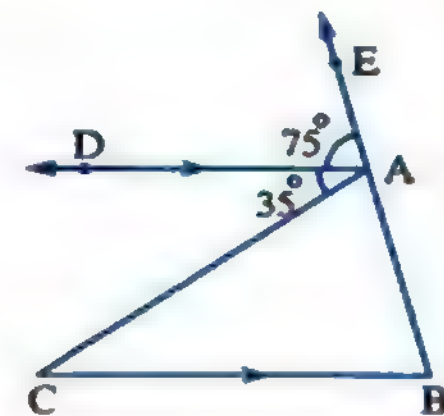
[b] In the opposite figure :

$E \in \overline{BA}$, $\overline{AD} \parallel \overline{BC}$

, $m(\angle DAE) = 75^\circ$

, $m(\angle DAC) = 35^\circ$

Prove that : $BC > AB$



10

El-Monofia Governorate

El-Shohadea Directorate
Mathe Supervision

Answer the following questions :

1 Choose the correct answer :

- 1 The intersecting point of the medians of the triangle divides each median in the ratio of from its base.
(a) 1 : 2 (b) 2 : 1 (c) 3 : 1 (d) 1 : 3
- 2 The number of symmetry axes of the isosceles triangle is
(a) 1 (b) 2 (c) 3 (d) 4
- 3 The sum of lengths of any two sides of a triangle the length of the third side.
(a) < (b) > (c) = (d) =
- 4 The diagonals are perpendicular in the
(a) trapezium. (b) parallelogram. (c) square. (d) rectangle.
- 5 If ΔABC is right-angled at B , $AB = 6$ cm. , $BC = 8$ cm. , then the length of the median drawn from B equals cm.
(a) 3 (b) 4 (c) 5 (d) 6
- 6 If 4 cm. , $(x + 3)$ cm. and 8 cm. are side lengths of an isosceles triangle , then $x =$
(a) 3 (b) 4 (c) 5 (d) 6

2 Complete each of the following :

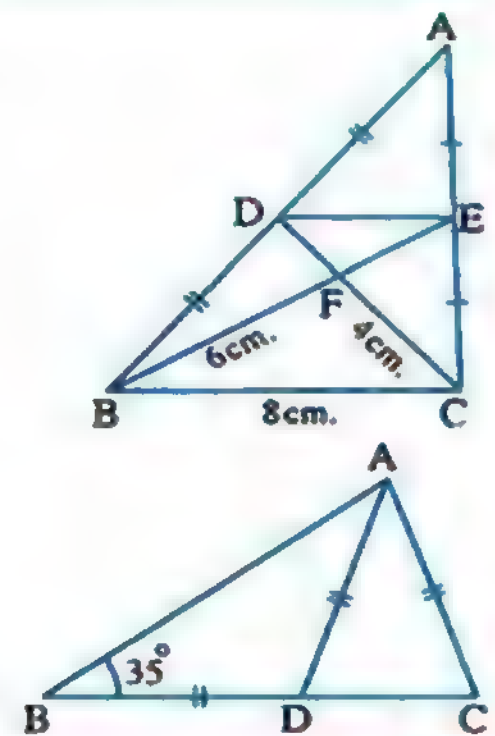
- 1 The base angles in an isosceles triangle are
- 2 If $m(\angle A) = 100^\circ$, then $m(\text{reflex } \angle A) =$ $^\circ$
- 3 The number of medians of the isosceles triangle is
- 4 In ΔABC , if $AB > BC$, then $m(\angle A)$ $m(\angle C)$
- 5 The bisector of the vertex angle of an isosceles triangle bisects the base and

3 [a] In the opposite figure :

ABC is a triangle in which D , E are the midpoints of \overline{AB} , \overline{AC}
 , $FC = 4$ cm. , $FB = 6$ cm. and $BC = 8$ cm.

Find : The perimeter of ΔDFE

[b] In the opposite figure :

 $AC = AD = BD$, $m(\angle B) = 35^\circ$ Find : $m(\angle BAC)$ 

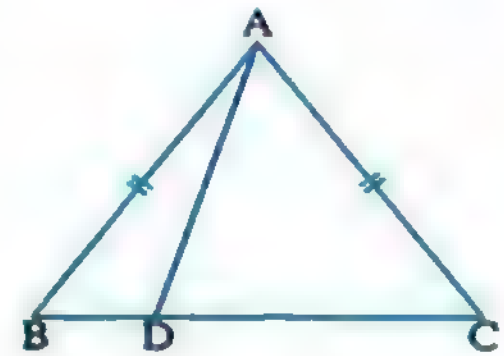
Geometry

4 [a] In the opposite figure :

$$AC = AB$$

Prove that :

$$AB > AD$$



[b] ABC is a triangle in which $m(\angle A) = 40^\circ$, $m(\angle B) = 80^\circ$ Arrange the lengths of the sides of the triangle descendingly.

5 In the opposite figure :

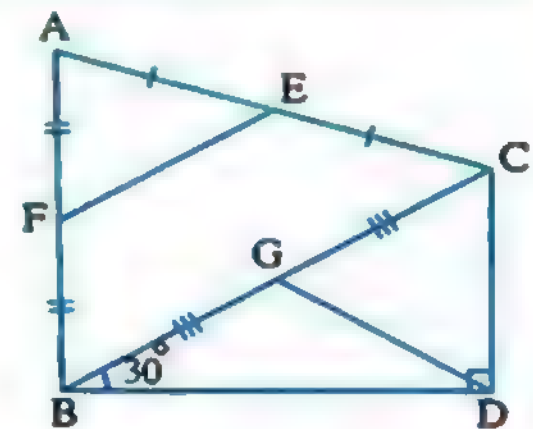
F, E, G are the midpoints of \overline{AB} , \overline{AC} , \overline{BC}

$$m(\angle BDC) = 90^\circ, m(\angle CBD) = 30^\circ$$

$$BC = 10 \text{ cm.}$$

1 Prove that : $FE = DC = GD$

2 Find : The perimeter of $\triangle GCD$



11

El-Dakahlia Governorate

Telkha Educational Directorate
A.M.D.I. School



Answer the following questions :

1 Choose the correct answer from the given ones :

1 The numbers 4, $x + 4$, 8 can be lengths of sides of an isosceles triangle if $x = \dots\dots\dots$

- (a) 4 (b) 0 (c) 3 (d) 8

2 In $\triangle LMN$, if $m(\angle M) = 55^\circ$, $m(\angle N) = 80^\circ$, then $LM \dots\dots\dots MN$

- (a) < (b) > (c) = (d) twice

3 The measure of the exterior angle of the equilateral triangle equals $\dots\dots\dots$

- (a) 30° (b) 60° (c) 90° (d) 120°

4 If \overline{AD} is a median of $\triangle ABC$, and M is the point of concurrence of the medians, then $AD = \dots\dots\dots AM$

- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

5 The base angles of the isosceles triangle are $\dots\dots\dots$

- (a) alternate (b) corresponding (c) congruent (d) supplementary

6 If $XA = XB$, $YA = YB$, then $\overline{XY} \dots\dots\dots \overline{AB}$

- (a) \perp (b) \equiv (c) \parallel (d) =

2 Complete the following :

- 1 The number of axes of symmetry of the isosceles triangle is
- 2 The bisector of the vertex angle of the isosceles triangle
- 3 The medians of the triangle intersect at
- 4 The longest side in the right-angled triangle is the
- 5 In $\triangle ABC$, if $AB = AC$, $m(\angle C) = 40^\circ$, then $m(\angle A) = \dots\dots\dots^\circ$

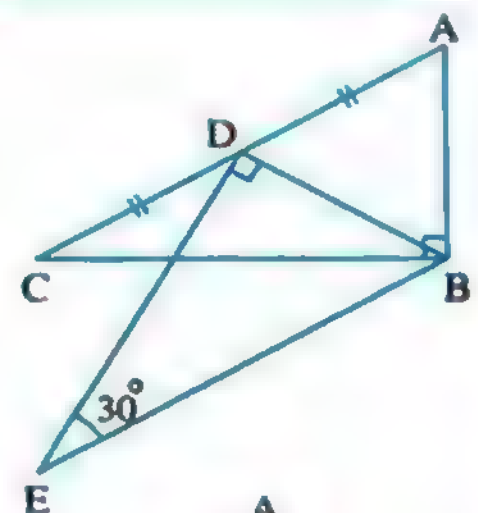
3 [a] In the opposite figure :

$$m(\angle ABC) = m(\angle BDE) = 90^\circ$$

$$, m(\angle E) = 30^\circ$$

, D is the midpoint of \overline{AC}

Prove that : $AC = BE$

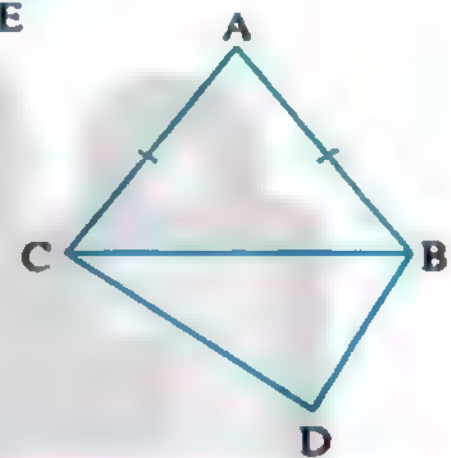


[b] In the opposite figure :

$$AB = AC , DC > DB$$

Prove that :

$$m(\angle ABD) > m(\angle ACD)$$

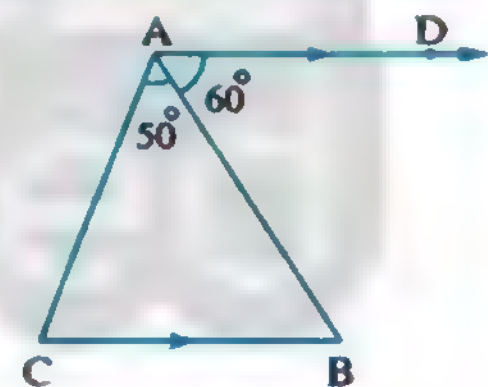


4 [a] In the opposite figure :

ABC is a triangle , $\overline{AD} \parallel \overline{CB}$

$$, m(\angle DAB) = 60^\circ , m(\angle BAC) = 50^\circ$$

Prove that : $AB > AC$



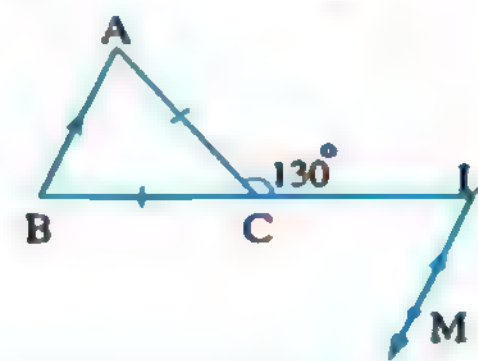
[b] In the opposite figure :

$$C \in \overline{LB} , AC = BC$$

$$, m(\angle LCA) = 130^\circ$$

$$, \overline{LM} \parallel \overline{AB}$$

Find : $m(\angle MLC)$



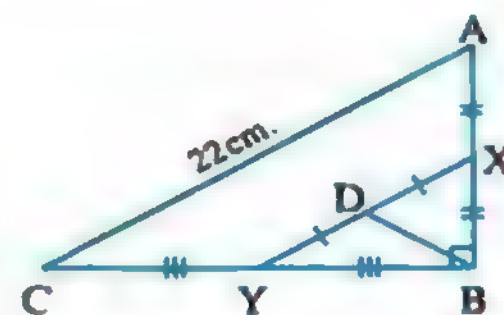
5 [a] In the opposite figure :

$$m(\angle ABC) = 90^\circ , X , Y , D$$

are the midpoints of $\overline{AB} , \overline{BC} , \overline{XY}$

respectively , if $AC = 22$ cm.

, find : BD

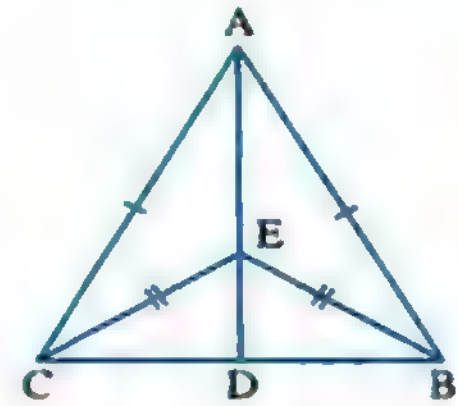


Geometry

[b] In the opposite figure :

$$AB = AC, EB = EC$$

Prove that : $BD = CD$



12

Suez Governorate

Directorate of Education
Inspection of Mathematics



Answer the following questions :

1 Complete :

- 1 The base angles in an isosceles triangle are
- 2 If the angles of a triangle are congruent, then the triangle is
- 3 In $\triangle ABC$, if $m(\angle A) = 70^\circ$, $m(\angle B) = 50^\circ$, then the longest side is
- 4 The point of concurrence of the medians of the triangle divides each median in the ratio of from its vertex.
- 5 In $\triangle ABC$, if $m(\angle A) = 30^\circ$ and $m(\angle B) = 90^\circ$, then $AC = \dots\dots\dots BC$

2 Choose the correct answer :

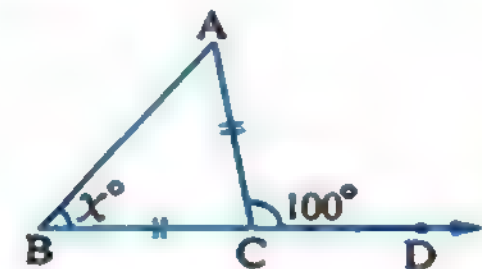
- 1 The triangle which has three axes of symmetry is
(a) scalene. (b) isosceles. (c) right-angled. (d) equilateral.
- 2 If the lengths of two sides in an isosceles triangle are 3 cm. and 7 cm., then the length of the third side equals cm.
(a) 3 (b) 4 (c) 6 (d) 7
- 3 XYZ is a triangle in which $m(\angle Z) = 70^\circ$ and $m(\angle Y) = 60^\circ$, then $YZ \dots\dots\dots XY$
(a) $>$ (b) $<$ (c) $=$ (d) twice

4 In the opposite figure :

$$CA = CB, m(\angle B) = x^\circ$$

$$m(\angle ACD) = 100^\circ \text{ where } C \in \overline{BD}$$

, then $x = \dots\dots\dots$



- (a) 50° (b) 100° (c) 150° (d) 200°
- 5 In $\triangle ABC$, if $AB = AC$ and \overline{AD} is a median, then $\overline{AD} \dots\dots\dots \overline{BC}$
(a) \equiv (b) \perp (c) \subset (d) \parallel
- 6 In $\triangle ABC$, if $AB = 3$ cm., $BC = 5$ cm., then $AC \in \dots\dots\dots$
(a) $]2, 8[$ (b) $]2, 7[$ (c) $]2, 15[$ (d) $]8, 15[$

Final Examinations

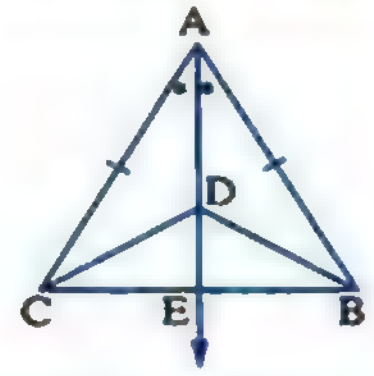
- 3 [a] ABC is a triangle in which $m(\angle A) = 40^\circ$, $m(\angle B) = 75^\circ$ Arrange the lengths of sides of the triangle descendingly.

[b] In the opposite figure :

$AB = AC$, \overline{AE} bisects $\angle BAC$

$\overline{AE} \cap \overline{BC} = \{E\}$, $D \in \overline{AE}$

Prove that : $BD = CD$



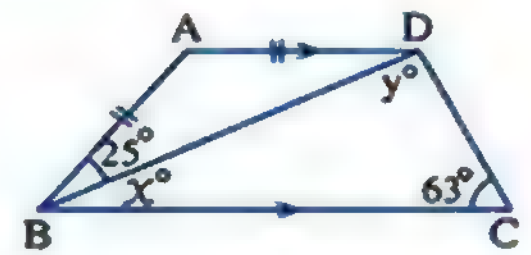
- 4 [a] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $AD = AB$

$m(\angle ABD) = 25^\circ$, $m(\angle C) = 63^\circ$

$m(\angle DBC) = x^\circ$, $m(\angle CDB) = y^\circ$

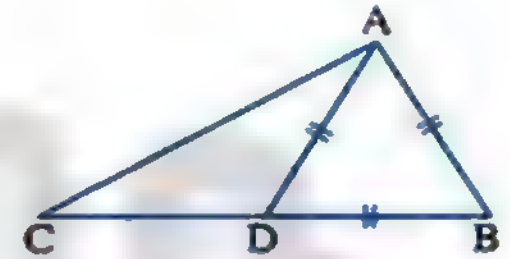
Find the value of each of : x and y



[b] In the opposite figure :

$AB = BD = DA$

Prove that : $BC > AC$



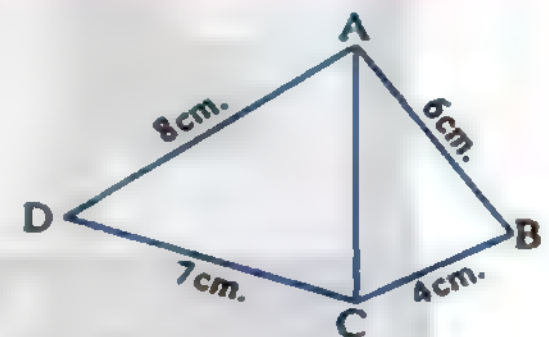
- 5 [a] In the opposite figure :

ABCD is a quadrilateral

$AB = 6 \text{ cm}$, $BC = 4 \text{ cm}$.

$CD = 7 \text{ cm}$, $AD = 8 \text{ cm}$.

Prove that : $m(\angle BCD) > m(\angle BAD)$



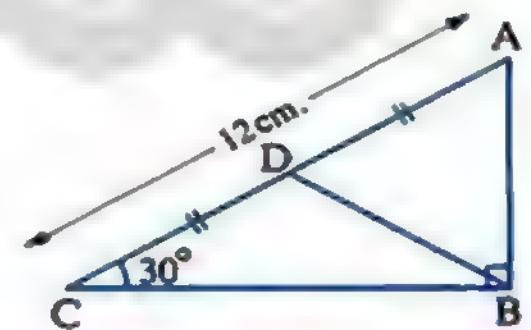
[b] In the opposite figure :

ABC is a triangle, $m(\angle ABC) = 90^\circ$

D is the midpoint of \overline{AC}

$AC = 12 \text{ cm}$, $m(\angle C) = 30^\circ$

then find : The perimeter of $\triangle ABD$



13

El-Beheira Governorate

Damanhur Directorate
Al-Farabi Language School

Answer the following questions :

- 1 Complete the following :

- 1 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals the length of the hypotenuse.

Geometry

- 2 If \overline{AD} is a median in $\triangle ABC$, M is the point of intersection of its medians and $AM = 12$ cm., then $AD = \dots\dots\dots$
- 3 The number of axes of symmetry of the isosceles triangle equals $\dots\dots\dots$
- 4 In a triangle, if two angles are unequal in measure, then the greater angle in measure is opposite to $\dots\dots\dots$
- 5 If $\overline{AB} \equiv \overline{XY}$ and $AB = 5$ cm., then $2AB - XY = \dots\dots\dots$

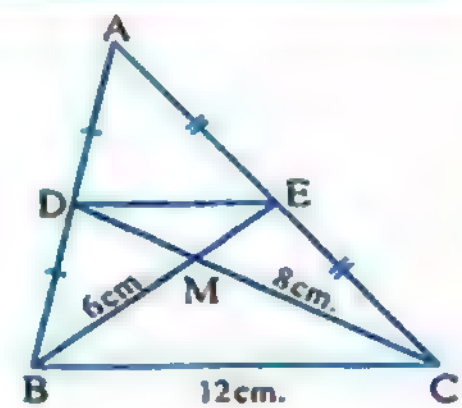
2 Choose the correct answer :

- 1 The measure of one of the base angles in the isosceles triangle is 65° , then the measure of its vertex angle equals $\dots\dots\dots^\circ$
 (a) 65 (b) 50 (c) 130 (d) 55
- 2 If 4 cm., $(X + 3)$ cm. and 8 cm. are side lengths of an isosceles triangle, then $X = \dots\dots\dots$
 (a) 4 (b) 3 (c) 5 (d) 8
- 3 If $\triangle ABC$ is right-angled at B , $AB = 6$ cm., $BC = 8$ cm., then the length of the median drawn from B equals $\dots\dots\dots$ cm.
 (a) 10 (b) 8 (c) 6 (d) 5
- 4 The diagonals are perpendicular in the $\dots\dots\dots$
 (a) trapezium. (b) parallelogram. (c) square. (d) triangle.
- 5 The point of concurrence of the medians of the triangle divides each median in the ratio of $\dots\dots\dots$ from the base.
 (a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 3 : 1
- 6 The acute angle supplements $\dots\dots\dots$ angle.
 (a) an acute (b) an obtuse (c) a right (d) a reflex

3 [a] In the opposite figure :

\overline{BE} , \overline{CD} are medians in $\triangle ABC$
 $MB = 6$ cm., $MC = 8$ cm.
 $BC = 12$ cm.

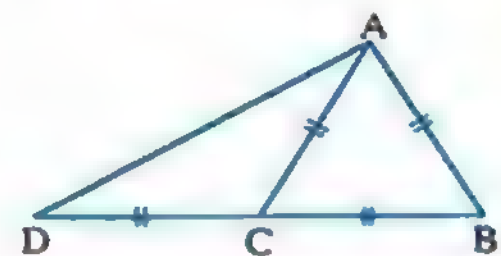
Find : The perimeter of $\triangle MDE$



[b] In the opposite figure :

$AB = BC = AC = DC$

Prove that : $m(\angle BAD) = 90^\circ$



Final Examinations

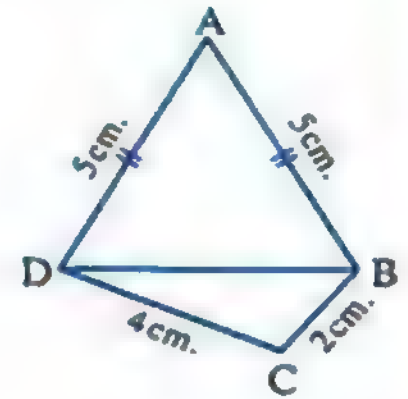
4 [a] In the opposite figure :

ABCD is a quadrilateral in which $AB = AD = 5 \text{ cm}$.

, $BC = 2 \text{ cm}$, $DC = 4 \text{ cm}$.

Prove that :

$m(\angle ABC) > m(\angle ADC)$

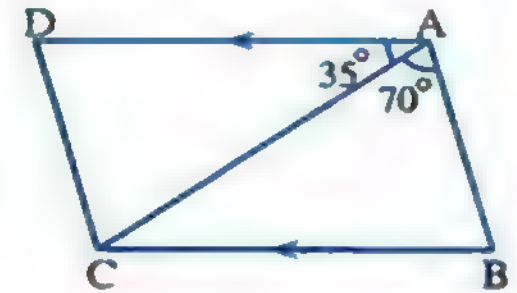


[b] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle BAC) = 70^\circ$

and $m(\angle DAC) = 35^\circ$

Prove that : $AC > BC$



5 In the opposite figure :

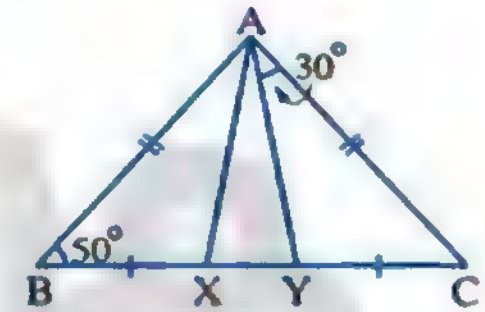
ABC is a triangle in which

$AB = AC$, $BX = CY$

If $m(\angle B) = 50^\circ$, $m(\angle CAY) = 30^\circ$

1 Prove that : $\triangle AYX$ is an isosceles triangle.

2 Find : $m(\angle AXY)$



14

El-Menia Governorate

El-Menia Directorate of Education
Kefr El-Mensoura Formal Language School



Answer the following questions :

1 Choose the correct answer :

1 The triangle in which the measures of two angles of it are 42° and 69° is

(a) an isosceles triangle.

(b) an equilateral triangle.

(c) a scalene triangle.

(d) a right-angled triangle.

2 In $\triangle ABC$ which is right-angled at B , if $AC = 20 \text{ cm}$, then the length of the median drawn from B equals

(a) 10 cm.

(b) 8 cm.

(c) 6 cm.

(d) 5 cm.

3 In $\triangle ABC$, if $m(\angle B) = 130^\circ$, then the longest side of it is

(a) \overline{BC}

(b) \overline{AC}

(c) \overline{AB}

(d) its median.

4 The two angles are said to be supplementary if the sum of their measures is

(a) zero°

(b) 90°

(c) 180°

(d) 360°

Geometry

- 5 The lengths which can be lengths of sides of a triangle are
- (a) (0 , 3 , 5) (b) (3 , 3 , 5) (c) (3 , 3 , 6) (d) (3 , 3 , 7)
- 6 ΔXYZ is an isosceles triangle in which $m(\angle X) = 100^\circ$, then $m(\angle Y) = \dots\dots\dots$
- (a) 100° (b) 80° (c) 60° (d) 40°

2 Complete :

- 1 The sum of measures of the accumulative angles at a point is
- 2 The ray drawn from the midpoint of a side of a triangle parallel to another side the third side.
- 3 If the measure of an angle in an isosceles triangle equals 60° , then the triangle is
- 4 The point of concurrence of the medians of the triangle divides each median in the ratio of from the base.
- 5 In ΔABC , $m(\angle B) = 70^\circ$, $m(\angle C) = 50^\circ$, then $AC \dots\dots\dots AB$

3 [a] In the opposite figure :

$$\overline{AB} \cap \overline{CD} = \{M\}, \overline{AC} \perp \overline{CD},$$

$$\overline{BD} \perp \overline{CD}$$

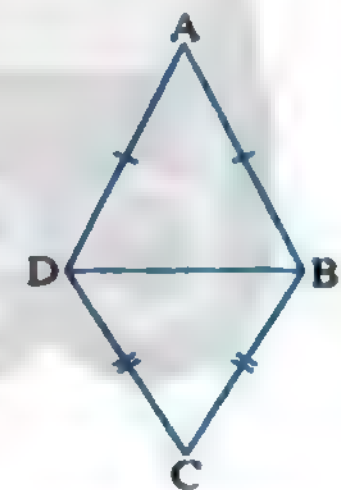
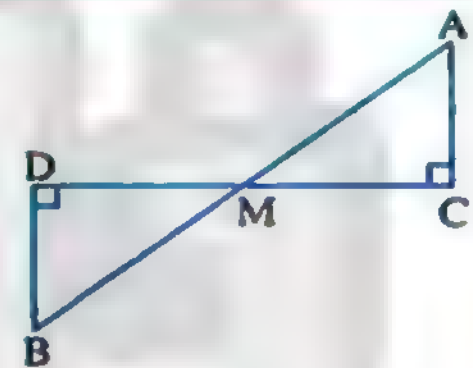
Prove that : $AB > CD$

[b] In the opposite figure :

$$AB = AD, BC = CD$$

Prove that :

$$m(\angle ABC) = m(\angle ADC)$$



4 [a] In the opposite figure :

$$AB > BC, \overline{XY} \parallel \overline{BC}$$

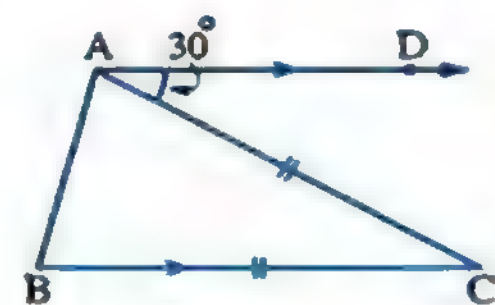
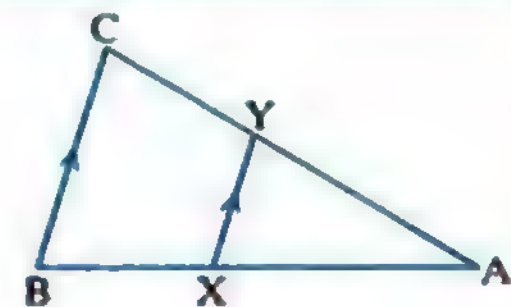
Prove that : $AX > XY$

[b] In the opposite figure :

ABC is a triangle in which $AC = BC$
 $\overline{AD} \parallel \overline{BC}$, $m(\angle DAC) = 30^\circ$

Find with proof :

The measures of the angles of ΔABC



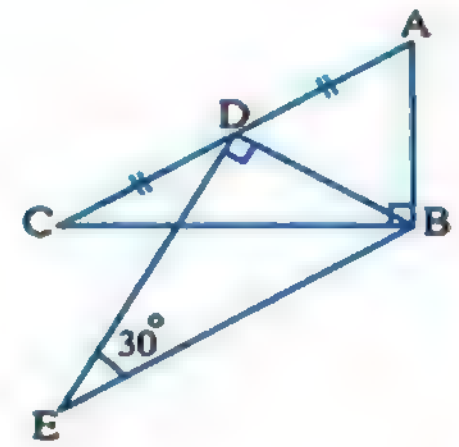
5 [a] In the opposite figure :

$$m(\angle ABC) = m(\angle BDE) = 90^\circ$$

$$, m(\angle E) = 30^\circ$$

, D is the midpoint of \overline{AC}

Prove that : $AC = BE$



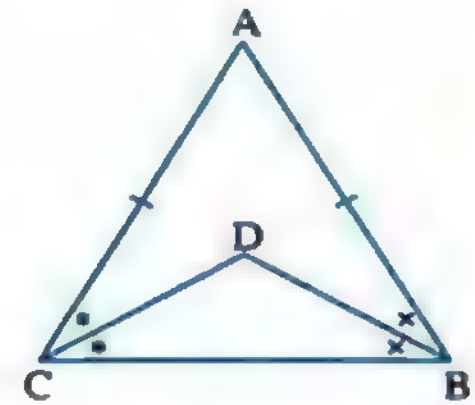
[b] In the opposite figure :

$AB = AC$, \overline{BD} bisects $\angle ABC$

and \overline{CD} bisects $\angle ACB$

Prove that :

$\triangle DBC$ is isosceles.



15

Qena Governorate

Qena Directorate of Education
Math's Supervision

Answer the following questions :

1 Complete each of the following :

- 1 The number of axes of symmetry of the equilateral triangle equals
- 2 In the triangle ABC , if $AC = BC$ and $m(\angle C) = 80^\circ$, then $m(\angle A) = \dots\dots\dots^\circ$
- 3 XYZ is a triangle , $m(\angle X) = 60^\circ$, $m(\angle Y) = 40^\circ$, then $XZ \dots\dots\dots ZY$
- 4 The point of intersection of the medians of the triangle divides each of them with the ratio of from the vertex.
- 5 The perpendicular bisector of a line segment is called

2 Choose the correct answer from those given :

- 1 The lengths 9 cm. , 4 cm. and may be the side lengths of an isosceles triangle.
(a) 9 cm. (b) 13 cm. (c) 5 cm. (d) 4 cm.
- 2 \overline{AD} is a median of $\triangle ABC$, and M is the point of concurrence of the medians , then $AM = \dots\dots\dots AD$
(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) 2
- 3 The measure of the exterior angle of an equilateral triangle equals
(a) 30° (b) 60° (c) 120° (d) 90°

Geometry

4 In the triangle ABC , if $m(\angle B) = 90^\circ$, then the greatest side in length is

- (a) \overline{AB} (b) \overline{AC} (c) \overline{CB} (d) \overline{XY}

5 In $\triangle XYZ$, if $XY > ZX$, then $m(\angle Y)$ $m(\angle Z)$

- (a) $>$ (b) $<$ (c) $=$ (d) \equiv

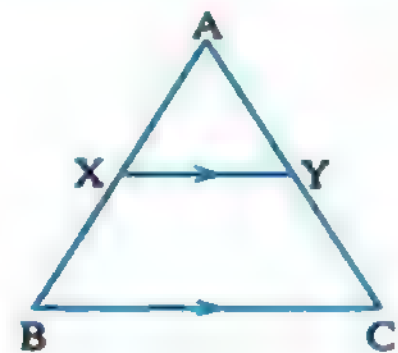
3 [a] In the opposite figure :

ABC is a triangle in which $AB = AC$

, $\overline{XY} \parallel \overline{BC}$

Prove that :

$\triangle AXY$ is an isosceles triangle.



[b] In $\triangle ABC$, $m(\angle A) = 40^\circ$, $m(\angle B) = 75^\circ$ Arrange the lengths of sides of $\triangle ABC$ in an ascending order.

4 [a] In the opposite figure :

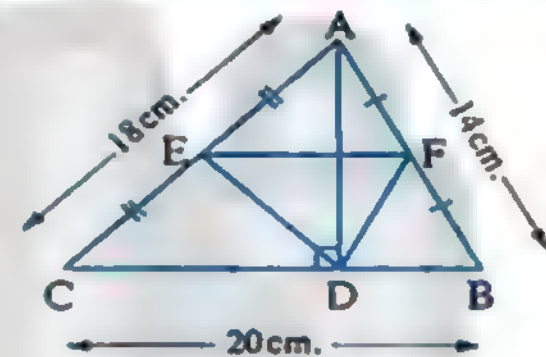
ABC is a triangle in which $AB = 14$ cm.

, $AC = 18$ cm. , $BC = 20$ cm.

, E is the midpoint of \overline{AC}

, F is the midpoint of \overline{AB} , and $\overline{AD} \perp \overline{BC}$

Find : The perimeter of $\triangle DEF$



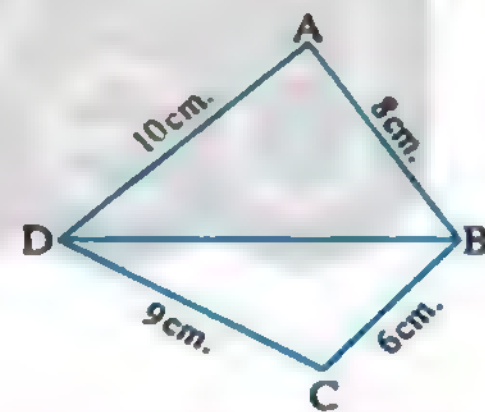
[b] In the opposite figure :

ABCD is a quadrilateral in which $AB = 8$ cm.

, $BC = 6$ cm. , $CD = 9$ cm.

and $DA = 10$ cm.

Prove that : $m(\angle ABC) > m(\angle ADC)$

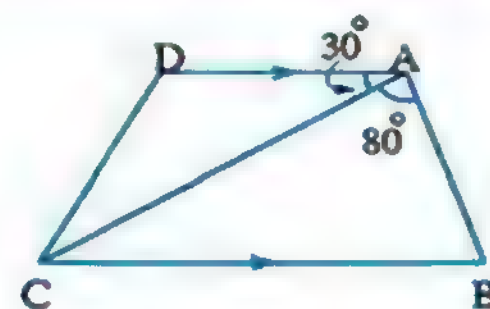


5 [a] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle BAC) = 80^\circ$

, $m(\angle DAC) = 30^\circ$

Prove that : $BC > AB$



[b] Complete : In $\triangle ABC$, if $AB = 7$ cm. , $AC = 5$ cm. , then $< BC <$

Final
Examinations of

Geometry
2019



Some Schools Examinations on Geometry

1

Cairo Governorate

East Nasr city administration
Heliopolis Language School
Mathematics Department

Answer the following questions :

1 Complete :

- (1) The intersection point of the three medians of the triangle divide the median in the ratio from the vertex.
- (2) In $\triangle ABC$: If $CA = CB$ and $m(\angle C) = m(\angle A)$, then $m(\angle B) = \dots\dots\dots^\circ$
- (3) The bisector of the vertex angle of the isosceles triangle is and
- (4) If the measure of an angle in the isosceles triangle is 100° , then the number of axes of symmetry of $\triangle ABC$ is
- (5) The longest side in the right-angled triangle is

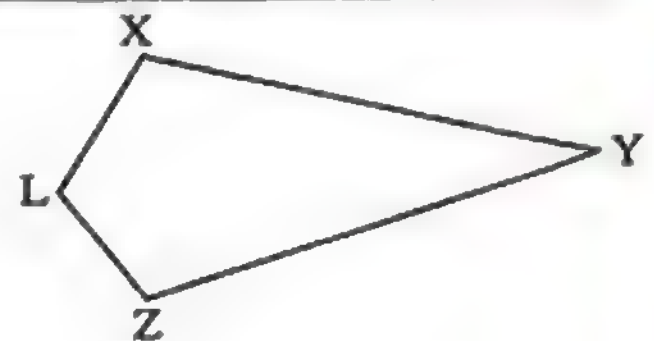
2 Choose the correct answer :

- (1) In $\triangle ABC$: If $m(\angle B) = 90^\circ$, then
 (a) $AC > CB$ (b) $AB > AC$ (c) $BC > AC$ (d) $AB = AC$
- (2) If the lengths of two sides of an isosceles triangle are 3 cm. and 7 cm. , then the length of the third side is
 (a) 3 (b) 4 (c) 7 (d) 10
- (3) In $\triangle ABC$: If $AB = AC$ and $m(\angle A) = 60^\circ$, then the number of axes of symmetry of the triangle ABC is
 (a) 0 (b) 1 (c) 2 (d) 3
- (4) Any triangle has medians.
 (a) 0 (b) 1 (c) 2 (d) 3
- (5) If ABCD is a square , then the axes of symmetry of \overline{AC} is
 (a) \overrightarrow{AD} (b) \overrightarrow{BC} (c) \overrightarrow{BD} (d) \overrightarrow{AB}

3 [a] In the opposite figure :

$XY > XL$

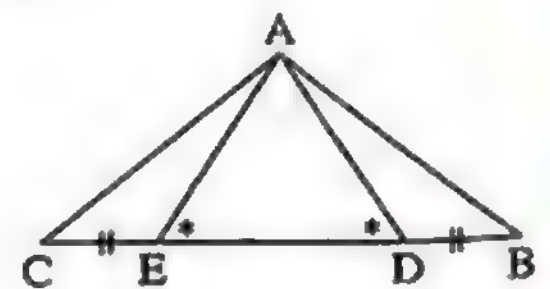
and $YZ > ZL$

Prove that : $m(\angle XLZ) > m(\angle XYZ)$ 

[b] In the opposite figure :t

$\angle ADC \equiv \angle AED$ and $BD = CE$

, B , D , E and C are collinear.

Prove that : $\triangle ABC$ is an isosceles triangle.

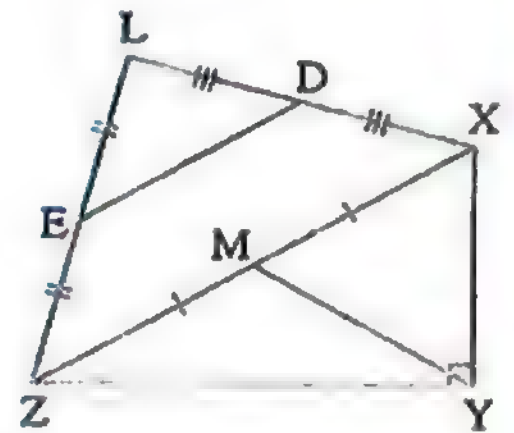
4 [a] In the opposite figure :

$$m(\angle XYZ) = 90^\circ$$

, D is midpoint of \overline{XL}

, E is midpoint of \overline{ZL} and M is the midpoint of \overline{XZ}

Prove that : $DE = YM$



[b] In the opposite figure :

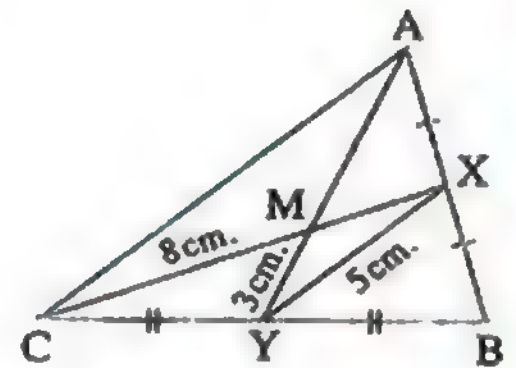
ABC is a triangle , X is the midpoint of \overline{AB}

, Y is midpoint of \overline{BC} , $XY = 5$ cm. and $\overline{XC} \cap \overline{AY} = \{M\}$

where $CM = 8$ cm. , $YM = 3$ cm.

Find : (1) The perimeter of $\triangle MXY$

(2) The perimeter of $\triangle MAC$



5 [a] In the opposite figure :

$AC > AB$ and $DB = DC$

Prove that : $m(\angle ABD) > m(\angle ACD)$

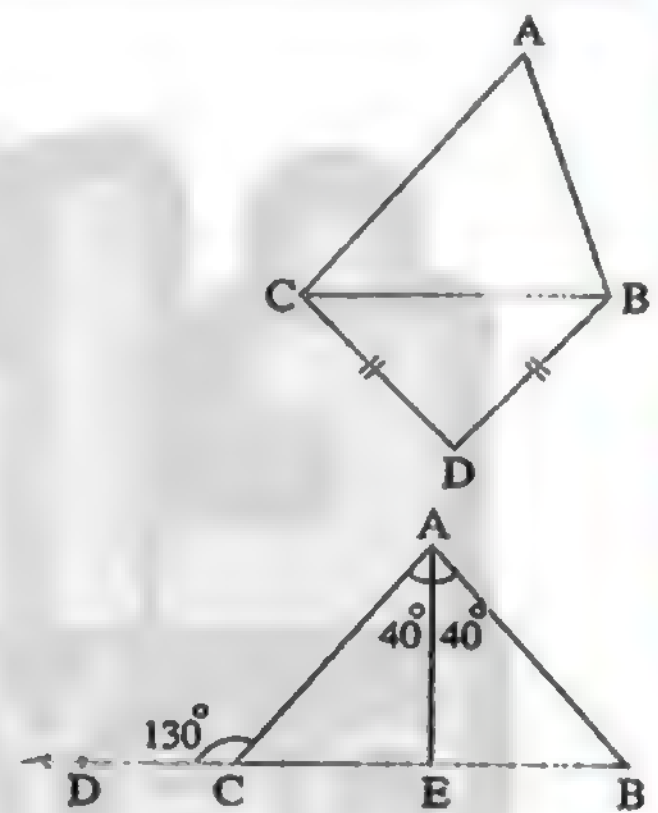
[b] In the opposite figure :

$C \in \overline{BD}$, $m(\angle ACD) = 130^\circ$

and $m(\angle BAE) = m(\angle CAE) = 40^\circ$

Prove that : (1) $\overline{AE} \perp \overline{BC}$

(2) E bisects \overline{BC}



2

Cairo Governorate

Maadi Educational Zone
Sakkara Language School



Answer the following questions :

1 Complete :

(1) In $\triangle XYZ$, $m(\angle X) = 90^\circ$, then the longest side is

(2) The base angles of the isosceles triangle are

(3) ABC is a triangle in which $AB = 4$ cm. , $CB = 7$ cm. , then $AC \in].....,[$

(4) If A \in the axis of symmetry of \overline{XY} , then =

(5) If the measure of an angle in the isosceles triangle equals 60° , then the triangle has axes of symmetry.

Geometry

2 Choose the correct answer :

- (1) The measure of the exterior angle of equilateral triangle =
- (a) 90° (b) 120° (c) 45° (d) 60°
- (2) If \overline{AD} is a median in $\triangle ABC$ and M is the point of intersection of the medians , then $AM = \dots\dots\dots AD$
- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{1}{2}$
- (3) In $\triangle XYZ$, if $m(\angle Z) = 70^\circ$ and $m(\angle Y) = 60^\circ$, then $YZ \dots\dots\dots XY$
- (a) $<$ (b) $=$ (c) $>$ (d) is twice
- (4) The numbers 4 , 8 , can be lengths of sides of an isosceles triangle.
- (a) 4 (b) 8 (c) 12 (d) 3
- (5) In $\triangle ABC$, if $m(\angle B) = 90^\circ$ and $m(\angle C) = 30^\circ$, then $AB \dots\dots\dots AC$
- (a) $\frac{1}{3}$ (b) 2 (c) equals (d) $\frac{1}{2}$

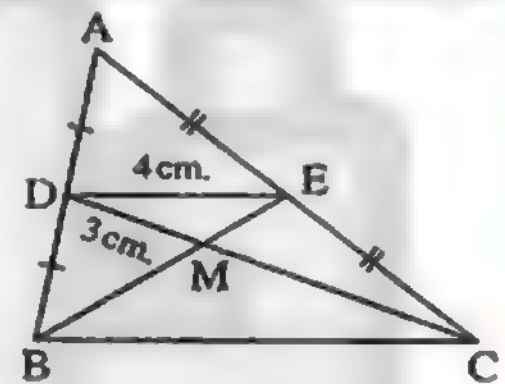
3 [a] In the opposite figure :

D is the midpoint of \overline{AB} , E is the midpoint of \overline{AC}

$\overline{CD} \cap \overline{BE} = \{M\}$

If $DE = 4 \text{ cm.}$, $DM = 3 \text{ cm.}$, $BE = 6 \text{ cm.}$

Find : The perimeter of $\triangle BMC$



- [b] In $\triangle ABC$, if $AB = 5 \text{ cm.}$, $BC = 7 \text{ cm.}$ and $AC = 9 \text{ cm.}$
Arrange the measures of its angles in a descending order.

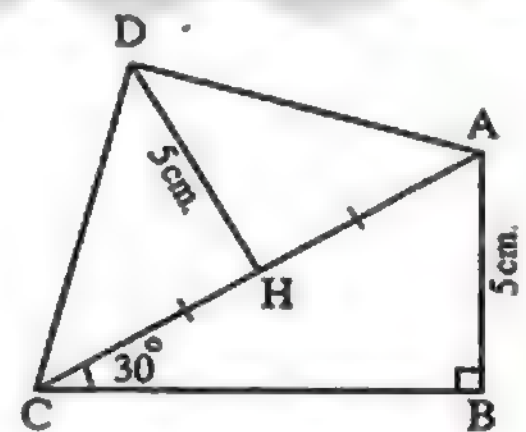
4 [a] In the opposite figure :

ABC is a right angled triangle at B

$m(\angle ACB) = 30^\circ$, $AB = 5 \text{ cm.}$

$DH = 5 \text{ cm.}$ and H is the midpoint of \overline{AC}

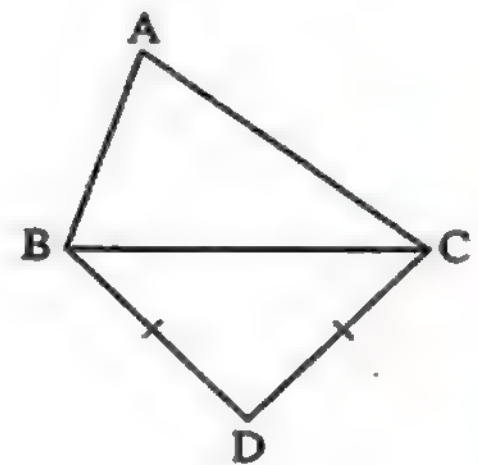
Prove that : $m(\angle ADC) = 90^\circ$



[b] In the opposite figure :

If $AC > AB$ and $DC = DB$

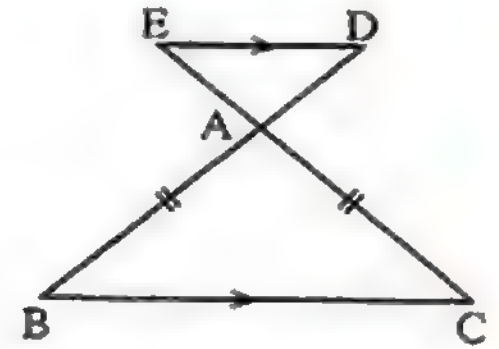
Prove that : $m(\angle ABD) > m(\angle ACD)$



5 [a] In the opposite figure :

If $AB = AC$

Prove that : $AD = AE$



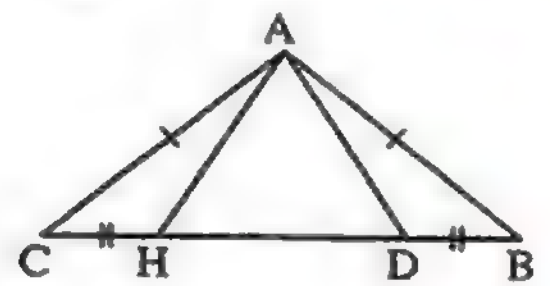
[b] In the opposite figure :

ABC is a triangle in which :

$AB = AC$, $BD = CH$

Prove that : ① $\triangle ADH$ is an isosceles triangle.

② $\angle AHD \equiv \angle ADH$



3

Cairo Governorate

El-Sayda Zinab Educational Zone



Answer the following questions :

1 Choose the suitable answer :

① The number of axes of symmetry of an equilateral triangle is

- (a) 0 (b) 1 (c) 2 (d) 3

② An isosceles triangle , one of its base angles has measure 50° , then the measure of the vertex angle =

- (a) 50° (b) 60° (c) 70° (d) 80°

③ \overline{AD} is a median of triangle ABC , and M is the point of intersection of the medians , then $AM = \dots\dots\dots AD$

- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

④ If the lengths of two sides of a triangle are 4 cm. and 8 cm. , then the length of the third side = cm.

- (a) 3 (b) 4 (c) 8 (d) 12

⑤ In a triangle ABC , if $m(\angle A) = 80^\circ$ and $m(\angle C) = 60^\circ$, then $AB \dots\dots\dots BC$

- (a) $<$ (b) $>$ (c) $=$ (d) \geq

2 Complete :

① If XYZ is a right-angled triangle at Y , then the longest side is

② The sum of measures of any two consecutive angles in the parallelogram = $^\circ$

③ The straight line perpendicular to the midpoint of a line segment is called

④ The bisectors of the vertex angle of an isosceles triangle and

⑤ The measure of the exterior angle of the equilateral triangle = $^\circ$

Geometry

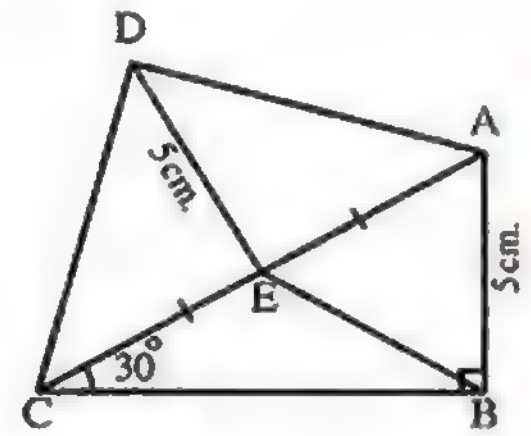
3 [a] In the opposite figure :

ABC is a right-angled triangle at B

, $m(\angle ACB) = 30^\circ$, $AB = 5$ cm.

, E is midpoint of \overline{AC}

If $DE = 5$ cm. then prove that : $m(\angle ADC) = 90^\circ$



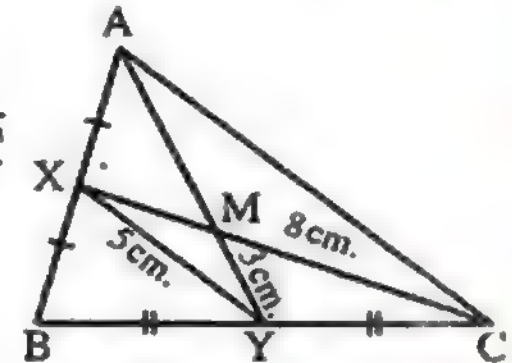
[b] In the opposite figure :

ABC is a triangle , X is the midpoint of \overline{AB} , Y is the midpoint of \overline{BC}

, $XY = 5$ cm. , $\overline{XC} \cap \overline{AY} = \{M\}$

where : $CM = 8$ cm. , $YM = 3$ cm.

Find with proof : The length of each of : ① \overline{AM} ② \overline{MX} ③ \overline{AC}

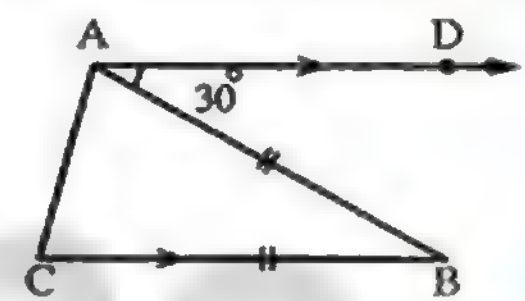


4 [a] In the opposite figure :

ABC is a triangle in which : $AB = BC$, $\overline{AD} \parallel \overline{BC}$

, $m(\angle DAB) = 30^\circ$

Find : The measures of the angles of $\triangle ABC$

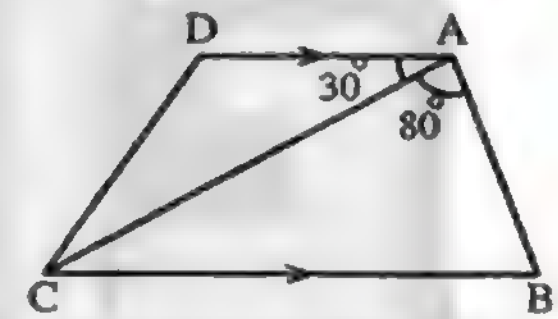


[b] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle BAC) = 80^\circ$

, $m(\angle DAC) = 30^\circ$

Prove that : $BC > AB$

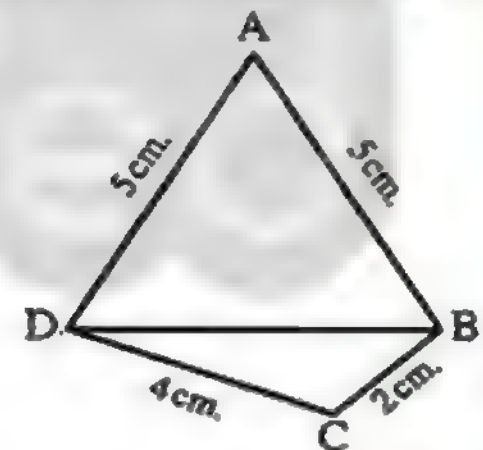


5 In the opposite figure :

ABCD is a quadrilateral in which : $AB = AD = 5$ cm.

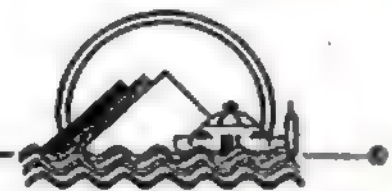
, $BC = 2$ cm. , $DC = 4$ cm.

Prove that : $m(\angle ABC) > m(\angle ADC)$



Giza Governorate

Dokki District
Modern Narmar Language School



Answer the following questions :

1 Choose the correct answer from those given :

① In the opposite figure :

$\triangle ADB$, $m(\angle ADB) = 90^\circ$, $BD = 5$ cm.

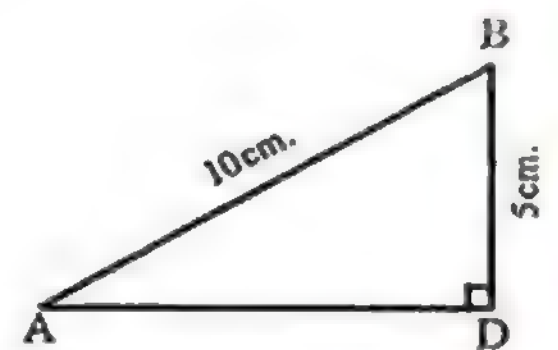
and $AB = 10$ cm. , then $m(\angle A) = \dots\dots\dots^\circ$

(a) 30

(b) 50

(c) 70

(d) 90



(2) In the opposite figure :

If $AB = AC$ and $BE = BC$

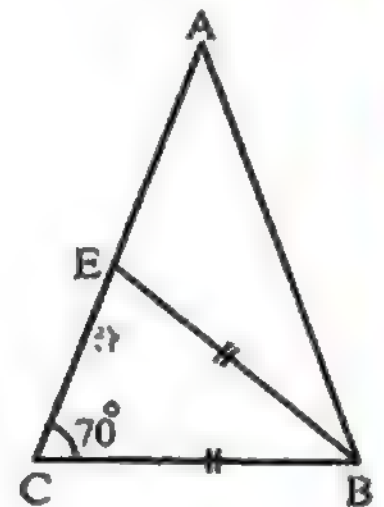
, then : $m(\angle ABE) = \dots\dots\dots$

(a) 30°

(b) 40°

(c) 70°

(d) 110°



(3) In the opposite figure :

$\triangle ABC$, $AB = BC$

, an altitude is drawn from B to \overline{AC} and intersects \overline{AC} at D

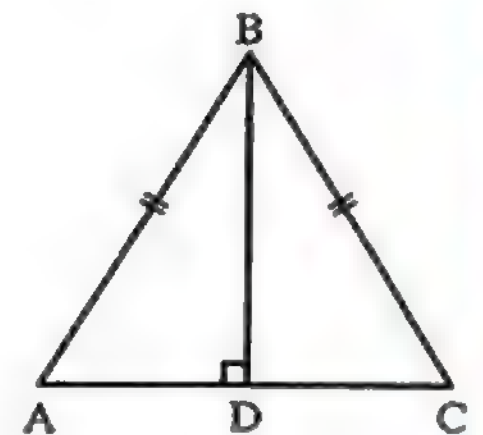
which conclusion is not always true ?

(a) $m(\angle ABD) = m(\angle CBD)$

(b) $m(\angle BDA) = m(\angle BDC)$

(c) $AD = BD$

(d) $AD = DC$



(4) Which set of numbers represents the lengths of the sides of a triangle ?

(a) $\{5, 18, 13\}$

(b) $\{6, 17, 22\}$

(c) $\{16, 24, 7\}$

(d) $\{26, 8, 15\}$

(5) The point of concurrency of medians divides each median in the ratio from the base.

(a) $1 : 2$

(b) $2 : 1$

(c) $3 : 1$

(d) $2 : 3$

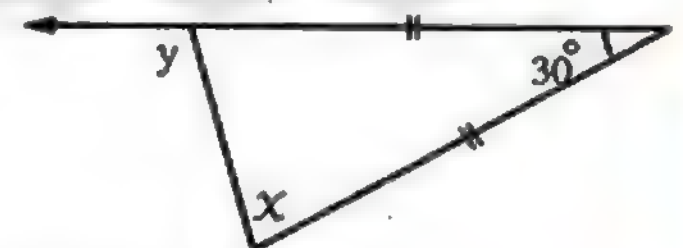
2 Complete :

(1) The longest side in the right-angled triangle is

(2) If the measure of an angle in the isosceles triangle equals 60° , then the triangle is

(3) In the opposite figure :

$x = \dots\dots\dots^\circ$ and $y = \dots\dots\dots^\circ$



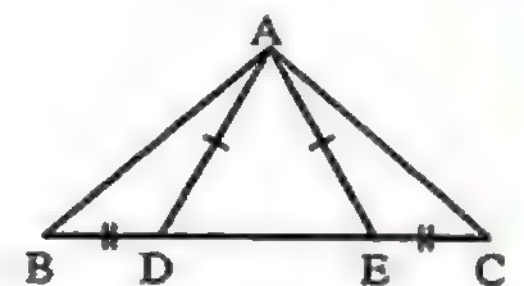
(4) If the length of the median drawn from the right vertex of a triangle is 6 cm. , then the length of the hypotenuse is cm.

(5) In $\triangle ABC$, $m(\angle A) = 60^\circ$, $m(\angle B) = 50^\circ$, then the longest side is

3 [a] In the opposite figure :

$AD = AE$ and $BD = CE$

Prove that : $\triangle ABC$ is an isosceles triangle.



Geometry

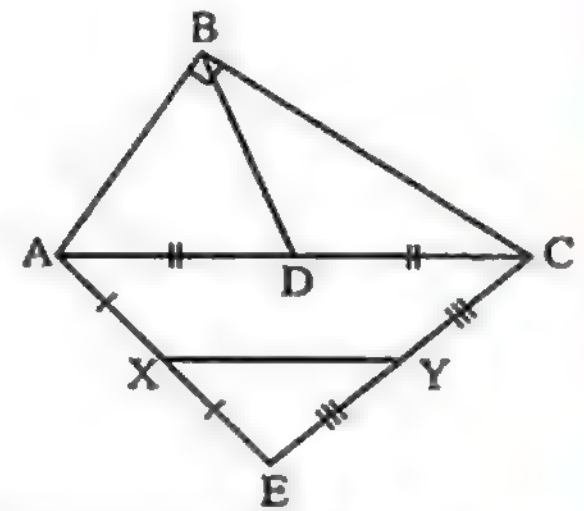
[b] In the opposite figure :

$\triangle ABC$ is right-angled at B

, D is the midpoint of \overline{AC}

, X and Y are the midpoints of \overline{AE} and \overline{CE} respectively.

Prove that : $BD = XY$

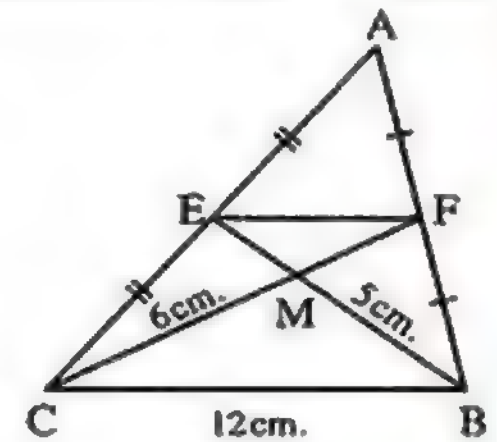


[4] [a] In the opposite figure :

$\triangle ABC$, F and E are the midpoints of \overline{AB} and \overline{AC} respectively.

If $BM = 5$ cm. , $CM = 6$ cm. , $BC = 12$ cm. ,

then find : The perimeter of $\triangle MEF$



[b] In $\triangle ABC$, $m(\angle A) = 3x^\circ$, $m(\angle B) = (4x - 9)^\circ$

and $m(\angle C) = (2x + 9)^\circ$

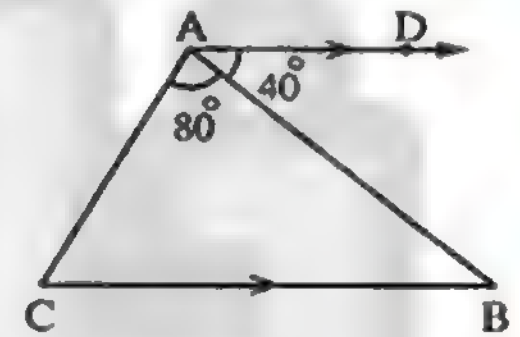
Find the measure of each angle and arrange the sides in a descending order according to their lengths.

[5] [a] In the opposite figure :

$\triangle ABC$, in which : $\overline{AD} \parallel \overline{BC}$

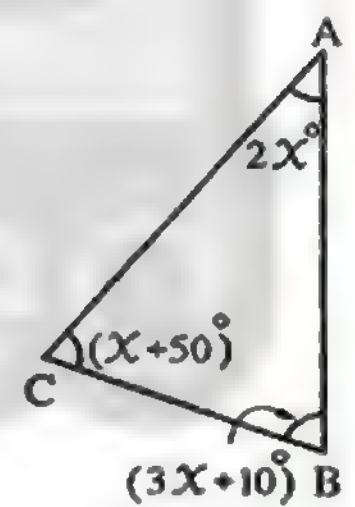
$m(\angle DAB) = 40^\circ$ and $m(\angle BAC) = 80^\circ$

Prove that : $AB > AC$



[b] In the opposite figure :

Show with proof , which sides are equal in length.

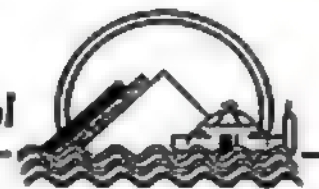


5

Giza Governorate

Omrania Directorate

El sadat Governmental Language School



Answer the following questions :

[1] Complete each of the following :

- ① The point of concurrence of medians of a triangle divides each median in ratio : from the vertex.
- ② The longest side in the right-angled triangle is
- ③ The straight line perpendicular to the midpoint of a line segment is called
- ④ The base angles of the isosceles triangle are
- ⑤ In $\triangle ABC$, if $AB < BC < AC$, then the greatest angle in measure is

2 Choose the correct answer from given ones :

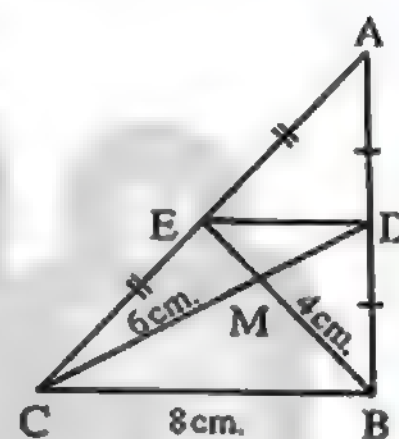
- ① The number of axes of symmetry in the scalene triangle is
 (a) 1 (b) 2 (c) 3 (d) zero
- ② The measure of the exterior angle of an equilateral triangle is
 (a) 90° (b) 120° (c) 60° (d) 30°
- ③ The numbers 5 , 4 , can be lengths of sides of a triangle.
 (a) 8 (b) 9 (c) 10 (d) 12
- ④ In $\triangle ABC$, $AB = AC$ and $m(\angle B) = 70^\circ$, then $m(\angle A) =$
 (a) 140° (b) 70° (c) 40° (d) 110°
- ⑤ $\triangle ABC$ in which : $m(\angle B) > m(\angle C)$, then AC AB
 (a) $>$ (b) $<$ (c) $=$ (d) \leq

3 [a] In the opposite figure :

ABC is a triangle in which D , E are midpoints of \overline{AB} and \overline{AC} respectively ,

$MC = 6$ cm. , $MB = 4$ cm. and $BC = 8$ cm.

Find : The perimeter of $\triangle DME$



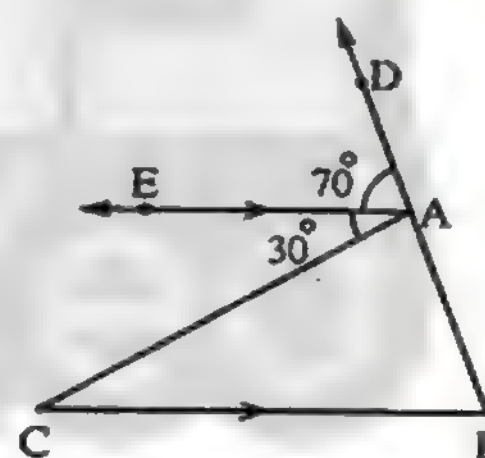
[b] In the opposite figure :

$\overline{AE} \parallel \overline{BC}$

, $m(\angle DAE) = 70^\circ$

, $m(\angle EAC) = 30^\circ$

Prove that : $AC > AB$

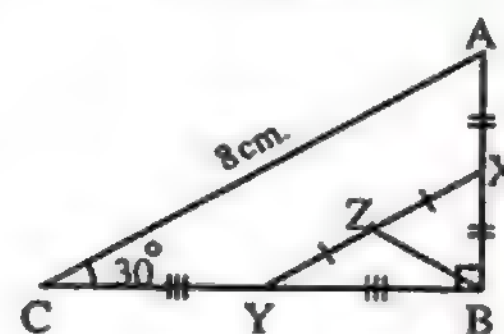


4 [a] In the opposite figure :

ABC is a triangle in which : $m(\angle ABC) = 90^\circ$

, $m(\angle C) = 30^\circ$, X , Y and Z are midpoints of \overline{AB} , \overline{BC} and \overline{XY} respectively and $AC = 8$ cm.

Find : The length of each of \overline{AB} , \overline{XY} , \overline{BZ}

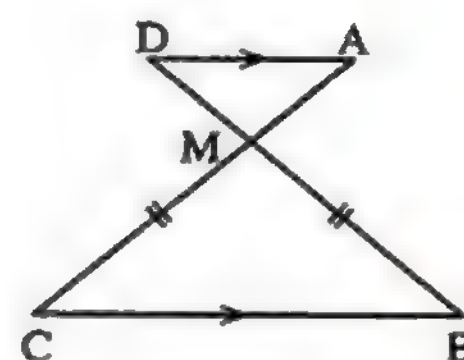


[b] In the opposite figure :

$\overline{AC} \cap \overline{BD} = \{M\}$

, $MB = MC$ and $\overline{AD} \parallel \overline{BC}$

Prove that : $MA = MD$



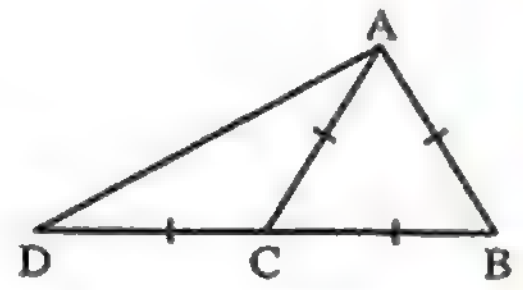
Geometry

5 In the opposite figure :

ABC is an equilateral triangle

, $D \in \overline{BC}$ such that $BC = CD$

Prove that : $\overline{BA} \perp \overline{AD}$



Alexandria Governorate

Middle Educational Directorate
Math's Supervision



Answer the following questions :

1 Choose the correct answer :

- ① The isosceles triangle has of symmetry.
(a) one axis (b) two axes (c) three axes (d) zero axes
- ② In $\triangle ABC$, if $m(\angle A) = 125^\circ$, then the longest side of it is
(a) \overline{AB} (b) \overline{AC} (c) \overline{BC} (d) its median
- ③ If XYZ is an isosceles triangle , $m(\angle Y) = 100^\circ$, then $m(\angle X) =$
(a) 80° (b) 40° (c) 20° (d) 100°
- ④ In $\triangle ABC$ if $m(\angle A) = 30^\circ$, $m(\angle B) = 90^\circ$, then $BC =$ AC
(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) 2
- ⑤ The measure of each exterior angle of equilateral triangle is
(a) 180° (b) 360° (c) 60° (d) 120°

2 Complete :

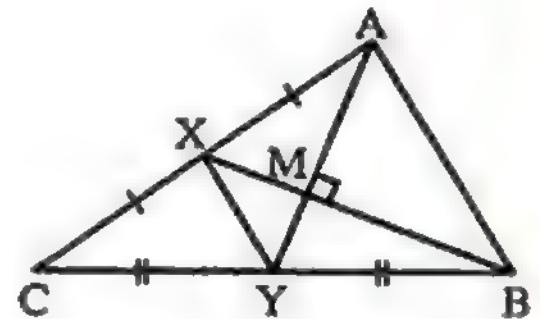
- ① The point of concurrence divides each median in the ratio from the base.
- ② The longest side in the right angled triangle is
- ③ The sum of measures of the exterior angles of a square is °
- ④ The numbers 8 , 4 , can be lengths of sides of an isosceles triangle.
- ⑤ The axis of symmetry of a line segment is the straight line which is

3 [a] In the opposite figure :

\overline{AY} and \overline{BX} are two medians where $\overline{AY} \perp \overline{BX}$

, if $AY = 12$ cm. and $XM = 5$ cm.

Find : The area of $\triangle ABM$

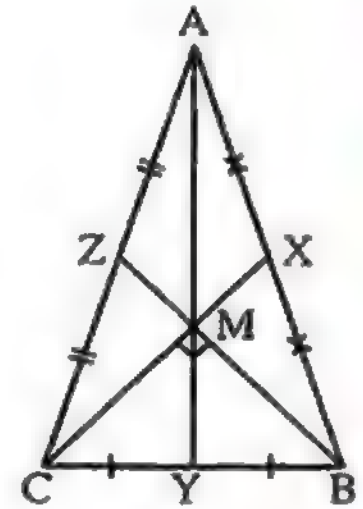


[b] ABC is a triangle in which : $m(\angle A) = 6x^\circ$, $m(\angle B) = (4x - 9)^\circ$ and $m(\angle C) = 3(x - 2)^\circ$ Arrange the lengths of sides descendingly.

4 [a] In the opposite figure :

\overline{BZ} and \overline{CX} are two medians of $\triangle ABC$
 $\overline{CX} \perp \overline{BZ}$

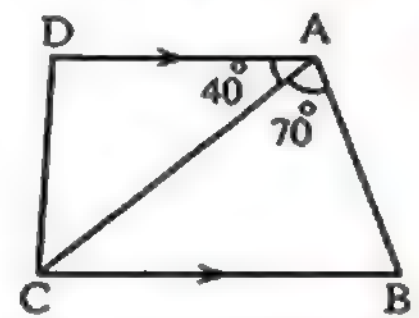
Prove that : $AM = BC$



[b] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle DAC) = 40^\circ$
 $m(\angle BAC) = 70^\circ$

Prove that : $BC = AC$



5 [a] In the opposite figure :

$AB = AC$

Prove that : $EC > EF$

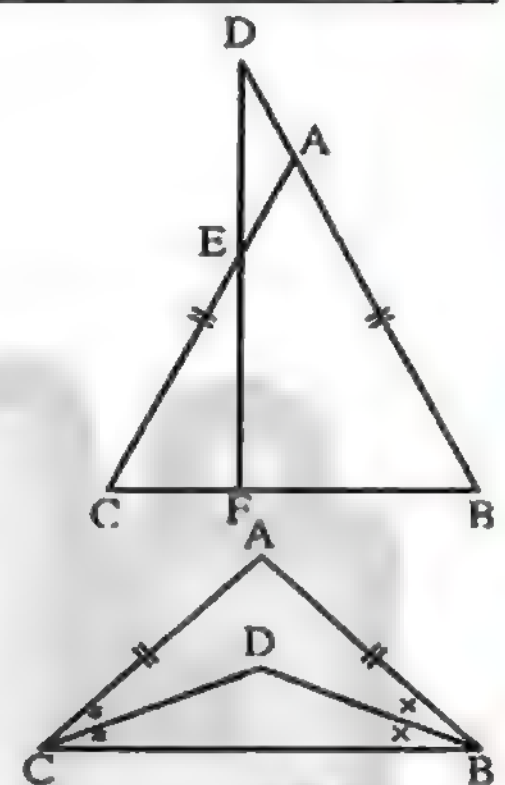
[b] In the opposite figure :

$AB = AC$

\overline{BD} bisects $\angle B$

\overline{CD} bisects $\angle C$

Prove that : $BD = CD$



Alexandria Governorate

East Educational Zone
 Mathematics Directing



Answer the following questions :

1 Complete the following :

- ① If ABCD is a parallelogram and $m(\angle A) = 70^\circ$, then $m(\angle B) = \dots\dots\dots^\circ$
- ② The measure of the exterior angle in the equilateral triangle = $\dots\dots\dots^\circ$
- ③ The length of the median from the vertex of the right angle in the right-angled triangle = $\dots\dots\dots$
- ④ If $AB = AC$ in $\triangle ABC$ and $m(\angle B) = 40^\circ$, then $m(\angle C) = \dots\dots\dots^\circ$
- ⑤ In $\triangle XYZ$, if $XY < YZ < ZX$, then the greatest angle in measure is $\angle \dots\dots\dots$

2 Choose the correct answer from those given :

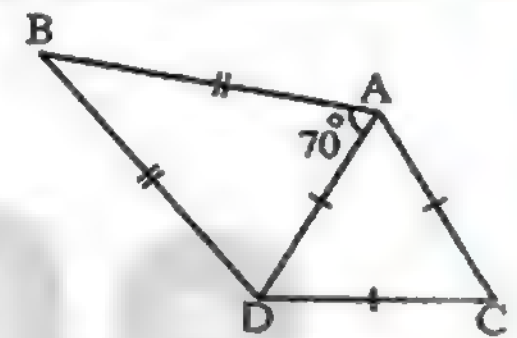
- ① The diagonals are perpendicular in $\dots\dots\dots$
 - (a) square and rectangle.
 - (b) rectangle and rhombus.
 - (c) square and rhombus.
 - (d) parallelogram and rectangle.

Geometry

- (2) The point of the intersection of the medians in triangle divides each median from the base into the ratio
- (a) 1 : 2 (b) 2 : 1 (c) 3 : 1 (d) 2 : 3
- (3) The isosceles triangle has axis of symmetry.
- (a) 0 (b) 1 (c) 2 (d) 3
- (4) If the lengths of two sides in an isosceles triangle 3 cm. and 7 cm. , then the length of the third side = cm.
- (a) 3 (b) 4 (c) 7 (d) 10
- (5) In $\triangle ABC$, if $m(\angle A) < m(\angle B)$, then
- (a) $AC < BC$ (b) $AC > BC$ (c) $AC = BC$ (d) $\overline{AC} \parallel \overline{BC}$

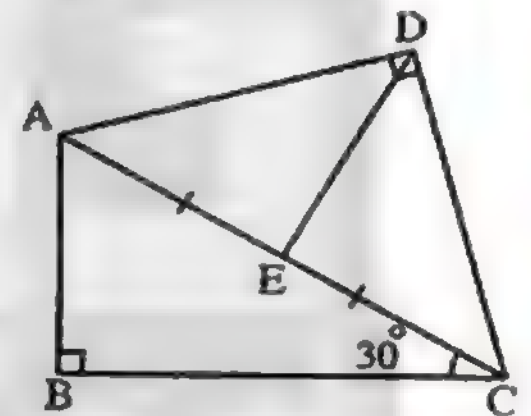
3 [a] In the opposite figure :

$AB = BD$, $m(\angle BAD) = 70^\circ$
 , $\triangle ADC$ is an equilateral triangle.
 Find : $m(\angle BDC)$



[b] In the opposite figure :

$m(\angle ABC) = m(\angle ADC) = 90^\circ$
 , $m(\angle ACB) = 30^\circ$
 , E is the midpoint of \overline{AC}
 Prove that : $AB = ED$

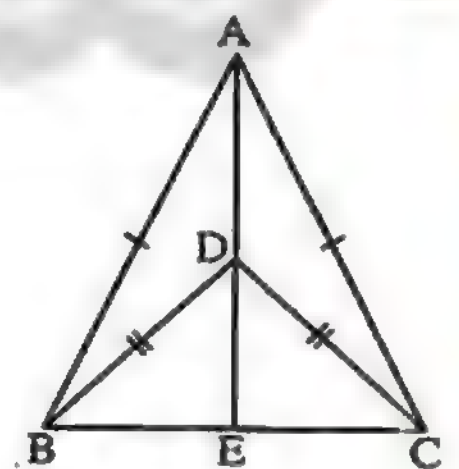


4 [a] In the opposite figure :

$AB = AC$, $DB = DC$, $D \in \overline{AE}$

Prove that :

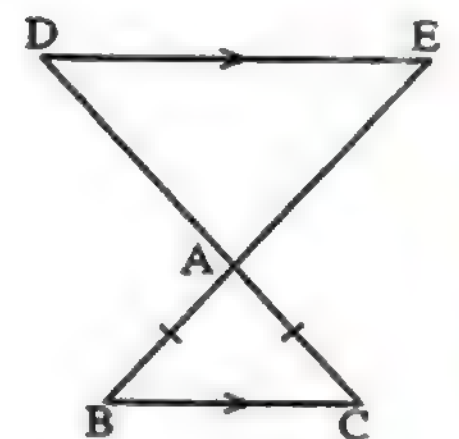
- ① $\overline{AE} \perp \overline{BC}$
 ② $BE = EC$



[b] In the opposite figure :

$AB = AC$ and $\overline{DE} \parallel \overline{BC}$

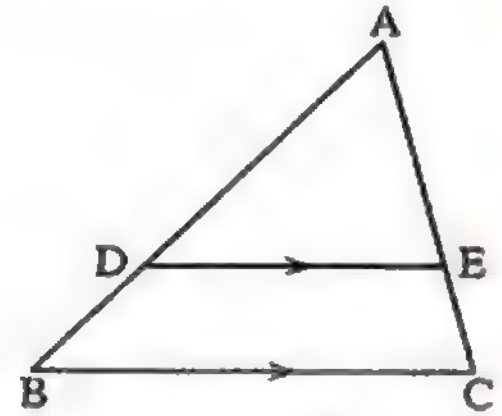
Prove that : $AD = AE$



5 [a] In the opposite figure :

$$AB > AC, \overline{DE} \parallel \overline{BC}$$

Prove that : $AD > AE$



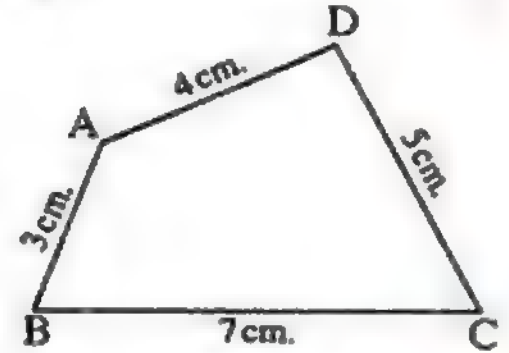
[b] In the opposite figure :

ABCD is a quadrilateral in which :

$$AB = 3 \text{ cm.}, BC = 7 \text{ cm.}$$

$$, CD = 5 \text{ cm. and } DA = 4 \text{ cm.}$$

Prove that : $m(\angle BAD) > m(\angle BCD)$



8 El-Kalyoubia Governorate

Al-Obour Educational Zone
Al-Resala Language School



Answer the following questions :

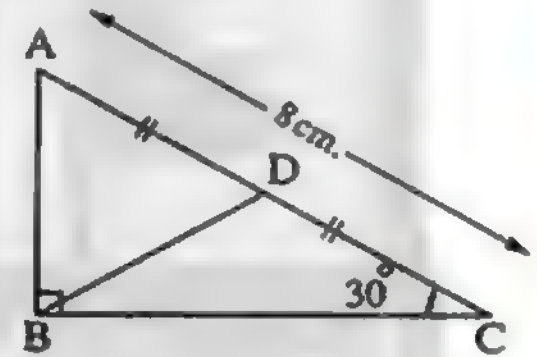
1 Complete the following :

(1) The bisector of the vertex angle of an isosceles triangle bisect the base and

(2) 3 cm. , 8 cm. and cm. are three sides of an isosceles triangle.

(3) In the opposite figure :

The perimeter of $\triangle ABD = \dots\dots\dots$ cm.



(4) The measure of the exterior angle of the equilateral triangle =°

(5) In $\triangle ABC$, $m(\angle A) = 100^\circ$, then the longest side is

2 Choose the correct answer :

(1) In $\triangle ABC$, if $m(\angle B) = 90^\circ$ and $m(\angle A) = 30^\circ$, then $BC = \dots\dots\dots$

(a) $\frac{1}{2} AC$

(b) $2 AC$

(c) $2 AB$

(d) $\frac{1}{2} AB$

(2) If A \in the axis of symmetry of \overline{BC} , then $AB = \dots\dots\dots$

(a) XY

(b) XZ

(c) AC

(d) BC

(3) The triangle whose side length are 2 cm. , $(X + 3)$ cm. and 5 cm. becomes an isosceles triangle when $X = \dots\dots\dots$ cm.

(a) zero

(b) 1

(c) 2

(d) 3

(4) The number of axis of symmetry of the equilateral triangle =

(a) zero

(b) 1

(c) 2

(d) 3

Geometry

(5) The sum of the lengths of any two sides in the triangle the length of the third side.

(a) <

(b) \leq (c) \geq

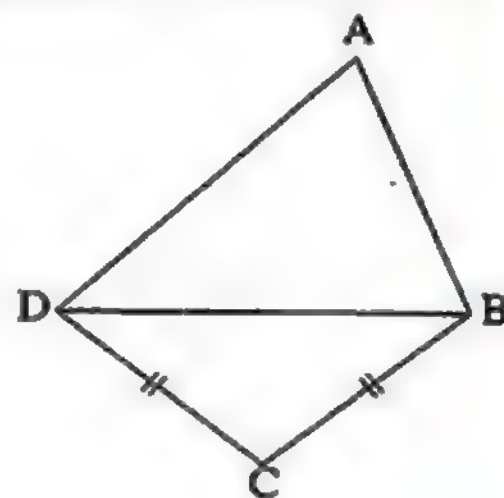
(d) >

(e) =

3 [a] In the opposite figure :

ABCD is a quadrilateral in which $AD > AB$ and $BC = CD$

Prove that : $m(\angle ABC) > m(\angle ADC)$



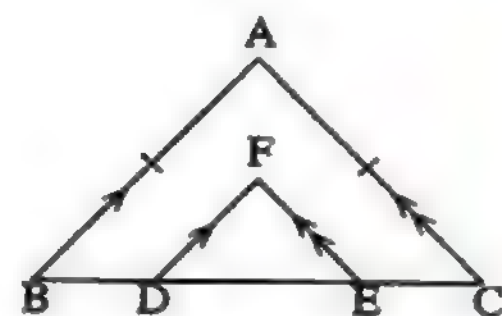
[b] In the opposite figure :

$D \in \overline{BC}$, $E \in \overline{BC}$

, $\overline{AB} \parallel \overline{FD}$ and $\overline{AC} \parallel \overline{FE}$

, if $AB = AC$

Prove that : FDE is an isosceles triangle.



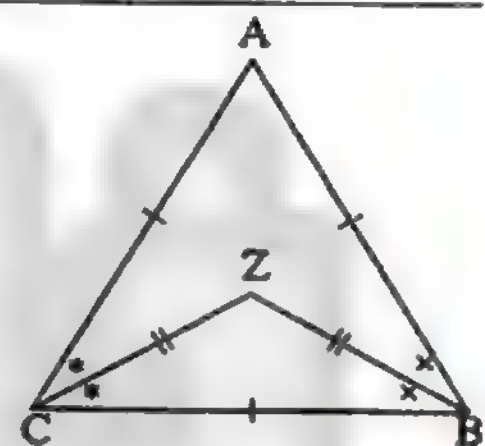
4 [a] In the opposite figure :

$\triangle ABC$ is an equilateral triangle

, \overline{BZ} bisects $\angle B$

, \overline{CZ} bisects $\angle C$

Find : The measure of the angles in triangle CZB



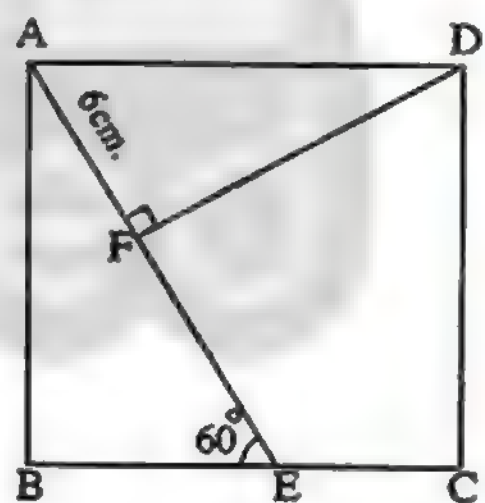
[b] In the opposite figure :

ABCD is a square

, $m(\angle AEB) = 60^\circ$

, $AF = 6 \text{ cm}$, $\overline{DF} \perp \overline{AE}$

Find : The perimeter of the square ABCD



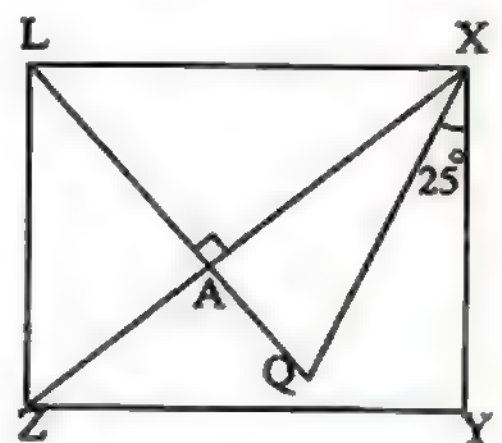
5 [a] In the opposite figure :

XYZL is a rectangle in which $m(\angle YXQ) = 25^\circ$

, $\overline{LQ} \perp \overline{XZ}$

, \overline{XQ} bisects angle YXZ

Prove that : $LQ = XL$



[b] In $\triangle ABC$, $m(\angle A) = 40^\circ$, $m(\angle B) = 80^\circ$

Arrange the length of the sides of the triangle ABC in a descending order.

9

El-Monofia Governorate

Maths Supervision



Answer the following questions :

1 Complete :

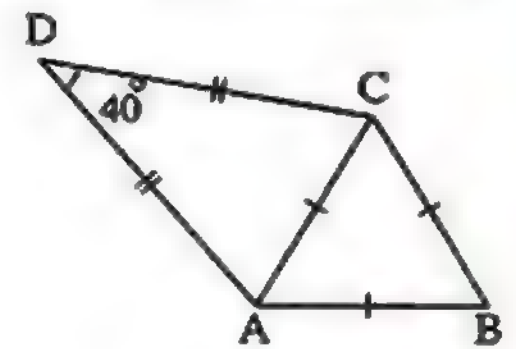
- (1) The perpendicular which is drawn from vertex of an isosceles triangle to its base and
- (2) The length of the median from the vertex of the right-angled triangle equals
- (3) In $\triangle ABC$, if $AB = AC$ and $m(\angle A) = 80^\circ$, then $m(\angle B) = \dots\dots\dots^\circ$
- (4) The measure of the exterior angle of the equilateral triangle = $\dots\dots\dots^\circ$
- (5) In $\triangle DEF$, if $DE > DF$, then $m(\angle F) > \dots\dots\dots$

2 Choose the correct answer :

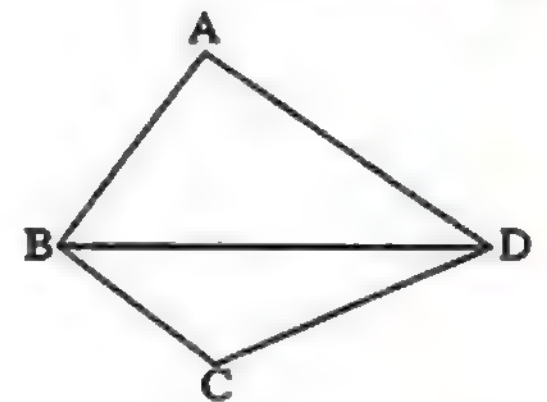
- (1) If the length of two sides in an isosceles triangle are 8 cm. and 4 cm. , then the length of the third side is cm.
 (a) 4 (b) 8 (c) 3 (d) 12
- (2) The number of axes of symmetry in the isosceles triangle =
 (a) 1 (b) 0 (c) 2 (d) 3
- (3) \overline{AD} is a median in $\triangle ABC$, M is the point of intersection of the medians , $MD = 2$ cm. , then $AD = \dots\dots\dots$ cm.
 (a) 2 (b) 4 (c) 6 (d) 8
- (4) $\triangle ABC$: $m(\angle B) = 125^\circ$, then the longest side of it is
 (a) \overline{BC} (b) \overline{AC} (c) \overline{AB} (d) its median
- (5) In $\triangle XYZ$, if $m(\angle Y) = 90^\circ$, $m(\angle X) = 30^\circ$ and $XZ = 20$ cm. , then $ZY = \dots\dots\dots$ cm.
 (a) 12 (b) 6 (c) 24 (d) 10

3 [a] In the opposite figure :

$m(\angle D) = 40^\circ$, $DA = DC$
 and $\triangle ABC$ is an equilateral triangle
 Find : $m(\angle DCB)$

**[b] In the opposite figure :**

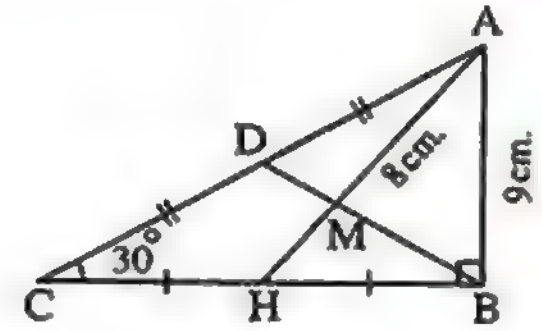
$AB < AD$ and $BC < CD$
 Prove that : $m(\angle ABC) > m(\angle ADC)$



Geometry

4 [a] In the opposite figure :

D and H are the midpoints of \overline{AC} and \overline{CB} respectively
 $m(\angle C) = 30^\circ$, $m(\angle B) = 90^\circ$, $AB = 9 \text{ cm.}$, $AM = 8 \text{ cm.}$
Find : The length of each of \overline{BD} , \overline{AH} and \overline{MD}

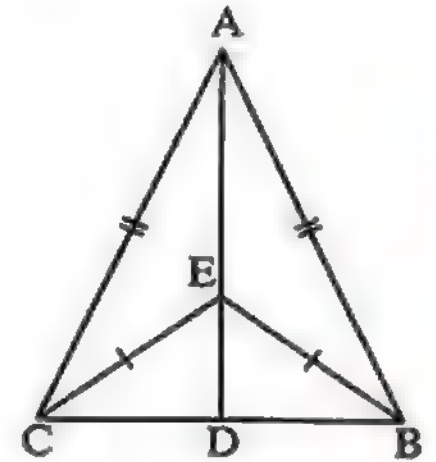


[b] In the opposite figure :

$AB = AC$ and $EB = EC$

Prove that :

- ① \overrightarrow{AE} is the axis of \overline{BC}
- ② $BD = CB$

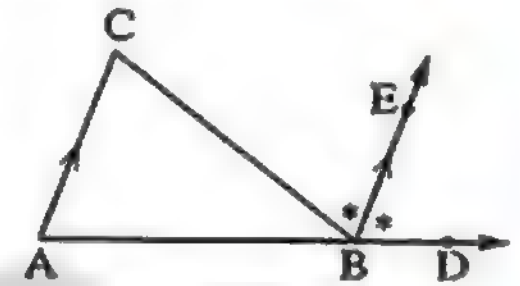


5 [a] In the opposite figure :

$D \in \overline{AB}$, \overrightarrow{BE} bisects $\angle CBD$
 and $\overrightarrow{BE} \parallel \overline{AC}$

Prove that :

$\triangle ABC$ is an isosceles triangle,



[b] In $\triangle ABC$: $m(\angle A) = 40^\circ$ and $m(\angle B) = 80^\circ$

Arrange the lengths of the sides of the triangle ABC descendingly.

10 El-Dakahlia Governorate

Math's Supervision (L.E.S.)



Answer the following questions :

1 Complete :

- ① The number of axes of symmetry of isosceles triangle is
- ② The bisector of the vertex angle of the isosceles triangle
- ③ The medians of the triangle at one point.
- ④ The longest side of the right-angled triangle is the
- ⑤ In $\triangle ABC$, if $AB = AC$ and $m(\angle C) = 40^\circ$, then $m(\angle A) = \dots\dots\dots^\circ$

2 Choose the correct answer :

- ① Isosceles triangle whose side lengths are 4 cm. , $(x + 3)$ cm. and 8 cm. , then $x = \dots\dots\dots$
 (a) 4 (b) 5 (c) 3 (d) 8
- ② In $\triangle LMN$, if $m(\angle M) = 55^\circ$ and $m(\angle N) = 80^\circ$, then $LM \dots\dots\dots MN$
 (a) < (b) > (c) = (d) twice

- (3) The measure of the exterior angle of the equilateral triangle =°
 (a) 30 (b) 60 (c) 90 (d) 120
- (4) The base angles of the isosceles triangle are
 (a) alternating (b) corresponding (c) congruent (d) supplementary
- (5) If \overline{AD} is a median of $\triangle ABC$ and M is the point of concurrence of the medians, then $MD = \dots\dots\dots AD$
 (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

3 [a] In the opposite figure :

$$m(\angle ABC) = m(\angle BDE) = 90^\circ$$

$$, m(\angle E) = 30^\circ$$

, D is the midpoint of \overline{AC}

Prove that : $AC = BE$

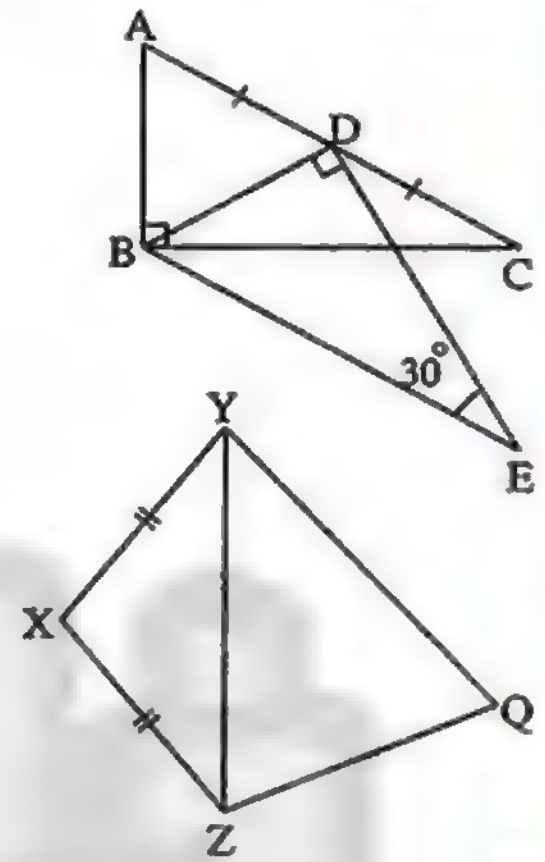
[b] In the opposite figure :

$$XY = XZ$$

$$, QY > QZ$$

Prove that :

$$m(\angle XZQ) > m(\angle XYQ)$$



4 [a] In the opposite figure :

$$X \in \overline{BC}, \overline{BC} \parallel \overline{PQ}$$

$$, m(\angle P) = 110^\circ$$

$$, m(\angle A) = 40^\circ$$

Prove that : $AB = AC$

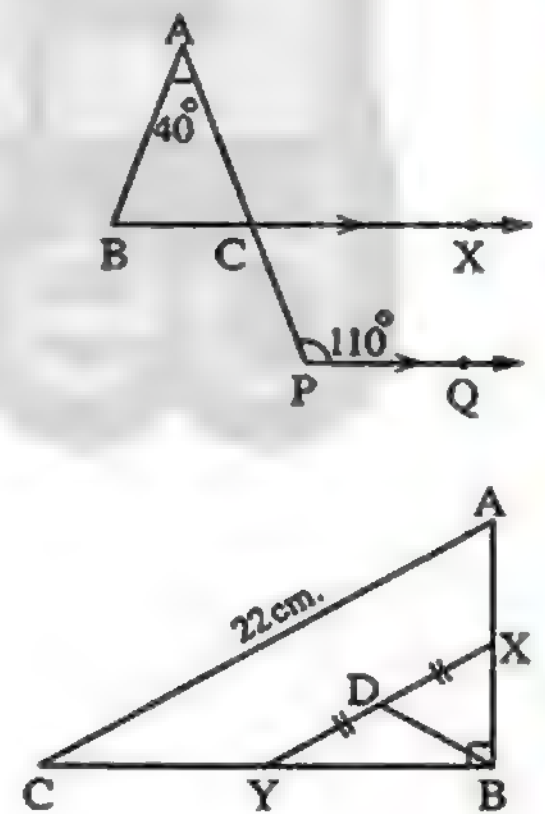
[b] In the opposite figure :

$$m(\angle ABC) = 90^\circ$$

X, Y, D are midpoints of \overline{AB} , \overline{BC} , \overline{XY} respectively.

$$AC = 22 \text{ cm.}$$

Find : BD

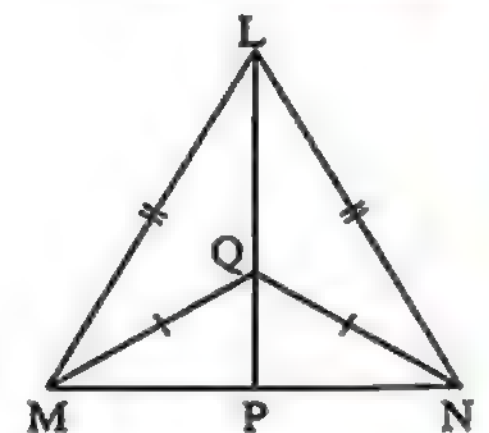


5 [a] In the opposite figure :

$$LM = LN$$

$$, QM = QN$$

Prove that : $MP = NP$



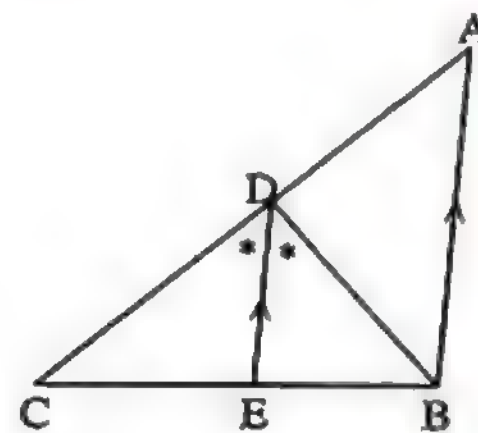
Geometry

[b] In the opposite figure :

\overline{DE} bisects $\angle BDC$ and $\overline{DE} \parallel \overline{AB}$

Prove that :

$AC > BC$



11

Ismailia Governorate

Directorate of Education
Directorate of Math's



Answer the following questions :

1 Choose the correct answer :

(1) In the opposite figure :

If $m(\angle A) = 90^\circ$, \overline{AD} is a median,

M is the point of intersection of its medians

and $BC = 18$ cm., then $MA = \dots\dots\dots$ cm.

(a) 9 cm.

(b) 3 cm.

(c) 6 cm.

(d) 18 cm.

(2) In $\triangle XYZ$, if $m(\angle Y) < m(\angle Z)$, then $XY \dots\dots\dots XZ$

(a) =

(b) <

(c) >

(d) twice

(3) If the measures of two angles of a triangle are 65° and 50° , then the triangle is

(a) scalene

(d) equilateral

(c) isosceles

(d) right angled

(4) If ABCD is a parallelogram, $x : y = 1 : 2$

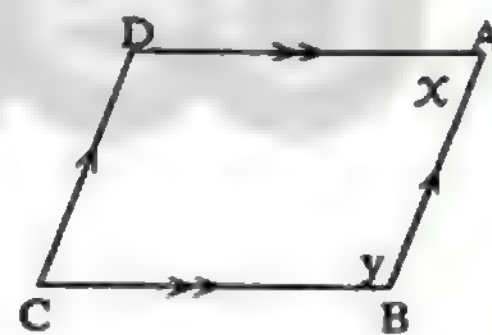
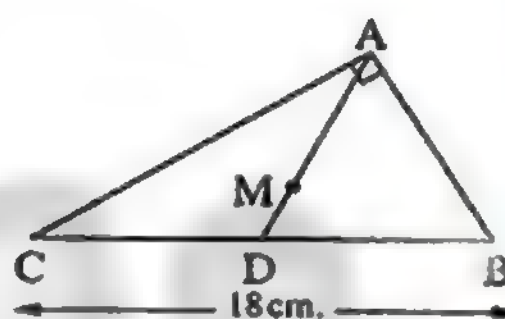
, then $m(\angle C) = \dots\dots\dots^\circ$

(a) 60°

(b) 120°

(c) 180°

(d) 360°



(5) If 10 cm., 5 cm. and x cm. are side lengths of an isosceles triangle, then $x = \dots\dots\dots$ cm.

(a) 10

(b) 5

(c) 15

(d) 4

2 Complete :

(1) Number of axes of symmetry of an equilateral triangle =

(2) The perpendicular from the vertex angle of an isosceles triangle bisects each of
and

(3) In $\triangle ABC$, if $AB = 3$ cm. and $BC = 5$ cm., then $AC \in] \dots\dots\dots , \dots\dots\dots [$

(4) If ABCD is a square , then $m(\angle ACB) = \dots\dots\dots^\circ$

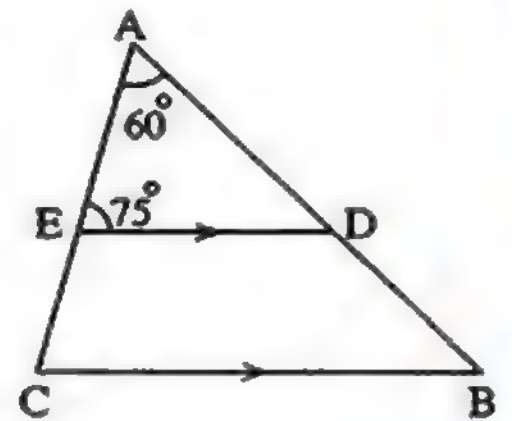
(5) If $A \in L$ where L is the axis of symmetry of \overline{BC} , then $AB \dots\dots\dots AC$

3 [a] In the opposite figure :

$$\overline{ED} \parallel \overline{BC}$$

$$, m(\angle A) = 60^\circ \text{ and } m(\angle AED) = 75^\circ$$

Prove that : $AB > AC$



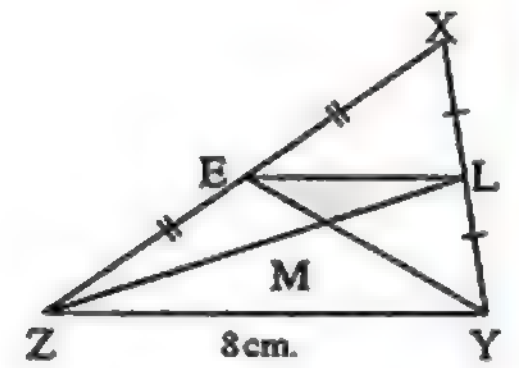
[b] In the opposite figure :

$\triangle XYZ$ in which : L and E are midpoints

of \overline{XY} and \overline{XZ} respectively.

$$\overline{YE} \cap \overline{ZL} = \{M\} , YZ = 8 \text{ cm.} , YM = 4 \text{ cm. and } ZL = 9 \text{ cm.}$$

Find : The perimeter of $\triangle EML$



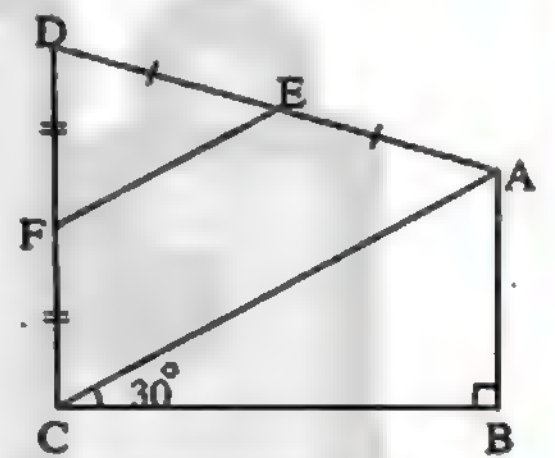
4 [a] In the opposite figure :

$$m(\angle B) = 90^\circ , m(\angle ACB) = 30^\circ$$

E is the midpoint of \overline{AD}

and F is the midpoint of \overline{CD}

Prove that : $AB = EF$

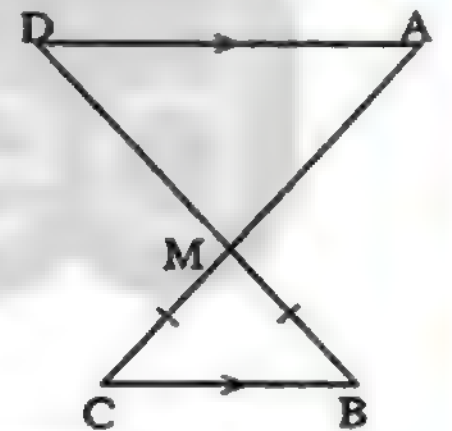


[b] In the opposite figure :

$$\text{If } \overline{AC} \cap \overline{BD} = \{M\}$$

$$, \overline{AD} \parallel \overline{BC} \text{ and } MB = MC$$

Prove that : $\triangle MAD$ is an isosceles.



5 [a] In $\triangle ABC$: If $m(\angle A) = 50^\circ$ and $m(\angle B) = 85^\circ$

Find : $m(\angle C)$, then arrange the lengths of its sides ascendingly.

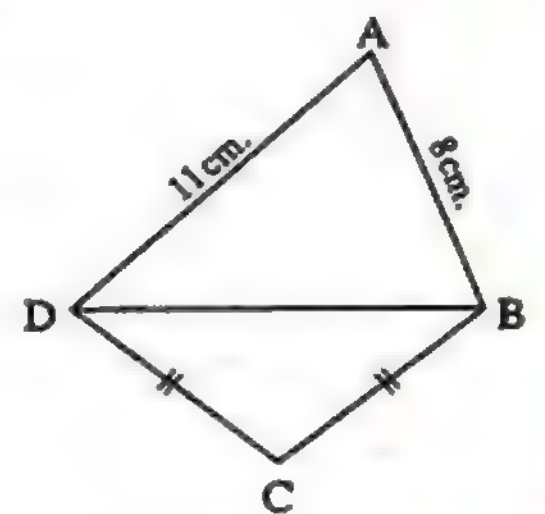
[b] In the opposite figure :

ABCD is a quadrilateral

$$, AD = 11 \text{ cm.} , AB = 8 \text{ cm.}$$

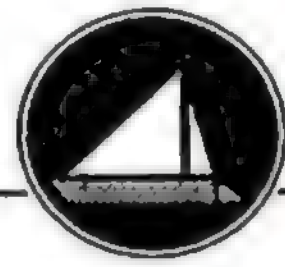
$$\text{and } CB = CD$$

Prove that : $m(\angle ABC) > m(\angle ADC)$



Geometry

12 Damietta Governorate

Damietta Inspection of Mathematic
Official Language Schools

Answer the following questions :

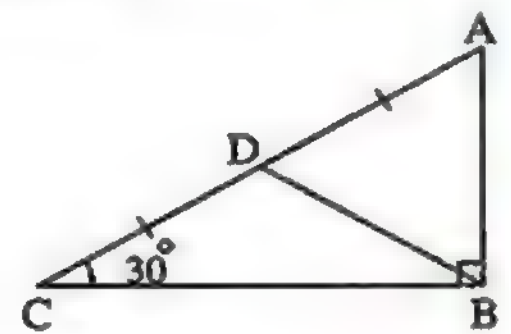
1 Choose the correct answer :

- (1) In $\triangle ABC$: $m(\angle B) = 80^\circ$ and $m(\angle C) = 50^\circ$, then $AB = \dots\dots\dots$
 (a) BC (b) AC (c) $2AC$ (d) $\frac{1}{2}BC$
- (2) The lengths 6 cm. , 7 cm. and $\dots\dots\dots$ can be lengths of the sides of a triangle.
 (a) 15 cm. (b) 13 cm. (c) 18 cm. (d) 11 cm.
- (3) In $\triangle ABC$, if $m(\angle A) = 30^\circ$ and $m(\angle B) = 90^\circ$, then $AC = \dots\dots\dots$
 (a) $\frac{1}{2}BC$ (b) $2BC$ (c) $2AB$ (d) BC
- (4) The point of intersection of the medians of the triangle divides each of them with ratio $\dots\dots\dots$ from the vertex.
 (a) 1 : 2 (b) 3 : 1 (c) 2 : 1 (d) 1 : 3
- (5) In $\triangle ABC$, $m(\angle A) = 50^\circ$ and $m(\angle B) = 100^\circ$ then $\dots\dots\dots$
 (a) $AB > AC$ (b) $AC < AB$ (c) $BC < AC$ (d) $AB = BC$

2 Complete :

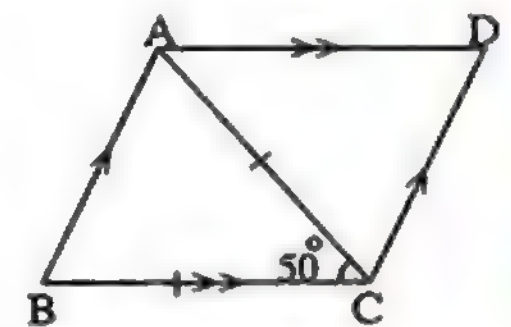
- (1) The measure of exterior angle of the equilateral triangle = $\dots\dots\dots^\circ$
- (2) If $\triangle ABC \equiv \triangle XYZ$, then $\angle A \equiv \dots\dots\dots$
- (3) The longest side in a right-angled triangle is $\dots\dots\dots$
- (4) If \overleftrightarrow{XY} is an axis of symmetry of \overline{AB} , $D \in \overleftrightarrow{XY}$, then $AD = \dots\dots\dots$
- (5) Square with side length 5 cm. , then its area = $\dots\dots\dots \text{cm}^2$

3 [a] In the opposite figure :

D is a midpoint of \overline{AC} $m(\angle B) = 90^\circ$, $m(\angle ACB) = 30^\circ$ Prove that : $\triangle ABD$ is an equilateral triangle

[b] In the opposite figure :

ABCD is a parallelogram

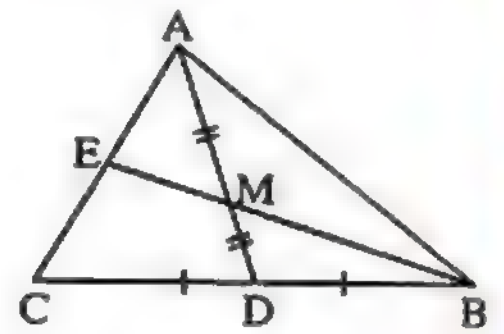
 $CA = CB$ and $m(\angle ACB) = 50^\circ$ Find with proof : $m(\angle D)$ 

4 [a] In the opposite figure :

E and D are the midpoints of \overline{AC} and \overline{CB} respectively

If $AD = 4.5$ cm and $BM = 4$ cm.

Find : The length of each of \overline{MD} and \overline{BE}



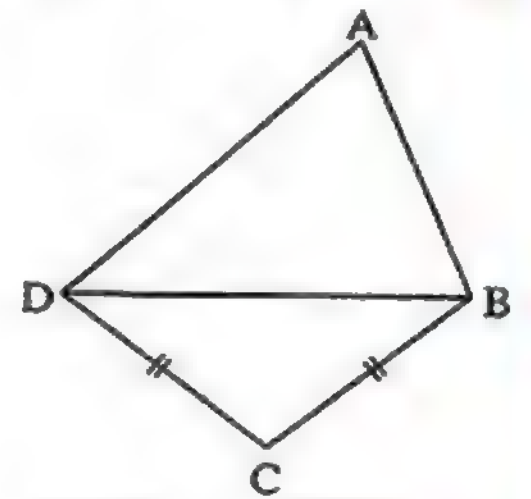
[b] In the opposite figure :

ABCD is a quadrilateral in which : $AD > AB$

and $BC = CD$

Prove that :

$m(\angle ABC) > m(\angle ADC)$



5 [a] ABC is a triangle in which : $m(\angle A) = 40^\circ$ and $m(\angle B) = 75^\circ$

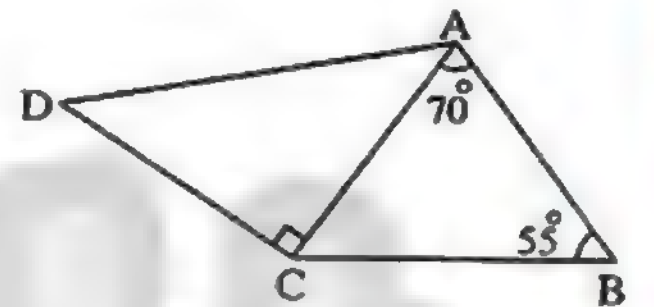
Arrange the lengths of sides of $\triangle ABC$ in ascending order.

[b] In the opposite figure :

$m(\angle BAC) = 70^\circ$, $m(\angle B) = 55^\circ$

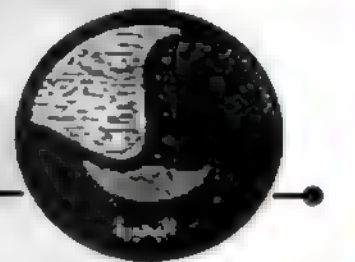
and $m(\angle ACD) = 90^\circ$

Prove that : $AD > AB$



13 El-Behira Governorate

Maths Inspection



Answer the following questions :

1 Complete the following :

- (1) If the length of two sides of isosceles triangle are 8 cm. and 4 cm. , then the length of the third side is
- (2) The number of axis of symmetry of scalene triangle is
- (3) The length of the median of the right-angled triangle from the vertex of right angle equals the length of the hypotenuse.
- (4) The base angles of the isosceles triangle are in measure.
- (5) In $\triangle ABC$, if $m(\angle A) = 40^\circ$ and $m(\angle B) = 60^\circ$, then the longest side is

2 Choose the correct answer :

- (1) If A lies on the line of symmetry of \overline{BC} then AB AC
 (a) $>$ (b) $<$ (c) $=$ (d) $//$
- (2) The measure of the exterior angle of the equilateral triangle =
 (a) 90° (b) 60° (c) 120° (d) 180°
- (3) In $\triangle ABC$, if $BC > AC$, then $m(\angle A)$ $m(\angle B)$
 (a) $>$ (b) $<$ (c) $=$ (d) \geq

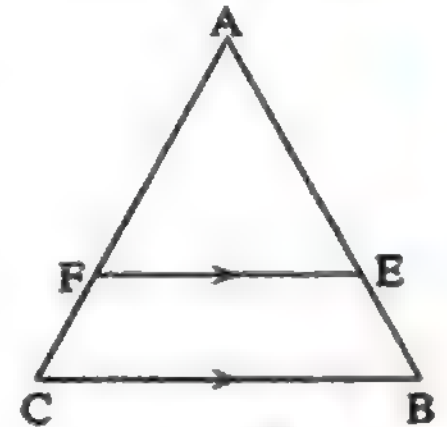
Geometry

- (4) If $\triangle ABC$ is a right-angled triangle at B and $m(\angle C) = 30^\circ$, then $AB = \dots\dots\dots AC$
 (a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) 3
- (5) The sum of lengths of two sides of a triangle is $\dots\dots\dots$ the length of the third side.
 (a) greater than (b) less than (c) equal (d) greater than or equal

3 [a] In the opposite figure :

$$AB = AC, \overline{EF} \parallel \overline{CB}$$

Prove that : $AE = AF$



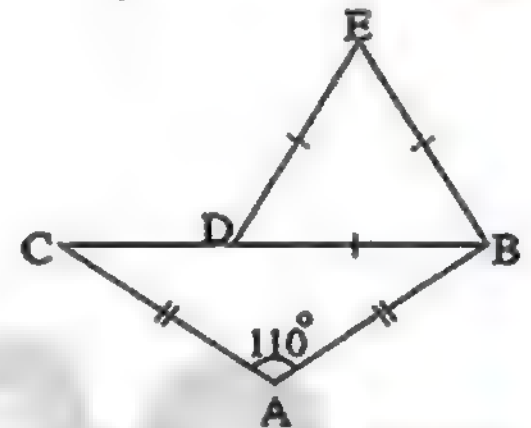
[b] In the opposite figure :

$$EB = ED = DB$$

$$, AB = AC$$

$$\text{and } m(\angle A) = 110^\circ$$

Find : $m(\angle ABE)$

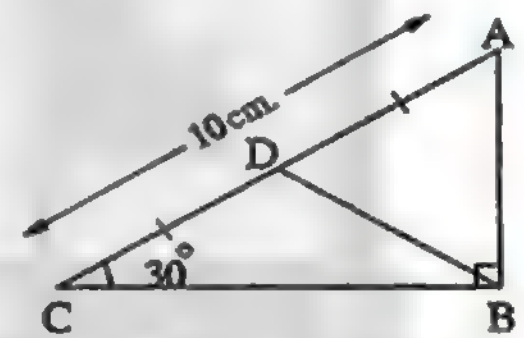


- 4 [a] In $\triangle ABC$, if $m(\angle A) = 50^\circ$ and $m(\angle B) = 60^\circ$
 Arrange the side lengths of $\triangle ABC$ ascendingly.

[b] In the opposite figure :

$$m(\angle ABC) = 90^\circ, m(\angle C) = 30^\circ, AD = DC \text{ and } AC = 10 \text{ cm.}$$

Find : The perimeter of $\triangle ABD$



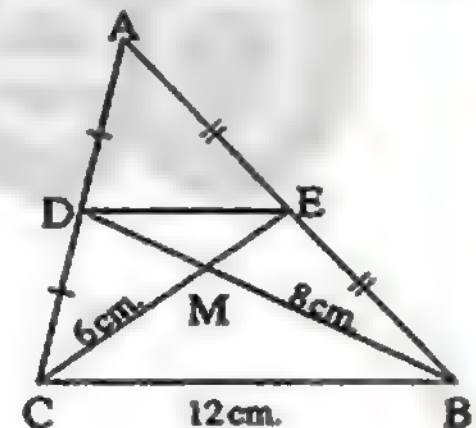
5 In the opposite figure :

$$AE = EB, AD = DC$$

$$, MB = 8 \text{ cm.}, MC = 6 \text{ cm.}$$

$$\text{and } BC = 12$$

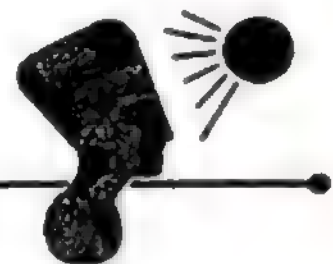
Find : The perimeter of $\triangle MED$



14

El-Minia Governorate

El-Minia Directorate of Education
 Governmental languages schools



Answer the following questions :

1 Complete the following : (Calculator is allowed)

- (1) The number of axes of symmetry in the equilateral triangle equals $\dots\dots\dots$
- (2) If the length of two sides in a triangle are 2 cm. and 7 cm.
 , then $\dots\dots\dots < \text{length of third side} < \dots\dots\dots$

- (3) The length of median which drawn from the vertex of the right-angle in the right-angled triangle equals
- (4) If the measure of an angle in an isosceles triangle is 60° , then the triangle is
- (5) The length of the side opposite to the angle of measure 30° in the right-angled triangle equals

2 Choose the correct answer :

- (1) XYZ is a triangle in which : $m(\angle Z) = 70^\circ$ and $m(\angle Y) = 60^\circ$ then YZ XY
 (a) $>$ (b) $<$ (c) $=$ (d) twice
- (2) The numbers which can be lengths of sides of triangle are
 (a) 0 , 3 , 5 (b) 3 , 3 , 5 (c) 3 , 3 , 6 (d) 3 , 3 , 7
- (3) The measure of the exterior angle of the equilateral triangle equals $^\circ$
 (a) 60 (b) 30 (c) 100 (d) 120
- (4) If the length of two sides in an isosceles triangle are 8 cm. and 4 cm. , then the length of the third side is cm.
 (a) 4 (b) 8 (c) 3 (d) 12
- (5) If ΔABC is a right-angled at B , $AB = 6$ cm. and $BC = 8$ cm. , then the length of the median drawn from B is cm.
 (a) 10 (b) 8 (c) 6 (d) 5

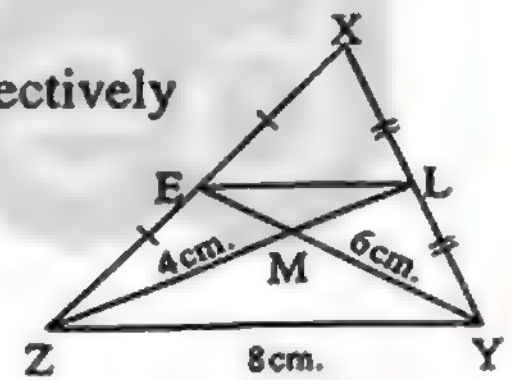
- 3 [a]** In ΔABC , $AB = 7$ cm. , $BC = 5$ cm. and $AC = 6$ cm.

Arrange its angles measures ascendingly.

[b] In the opposite figure :

ΔXYZ in which : L and E are the midpoints of \overline{XY} and \overline{XZ} respectively
 $\overline{YE} \cap \overline{ZL} = \{M\}$
 $YZ = 8$ cm. , $YM = 6$ cm. , $ZM = 4$ cm.

Find : The perimeter of ΔMLE



- 4 [a] In the opposite figure :**

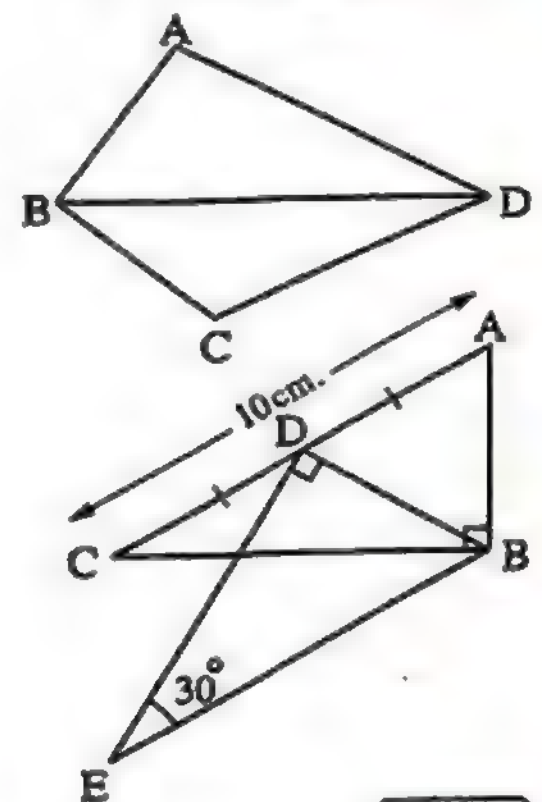
$AB < AD$, $BC < CD$

Prove that : $m(\angle ABC) > m(\angle ADC)$

[b] In the opposite figure :

$m(\angle ABC) = m(\angle BDE) = 90^\circ$
 D is the midpoint of \overline{AC}
 $m(\angle E) = 30^\circ$ and $AC = 10$ cm.

Find : The length of \overline{BE}



Geometry

5 [a] In the opposite figure :

$AB = AC$, \overline{BD} bisects $\angle B$

and \overline{CD} bisects $\angle C$

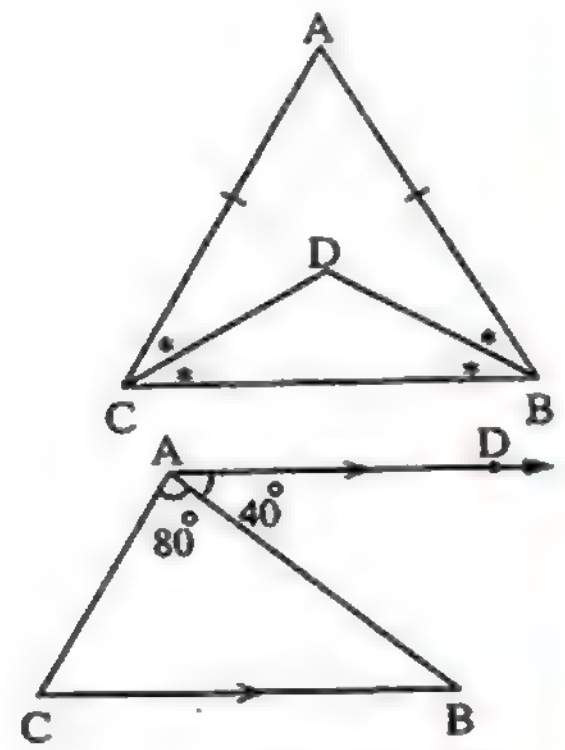
Prove that : $\triangle DBC$ is an isosceles triangle.

[b] In the opposite figure :

$\triangle ABC$ in which : $\overline{AD} \parallel \overline{CB}$

, $m(\angle DAB) = 40^\circ$ and $m(\angle BAC) = 80^\circ$

Prove that : $AB > AC$



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Answer the following questions :

1 Choose the correct answer from given answers :

(1) In isosceles triangle the base angles are

(a) complementary. (b) supplementary. (c) adjacent. (d) congruent.

(2) The sum of the lengths of the two sides of the triangle the length of the third side.

(a) double (b) equals (c) greater than (d) less than

(3) In the opposite figure :

If $AB = 12$ cm.

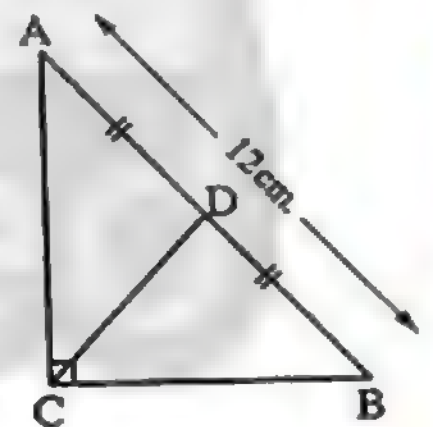
, then $CD = \dots\dots\dots$ cm.

(a) 12

(b) 9

(c) 6

(d) 3



(4) The triangle that has one axis of symmetry is triangle.

(a) an equilateral (b) an isosceles (c) a scalene (d) a right-angled

(5) The is a parallelogram where one of its angles is right angle.

(a) a rectangle (b) a square (c) a rhombus (d) a trapezium

2 Complete the following :

(1) The point that divides the median of the triangle in the ratio 1 : 2 from the base is the point of intersection of

(2) In $\triangle ABC$, if $AB > BC$, then $m(\angle A) < m(\angle \dots\dots\dots)$

(3) The sum of the measures of accumulative angles at point is°

- (4) ABC is a triangle in which : $m(\angle B) = 130^\circ$, then the longest side of its sides is
- (5) In the right-angled triangle , the length of the side that opposite to the angle of measure $30^\circ = \dots\dots\dots$ the length of the hypotenuse.

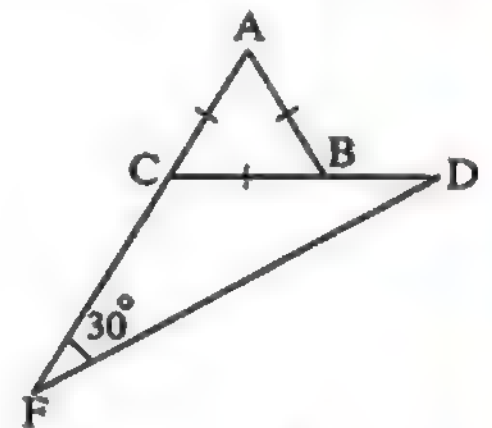
3 [a] In the opposite figure :

ABC is an equilateral triangle

, $F \in \overline{AC}$, $D \in \overline{CB}$

, $m(\angle DFC) = 30^\circ$

Prove that : $\triangle DCF$ is an isosceles triangle.



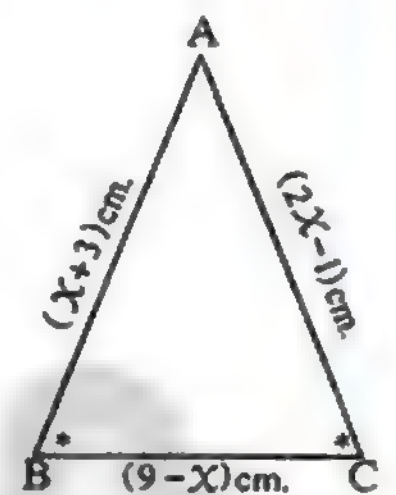
[b] In the opposite figure :

ABC is a triangle in which :

$m(\angle B) = m(\angle C)$

Find :

The perimeter of $\triangle ABC$

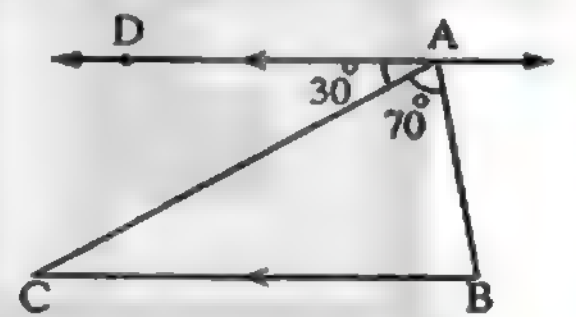


4 [a] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, $m(\angle BAC) = 70^\circ$

and $m(\angle DAC) = 30^\circ$

Prove that : $AC > BC$



[b] ABC is a triangle in which : $AB = 7$ cm. , $BC = 5$ cm. and $AC = 6$ cm.

Arrange the measures of its angles in an ascending order.

5 [a] In the opposite figure :

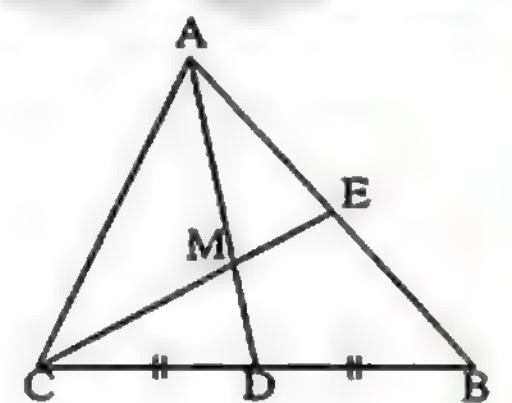
ABC is a triangle

, D is the midpoint of \overline{BC} , $M \in \overline{AD}$

, where $AM = 2 MD$

Draw \overline{CM} cuts \overline{AB} at E , if $EC = 12$ cm.

, find : The length of \overline{EM}

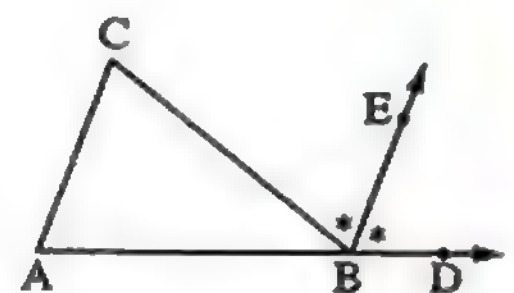


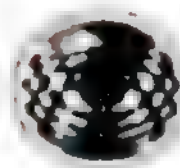
[b] In the opposite figure :

$BA = BC$

and \overline{BE} bisects $\angle CBD$

Prove that : $\overline{BE} \parallel \overline{AC}$



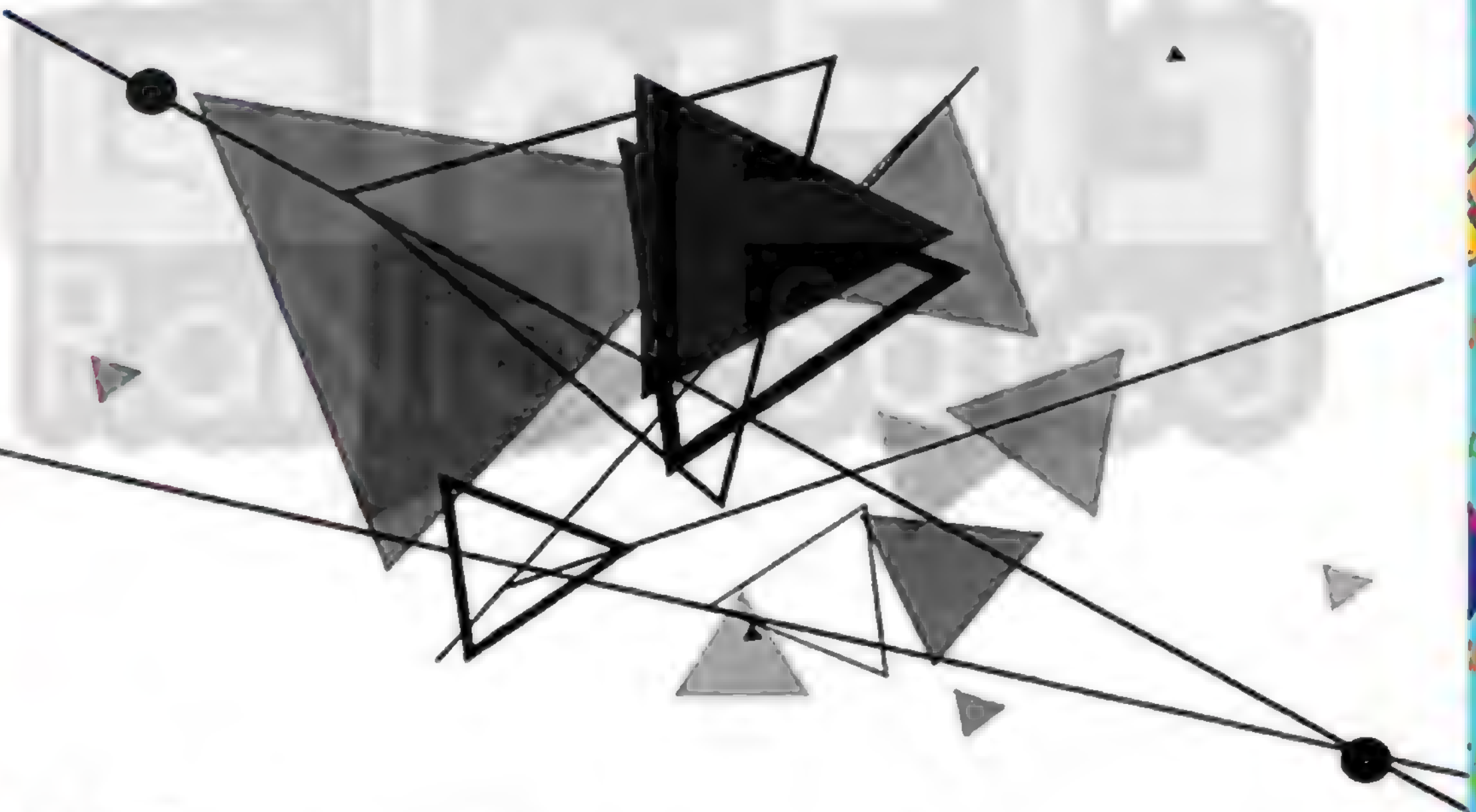


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In Mathematics

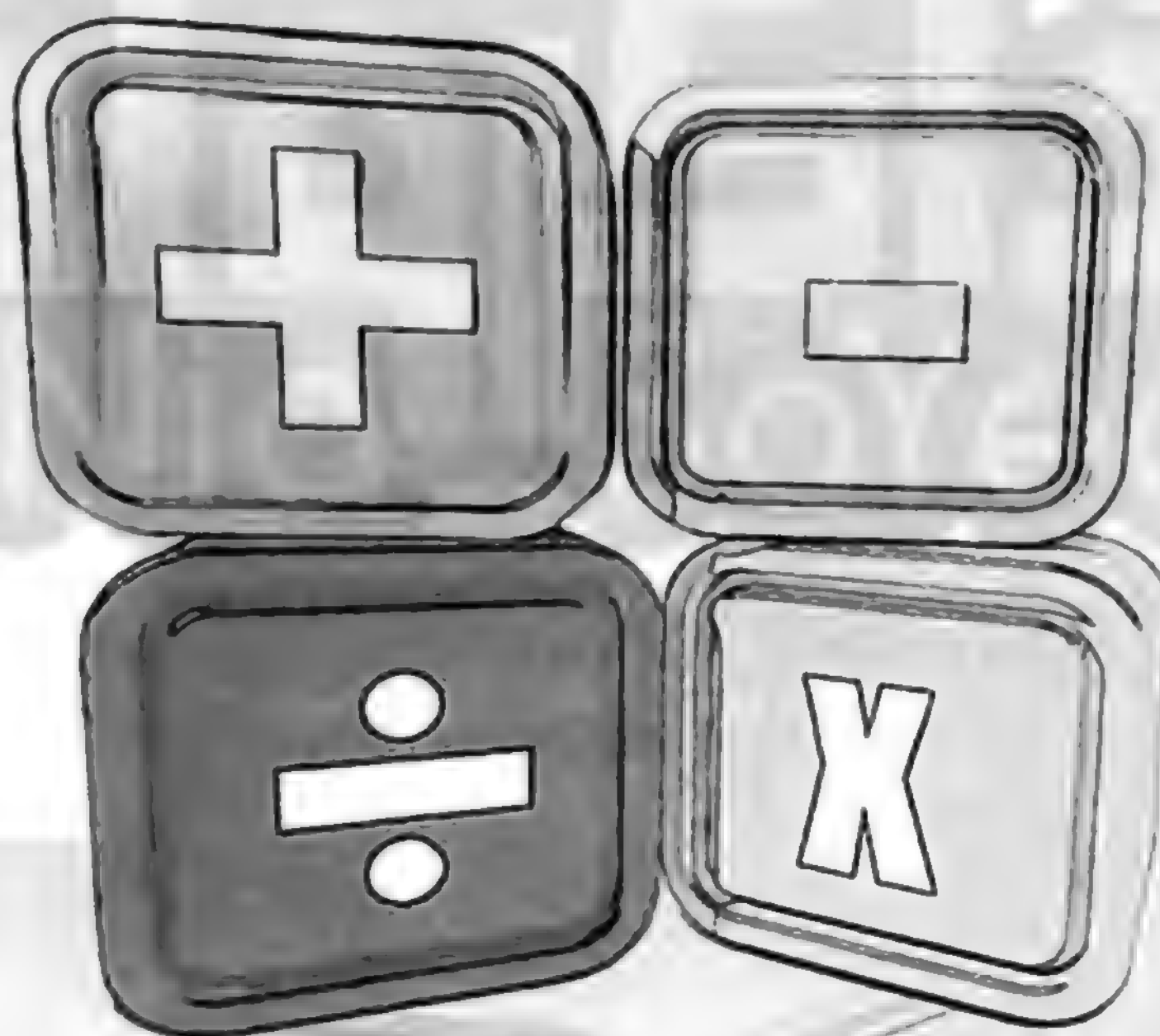
Guide Answers

For **2nd** Prep.
First Term



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By
A group of supervisors

**Guide
Answers****of Algebra and Statistics Exercises**

Answers of Unit 1

Answers of revision exercise

- 1 $\frac{1}{3}$ 2 $\frac{3}{10}$ 3 $\frac{1}{4}$ 4 $\frac{3}{4}$ 5 $-\frac{6}{1}$ 6 $\frac{5}{4}$

- 2
1 b 2 b 3 c
4 a 5 c 6 d
7 d 8 b 9 b

- 3
1 13 2 $\frac{1}{2}$ 3 -7
4 zero 5 1 6 2
7 1.5×10^{-4} 8 4.21×10^5 9 15
10 zero 11 1

- 4
1 $\because 5X = 20 - 3 = 17 \therefore X = \frac{17}{5}$
2 $\because 7X = 12 - 11 = 1 \therefore X = \frac{1}{7}$
3 $\because 3X = 1 - 5 = -4 \therefore X = -\frac{4}{3}$
4 $X = 7 - 3 = 4$

- 5
1 $\because X^2 + 12 = 21 \therefore X^2 = 21 - 12$
 $\therefore X^2 = 9 \therefore X = \pm\sqrt{9}$
 $\therefore X = \pm 3 \therefore \text{The S.S.} = \{3, -3\}$
2 $\because 2X^2 - 1 = -9 \therefore 2X^2 = -9 + 1$
 $\therefore 2X^2 = -8 \therefore X^2 = \frac{-8}{2}$
 $\therefore X^2 = -4 \therefore X = \pm\sqrt{-4} \notin \mathbb{R}$
 $\therefore \text{The S.S.} = \emptyset$
3 $\because |X| = 2 \therefore X = \pm 2$
 $\therefore \text{The S.S.} = \{2, -2\}$
4 $\because \sqrt{X^2} = 4 \therefore |X| = 4$
 $\therefore X = \pm 4 \therefore \text{The S.S.} = \{4, -4\}$

Answers of unit one

Answers of Exercise 1

Number a	8	125	-27	-1000	$3\frac{3}{8}$	$-\frac{8}{125}$	216	-64
$\sqrt[3]{a}$	2	5	-3	-10	$\frac{3}{2}$	$-\frac{2}{5}$	6	-4

- 2
1 6 2 -7 3 $\frac{4}{5}$ 4 $-\frac{2}{3}$
5 0.1 6 $-\frac{4}{3}$ 7 $2X$ 8 $-3a^2$

- 3
1 a 2 64 3 64 4 25
5 zero 6 -1 7 6 8 1
9 4 10 zero 11 $-\frac{1}{2}$ 12 61

- 4
1 c 2 b 3 a 4 a 5 d
6 c 7 c 8 d 9 a 10 d
11 c 12 d 13 b

- 5
1 125 2 $-\frac{1}{64}$
3 $\because \sqrt[3]{X} = -2 \therefore X = -8$
4 $\because \sqrt[3]{X} = -1 + 3 = 2 \therefore X = 8$
5 -2 6 4
7 $\because X^3 = 32 - 5 = 27 \therefore X = 3$
8 $\because X^3 = 54 + 2 = 27 \therefore X = 3$
9 $\because X^3 = -200 + \frac{1}{5} = -1000 \therefore X = -10$

- 6
1 $\because X^3 = -27 \therefore X = \sqrt[3]{-27} = -3$
 $\therefore \text{The S.S.} = \{-3\}$
2 $\because 8X^3 = 8 - 7 = 1 \therefore X^3 = \frac{1}{8}$
 $\therefore X = \sqrt[3]{\frac{1}{8}} = \frac{1}{2} \therefore \text{The S.S.} = \{\frac{1}{2}\}$
3 $\because X^3 = \frac{3}{8} - 16 = -\frac{125}{8} \therefore X = \sqrt[3]{-\frac{125}{8}} = -\frac{5}{2}$
 $\therefore \text{The S.S.} = \{-\frac{5}{2}\}$
4 $\because 2X^3 - X^3 = 3 + 5 \therefore X^3 = 8$
 $\therefore X = \sqrt[3]{8} = 2 \therefore \text{The S.S.} = \{2\}$
5 $\because X + 3 = \sqrt[3]{343} = 7 \therefore X = 7 - 3 = 4$
 $\therefore \text{The S.S.} = \{4\}$
6 $\because 3X + 1 = \sqrt[3]{-8} = -2 \therefore 3X = -2 - 1 = -3$
 $\therefore X = -3 + 3 = 0 \therefore \text{The S.S.} = \{0\}$
7 $\because (2X + 1)^3 = 20 + 7 = 27 \therefore 2X + 1 = \sqrt[3]{27} = 3$
 $\therefore 2X = 3 - 1 = 2 \therefore X = 2 + 2 = 1$
 $\therefore \text{The S.S.} = \{1\}$

Algebra and Statistics

$$\begin{aligned} \text{8} \quad \because (5x-2)^3 &= 18-10=8 \quad \therefore 5x-2=\sqrt[3]{8}=2 \\ \therefore 5x &= 2+2=4 \quad \therefore x=\frac{4}{5} \\ \therefore \text{The S.S.} &= \left\{\frac{4}{5}\right\} \end{aligned}$$

$$\begin{aligned} \text{7} \quad \text{1} \quad \sqrt[3]{2\frac{1}{4} + \frac{2}{3}} &= \sqrt[3]{\frac{9}{4} \times \frac{3}{2}} = \sqrt[3]{\frac{27}{8}} = \frac{3}{2} \\ \text{2} \quad -\sqrt[3]{2^9 \times 3^6} &= -\sqrt[3]{(2^3 \times 3^2)^3} = -2^3 \times 3^2 = -8 \times 9 = -72 \\ \text{3} \quad \sqrt[3]{\sqrt[3]{729}} &= \sqrt[3]{9} = 3 \\ \text{4} \quad \sqrt[3]{\sqrt[3]{512}} &= \sqrt[3]{8} = 2 \\ \text{5} \quad \sqrt[3]{27} \sqrt[3]{27} &= \sqrt[3]{27 \times 27} = \sqrt[3]{81} = 9 \end{aligned}$$

$$\text{8} \quad \text{The edge length of the cube} = \sqrt[3]{15\frac{5}{8}} = \sqrt[3]{\frac{125}{8}} = 2.5 \text{ cm.}$$

$$\begin{aligned} \text{9} \quad \text{The edge length of the cube} &= \sqrt[3]{216} = 6 \text{ cm.} \\ \therefore \text{Its total area} &= 6 \times 6^2 = 216 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{10} \quad \text{Let the number be } x \quad \therefore x^3 &= 27 \\ \therefore x &= 3 \quad \therefore x^2 = 9 \\ \therefore \text{The square of the number} &= 9 \end{aligned}$$

$$\begin{aligned} \text{11} \quad \text{Let the number be } x \quad \therefore \frac{1}{2}x^3 &= 32 \\ \therefore x^3 &= 64 \quad \therefore x = 4 \\ \therefore \text{The number} &= 4 \end{aligned}$$

$$\text{12} \quad \text{The length of the inner edge} = \sqrt[3]{1000} = 10 \text{ cm.}$$

$$\begin{aligned} \text{13} \quad \text{The volume of the sphere} &= \frac{4}{3} \pi r^3 = \frac{1372}{81} \pi \\ \therefore r^3 &= \frac{1372}{81} \times \frac{3}{4} = \frac{343}{27} \\ \therefore r &= \sqrt[3]{\frac{343}{27}} = \frac{7}{3} \\ \therefore \text{The diameter length of the sphere} &= 2 \times \frac{7}{3} = \frac{14}{3} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} \text{14} \quad \therefore \text{The volume of the sphere} &= \frac{4}{3} \pi r^3 = 113.04 \\ \therefore \frac{4}{3} \times 3.14 \times r^3 &= 113.04 \end{aligned}$$

$$\begin{aligned} \therefore r^3 &= 27 \\ \therefore r &= \sqrt[3]{27} = 3 \text{ cm.} \\ \therefore \text{The diameter length of the sphere} &= 2 \times 3 = 6 \text{ cm.} \end{aligned}$$

$$\begin{aligned} \text{15} \quad \text{1} \quad \because (x^2+6)^3 &= 1000 \quad \therefore x^2+6=10 \\ \therefore x^2 &= 4 \quad \therefore x=\pm 2 \\ \therefore \text{The S.S.} &= \{2, -2\} \\ \text{2} \quad \because (x^3-14)^2 &= 169 \quad \therefore x^3-14=\pm 13 \\ \therefore x^3 &= 14 \pm 13 \quad \therefore x^3=27 \quad \therefore x=3 \\ \text{or } x^3 &= 1 \quad \therefore x=1 \\ \therefore \text{The S.S.} &= \{3, 1\} \\ \text{3} \quad \text{Cubing the two sides} \quad \therefore (x-1)^2 &= 25 \\ \therefore x-1 &= \pm 5 \quad \therefore x=6 \text{ or } x=-4 \\ \therefore \text{The S.S.} &= \{6, -4\} \\ \text{4} \quad \because \sqrt[3]{(x-2)(x-2)^2} &= 3 \quad \therefore \sqrt[3]{(x-2)^3} = 3 \\ \therefore x-2 &= 3 \quad \therefore x=5 \\ \therefore \text{The S.S.} &= \{5\} \end{aligned}$$

$$\begin{aligned} \text{16} \quad \text{Cubing the two sides} \\ \therefore \sqrt{x} + 19 &= 27 \quad \therefore \sqrt{x} = 8 \\ \text{Squaring the two sides} \\ \therefore x &= 64 \quad \therefore \sqrt[3]{x} = \sqrt[3]{64} = 4 \end{aligned}$$

$$\begin{aligned} \text{17} \quad \text{Let the age of the grandfather be } x \text{ year} \\ \therefore \text{The age of the man} &= \frac{1}{2}x \text{ year} \\ \text{The age of the grandson (the elder)} &= \sqrt{x} \text{ year} \\ \text{The age of the grandson (the middle)} &= \sqrt[3]{x} \text{ year} \\ \text{The age of the granddaughter} &= \frac{\sqrt{x}}{\sqrt[3]{x}} \text{ year} \\ \therefore \sqrt{x} &= 2\sqrt[3]{x}, \text{ then cubing the two sides} \\ \therefore x\sqrt{x} &= 8x, \text{ then squaring the two sides} \\ \therefore x^3 &= 64x^2 \quad \therefore x=64 \\ \therefore \text{The age of the grandfather} &= 64 \text{ years} \\ \text{The age of the grandson (the elder)} &= \sqrt{64} = 8 \text{ years} \\ \text{The age of the grandson (the middle)} &= \sqrt[3]{64} = 4 \text{ years} \\ \text{The age of the granddaughter} &= 8 \div 4 = 2 \text{ years} \end{aligned}$$

Answers of Exercise 2

1

The rational numbers are No.

1, 2, 3, 4, 5, 6, 9,
11, 13, 14, 16, 17, 18, 19

The remained numbers are irrational.

2

1) 3.32 2) 1.9 3) -2.1

3

$$1) \because \sqrt{4} < \sqrt{5} < \sqrt{9} \quad \therefore 2 < \sqrt{5} < 3$$

 \therefore The two numbers are 2, 3

$$2) \because \sqrt{9} < \sqrt{12} < \sqrt{16} \quad \therefore 3 < \sqrt{12} < 4$$

 \therefore The two numbers are 3 and 4

$$3) \because \sqrt[3]{8} < \sqrt[3]{10} < \sqrt[3]{27} \quad \therefore 2 < \sqrt[3]{10} < 3$$

 \therefore The two numbers are 2 and 3

$$4) \because \sqrt[3]{-27} < \sqrt[3]{-20} < \sqrt[3]{-8} \quad \therefore -3 < \sqrt[3]{-20} < -2$$

 \therefore The two numbers are -2 and -3

4

$$1) \because \sqrt{1} < \sqrt{2} < \sqrt{4} \quad \therefore 1 < \sqrt{2} < 2 \quad \therefore x = 1$$

$$2) \because \sqrt{64} < \sqrt{80} < \sqrt{81} \quad \therefore 8 < \sqrt{80} < 9 \quad \therefore x = 8$$

$$3) \because \sqrt[3]{1} < \sqrt[3]{5} < \sqrt[3]{8} \quad \therefore 1 < \sqrt[3]{5} < 2 \quad \therefore x = 1$$

$$4) \because \sqrt[3]{27} < \sqrt[3]{50} < \sqrt[3]{64} \quad \therefore 3 < \sqrt[3]{50} < 4 \quad \therefore x = 3$$

$$5) \because \sqrt[3]{-125} < \sqrt[3]{-100} < \sqrt[3]{-64}$$

$$\therefore -5 < \sqrt[3]{-100} < -4 \quad \therefore x = -5$$

$$6) \because \sqrt{25} < \sqrt{35} < \sqrt{36} \quad \therefore 5 < \sqrt{35} < 6 \quad \therefore x = 5$$

5

$$1) \because \sqrt{16} < \sqrt{20} < \sqrt{25} \quad \therefore 4 < \sqrt{20} < 5$$

$$\therefore (4.1)^2 = 16.81, (4.2)^2 = 17.64, (4.3)^2 = 18.49, \\ (4.4)^2 = 19.36, (4.5)^2 = 20.25$$

$$\therefore 4.4 < \sqrt{20} < 4.5 \quad \therefore \sqrt{20} = 4.4 \text{ or } 4.5$$

Using the calculator $\sqrt{20} = 4.47$

$$2) \because \sqrt[3]{8} < \sqrt[3]{17} < \sqrt[3]{27} \quad \therefore 2 < \sqrt[3]{17} < 3$$

$$\therefore (2.1)^3 = 9.261, (2.2)^3 = 10.648, (2.3)^3 = 12.167, \\ (2.4)^3 = 13.824, (2.5)^3 = 15.625, (2.6)^3 = 17.576$$

$$\therefore 2.5 < \sqrt[3]{17} < 2.6$$

$$\sqrt[3]{17} \approx 2.5 \text{ or } 2.6$$

Using the calculator $\sqrt[3]{17} = 2.57$

$$3) \because \sqrt{4} < \sqrt{5} < \sqrt{9} \quad \therefore 2 < \sqrt{5} < 3$$

$$\therefore (2.1)^2 = 4.41, (2.2)^2 = 4.84, (2.3)^2 = 5.29$$

$$\therefore 2.2 < \sqrt{5} < 2.3 \quad \therefore 3.2 < \sqrt{5} + 1 < 3.3$$

$$\therefore \sqrt{5} + 1 = 3.2 \text{ or } 3.3$$

Using the calculator $\sqrt{5} + 1 = 3.24$

$$4) \because \sqrt[3]{8} < \sqrt[3]{9} < \sqrt[3]{27} \quad \therefore 2 < \sqrt[3]{9} < 3$$

$$\therefore (2.1)^3 = 9.261 \quad \therefore 2 < \sqrt[3]{9} < 2.1$$

$$\therefore 1 < \sqrt[3]{9} - 1 < 1.1 \quad \therefore \sqrt[3]{9} - 1 \approx 1 \text{ or } 1.1$$

Using the calculator $\sqrt[3]{9} - 1 \approx 1.08$

6

$$1) d \quad 2) b \quad 3) b \quad 4) c$$

$$5) c \quad 6) b \quad 7) b \quad 8) d$$

$$9) c \quad 10) c \quad 11) d$$

7

$$1) x^2 = \frac{10}{5} = 2 \quad \therefore x = \pm\sqrt{2} \quad \therefore x \in \mathbb{Q}$$

$$2) x^2 = \frac{9}{4} \quad \therefore x = \pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2} \quad \therefore x \in \mathbb{Q}$$

$$3) x = \sqrt[3]{125} \quad \therefore x = 5 \quad \therefore x \in \mathbb{Q}$$

$$4) x^3 = \frac{27}{3} = 9 \quad \therefore x = \sqrt[3]{9} \quad \therefore x \in \mathbb{Q}$$

$$5) x^2 = \frac{10}{0.1} = 100 \quad \therefore x = \pm\sqrt{100} = \pm 10 \quad \therefore x \in \mathbb{Q}$$

$$6) x^3 = \frac{-8}{0.001} = -8000$$

$$\therefore x = \sqrt[3]{-8000} = -20 \quad \therefore x \in \mathbb{Q}$$

$$7) x - 1 = \pm\sqrt{4} = \pm 2 \quad \therefore x = 2 + 1 = 3$$

$$\text{or } x = -2 + 1 = -1 \quad \therefore x \in \mathbb{Q}$$

$$8) x - 5 = \sqrt[3]{1} = 1 \quad \therefore x = 1 + 5 = 6 \quad \therefore x \in \mathbb{Q}$$

8

$$1) x^2 = 13 \quad \therefore x = \pm\sqrt{13}$$

$$\therefore \text{The S.S.} = \{\sqrt{13}, -\sqrt{13}\}$$

$$2) x^3 = 16 \quad \therefore x = \sqrt[3]{16} \quad \therefore \text{The S.S.} = \{\sqrt[3]{16}\}$$

$$3) x^2 = \frac{25}{2} \times \frac{5}{2} = \frac{125}{4} \quad \therefore x = \pm\sqrt{\frac{125}{4}}$$

$$\therefore \text{The S.S.} = \{\sqrt{\frac{125}{4}}, -\sqrt{\frac{125}{4}}\}$$

Algebra and Statistics

$$4) x^3 = -2 \times \frac{4}{3} = -\frac{8}{3} \quad \therefore x = \sqrt[3]{-\frac{8}{3}}$$

$$\therefore \text{The S.S.} = \{\sqrt[3]{-\frac{8}{3}}\}$$

$$5) 125x^3 = 27 \quad \therefore x^3 = \frac{27}{125}$$

$$\therefore x = \sqrt[3]{\frac{27}{125}} = \frac{3}{5}$$

$$\therefore \text{The S.S.} = \emptyset \text{ because } \frac{3}{5} \notin \mathbb{Q}$$

$$6) \frac{1}{4}x^2 = 64 \quad \therefore x^2 = 64 \times 4 = 256$$

$$\therefore x = \pm\sqrt{256} = \pm 16$$

$$\therefore \text{The S.S.} = \emptyset \text{ because } 16 \notin \mathbb{Q}, -16 \notin \mathbb{Q}$$

$$7) \therefore (x^3 + 5)(x^2 - 3) = 0$$

$$\therefore x^3 + 5 = 0 \quad \therefore x^3 = -5 \quad \therefore x = -\sqrt[3]{5}$$

$$\text{or } x^2 - 3 = 0 \quad \therefore x^2 = 3 \quad \therefore x = \pm\sqrt{3}$$

$$\therefore \text{The S.S.} = \{-\sqrt[3]{5}, \sqrt{3}, -\sqrt{3}\}$$

$$8) \therefore (x + \sqrt{7})(x^3 - 6) = 0$$

$$\therefore x + \sqrt{7} = 0 \quad \therefore x = -\sqrt{7}$$

$$\text{or } x^3 - 6 = 0 \quad \therefore x^3 = 6$$

$$\therefore x = \sqrt[3]{6} \quad \therefore \text{The S.S.} = \{-\sqrt{7}, \sqrt[3]{6}\}$$

9

$$1) \therefore (1.4)^2 = 1.96, (1.5)^2 = 2.25, (\sqrt{2})^2 = 2$$

$$\therefore \sqrt{2} \text{ is included between } 1.4, 1.5$$

$$2) \therefore (3.31)^2 = 10.96, (3.32)^2 = 11.02, (\sqrt{11})^2 = 11$$

$$\therefore \sqrt{11} \text{ is included between } 3.31, 3.32$$

$$3) \therefore (1.2)^3 = 1.728, (1.3)^3 = 2.197, (\sqrt[3]{2})^3 = 2$$

$$\therefore \sqrt[3]{2} \text{ is included between } 1.2, 1.3$$

$$4) \therefore (2.4)^3 = 13.824, (2.5)^3 = 15.625$$

$$(\sqrt[3]{15})^3 = 15$$

$$\therefore \sqrt[3]{15} \text{ is included between } 2.4, 2.5$$

$$5) \therefore (-2.6)^3 = -17.576, (-2.5)^3 = -15.625$$

$$(\sqrt[3]{-17})^3 = -17$$

$$\therefore \sqrt[3]{-17} \text{ is included between } -2.6, -2.5$$

$$6) \therefore 2.7 - 1 = 1.7, (1.7)^2 = 2.89$$

$$2.8 - 1 = 1.8, (1.8)^2 = 3.24$$

$$\sqrt{3} + 1 - 1 = \sqrt{3}, (\sqrt{3})^2 = 3$$

$$\therefore \sqrt{3} \text{ is included between } 1.7, 1.8$$

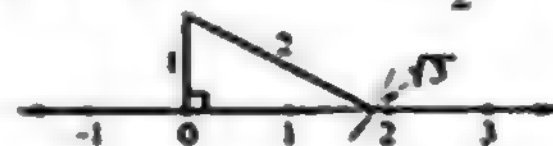
$$\therefore \sqrt{3} + 1 \text{ is included between } 2.7, 2.8$$

6

10

$$1) \text{ The length of one side of the right angle} = \frac{3-1}{2} = 1$$

$$\text{The length of the hypotenuse} = \frac{3+1}{2} = 2$$



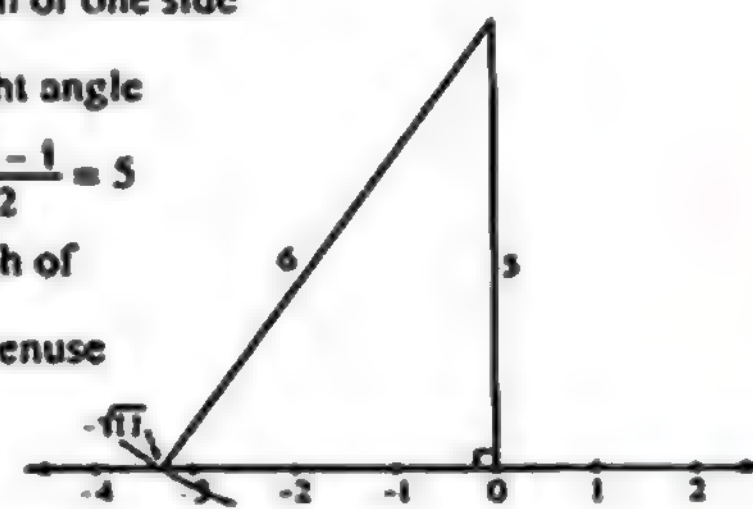
$$2) \text{ The length of one side}$$

$$\text{of the right angle}$$

$$\text{equals } \frac{11-1}{2} = 5$$

$$\text{The length of the hypotenuse}$$

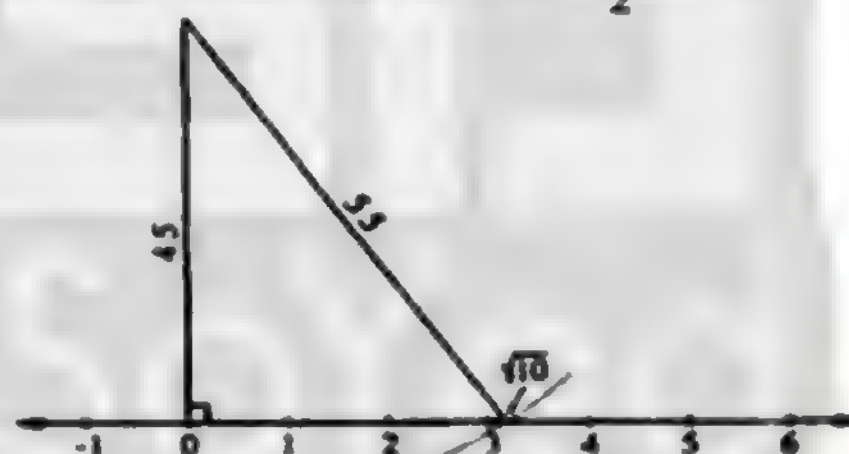
$$= \frac{11+1}{2} = 6$$



$$3) \text{ The length of one side of the right angle}$$

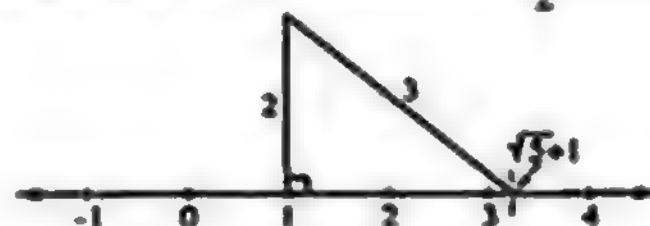
$$= \frac{10-1}{2} = 4.5$$

$$\text{The length of the hypotenuse} = \frac{10+1}{2} = 5.5$$



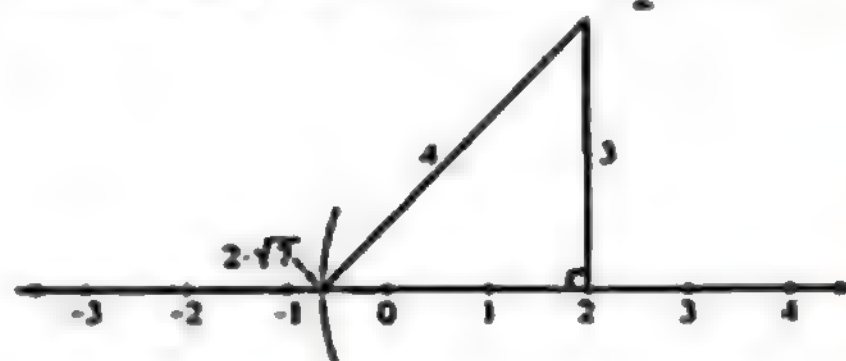
$$4) \text{ The length of one side of the right angle} = \frac{5-1}{2} = 2$$

$$\text{The length of the hypotenuse} = \frac{5+1}{2} = 3$$



$$5) \text{ The length of one side of the right angle} = \frac{7-1}{2} = 3$$

$$\text{The length of the hypotenuse} = \frac{7+1}{2} = 4$$

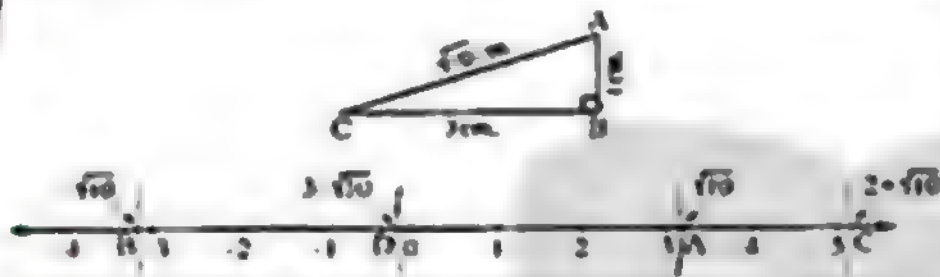


Answers of Unit 1

11

The length of one side of the right angle = $\frac{2-1}{2} = 0.5$ The length of the hypotenuse = $\frac{2+1}{2} = 1.5$ 

12



13

The length of the side of the square = $\sqrt{10}$ cm.

The square of the length of the diagonal

$$= (\sqrt{10})^2 + (\sqrt{10})^2 = 10 + 10 = 20$$

∴ The length of the diagonal = $\sqrt{20}$ cm.

14

∴ The length of the tree = 3 m.

∴ AB + BC = 3 m.

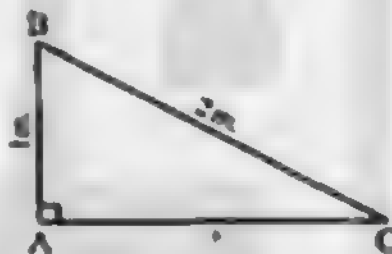
∴ the length of the left part of the tree = 1 m.

∴ BC = 3 - 1 = 2 m.

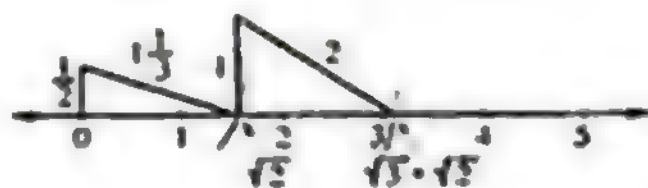
∴ In $\triangle ABC$: $m(\angle A) = 90^\circ$

$$\therefore (AC)^2 = (BC)^2 - (AB)^2 = 4 - 1 = 3$$

$$\therefore AC = \sqrt{3} \text{ m.}$$

∴ The distance between the base of the tree and the point of touching of its top with the ground = $\sqrt{3}$ m.

15

We represent on the number line the point representing the number $\sqrt{3} + \sqrt{2}$ as shown in the figure :

We find that the point representing the number $\sqrt{3} + \sqrt{2}$ lies between the point representing the number 3 and the point representing the number 4
i.e. $\sqrt{3} + \sqrt{2}$ lies between 3 and 4

Answers of Exercise 3

1

The number	Natural	Integer	Rational	Irrational	Real
-5	x	✓	✓	x	✓
$\sqrt{2}$	x	x	x	✓	✓
$1\frac{1}{2}$	x	x	✓	x	✓
$\sqrt[3]{9}$	x	x	x	✓	✓
$ -2 $	✓	✓	✓	x	✓
$-\sqrt{4}$	x	✓	✓	x	✓
$\frac{5}{2}$	x	x	✓	x	✓
0.3	x	x	✓	x	✓
$\sqrt{-1}$	x	x	x	x	x

2

1) \emptyset	2) \mathbb{R}	3) \emptyset
4) \mathbb{R}^+	5) \mathbb{Q}	6) \mathbb{Q}

3

1) positive	2) negative	3) positive
4) positive	5) negative	6) positive

4

1) >	2) >	3) <
4) <	5) >	6) =
7) >	8) >	9) >

5

1) a	2) b	3) a	4) d
5) a	6) c	7) d	

6

- 1) The ascending order is :
 $-\sqrt{11}, -\sqrt{7}, -\sqrt{3}, \sqrt{5}, \sqrt{8}$ and $\sqrt{15}$
- 2) $\therefore 0.6 = \sqrt{0.36}, \sqrt{-1} = -1 = -\sqrt{1}$
 ∴ The ascending order is :
 $-\sqrt{45}, -\sqrt{1}, \sqrt{0.36}, \sqrt{20}$ and $\sqrt{27}$
 i.e. $-\sqrt{45}, \sqrt{-1}, 0.6, \sqrt{20}$ and $\sqrt{27}$

Algebra and Statistics

7

$$1 \because 8 = \sqrt[3]{64}$$

\therefore The descending order is :

$$\sqrt{70}, \sqrt{64}, \sqrt{62} \text{ and } -\sqrt{50}$$

i.e. $\sqrt{70}, 8, \sqrt{62} \text{ and } -\sqrt{50}$

$$2 \because 9 = \sqrt{81}$$

\therefore The descending order is :

$$\sqrt{101}, \sqrt{81}, \sqrt{6}, -\sqrt{7}, -\sqrt{10} \text{ and } -\sqrt{50}$$

i.e. $\sqrt{101}, 9, \sqrt{6}, -\sqrt{7}, -\sqrt{10} \text{ and } -\sqrt{50}$

8

$$\because 2^2 = 4 \quad \therefore 4 > 3 > 2 > \frac{3}{2} > 0$$

$$\therefore 2 > \sqrt{3} > \sqrt{2} > \sqrt{\frac{3}{2}} > 0$$

\therefore The positive irrational numbers are

$$\sqrt{3}, \sqrt{2} \text{ and } \sqrt{\frac{3}{2}}$$

(There are other solutions)

9

The irrational numbers are

$$-\sqrt{5}, -\sqrt{3} \text{ and } -\sqrt{2} \text{ (There are other solutions)}$$

10

$$\because (15)^2 = 225, (17)^2 = 289$$

Then choosing 4 integers included between 225, 289

$$\text{(except 256 because } \sqrt{256} = 16 \in \mathbb{Q})$$

$$\therefore 225 < 235 < 245 < 255 < 265 < 289$$

$$\therefore 15 < \sqrt{235} < \sqrt{245} < \sqrt{255} < \sqrt{265} < 17$$

\therefore The four irrational numbers are

$$\sqrt{235}, \sqrt{245}, \sqrt{255} \text{ and } \sqrt{265}$$

(There are other solutions)

11

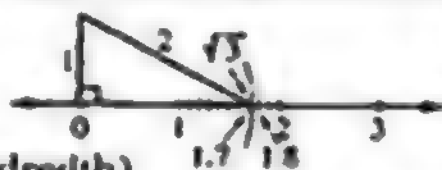
Using the calculator

$$\sqrt{3} \approx 1.73 \text{ (to the nearest hundredth)}$$

$$\therefore 1.7 < \sqrt{3} < 1.8 \text{ for representing } \sqrt{3}$$

$$\therefore \text{The length of the hypotenuse} = \frac{3+1}{2} = 2$$

$$\text{the length of one side of the right angle} = \frac{3-1}{2} = 1$$



12

$$1 \quad X^2 = 6 \quad \therefore X = \pm\sqrt{6} \approx \pm 2.45$$

$$2 \quad X^2 = 24 \times \frac{4}{3} = 32 \quad \therefore X = \pm\sqrt{32} \approx \pm 5.66$$

8

$$3 \quad \frac{1}{2} X^2 = 5$$

$$\therefore X = \pm\sqrt{10}$$

$$\therefore X^2 = 5 \times 2 = 10$$

$$\therefore X = \pm 3.16$$

$$4 \quad 5 X^3 = -1$$

$$\therefore X = \sqrt[3]{-\frac{1}{5}} \approx -0.58$$

$$\therefore X^3 = -\frac{1}{5}$$

$$5 \quad \frac{3}{4} X^2 = -13$$

(has no solution in \mathbb{R})

$$\therefore X^2 = -13 \times \frac{4}{3} = -\frac{52}{3}$$

$$6 \quad \frac{2}{X^3} = 16$$

$$\therefore X^3 = \frac{1}{8}$$

$$\therefore X = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$$

$$7 \quad \because (X^2 - 9)(X^3 - 5) = 0$$

$$\therefore X^2 - 9 = 0$$

$$\therefore X^2 = 9$$

$$\therefore X = \pm\sqrt{9} = \pm 3$$

$$\text{or } X^3 - 5 = 0$$

$$\therefore X^3 = 5$$

$$\therefore X = \sqrt[3]{5} \approx 1.71$$

$$8 \quad \because (2X^3 - 5)(X^2 + 1) = 0$$

$$\therefore 2X^3 - 5 = 0$$

$$\therefore 2X^3 = 5$$

$$\therefore X^3 = \frac{5}{2}$$

$$\therefore X = \sqrt[3]{\frac{5}{2}} \approx 1.36$$

$$\text{or } X^2 + 1 = 0$$

$$\therefore X^2 = -1 \text{ (has no solution in } \mathbb{R})$$

13

$$\text{The side length} = \sqrt{5} \text{ cm. } \sqrt{5} \notin \mathbb{Q}$$

14

$$\text{The edge length} = \sqrt[3]{1.728} = \frac{6}{5} \text{ cm. } \frac{6}{5} \in \mathbb{Q}$$

15

$$\therefore \text{The total area of the cube} = 6l^2$$

$$\therefore 13.5 = 6l^2$$

$$\therefore \frac{13.5}{6} = l^2$$

$$\therefore l = \sqrt{\frac{13.5}{6}} = 1.5 \text{ cm. } 1.5 \in \mathbb{Q}$$

16

$$\text{The diagonal length} = \sqrt{6^2 + 6^2} = \sqrt{72} \text{ cm.}$$

17

$$\text{The side length} = \sqrt{32} \text{ cm.}$$

$$\therefore \text{The diagonal length} = \sqrt{(\sqrt{32})^2 + (\sqrt{32})^2}$$

$$= \sqrt{32 + 32} = \sqrt{64} = 8 \text{ cm.}$$

18

$$\text{The length of the hypotenuse} = \sqrt{5^2 + 5^2} = \sqrt{50} \text{ cm.}$$

19

$$\text{The diagonal length of the rectangle}$$

$$= \sqrt{(5)^2 + (7)^2} = \sqrt{74} \text{ cm.}$$

Answers of Unit 1

∴ The area of the square = The area of the rectangle = $5 \times 7 = 35 \text{ cm}^2$

∴ The side length of the square = $\sqrt{35} \text{ cm}$.

∴ The diagonal length of the square = $\sqrt{35 + 35} = \sqrt{70} \text{ cm}$.

20

Cubing the two sides then squaring them we find that

$$(\sqrt[3]{3})^3 = 3, 3^2 = 9, (\sqrt{2})^2 = 2, (2\sqrt{2})^2 = 8$$

$$\therefore 9 > 8 \quad \therefore \sqrt[3]{3} > \sqrt{2}$$

21

Let the other number = X ∴ $X^2 + 2^2 = 7$

$$\therefore X^2 = 7 - 4 = 3 \quad \therefore X = \pm\sqrt{3}$$

∴ The other number is $\sqrt{3}$ or $-\sqrt{3}$

Answers of Exercise 4

1

$$2 \{x: 1 \leq x < 3, x \in \mathbb{R}\}$$



$$3]0, 3]$$



$$4]-2, 3[, \{x: -2 < x < 3, x \in \mathbb{R}\}$$



$$5 \{x: x \leq 1, x \in \mathbb{R}\}$$



$$6]0, \infty[, \{x: x > 0, x \in \mathbb{R}\}$$



$$7]-\infty, 4[$$



$$8 \{x: x \geq -2, x \in \mathbb{R}\}$$



2

1 c

2 a

3 b

4 c

5 d

3

1 ∈

2 ∉

3 ∈

4 ∈

5 ∈

6 ∈

7 ∈

8 ∉

9 ∉

10 ∉

4

$$1]-1, 5[$$



$$2]2, 3[$$



$$3]3, 5[$$



$$4]-1, 2[$$



$$5]-\infty, 2[\cup]5, \infty[$$



$$6]-\infty, -1[\cup]3, \infty[$$



5

1 ℝ



$$2]-4, 3]$$



$$3]-\infty, -4[$$



$$4]3, \infty[$$



$$5]3, \infty[$$



$$6]-\infty, -4[$$



6 Use the number line to get the following results :

$$1]-1, \infty[$$

$$2]3, 4]$$

$$3]-1, 3[$$

$$4]-1, 4[= \{3\}$$

$$5 \{3, 4\}$$

$$6]4, \infty[$$

$$7]-\infty, -1[\cup]4, \infty[$$

$$8]-\infty, 3[$$

7 Use the number line to get the following results :

$$1]2, 4]$$

$$2]-1, 5]$$

$$3]0, 1[$$

$$4]-2, 3]$$

$$5]3, 6]$$

$$6]-1, 2[$$

$$7]-3, 2] - \{0\}$$

$$8 \emptyset$$

$$9 \emptyset$$

$$10]-2, 1[\cup]2, 4]$$

$$11 \emptyset$$

$$12 \{-1, 5\}$$

8 Use the number line to get the following results :

$$1]-3, \infty[$$

$$2]2, 3[$$

$$3]-4, 3]$$

$$4 \mathbb{R}$$

$$5]-\infty, -1[$$

$$6]-\infty, -3[$$

$$7]0, 2]$$

$$8 \mathbb{R} - [3, 4]$$

9

$$1]3, 5]$$

$$2]3, 5]$$

$$3 \{3, 5\}$$

$$4 \emptyset$$

$$5]3, 5[$$

$$6]3, 5[$$

$$7 \emptyset$$

$$8 \{3, 5\}$$

$$9]3, 5[$$

$$10]3, 5[$$

$$11 \{3, 4\}$$

$$12]-3, 5]$$

10

$$1]1, 7[$$

$$2]-3, 0[$$

$$3]3, 4]$$

$$4]2, 5[$$

$$5 \{5\}$$

$$6]3, 4[$$

$$7 \{2, 7\}$$

$$8 \{4\}$$

$$9]3, 4[$$

$$10]0, 1[$$

Algebra and Statistics

11

1 b

2 d

3 c

4 b

5 b

6 d

12

1 $[-3, 3]$ 2 \mathbb{R} 3 $]-\infty, -1[$ 4 $]-\infty, -3[$ 5 $]-2, 0]$ 6 $[0, 2]$ 7 $\{1, 2\}$ 8 $\{0, 1\}$ 9 $\{-1, 0, 1, 2\}$ 10 $]0, 5]$ 11 $[-3, 0[$

13

1 $[-3, 1[$ 2 $\mathbb{R} -]-3, 1[$ 3 $]-3, 1[$ 4 $\mathbb{R} -]-3, 1[$ 5 $[-3, 3[- \{1\}$

14

Let X be the temperature degrees needed to keep the first kind.

Y be the temperature degrees needed to keep the second kind.

$\therefore X = [-3, 4], Y = [2, 10]$

\therefore The temperature needed to keep the two kinds altogether at the same place $= X \cap Y = [2, 4]$

15

1 d

2 c

3 c

4 c

5 b

6 d

7 c

8 c

9 c

16

$\therefore X \subset Y \therefore X = X \cap Y = [4, 7]$

$Y = X \cup Y = [3, 7], Y - X = [3, 4[$

Answers of Exercise 5

1

1 $3\sqrt{3}$ 2 $-2\sqrt{2}$

3 zero

4 $-\sqrt[3]{7}$ 5 $6\sqrt{5}$

6 zero

2

1 $3\sqrt{5}$ 2 $3\sqrt{3} - 1$ 3 $8\sqrt{7} - 3\sqrt{2}$ 4 $7\sqrt{2} - 2\sqrt{2}$ 5 $\sqrt{2}$

6 $8 \times \frac{1}{2} + 2\sqrt[3]{3} - 4 - 5\sqrt[3]{3} = 4 + 2\sqrt[3]{3} - 4 - 5\sqrt[3]{3}$
 $= -3\sqrt[3]{3}$

3

1 3

2 -30

3 $6\sqrt{2}$

4 1

5 $15\sqrt{3}$

6 $2\sqrt{3} \times \frac{2\sqrt{7}}{7} \times \frac{5\sqrt{7}}{20\sqrt{3}} = 1$

4

1 $2\sqrt{2} + 2\sqrt{5}$ 2 $5\sqrt{2} + 2$ 3 $7 + 2\sqrt{7}$ 4 $5\sqrt{3} + 3$ 5 $-6\sqrt{5} + 10$ 6 $2 - 7 + 3\sqrt{7} = -5 + 3\sqrt{7}$ 7 $-24 - 6\sqrt{3} + 6\sqrt{3} = -24$ 8 $3\sqrt{5} - 5 - 2 - 2\sqrt{5} = \sqrt{5} - 7$

5

1 $(\sqrt{2})^2 - (1)^2 = 2 - 1 = 1$ 2 $(4)^2 - (3\sqrt{2})^2 = 16 - 18 = -2$

3 $(\sqrt{5})^2 - 2 \times 1 \times \sqrt{5} + (-1)^2 = 5 - 2\sqrt{5} + 1$
 $= 6 - 2\sqrt{5}$

4 $(2\sqrt{3})^2 + 2 \times 4 \times 2\sqrt{3} + (4)^2 = 12 + 16\sqrt{3} + 16$
 $= 28 + 16\sqrt{3}$

5 $3 + \sqrt{3} - 2 = 1 + \sqrt{3}$

6 $(5)^2 - 2 \times 5 \times \sqrt{3} + (-\sqrt{3})^2 - 28$
 $= 25 - 10\sqrt{3} + 3 - 28 = -10\sqrt{3}$

6

1 $\frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$ 2 $\frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$ 3 $-\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\frac{6\sqrt{3}}{3} = -2\sqrt{3}$ 4 $\frac{8}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{8\sqrt{6}}{6} = \frac{4\sqrt{6}}{3}$ 5 $\frac{2}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{6} = \frac{\sqrt{2}}{3}$ 6 $\frac{6}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{6} = \sqrt{3}$ 7 $\frac{25}{2\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{25\sqrt{10}}{20} = \frac{5\sqrt{10}}{4}$ 8 $\frac{\sqrt{2} + 3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2 + 3\sqrt{2}}{2}$

9 $\frac{\sqrt{5} - 15}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5 - 15\sqrt{5}}{10} = \frac{1 - 3\sqrt{5}}{2}$

Answers of Unit 1

7

- 1 c 2 d 3 b 4 c 5 a 6 d
7 c 8 b 9 d 10 b 11 d 12 c

8

- 1 1, zero 2 $\sqrt{2} - 1$ 3 $5\sqrt{3}$
4 1 5 $2 + \sqrt{3}$ 6 $\frac{2}{3}$
7 $4\sqrt{3}$ 8 $3 + 2\sqrt{2}$ 9 $\pm\sqrt{5}$
10 $8\sqrt{2}$ 11 60 cm^2
12 The additive inverse 13 R

9

- 1 $\sqrt{5} - 2 + \sqrt{5} + 2 = 2\sqrt{5}$
2 $\sqrt{5} - 2 - \sqrt{5} - 2 = -4$
3 $(\sqrt{5} - 2)(\sqrt{5} + 2) = 5 - 4 = 1$
4 $x^2 - y^2 = (x - y)(x + y) = (-4)(2\sqrt{5}) = -8\sqrt{5}$
5 $x^2 + 2xy + y^2 = (x + y)^2 = (2\sqrt{5})^2 = 20$
6 $x^2 - 2xy + y^2 = (x - y)^2 = (-4)^2 = 16$

10

$$\begin{aligned}\text{The expression} &= a(a - b)^3 + b(b - a)^3 \\ &= a(a - b)^3 - b(a - b)^3 \\ &= (a - b)^3(a - b) = (a - b)^4 \\ &= (2\sqrt{3})^4 = 144\end{aligned}$$

11

$$\begin{aligned}\therefore x &= \sqrt{3 + \sqrt{2}} & \therefore x^2 &= 3 + \sqrt{2} \\ \therefore \text{The expression} &= x^4 - 2x^2 + 1 \\ &= (x^2 - 1)^2 = (3 + \sqrt{2} - 1)^2 \\ &= (2 + \sqrt{2})^2 = 4 + 4\sqrt{2} + 2 \\ &= 6 + 4\sqrt{2}\end{aligned}$$

12

- 1 $x \approx 3 + 2 = 5$
and using the calculator
 $\therefore x \approx 5.2$
(accepted estimation)
 $y \approx 1 + 3 = 4$
and using the calculator $\therefore y \approx 3.8$ (accepted estimation)
2 $x + y \approx 5 + 4 = 9$
and using the calculator, the expression ≈ 9.06
(accepted estimation)

$$3 \quad x - y \approx 5 - 4 = 1$$

and using the calculator, the expression ≈ 1.4
(accepted estimation)

$$4 \quad xy \approx 5 \times 4 = 20$$

and using the calculator, the expression equals 20.05
(accepted estimation)

13

$$1 \quad x \approx 4 + 2 = 6$$

and using the calculator $x \approx 5.9$ (accepted estimation)

$$y \approx 4 - 3 = 1$$

and using the calculator $y \approx 1.08$ (accepted estimation)

$$2 \quad x \times y \approx 6 \times 1 = 6$$

and using the calculator, the expression ≈ 6.3
(accepted estimation)

$$3 \quad x + y \approx 6 + 1 = 7$$

and using the calculator, the expression ≈ 6.9
(accepted estimation)

14

$$\text{The perimeter} = 2(6 + \sqrt{5} + 6 - \sqrt{5}) \approx 2 \times 12 = 24 \text{ cm.}$$

$$\text{The area} = (6 + \sqrt{5})(6 - \sqrt{5}) \approx 36 - 5 = 31 \text{ cm}^2$$

15

$$1 \quad \therefore \text{The area of the small square} = 13 \text{ cm}^2$$

$$\therefore \text{The side length of the small square} = \sqrt{13} \text{ cm.}$$

\therefore the side of the chess board consists of 8 small squares.

$$\therefore \text{The side length of the chess board} = 8 \times \sqrt{13} = 8\sqrt{13} \text{ cm.}$$

$$2 \quad \therefore (\text{The diagonal length of the square})^2$$

$$= (\text{its side length})^2 + (\text{its side length})^2$$

= Pythagoras' theorem

$$\therefore \text{The diagonal length of the square}$$

$$= \sqrt{(8\sqrt{13})^2 + (8\sqrt{13})^2}$$

$$= \sqrt{64 \times 13 + 64 \times 13} = \sqrt{1664} \text{ cm.}$$

16

$$(\sqrt{a} - 1) \times \frac{\sqrt{a} + 1}{4} = 1$$

$$\therefore \frac{a - 1}{4} = 1$$

$$\therefore a - 1 = 4$$

$$\therefore a = 5$$

Algebra and Statistics

17

$$x = \sqrt{2}, y = \frac{\sqrt{2}}{2}, z = \frac{\sqrt{2}}{4}$$

$$\therefore x^2 + 2y^2 + 4z^2 = (\sqrt{2})^2 + 2 \times \left(\frac{\sqrt{2}}{2}\right)^2 + 4 \times \left(\frac{\sqrt{2}}{4}\right)^2$$

$$= 2 + 2 \times \frac{2}{4} + 4 \times \frac{2}{16} = 3\frac{1}{2}$$

18

$$\frac{1}{2}(2y) = 1 - \sqrt{2} \therefore y = 1 - \sqrt{2}$$

$$\therefore x = -1 + \sqrt{2}$$

$$\therefore xy - 2\sqrt{2} = (-1 + \sqrt{2})(1 - \sqrt{2}) - 2\sqrt{2}$$

$$= -1 + \sqrt{2} + \sqrt{2} - 2 - 2\sqrt{2} = -3$$

Answers of exams on the first part of unit one

Model 1

1

- 1 c 2 d 3 a 4 b 5 d 6 b

2

- 1 23 2 5 3 Q
4 $]-2, 7[$ 5 $\sqrt{3} - 5$

3

- (a) 1 $X \cap Y = [1, 3]$
2 $X \cup Y = [-2, 5[$
3 $Y - X =]3, 5[$
(b) The S.S. = $\{2, -2, \sqrt{7}\}$

4

- (a) Prove by yourself. (b) $\sqrt{50}$ cm.

5

- (a) Determine by yourself.
(b) 1 2 2 $5 + \sqrt{7}$

Model 2

1

- 1 c 2 c 3 b 4 b 5 a 6 a

2

- 1 $\sqrt{2} - \sqrt{7}$ 2 \mathbb{R}^* 3 $[-4, 0]$
4 -3 5 \emptyset

3

- (a) $3\sqrt{7}$

(b) 1 $X \cup Y =]-\infty, 4[$

2 $X \cap Y = [-2, 1[$

3 $X^c = [1, \infty[$

4

(a) 1 The S.S. = $\{\sqrt{20}, -\sqrt{20}\}$

2 The S.S. = \emptyset

(b) Prove by yourself.

5

(a) 1 $37 - 20\sqrt{3}$

2 $5 + 2\sqrt{5}$

(b) $\sqrt{122}, \sqrt{123}, \sqrt{124}$ and $\sqrt{125}$

(There are other numbers)

Answers of Exercise 6

1

1 $\sqrt{4 \times 3} = 2\sqrt{3}$ 2 $\sqrt{4 \times 7} = 2\sqrt{7}$

3 $2\sqrt{36 \times 2} = 2 \times 6\sqrt{2} = 12\sqrt{2}$

4 $\frac{2}{3}\sqrt{100 \times 10} = \frac{2}{3} \times 10\sqrt{10} = 4\sqrt{10}$

5 $\sqrt{4 \times \frac{1}{2}} = \sqrt{2}$ 6 $2\sqrt{\frac{2}{3} \times 9} = 2\sqrt{6}$

2

1 $5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$

2 $2\sqrt{5} - 3\sqrt{5} = -\sqrt{5}$

3 $3\sqrt{2} + 2\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

4 $7\sqrt{2} - 8\sqrt{2} - 3\sqrt{2} + 4\sqrt{2} = \text{zero}$

5 $2 \times 3\sqrt{2} + 5\sqrt{2} + \frac{1}{3} \times 9\sqrt{2}$
 $= 6\sqrt{2} + 5\sqrt{2} + 3\sqrt{2} = 14\sqrt{2}$

6 $7\sqrt{2} + 5\sqrt{2} - \frac{1}{2} \times 10\sqrt{2} - \sqrt{2}$
 $= 7\sqrt{2} + 5\sqrt{2} - 5\sqrt{2} - \sqrt{2} = 6\sqrt{2}$

7 $3\sqrt{3} + 5 \times 3\sqrt{2} - 10\sqrt{3} = 15\sqrt{2} - 7\sqrt{3}$

3

1 $2\sqrt{5} + 4 \times 2\sqrt{5} - \sqrt{25 \times \frac{1}{5}} = 2\sqrt{5} + 8\sqrt{5} - \sqrt{5}$
 $= 9\sqrt{5}$

2 $4\sqrt{2} - 6\sqrt{2} + 3\sqrt{4 \times \frac{1}{2}} = 4\sqrt{2} - 6\sqrt{2} + 3\sqrt{2}$
 $= \sqrt{2}$

Answers of Unit 1

$$\textcircled{3} 2\sqrt{5} + 2\sqrt{9 \times \frac{1}{3}} - 2\sqrt{3} - \sqrt{25 \times \frac{1}{5}}$$

$$= 2\sqrt{5} + 2\sqrt{3} - 2\sqrt{3} - \sqrt{5} = \sqrt{5}$$

$$\textcircled{4} \sqrt{3} + \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} - \sqrt{12} = \sqrt{3} + \sqrt{3} - 2\sqrt{3} = \text{zero}$$

$$\textcircled{5} 3\sqrt{2} - \sqrt{\frac{12}{6}} = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$$

$$\textcircled{6} 5 + 3\sqrt{2} - \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 5 + 3\sqrt{2} - 3\sqrt{2} = 5$$

4

$$\textcircled{1} 10\sqrt{6} \quad \textcircled{2} 6\sqrt{36} = 6 \times 6 = 36$$

$$\textcircled{3} 2\sqrt{50} = 2 \times 5\sqrt{2} = 10\sqrt{2}$$

$$\textcircled{4} \sqrt{\frac{2}{7}} \times \frac{7}{2} = \sqrt{1} = 1 \quad \textcircled{5} 3\sqrt{\frac{15}{3}} = 3\sqrt{5}$$

$$\textcircled{6} 12 \times \sqrt{\frac{2}{3}} \times 54 = 12\sqrt{36} = 12 \times 6 = 72$$

5

$$\textcircled{1} \sqrt{18} - \sqrt{12} = 3\sqrt{2} - 2\sqrt{3}$$

$$\textcircled{2} 20 + 5\sqrt{24} = 20 + 5 \times 2\sqrt{6} = 20 + 10\sqrt{6}$$

$$\textcircled{3} (3\sqrt{5})^2 - (\sqrt{7})^2 = 45 - 7 = 38$$

$$\textcircled{4} (\sqrt{3})^2 - 2 \times \sqrt{3} \times \sqrt{2} + (-\sqrt{2})^2 = 3 - 2\sqrt{6} + 2$$

$$= 5 - 2\sqrt{6}$$

$$\textcircled{5} (\sqrt{3})^2 + 2 \times \sqrt{3} \times \sqrt{5} + (\sqrt{5})^2 - 2\sqrt{15}$$

$$= 3 + 2\sqrt{15} + 5 - 2\sqrt{15} = 8$$

$$\textcircled{6} 3\sqrt{2} - \frac{12}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} + 2\sqrt{6} - 3\sqrt{2}$$

$$= 3\sqrt{2} - 2\sqrt{6} + 2\sqrt{6} - 3\sqrt{2} = \text{zero}$$

6

$$\textcircled{1} \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$$

$$\textcircled{2} \sqrt{\frac{5}{3}} = \frac{\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{15}}{3}$$

$$\textcircled{3} \frac{5\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{15}}{5} = \sqrt{15}$$

$$\textcircled{4} \frac{4\sqrt{3} - \sqrt{2}}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{12 - \sqrt{6}}{6}$$

7

$$\textcircled{1} a \quad \textcircled{2} b \quad \textcircled{3} c \quad \textcircled{4} a \quad \textcircled{5} c$$

$$\textcircled{6} a \quad \textcircled{7} a \quad \textcircled{8} c \quad \textcircled{9} b$$

8

$$\textcircled{1} \frac{1}{2} \quad \textcircled{2} \sqrt{2} \quad \textcircled{3} \sqrt{3} \quad \textcircled{4} -2$$

$$\textcircled{5} \sqrt{125} \quad \textcircled{6} \pm \frac{2\sqrt{2}}{3} \quad \textcircled{7} 20, \text{zero}$$

9

$$\textcircled{1} x + y = 3 + \sqrt{5} + 1 - \sqrt{5} = 4$$

$$x \times y = (3 + \sqrt{5})(1 - \sqrt{5}) = 3 - 2\sqrt{5} - 5$$

$$= -2 - 2\sqrt{5}$$

$$\textcircled{2} x + y = \sqrt{3} - \sqrt{2} + \sqrt{3} + \sqrt{2} = 2\sqrt{3}$$

$$x \times y = (\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2}) = 3 - 2 = 1$$

$$\textcircled{3} x + y = 5 - 3\sqrt{2} + 5 - 3\sqrt{2} = 10 - 6\sqrt{2}$$

$$x \times y = (5 - 3\sqrt{2})(5 - 3\sqrt{2})$$

$$= 25 - 30\sqrt{2} + 18 = 43 - 30\sqrt{2}$$

10

$$\therefore x = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}, y = \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$$

$$\therefore 6(x + y) = 6\left(\frac{\sqrt{6}}{3} + \frac{\sqrt{6}}{2}\right) = 6 \times \frac{\sqrt{6}}{3} + 6 \times \frac{\sqrt{6}}{2}$$

$$= 2\sqrt{6} + 3\sqrt{6} = 5\sqrt{6}$$

11

$$\therefore x = \frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$$

$$y = 3\sqrt{5} + \sqrt{2}, z = 2\sqrt{2} + \sqrt{5}$$

$$\therefore (x - y + z)^2$$

$$= (2\sqrt{5} - 3\sqrt{5} - \sqrt{2} + 2\sqrt{2} + \sqrt{5})^2 = (\sqrt{2})^2 = 2$$

12

We know that $(x + y)^2 = x^2 + 2xy + y^2$

$$\therefore x^2 + 2xy + y^2 = (2\sqrt{5} + \sqrt{2} + 2\sqrt{5} - \sqrt{2})^2$$

$$= (4\sqrt{5})^2 = 16 \times 5 = 80$$

13

$$\therefore x = \sqrt{7} + \frac{1}{2} \times 2\sqrt{3} = \sqrt{7} + \sqrt{3}$$

$$y = \frac{1}{3} \times 3\sqrt{7} - \sqrt{3} = \sqrt{7} - \sqrt{3}$$

$$\therefore x^2 y^2 = (xy)^2 = ((\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3}))^2$$

$$= (7 - 3)^2 = 4^2 = 16$$

Algebra and Statistics

14

The perimeter of $\triangle ABC$

$$\begin{aligned}
 &= \sqrt{28} + 28\sqrt{\frac{1}{7}} + 5\sqrt{7} \\
 &= \sqrt{4 \times 7} + 4\sqrt{49 \times \frac{1}{7}} + 5\sqrt{7} \\
 &= 2\sqrt{7} + 4\sqrt{7} + 5\sqrt{7} = 11\sqrt{7} \text{ cm.}
 \end{aligned}$$

15

1 The area of one square = $\frac{300}{6} = 50 \text{ cm}^2$ \therefore The side length of one square = $\sqrt{50} = 5\sqrt{2} \text{ cm.}$
 \therefore The perimeter of the figure = $14 \times 5\sqrt{2}$
 $= 70\sqrt{2} \text{ cm.}$
2 The area of one square = $\frac{72}{6} = 12 \text{ cm}^2$ \therefore The side length of one square = $\sqrt{12} = 2\sqrt{3} \text{ cm.}$
 \therefore The perimeter of the figure = $14 \times 2\sqrt{3}$
 $= 28\sqrt{3} \text{ cm.}$
3 The area of one square = $\frac{40}{5} = 8 \text{ cm}^2$ \therefore The side length of one square = $\sqrt{8} = 2\sqrt{2} \text{ cm.}$
 \therefore The perimeter of the figure = $12 \times 2\sqrt{2}$
 $= 24\sqrt{2} \text{ cm.}$

16

$$\begin{aligned}
 a^{x+y} &= a^x \times a^y = a^x + a^{-y} = 6 + \sqrt{3} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
 &= \frac{6\sqrt{3}}{3} = 2\sqrt{3}
 \end{aligned}$$

17

$$\frac{(\sqrt{5})^3 \times (\sqrt{5})^5}{(\sqrt{2})^6 \times (\sqrt{5})^6} = \frac{(\sqrt{5})^{3+5-6}}{(\sqrt{2})^6} = \frac{5}{8}$$

$$\frac{2\sqrt{2} \times (\sqrt{2})^{-3} \times (\sqrt{3})^{-3}}{(\sqrt{3})^{-3}} = 2 \times (\sqrt{2})^{-2} = \frac{2}{(\sqrt{2})^2} = \frac{2}{2} = 1$$

18

$$\begin{aligned}
 \therefore 3\sqrt{3} + \sqrt{4 \times \frac{1}{2}} + 3\sqrt{2} + 2\sqrt{3} - 5\sqrt{2} &= x\sqrt{2} + y\sqrt{3} \\
 \therefore 3\sqrt{3} + \sqrt{2} + 3\sqrt{2} + 2\sqrt{3} - 5\sqrt{2} &= x\sqrt{2} + y\sqrt{3} \\
 \therefore -\sqrt{2} + 5\sqrt{3} &= x\sqrt{2} + y\sqrt{3} \therefore x = -1, y = 5
 \end{aligned}$$

Answers of Exercise 7

1

$$\text{1) } \sqrt{5} - \sqrt{3} \quad \text{2) } 5 + 2\sqrt{7}$$

$$\text{3) The number is } \sqrt{5} + \frac{2}{\sqrt{2}} = \sqrt{5} + \sqrt{2}$$

$$\therefore \text{The conjugate number} = \sqrt{5} - \sqrt{2}$$

2

$$\text{1) } \frac{5}{\sqrt{7}-\sqrt{2}} \times \frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}} = \frac{5(\sqrt{7}+\sqrt{2})}{7-2} = \sqrt{7} + \sqrt{2}$$

$$\text{2) } \frac{\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{\sqrt{3}(2+\sqrt{3})}{4-3} = 2\sqrt{3} + 3$$

$$\text{3) } \frac{\sqrt{7}+3}{\sqrt{7}-3} \times \frac{\sqrt{7}+3}{\sqrt{7}+3} = \frac{16+6\sqrt{7}}{7-9} = -8-3\sqrt{7}$$

3

$$\begin{aligned}
 \therefore x &= \frac{2}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{2(\sqrt{7}+\sqrt{5})}{7-5} \\
 &= \sqrt{7} + \sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 \therefore (x+y)^2 &= (\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5})^2 \\
 &= (2\sqrt{7})^2 = 28
 \end{aligned}$$

4

$$\begin{aligned}
 x^2 y^2 &= (xy)^2 = \left(\frac{4}{\sqrt{7}-\sqrt{3}} \times \frac{4}{\sqrt{7}+\sqrt{3}} \right)^2 \\
 &= \left(\frac{16}{7-3} \right)^2 = 4^2 = 16
 \end{aligned}$$

5

$$\begin{aligned}
 \text{L.H.S.} &= \frac{4}{x} + 2x \\
 &= \frac{4}{\sqrt{5}+\sqrt{3}} + 2(\sqrt{5}+\sqrt{3}) \\
 &= \frac{4(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} + 2(\sqrt{5}+\sqrt{3}) \\
 &= \frac{4(\sqrt{5}-\sqrt{3})}{2} + 2(\sqrt{5}+\sqrt{3}) \\
 &= 2(\sqrt{5}-\sqrt{3}) + 2(\sqrt{5}+\sqrt{3}) \\
 &= 2\sqrt{5} - 2\sqrt{3} + 2\sqrt{5} + 2\sqrt{3} \\
 &= 4\sqrt{5} = \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned} \therefore b &= \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{3-2} \\ &= \sqrt{3}-\sqrt{2} \end{aligned}$$

We know that : $(a-b)(a+b) = a^2 - b^2$

$$\begin{aligned} \therefore (\sqrt{3}+\sqrt{2}-\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}) \\ = 2\sqrt{2} \times 2\sqrt{3} = 4\sqrt{6} \end{aligned}$$

$$\begin{aligned} \therefore y &= \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3} \\ &= \sqrt{5}+\sqrt{3} \end{aligned}$$

$$\begin{aligned} \therefore x^2 + 2xy + y^2 &= (x+y)^2 \\ &= (\sqrt{5}-\sqrt{3}+\sqrt{5}+\sqrt{3})^2 = (2\sqrt{5})^2 = 20 \end{aligned}$$

$$\begin{aligned} \therefore y &= \frac{3}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{3(\sqrt{5}+\sqrt{2})}{5-2} \\ &= \sqrt{5}+\sqrt{2} \end{aligned}$$

$$x = \sqrt{5}-\sqrt{2}$$

$\therefore x$ and y are two conjugate numbers.

$$\begin{aligned} \therefore x^2 - 2xy + y^2 &= (x-y)^2 = (\sqrt{5}-\sqrt{2}-\sqrt{5}-\sqrt{2})^2 \\ &= (-2\sqrt{2})^2 = 8 \end{aligned}$$

$$\begin{aligned} \therefore x &= 3+\sqrt{5}, y = \frac{4}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{4(3-\sqrt{5})}{9-5} \\ &= 3-\sqrt{5} \end{aligned}$$

$\therefore x$ and y are two conjugate numbers

$$\textcircled{1} xy = (3+\sqrt{5})(3-\sqrt{5}) = 9-5 = 4$$

$$\begin{aligned} \textcircled{2} x^2 + y^2 &= (x+y)^2 - 2xy \\ &= (3+\sqrt{5}+3-\sqrt{5})^2 - 2 \times 4 = 36-8 = 28 \end{aligned}$$

$$\begin{aligned} \therefore x &= \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3} \\ &= \sqrt{5}+\sqrt{3} \end{aligned}$$

$$\begin{aligned} y &= \frac{2}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{2(\sqrt{5}-\sqrt{3})}{5-3} \\ &= \sqrt{5}-\sqrt{3} \end{aligned}$$

$$\begin{aligned} \therefore x^2 - xy + y^2 &= (x-y)^2 + xy \\ &= (\sqrt{5}+\sqrt{3}-\sqrt{5}+\sqrt{3})^2 + (\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3}) \\ &= (2\sqrt{3})^2 + 2 = 14 \end{aligned}$$

$$\begin{aligned} \frac{x+y}{xy-1} &= \frac{\sqrt{5}+\sqrt{2}+\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})-1} \\ &= \frac{2\sqrt{5}}{5-2-1} = \frac{2\sqrt{5}}{2} = \sqrt{5} \end{aligned}$$

$$\begin{aligned} \therefore a &= \frac{4}{\sqrt{7}-\sqrt{3}} \times \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} = \frac{4(\sqrt{7}+\sqrt{3})}{7-3} \\ &= \sqrt{7}+\sqrt{3} \end{aligned}$$

$$\begin{aligned} b &= \frac{4}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} = \frac{4(\sqrt{7}-\sqrt{3})}{7-3} \\ &= \sqrt{7}-\sqrt{3} \end{aligned}$$

$$\therefore \frac{a-b}{ab} = \frac{\sqrt{7}+\sqrt{3}-\sqrt{7}+\sqrt{3}}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})} = \frac{2\sqrt{3}}{7-3} = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \therefore x &= 2\sqrt{2}-\sqrt{3}, y = \frac{5}{2\sqrt{2}-\sqrt{3}} \\ \therefore y &= \frac{5}{2\sqrt{2}-\sqrt{3}} \times \frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}} = \frac{5(2\sqrt{2}+\sqrt{3})}{8-3} \\ &= 2\sqrt{2}+\sqrt{3} \end{aligned}$$

$\therefore x$ and y are conjugate numbers

$$\begin{aligned} \therefore \frac{x+y}{xy} &= \frac{2\sqrt{2}-\sqrt{3}+2\sqrt{2}+\sqrt{3}}{(2\sqrt{2}-\sqrt{3})(2\sqrt{2}+\sqrt{3})} \\ &= \frac{4\sqrt{2}}{8-3} = \frac{4\sqrt{2}}{5} \end{aligned}$$

$$\begin{aligned} \therefore x &= \frac{5\sqrt{2}+3\sqrt{5}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{10}+15}{5} \\ &= \sqrt{10}+3 \\ y &= \frac{2\sqrt{5}-3\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{10}-6}{2} = \sqrt{10}-3 \end{aligned}$$

$$\begin{aligned} \textcircled{1} x^2 + y^2 &= (x+y)^2 - 2xy \\ &= (\sqrt{10}+3+\sqrt{10}-3)^2 - 2(\sqrt{10}+3)(\sqrt{10}-3) \\ &= (2\sqrt{10})^2 - 2 \times (10-9) = 40-2 = 38 \end{aligned}$$

Algebra and Statistics

$$2 \quad xy = (\sqrt{10} + 3)(\sqrt{10} - 3) = 10 - 9 = 1$$

$$\therefore x^2 + y^2 = 38xy$$

15

$$\therefore x = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{2 - \sqrt{3}}{4 - 3} = 2 - \sqrt{3}$$

$$y = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$$\therefore x^2 + y = (2 - \sqrt{3})^2 + 4\sqrt{3} \\ = 4 - 4\sqrt{3} + 3 + 4\sqrt{3} = 7$$

16

$$\therefore y = \sqrt{3} - \sqrt{2}$$

$$x = \frac{1}{\sqrt{3} - \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} = \sqrt{3} + \sqrt{2}$$

$$\therefore (x + y)^2 = (\sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2})^2 \\ = (2\sqrt{3})^2 = 12$$

17

$$\therefore xy = 1$$

$$\therefore y = \frac{1}{x} = \frac{1}{\sqrt{13} + \sqrt{6}} = \frac{1}{\sqrt{13} + \sqrt{6}} \times \frac{\sqrt{13} - \sqrt{6}}{\sqrt{13} - \sqrt{6}} \\ = \frac{\sqrt{13} - \sqrt{6}}{7}$$

$$\therefore x^2 - 49y^2 = (x - 7y)(x + 7y) \\ = \left(\sqrt{13} + \sqrt{6} - 7 \left(\frac{\sqrt{13} - \sqrt{6}}{7} \right) \right) \\ \left(\sqrt{13} + \sqrt{6} + 7 \left(\frac{\sqrt{13} - \sqrt{6}}{7} \right) \right) \\ = (\sqrt{13} + \sqrt{6} - \sqrt{13} + \sqrt{6})(\sqrt{13} + \sqrt{6} + \sqrt{13} - \sqrt{6}) \\ = 2\sqrt{6} \times 2\sqrt{13} = 4\sqrt{78}$$

18

$$\therefore x = \frac{4(\sqrt{7} + \sqrt{3})}{(\sqrt{7} - \sqrt{3})(\sqrt{7} + \sqrt{3})} = \frac{4(\sqrt{7} + \sqrt{3})}{7 - 3} \\ = \sqrt{7} + \sqrt{3}$$

$$y = \sqrt{7} - \sqrt{3}$$

$\therefore x$ and y are two conjugate numbers

$$\therefore x^2 y^2 = (xy)^2 = [(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3})]^2 \\ = (7 - 3)^2 = 4^2 = 16$$

19

$$\therefore y = \frac{2}{\sqrt{7} + \sqrt{5}} \times \frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} - \sqrt{5}} = \frac{2(\sqrt{7} - \sqrt{5})}{7 - 5} \\ = \sqrt{7} - \sqrt{5}$$

$$\therefore \frac{x + y}{xy} = \frac{\sqrt{7} + \sqrt{5} + \sqrt{7} - \sqrt{5}}{(\sqrt{7} + \sqrt{5})(\sqrt{7} - \sqrt{5})} = \frac{2\sqrt{7}}{7 - 5} = \sqrt{7}$$

20

$$\therefore x = \frac{\sqrt{6} + \sqrt{5}}{\sqrt{6} - \sqrt{5}} \times \frac{\sqrt{6} + \sqrt{5}}{\sqrt{6} + \sqrt{5}} = \frac{11 + 2\sqrt{30}}{6 - 5} \\ = 11 + 2\sqrt{30}$$

$$\therefore \frac{1}{x} = \frac{1}{11 + 2\sqrt{30}} \times \frac{11 - 2\sqrt{30}}{11 - 2\sqrt{30}} = \frac{11 - 2\sqrt{30}}{121 - 120} \\ = 11 - 2\sqrt{30}$$

$$\therefore x + \frac{1}{x} = 11 + 2\sqrt{30} + 11 - 2\sqrt{30} = 22$$

21

- | | | |
|------------------|-------------------------|-------------------------|
| 1 4 | 2 $3 - \sqrt{2}, 7$ | 3 $\sqrt{3} - \sqrt{2}$ |
| 4 $1 - \sqrt{7}$ | 5 $\sqrt{3} - \sqrt{2}$ | 6 20 |
| 7 20 | 8 $\sqrt{5} + 2$ | 9 $(-1, 2\sqrt{3})$ |
| 10 -1 | | |

22

$$1 \quad \frac{11}{2\sqrt{5} + 3} = \frac{11(2\sqrt{5} - 3)}{(2\sqrt{5} + 3)(2\sqrt{5} - 3)} \\ = \frac{11(2\sqrt{5} - 3)}{20 - 9} = 2\sqrt{5} - 3$$

$$\therefore a = 2, b = -3$$

$$2 \quad \frac{3}{2\sqrt{2} - \sqrt{5}} = \frac{3(2\sqrt{2} + \sqrt{5})}{(2\sqrt{2} - \sqrt{5})(2\sqrt{2} + \sqrt{5})} \\ = \frac{3(2\sqrt{2} + \sqrt{5})}{8 - 5} = 2\sqrt{2} + \sqrt{5}$$

$$\therefore a = 2, b = 1$$

$$3 \quad \frac{7}{\sqrt{8} + 1} = \frac{7(\sqrt{8} - 1)}{(\sqrt{8} + 1)(\sqrt{8} - 1)} \\ = \frac{7(\sqrt{8} - 1)}{8 - 1} = \sqrt{8} - 1 \\ = 2\sqrt{2} - 1 = a + b\sqrt{2}$$

$$\therefore a = -1, b = 2$$

Answers of Unit 1

23

$$\begin{aligned} \text{1 The expression} &= \frac{4(\sqrt{5}-\sqrt{3})+4(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} \\ &= \frac{4\sqrt{5}-4\sqrt{3}+4\sqrt{5}+4\sqrt{3}}{5-3} \\ &= 4\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{2 The expression} &= \frac{(\sqrt{6}-\sqrt{5})^2-(\sqrt{6}+\sqrt{5})^2}{(\sqrt{6}+\sqrt{5})(\sqrt{6}-\sqrt{5})} \\ &= \frac{6+5-2\sqrt{30}-6-5-2\sqrt{30}}{6-5} \\ &= -4\sqrt{30} \end{aligned}$$

$$\begin{aligned} \text{3 The expression} &= 5\sqrt{3}-5+\frac{10(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} \\ &= 5\sqrt{3}-5+5(\sqrt{3}+1) \\ &= 5\sqrt{3}-5+5\sqrt{3}+5=10\sqrt{3} \end{aligned}$$

24

$$BC = \sqrt{28} + 2 = 2\sqrt{7} + 2 = 2(\sqrt{7} + 1) \text{ cm.}$$

The area of ΔABC

$$\begin{aligned} &= \frac{1}{2} BC \times AD = \frac{1}{2} \times 2(\sqrt{7} + 1) \times (\sqrt{7} - 1) \\ &= 7 - 1 = 6 \text{ cm}^2 \end{aligned}$$

25

$$\begin{aligned} xy^{-1} + yx^{-1} &= \frac{x}{y} + \frac{y}{x} = \frac{x^2 + y^2}{xy} \\ &= \frac{(\sqrt{5}+1)^2 + (\sqrt{5}-1)^2}{(\sqrt{5}+1)(\sqrt{5}-1)} \\ &= \frac{6+2\sqrt{5}+6-2\sqrt{5}}{5-1} = \frac{12}{4} = 3 \end{aligned}$$

26

$$\therefore \frac{x^8 y^9 - y}{(x+y)^5} = \frac{y(x^8 y^8 - 1)}{(x+y)^5} \quad (1)$$

$$\begin{aligned} \therefore x^8 y^8 &= (xy)^8 = [(\sqrt{7}+\sqrt{6})(\sqrt{7}-\sqrt{6})]^8 \\ &= 1^8 = 1 \end{aligned}$$

$$\begin{aligned} \therefore \text{from (1)}: \therefore \frac{x^8 y^9 - y}{(x+y)^5} &= \frac{(\sqrt{7}-\sqrt{6})(1-1)}{(\sqrt{7}+\sqrt{6}+\sqrt{7}-\sqrt{6})^5} \\ &= \frac{\text{zero}}{(2\sqrt{7})^5} = \text{zero} \end{aligned}$$

Answers of Exercise 8

1

$$\text{1 } \sqrt[3]{8 \times 2} = 2\sqrt[3]{2} \quad \text{2 } \sqrt[3]{-27 \times 2} = -3\sqrt[3]{2}$$

$$\text{3 } 2\sqrt[3]{125 \times 2} = 2 \times 5\sqrt[3]{2} = 10\sqrt[3]{2}$$

$$\text{4 } \frac{2}{3}\sqrt[3]{-27 \times 5} = \frac{2}{3} \times -3\sqrt[3]{5} = -2\sqrt[3]{5}$$

$$\text{5 } \sqrt[3]{\frac{1}{3} \times 27} = \sqrt[3]{9}$$

$$\text{6 } -2\sqrt[3]{\frac{2}{5} \times 125} = -2\sqrt[3]{50}$$

2

$$\text{1 } \sqrt[3]{2 \times 32} = \sqrt[3]{64} = 4 \quad \text{2 } \sqrt[3]{\frac{72}{9}} = \sqrt[3]{8} = 2$$

$$\text{3 } \frac{4}{2}\sqrt[3]{\frac{-54}{-2}} = 2\sqrt[3]{27} = 2 \times 3 = 6$$

$$\text{4 } \frac{1}{2} \times 6\sqrt[3]{10 \times 100} = 3\sqrt[3]{1000} = 3 \times 10 = 30$$

$$\text{5 } \sqrt[3]{\frac{2}{3} \times \frac{4}{25}} = \sqrt[3]{\frac{8}{125}} = \frac{2}{5}$$

$$\text{6 } \sqrt[3]{\frac{3}{4} + \frac{2}{9}} = \sqrt[3]{\frac{3}{4} \times \frac{9}{2}} = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$$

3

$$\text{1 } 2\sqrt[3]{2} - \sqrt[3]{2} = \sqrt[3]{2} \quad \text{2 } 5 - 2\sqrt[3]{3}$$

$$\text{3 } 3\sqrt[3]{3} - 2\sqrt[3]{3} = \sqrt[3]{3}$$

$$\text{4 } 3\sqrt[3]{2} + 2\sqrt[3]{2} - 5\sqrt[3]{2} = \text{zero}$$

$$\text{5 } 2 \times 3\sqrt[3]{2} - 5\sqrt[3]{2} + 2\sqrt[3]{2} = 3\sqrt[3]{2}$$

$$\text{6 } 2\sqrt[3]{2} - \frac{1}{3} \times 3\sqrt[3]{2} - \sqrt[3]{2} = \text{zero}$$

$$\text{7 } 2\sqrt[3]{2} + \sqrt[3]{250} = 2\sqrt[3]{2} + 5\sqrt[3]{2} = 7\sqrt[3]{2}$$

$$\begin{aligned} \text{8 } 2\sqrt[3]{3} - 2 \times 3\sqrt[3]{\frac{125}{9}} &= 2\sqrt[3]{3} - 2\sqrt[3]{\frac{125}{9} \times 27} \\ &= 2\sqrt[3]{3} - 2 \times 5\sqrt[3]{3} = -8\sqrt[3]{3} \end{aligned}$$

4

$$\begin{aligned} \text{1 The left hand side} &= 4\sqrt[3]{2} + 2\sqrt[3]{2} - 2 \times 3\sqrt[3]{2} = \text{zero} \\ &= \text{The right hand side.} \end{aligned}$$

$$\begin{aligned} \text{2 The left hand side} &= 3\sqrt[3]{2} \times 2\sqrt[3]{2} + (6\sqrt[3]{4}) \\ &= 6\sqrt[3]{4} + 6\sqrt[3]{4} = 12\sqrt[3]{4} = 1 = \text{The right hand side} \end{aligned}$$

5

$$\text{1 } 3\sqrt[3]{3} - 2\sqrt[3]{3} - \sqrt[3]{27 \times \frac{1}{9}} = 3\sqrt[3]{3} - 2\sqrt[3]{3} - \sqrt[3]{3} = \text{zero}$$

Algebra and Statistics

$$\begin{aligned} \text{[2]} \quad & 3\sqrt[3]{2} - 4\sqrt[3]{\frac{1}{4} \times 8} + 5 \times 2\sqrt[3]{2} \\ & = 3\sqrt[3]{2} - 4\sqrt[3]{2} + 10\sqrt[3]{2} = 9\sqrt[3]{2} \end{aligned}$$

$$\text{[3]} \quad 3\sqrt[3]{4} - 2\sqrt[3]{4} - \sqrt[3]{\frac{4}{8}} = 3\sqrt[3]{4} - 2\sqrt[3]{4} - \frac{1}{2}\sqrt[3]{4} = \frac{1}{2}\sqrt[3]{4}$$

$$\text{[4]} \quad \sqrt[3]{3} - \sqrt[3]{24} + \sqrt[3]{27 \times \frac{1}{9}} = \sqrt[3]{3} - 2\sqrt[3]{3} + \sqrt[3]{3} = \text{zero}$$

[6]

$$\begin{aligned} \text{[1]} \quad & \frac{7}{3} \times 3\sqrt[3]{2} + 3\sqrt[3]{2} - 7\sqrt[3]{2} + 2\sqrt[3]{2} \\ & = 7\sqrt[3]{2} + 3\sqrt[3]{2} - 7\sqrt[3]{2} + 2\sqrt[3]{2} = 5\sqrt[3]{2} \end{aligned}$$

$$\begin{aligned} \text{[2]} \quad & 3\sqrt[3]{3} + \frac{1}{3} \times 3 - 3\sqrt[3]{9 \times \frac{1}{3}} - 1 \\ & = 3\sqrt[3]{3} + 1 - 3\sqrt[3]{3} - 1 = \text{zero} \end{aligned}$$

$$\begin{aligned} \text{[3]} \quad & -2\sqrt[3]{2} + \frac{14}{\sqrt[3]{7}} \times \frac{\sqrt[3]{7}}{\sqrt[3]{7}} - 2\sqrt[3]{7} + 3\sqrt[3]{2} \\ & = -2\sqrt[3]{2} + 2\sqrt[3]{7} - 2\sqrt[3]{7} + 3\sqrt[3]{2} = \sqrt[3]{2} \end{aligned}$$

$$\begin{aligned} \text{[4]} \quad & 3\sqrt[3]{2} + 3\sqrt[3]{2} - \sqrt[3]{\frac{216}{12}} - 2\sqrt[3]{2} \\ & = 3\sqrt[3]{2} + \sqrt[3]{2} - \sqrt[3]{18} = 3\sqrt[3]{2} + \sqrt[3]{2} - 3\sqrt[3]{2} = \sqrt[3]{2} \end{aligned}$$

$$\text{[5]} \quad 5\sqrt[3]{2} - \frac{1}{2} \times 10\sqrt[3]{2} + \sqrt[3]{125} = 5\sqrt[3]{2} - 5\sqrt[3]{2} + 5 = 5$$

[7]

$$\begin{aligned} & 4\sqrt[3]{2} (3\sqrt[3]{4} + 10\sqrt[3]{4} - \sqrt[3]{\frac{8}{2}}) \\ & = 4\sqrt[3]{2} (3\sqrt[3]{4} + 10\sqrt[3]{4} - \sqrt[3]{4}) \\ & = 4\sqrt[3]{2} \times 12\sqrt[3]{4} = 48\sqrt[3]{8} = 96 \end{aligned}$$

[8]

$$\text{[1]} \text{ c} \quad \text{[2]} \text{ a} \quad \text{[3]} \text{ c} \quad \text{[4]} \text{ d} \quad \text{[5]} \text{ a}$$

[9]

$$\begin{aligned} \text{[1]} \quad & -2 \quad \text{[2]} \quad 9 \quad \text{[3]} \quad 250 \\ \text{[4]} \quad & \frac{2}{3}\sqrt[3]{7} \quad \text{[5]} \quad -\frac{1}{2} \quad \text{[6]} \quad 1 \end{aligned}$$

[10]

$$\begin{aligned} \text{[1]} \quad & (\sqrt[3]{5} + 1 - \sqrt[3]{5} + 1)^3 = 2^3 = 32 \\ \text{[2]} \quad & (\sqrt[3]{5} + 1 + \sqrt[3]{5} - 1)^3 = (2\sqrt[3]{5})^3 = 8 \times 5 = 40 \end{aligned}$$

[11]

$$\therefore x - y = 3 + \sqrt[3]{6} - 3 + \sqrt[3]{6} = 2\sqrt[3]{6}$$

$$x + y = 3 + \sqrt[3]{6} + 3 - \sqrt[3]{6} = 6$$

$$\begin{aligned} \therefore \left(\frac{x-y}{x+y} \right)^3 &= \left(\frac{2\sqrt[3]{6}}{6} \right)^3 = \left(\frac{\sqrt[3]{6}}{3} \right)^3 = \frac{(\sqrt[3]{6})^3}{3^3} \\ &= \frac{6}{27} = \frac{2}{9} \end{aligned}$$

[12]

$$\begin{aligned} & 2\sqrt[3]{4} + 2\sqrt[3]{8 \times \frac{1}{2}} - 4\sqrt[3]{-2} \times \sqrt[3]{-2} + 1 - \frac{4}{2} \\ & = 2\sqrt[3]{4} + 2\sqrt[3]{4} - 4\sqrt[3]{4} + 1 - 2 = -1 \end{aligned}$$

[13]

$$\text{The edge length of one cube} = \sqrt[3]{24} = 2\sqrt[3]{3} \text{ dm.}$$

$$\therefore \text{The area of one face of one cube}$$

$$= 2\sqrt[3]{3} \times 2\sqrt[3]{3} = 4\sqrt[3]{9} \text{ dm}^2$$

$$\therefore \text{The area of the using ground}$$

$$= 5 \times 4\sqrt[3]{9} = 20\sqrt[3]{9} \text{ dm}^2$$

[14]

$$\text{L.H.S.} = x^2 + y^2 = (x+y)^2 - 2xy$$

$$= (\sqrt[3]{2} + 1 + \sqrt[3]{2} - 1)^2 - 2(\sqrt[3]{2} + 1)(\sqrt[3]{2} - 1)$$

$$= (2\sqrt[3]{2})^2 - 2((\sqrt[3]{2})^2 - 1)$$

$$= 4\sqrt[3]{4} - 2\sqrt[3]{4} + 2 = 2\sqrt[3]{4} + 2 = \text{R.H.S.}$$

[15]

$$\frac{2}{\sqrt[3]{2}} = \frac{2 \times \sqrt[3]{4}}{\sqrt[3]{2} \times \sqrt[3]{4}} = \frac{2\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{2\sqrt[3]{4}}{2} = \sqrt[3]{4}$$

$$\text{Another solution : } \frac{2}{\sqrt[3]{2}} = \frac{\sqrt[3]{8}}{\sqrt[3]{2}} = \sqrt[3]{\frac{8}{2}} = \sqrt[3]{4}$$

Answers of Exercise 9

[1]

$$\text{[1]} \quad 125 \quad \text{[2]} \quad 96 \quad \text{[3]} \quad 4l^2 \quad \text{[4]} \quad 6l^2 \quad \text{[5]} \quad 8l^3$$

[2]

$$\therefore \text{Area of one face} = \frac{36}{4} = 9 \text{ cm}^2$$

$$\therefore \text{The edge length of the cube} = \sqrt[3]{9} = 3 \text{ cm.}$$

$$\text{[1]} \quad \text{Its total area} = 6l^2 = 6 \times 3^2 = 54 \text{ cm}^2$$

$$\text{[2]} \quad \text{Its volume} = l^3 = 3^3 = 27 \text{ cm}^3$$

[3]

$$\text{The edge length of the cube} = \sqrt[3]{\frac{12}{4}} = 3 \text{ cm.}$$

$$\text{[1]} \quad \text{Its volume} = l^3 = 3^3 = 27 \text{ cm}^3$$

$$\text{[2]} \quad \text{Its lateral area} = 4l^2 = 4 \times 3^2 = 36 \text{ cm}^2$$

Answers of Unit 1

4

The edge length of the cube = $\frac{60}{12} = 5$ cm.

1 Its volume = $l^3 = 5^3 = 125$ cm³

2 Its total area = $6l^2 = 6 \times 5^2 = 150$ cm²

5

1 d 2 c 3 a 4 d 5 b 6 d 7 a

6

1 The volume of the cuboid = $X \times y \times z$
= $9 \times 10 \times 5 = 450$ cm³

2 Its lateral area = $2(X + y) \times z = 2(9 + 10) \times 5$
= 190 cm²

3 Its total area = $2(Xy + yz + zX)$
= $2(9 \times 10 + 10 \times 5 + 5 \times 9)$
= $2(90 + 50 + 45) = 370$ cm²

7

The volume = $X \times y \times z = \sqrt{2} \times \sqrt{3} \times \sqrt{6} = 6$ cm³

8

The area of the base = $\sqrt{3}(\sqrt{3} - 1) = (3 - \sqrt{3})$ cm²

The volume = the area of the base \times height
= $(3 - \sqrt{3})(3 + \sqrt{3}) = 9 - 3 = 6$ cm³

9

\therefore The lateral area = the perimeter of the base \times height

\therefore The height = $\frac{480}{4 \times 10} = 12$ cm.

10

The area of the base = $\frac{\text{volume}}{\text{height}} = \frac{720}{5} = 144$ cm²

\therefore The side length of the base = $\sqrt{144} = 12$ cm.

\therefore The total area = $2(Xy + yz + zX)$
= $2 \times (12 \times 12 + 12 \times 5 + 12 \times 5)$
= 528 cm².

11

\therefore The area of the face of the cube = $\frac{294}{6} = 49$ cm²

\therefore The edge length = $\sqrt{49} = 7$ cm.

\therefore The volume of the cube = $l^3 = 7 \times 7 \times 7 = 343$ cm³

\therefore the volume of the cuboid = $X \times y \times z$
= $7\sqrt{2} \times 5\sqrt{2} \times 5$
= 350 cm³

\therefore The volume of the cuboid is greater than the volume of the cube

12

The volume of the cuboid = $X \times y \times z = 17 \times 7 \times 4 = 476$ cm³.

The total area = $2(X + y) \times z + Xy$
= $2(17 + 7) \times 4 + 17 \times 7$
= $192 + 119 = 311$ cm²

13

The circumference of the circle = $2\pi r$
= $2 \times \frac{22}{7} \times 10.5$
= 66 cm.

The area of the circle = $\pi r^2 = \frac{22}{7} \times (10.5)^2$
= 346.5 cm²

14

\therefore The area of the circle = πr^2
 $\therefore 154 = \frac{22}{7} r^2 \quad \therefore r^2 = \frac{154 \times 7}{22} = 49$

$\therefore r = \sqrt{49} = 7$ cm.

The circumference = $2\pi r = 2 \times \frac{22}{7} \times 7 = 44$ cm.

The diameter length = $2 \times 7 = 14$ cm.

15

The area of the circle = πr^2

$\therefore 64\pi = \pi r^2 \quad \therefore r^2 = 64$

$\therefore r = \sqrt{64} = 8$ cm.

The circumference of the circle = $2\pi r$
= $2 \times 3.14 \times 8 = 50$ cm.

16

The area of the circle = $2 \times 12.32 = 24.64$ cm².

$\therefore \pi r^2 = 24.64 \quad \therefore r^2 = 24.64 \times \frac{7}{22} = 7.84$

$\therefore r = \sqrt{7.84} = 2.8$ cm.

\therefore The perimeter of the figure = $\pi r + 2r$
= $\frac{22}{7} \times 2.8 + 2 \times 2.8 = 14.4$ cm.

17

The area of the shaded part = the area of the great circle
- the area of the small circle = $\pi r_1^2 - \pi r_2^2$
= $\pi \times 25 - \pi \times 9 = 16\pi$ cm²

18

Let the radius length of the circle = X cm.

\therefore The side length of the square = $2X$ cm.

Algebra and Statistics

∴ The area of the shaded part

$$= \frac{\text{the area of the square} - \text{the area of the circle}}{2}$$

$$= \frac{4x^2 - \pi x^2}{2} = 10\frac{5}{7} = \frac{75}{7}$$

$$\therefore 4x^2 - \frac{22}{7}x^2 = \frac{75}{7} \times 2$$

$$\therefore \frac{6}{7}x^2 = \frac{150}{7} \quad \therefore x^2 = \frac{150}{7} \times \frac{7}{6} = 25$$

$$\therefore x = \sqrt{25} = 5 \text{ cm.}$$

∴ The perimeter of the shaded part

$$= \frac{1}{2} \text{ the circumference of the circle}$$

$$+ \frac{1}{2} \text{ the perimeter of the square.}$$

$$= \frac{22}{7} \times 5 + 20 = 35\frac{5}{7} \text{ cm.}$$

19

In the right-angled triangle ABD at A

$$\therefore (AB)^2 + (AD)^2 = (BD)^2 \text{ (but } AB = AD)$$

$$\therefore 2(AB)^2 = (14)^2 \quad \therefore (AB)^2 = \frac{196}{2} = 98$$

$$\therefore AB = \sqrt{98} = 7\sqrt{2} \text{ cm.}$$

∴ The area of the shaded part

$$= \frac{\text{the area of the circle} - \text{the area of the square}}{4}$$

$$= \frac{\frac{22}{7} \times (7)^2 - 7\sqrt{2} \times 7\sqrt{2}}{4} = \frac{154 - 98}{4} = 14 \text{ cm}^2$$

The perimeter of the shaded part

$$= \frac{1}{4} \text{ the circumference of the circle}$$

$$+ \text{side length of the square}$$

$$= \frac{1}{4} \times 2 \times \frac{22}{7} \times 7 + 7\sqrt{2} = (11 + 7\sqrt{2}) \text{ cm.}$$

20

The volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times (14)^2 \times 20 = 12320 \text{ cm}^3$$

The total area of the cylinder = $2\pi rh + 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 14 \times 20 + 2 \times \frac{22}{7} \times (14)^2 = 2992 \text{ cm}^2$$

21

∴ The volume of the cylinder = $\pi r^2 h$

$$\therefore 924 = \frac{22}{7} \times r^2 \times 6$$

$$\therefore r^2 = \frac{924 \times 7}{6 \times 22} = 49 \quad \therefore r = 7 \text{ cm.}$$

$$\therefore \text{The lateral area} = 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 6 = 264 \text{ cm}^2$$

22

∴ The volume of the cylinder = $\pi r^2 h$

$$\therefore 7536 = 3.14 \times r^2 \times 24$$

$$\therefore r^2 = \frac{7536}{3.14 \times 24} = 100 \quad \therefore r = 10 \text{ cm.}$$

∴ The total area = $2\pi rh + 2\pi r^2$

$$= 2 \times 3.14 \times 10 \times 24 + 2 \times 3.14 \times (10)^2 = 2135.2 \text{ cm}^2$$

23

The volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times (7)^2 \times 10 = 1540 \text{ cm}^3$$

The volume of the cube = $l^3 = (11)^3 = 1331 \text{ cm}^3$

∴ The volume of the cylinder is greater than the volume of the cube.

24

$$\text{① } 2\pi rh + \pi r^2 h$$

$$\text{② } 2 \text{ cm.}$$

$$\text{③ } 20 \text{ cm.}$$

$$\text{④ } r \text{ cm.}$$

$$\text{⑤ } r \text{ cm.}$$

25

The circumference of the base = $2\pi r$

$$\therefore 44 = 2 \times \frac{22}{7} \times r$$

$$\therefore r = \frac{44}{2} \times \frac{7}{22} = 7 \text{ cm.}$$

∴ The volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times (7)^2 \times 25 = 3850 \text{ cm}^3$$

26

The lateral area = $2\pi rh$

$$\therefore 52 = 2 \times \frac{22}{7} \times 4 \times h$$

$$\therefore h = \frac{52 \times 7}{2 \times 22 \times 4} = \frac{91}{44} \text{ cm.}$$

∴ The volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 4^2 \times \frac{91}{44} = 104 \text{ cm}^3$$

27

∴ The volume of the cylinder = $\pi r^2 h$

$$\therefore h = r$$

∴ The volume of the cylinder = πr^3

$$\therefore 72\pi = \pi r^3 \quad \therefore r^3 = 72 \quad \therefore r = 2\sqrt[3]{9}$$

$$\therefore \text{The height of the cylinder} = 2\sqrt[3]{9} \text{ cm.}$$

Answers of Unit 1

28

The volume of the tank = the volume of the cuboid
 + $\frac{1}{2}$ of the volume of the cylinder
 $= 7 \times 7 \times 14 + \frac{1}{2} \times \frac{22}{7} \times (3.5)^2 \times 14$
 $= 686 + 269.5 = 955.5 \text{ m}^3$

29

The circumference of the base of the cylinder = BC
 $\therefore 2\pi r = 44 \quad \therefore r = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm.}$
 The height = AB = 10 cm.
 The volume = $\pi r^2 h = \frac{22}{7} \times (7)^2 \times 10 = 1540 \text{ cm}^3$

30

The volume of the sphere = $\frac{4}{3} \pi r^3$
 $= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3 = 38.808 \text{ cm}^3$
 The surface area of the sphere = $4\pi r^2$
 $= 4 \times \frac{22}{7} \times (2.1)^2 = 55.44 \text{ cm}^2$

31

\therefore The volume of the sphere = $\frac{4}{3} \pi r^3$
 $\therefore 4188 = \frac{4}{3} \times 3.141 \times r^3$
 $\therefore r^3 = \frac{4188 \times 3}{4 \times 3.141} = 1000 \quad \therefore r = 10 \text{ cm.}$

32

The volume of the sphere = $\frac{4}{3} \pi r^3$
 $\therefore 562.5 \pi = \frac{4}{3} \pi r^3 \quad \therefore r^3 = \frac{562.5 \times 3}{4} = 421.875$
 $\therefore r = \sqrt[3]{421.875} = 7.5 \text{ cm.}$
 \therefore The surface area of the sphere
 $= 4\pi r^2 = 4 \times \pi \times (7.5)^2 = 225 \pi \text{ cm}^2$

33

1 b 2 a 3 c 4 c 5 b 6 d

34

The volume of the cylinder = $\pi r^2 h$
 $= \pi \times (4)^2 \times 18 = 288 \pi \text{ cm}^3$
 \therefore The volume of the cylinder = The volume of the sphere.
 \therefore The volume of the sphere = $288 \pi \text{ cm}^3$
 $\therefore \frac{4}{3} \pi r^3 = 288 \pi$
 $\therefore r^3 = \frac{288 \times 3}{4} = 216$
 \therefore The radius length of the sphere = 6 cm.

35

\therefore The volume of the cylinder = $\pi r^2 h$
 $\therefore 7536 = 3.14 \times r^2 \times 24$
 $\therefore r^2 = \frac{7536}{3.14 \times 24} = 100$
 $\therefore r = \sqrt{100} = 10 \text{ cm.}$
 \therefore the radius length of the sphere
 $=$ the radius length of the cylinder base
 \therefore The volume of the sphere = $\frac{4}{3} \times 3.14 \times (10)^3$
 $= 4186 \frac{2}{3} \text{ cm}^3$

36

The volume of the cuboid = $77 \times 24 \times 21 = 38808 \text{ cm}^3$
 \therefore The volume of the cuboid = the volume of the sphere
 $\therefore 38808 = \frac{4}{3} \pi r^3$
 $\therefore r^3 = \frac{38808 \times 3 \times 7}{4 \times 22} = 9261$
 $\therefore r = \sqrt[3]{9261} = 21 \text{ cm.}$

37

The volume of the sphere = $\frac{4}{3} \pi (3)^3 = 36 \pi \text{ cm}^3$
 \therefore The volume of the cylinder = The volume of the sphere
 \therefore The volume of the cylinder = $36 \pi \text{ cm}^3$
 $\therefore \pi r^2 h = 36 \pi \quad \therefore 9 \pi h = 36 \pi \quad \therefore h = 4 \text{ cm.}$

38

\therefore The sphere touches the six faces of the cube
 \therefore The edge length of the cube = $2r$
 \therefore The volume of the sphere = $\frac{4}{3} \pi r^3$
 $\therefore 36 \pi = \frac{4}{3} \pi r^3 \quad \therefore r^3 = \frac{36 \times 3}{4} = 27$
 $\therefore r = 3 \text{ cm.}$
 \therefore The edge length of the cube = $2 \times 3 = 6 \text{ cm.}$
 \therefore The volume of the cube = $l^3 = 6^3 = 216 \text{ cm}^3$

39

\therefore The volume of the sphere
 $=$ The volume of 8 small spheres
 $\therefore \frac{4}{3} \pi r_1^3 = 8 \times \frac{4}{3} \pi r_2^3$
 $\therefore (16.8)^3 = 8 r_2^3 \quad \therefore r_2^3 = \frac{(16.8)^3}{8}$
 $\therefore r_2 = \frac{16.8}{2} = 8.4 \text{ cm.}$

Algebra and Statistics

40

The volume of the sphere = $\frac{4}{3} \pi (15)^3 = 4500 \pi \text{ cm}^3$ \therefore The volume of the cylinder= $\frac{4}{9}$ The volume of the sphere

$$\therefore \pi r^2 h = \frac{4}{9} \times 4500 \pi \quad \therefore r^2 \times 20 = 2000$$

$$\therefore r^2 = \frac{2000}{20} = 100 \quad \therefore r = \sqrt{100} = 10 \text{ cm.}$$

41

 \therefore The sum of lengths of all edges = 52 cm. \therefore the sum of the 4 heights = $3 \times 4 = 12 \text{ cm.}$ \therefore The sum of the remained edges = $52 - 12 = 40 \text{ cm.}$ \therefore The base is a square \therefore The side length of the square = $\frac{40}{4} = 10 \text{ cm.}$ \therefore The volume = $10 \times 10 \times 3 = 300 \text{ cm}^3$

42

The volume of the metal = the outer volume

- the inner volume = $\frac{4}{3} \pi r_1^3 - \frac{4}{3} \pi r_2^3$

$$= \frac{4}{3} \times \pi ((3.5)^3 - (2.1)^3) = \frac{88}{21} \times 33.614 \approx 140.859 \text{ cm}^3$$

 \therefore The mass of the metal = $140.859 \times 20 = 2817 \text{ gm.}$

Answers of Exercise 10

1

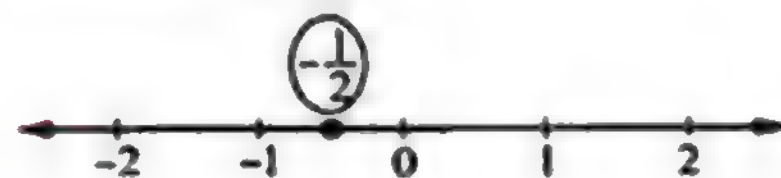
$$\textcircled{1} \therefore X = -5 \quad \therefore \text{The S.S.} = \{-5\}$$



$$\textcircled{2} \therefore 5X = 1 - 6 = -5 \quad \therefore X = \frac{-5}{5} = -1$$

 \therefore The S.S. = $\{-1\}$ 

$$\textcircled{3} \therefore 2X = 3 - 4 = -1 \quad \therefore X = -\frac{1}{2}$$

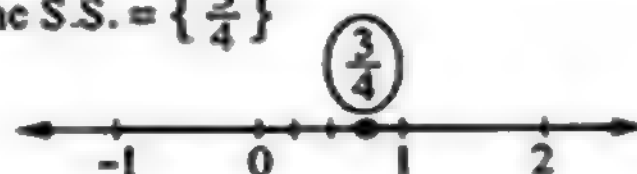
 \therefore The S.S. = $\{-\frac{1}{2}\}$ 

$$\textcircled{4} \therefore 2X = 4 + 3 = 7 \quad \therefore X = \frac{7}{2} = 3\frac{1}{2}$$

 \therefore The S.S. = $\{3\frac{1}{2}\}$ 

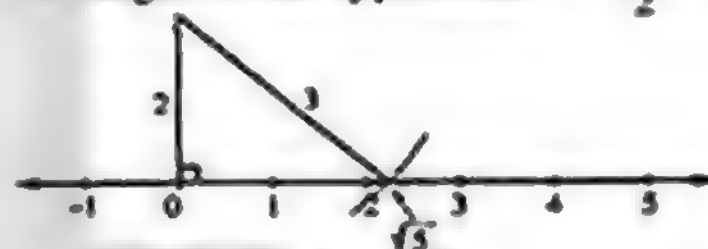
22

$$\textcircled{5} \therefore 4X - 1 = 2 \quad \therefore 4X = 2 + 1 = 3 \quad \therefore X = \frac{3}{4}$$

 \therefore The S.S. = $\{\frac{3}{4}\}$ 

$$\textcircled{6} \therefore \sqrt{5}X = 4 + 1 = 5$$

$$\therefore X = \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$$

 \therefore The S.S. = $\{\sqrt{5}\}$ The length of one side of the right angle = $\frac{5-1}{2} = 2$ \therefore The length of the hypotenuse = $\frac{5+1}{2} = 3$ 

$$\textcircled{7} \therefore X = \sqrt{3} + 1 \quad \therefore \text{The S.S.} = \{\sqrt{3} + 1\}$$

The length of one side of the right angle

$$= \frac{3-1}{2} = 1$$

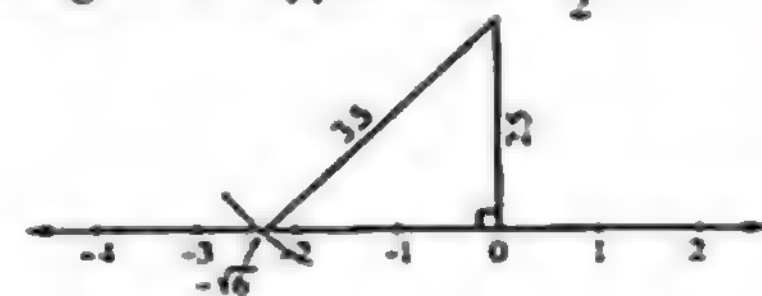
The length of the hypotenuse = $\frac{3+1}{2} = 2$ 

$$\textcircled{8} \therefore 2 - \sqrt{6}X = 8 \quad \therefore -\sqrt{6}X = 8 - 2 = 6$$

$$\therefore X = -\frac{6}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{-6\sqrt{6}}{6} = -\sqrt{6}$$

 \therefore The S.S. = $\{-\sqrt{6}\}$ The length of one side of the right angle = $\frac{6-1}{2}$

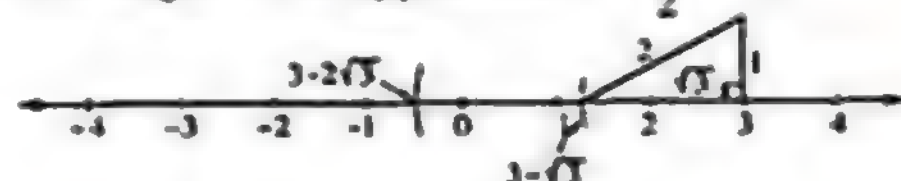
$$= 2.5$$

The length of the hypotenuse = $\frac{6+1}{2} = 3.5$ 

$$\textcircled{9} \therefore X = 3 - 2\sqrt{3} \quad \therefore \text{The S.S.} = \{3 - 2\sqrt{3}\}$$

The length of one side of the right angle = $\frac{3-1}{2}$

$$= 1$$

The length of the hypotenuse = $\frac{3+1}{2} = 2$ 

Answers of Unit 1

2

- 1 a 2 c 3 c 4 d 5 a 6 d

3

- 1 $\because X > 3 \therefore \text{The S.S.} =]3, \infty[$



- 2 $\because X \leq 2 \therefore \text{The S.S.} =]-\infty, 2]$



- 3 $\because X \leq 2 \therefore \text{The S.S.} =]-\infty, 2]$



- 4 $\because -X > -2 \therefore X < 2$

$\therefore \text{The S.S.} =]-\infty, 2[$



- 5 $\because 2X \geq -2 \therefore X \geq -1$

$\therefore \text{The S.S.} = [-1, \infty[$



- 6 $\because -5X < 5 \therefore X > -1$

$\therefore \text{The S.S.} =]-1, \infty[$



- 7 $\because \frac{1}{2}X \leq 1 \therefore X \leq 2$

$\therefore \text{The S.S.} =]-\infty, 2]$



- 8 $\because -2X \leq 4 \therefore X \geq -2$

$\therefore \text{The S.S.} = [-2, \infty[$



4

- 1 $\because 1 < X \leq 4 \therefore \text{The S.S.} =]1, 4]$



- 2 $\because -8 < X < 6 \therefore \text{The S.S.} =]-8, 6[$



- 3 $\because 3 \geq X > -3 \therefore \text{The S.S.} =]-3, 3]$



- 4 $\because -4 < -X \leq -2 \therefore 4 > X \geq 2$

$\therefore \text{The S.S.} = [2, 4[$



- 5 $\because -2 \leq X+1 \leq 3 \therefore -3 \leq X \leq 2$

$\therefore \text{The S.S.} = [-3, 2]$



- 6 $\because 2 < -X \leq 6 \therefore -2 > X \geq -6$

$\therefore \text{The S.S.} = [-6, -2[$



- 7 $\because -9 \leq 3X \leq 3 \therefore -3 \leq X \leq 1$

$\therefore \text{The S.S.} = [-3, 1]$



- 8 $\because 3 < 2X-1 < 5 \therefore 4 < 2X < 6$

$\therefore 2 < X < 3$

$\therefore \text{The S.S.} =]2, 3[$



- 9 $\because -1 < \frac{1}{2}X \leq 2 \therefore -2 < X \leq 4$

$\therefore \text{The S.S.} =]-2, 4]$



- 10 By multiplying by 3 $\therefore 0 \leq -2X+6 < 12$

$\therefore -6 \leq -2X < 6$

$\therefore 3 \geq X > -3$

$\therefore \text{The S.S.} =]-3, 3]$



- 5 Represent by yourself the S.S. on the number line :

- 1 $\because 3X-2X < 4 \therefore X < 4$

$\therefore \text{The S.S.} =]-\infty, 4[$

- 2 $\because 7X-4X \geq 9 \therefore 3X \geq 9 \therefore X \geq 3$

$\therefore \text{The S.S.} = [3, \infty[$

- 3 $\because 5X-2X < 9+3 \therefore 3X < 12 \therefore X < 4$

$\therefore \text{The S.S.} =]-\infty, 4[$

- 4 $\because 7X-5X \geq -8+12 \therefore 2X \geq 4 \therefore X \geq 2$

$\therefore \text{The S.S.} = [2, \infty[$

- 5 $\because X+X \leq 3+1 \therefore 2X \leq 4 \therefore X \leq 2$

$\therefore \text{The S.S.} =]-\infty, 2]$

- 6 $\because -X+2X \geq -3-1 \therefore X \geq -4$

$\therefore \text{The S.S.} = [-4, \infty[$

Algebra and Statistics

6

Represent by yourself the S.S. on the number line :

$$① \because X+3-X \geq 2X-X \geq X-2-X$$

$$\therefore 3 \geq X \geq -2 \quad \therefore \text{The S.S.} = [-2, 3]$$

$$② \because -X+X < X+X < 4-X+X \quad \therefore 0 < 2X < 4$$

$$\therefore 0 < X < 2 \quad \therefore \text{The S.S.} =]0, 2[$$

$$③ \because 4X-4X \leq 5X+2-4X < 4X+3-4X$$

$$\therefore 0 \leq X+2 < 3 \quad \therefore -2 \leq X < 1$$

$$\therefore \text{The S.S.} = [-2, 1[$$

$$④ \because X-1-X < 3X-1-X \leq X+1-X$$

$$\therefore -1 < 2X-1 \leq 1 \quad \therefore 0 < 2X \leq 2$$

$$\therefore 0 < X \leq 1 \quad \therefore \text{The S.S.} =]0, 1]$$

$$⑤ \because 2+2X-2X \leq 3X+3-2X < 5+2X-2X$$

$$\therefore 2 \leq X+3 < 5 \quad \therefore -1 \leq X < 2$$

$$\therefore \text{The S.S.} = [-1, 2[$$

⑥ By multiplying by 6

$$\therefore 3X-4 < 6X+6 < 3X+9$$

$$\therefore -4 < 3X+6 < 9$$

$$\therefore -10 < 3X < 3 \quad \therefore -\frac{10}{3} < X < 1$$

$$\therefore \text{The S.S.} =]-\frac{10}{3}, 1[$$

7

$$① \geq 3$$

$$② < 3$$

$$③ < -3$$

$$④ \geq -\frac{3}{2}$$

$$⑤ \leq 2\sqrt{2}$$

$$⑥]2, 4[$$

$$⑦]-2, 5]$$

$$⑧]2, \infty[$$

$$⑨ 6$$

8

$$① a$$

$$② b$$

$$③ c$$

$$④ c$$

$$⑤ c$$

9

 \therefore The weight of one box = 45 kg.Let the number of boxes be X \therefore the maximum weight that the lift can carry is 2200 kg. \therefore The weight of boxes \leq the maximum weight that the lift can carry

$$\therefore 45X \leq 2200 \quad \therefore X \leq 48\frac{8}{9}$$

 \therefore The maximum number of boxes can the lift carry in one time is 48 boxes.

10

$$\therefore -4 < -2X < 2 \quad \therefore 2 > X > -1 \quad \therefore \text{The S.S.} =]-1, 2[$$

$$\therefore \sqrt{3} = 1.7 \quad \therefore \sqrt{3} \in]-1, 2[$$

11

$$\therefore a+3 \leq X \leq b+3 \quad \therefore \text{The S.S.} = [a+3, b+3]$$

$$\therefore [4, 7] = [a+3, b+3]$$

$$\therefore a+3 = 4 \quad \therefore a = 1$$

$$\therefore b+3 = 7 \quad \therefore b = 4$$

12

$$\therefore \frac{1}{5} \leq \frac{2X+1}{5} \leq 1 \quad \therefore 1 \leq 2X+1 \leq 5$$

$$\therefore 0 \leq 2X \leq 4 \quad \therefore 0 \leq X \leq 2$$

$$\therefore \text{The S.S.} = [0, 2] \quad \therefore m=0, m+n=2 \quad \therefore n=2$$

13

$$\therefore 5 \leq \frac{2X}{3} + 1 \leq 7 \quad \therefore 4 \leq \frac{2X}{3} \leq 6$$

$$\therefore 12 \leq 2X \leq 18 \quad \therefore 6 \leq X \leq 9$$

$$\therefore 4 \leq X-2 \leq 7 \quad \therefore \text{The smallest value of } X-2 \text{ is } 4$$

14

Multiply both sides by $(\sqrt{3}-\sqrt{5})$

$$\therefore X \leq (\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})$$

*Note that the sign changed because $(\sqrt{3}-\sqrt{5})$ is a negative number because $\sqrt{3} < \sqrt{5}$

$$\therefore X \leq -2 \quad \therefore \text{The S.S.} =]-\infty, -2]$$

Answers of Unit 1

Answers of exams on
the second part of unit one

Model 1

1

1 a 2 c 3 b 4 d 5 a 6 c

2

1 $\sqrt{3} + \sqrt{2}$ 2 $\sqrt{6}$ 3 $2\sqrt{2}$
4 20 cm. 5 < -4

3

[a] $2\sqrt{2}$
[b] $36\pi \text{ cm}^2$

4

[a] The S.S. = $[-2, 3]$ and represent by yourself.
[b] $r = 3 \text{ cm}$.

5

[a] $\sqrt{2}$
[b] Prove by yourself $x^2 + 2xy + y^2 = 20$

Model 2

1

1 b 2 b 3 d 4 c 5 a 6 c

2

1 $2\sqrt{2} - \sqrt{5}$ 2 $[-2, \infty)$ 3 2
4 18 5 3

3

[a] Prove by yourself.
[b] $\frac{\sqrt{3}}{2}$

4

[a] $\sqrt{5}$
[b] 54 cm^2

5

[a] 132 cm^2
[b] The S.S. = $[-\infty, 4]$ and represent by yourself.

Algebra and Statistics

Answers of unit two

Answers of Exercise 11

1 (5, 14), (2, 5), (0, -1), (-3, -10)

2 The ordered pair (-1, 3) satisfies the relation.

3 1 (1, -3), (2, -1), (3, 1), (5, 5)

2 (0, 5), (2, 6), (4, 7), (6, 8)

3 (0, 2), (3, 2), (5, 2), (-4, 2)

4 (2.5, 7), (2.5, 3), (2.5, -7), (2.5, 4)

There are other solutions.

1	X	0	1	2	3
	y	1	5	9	13

2	X	-4	-3	-2
	y	-5	0	5

3	a	1	4	3
	b	-3	0	-1

4	a	2	5	-1
	b	-1	0	-2

5 1 7 2 -9 3 zero 4 -1

6 $\because (3, 6)$ satisfies the relation $y = kx$
 $\therefore 6 = 3k \quad \therefore k = 2$

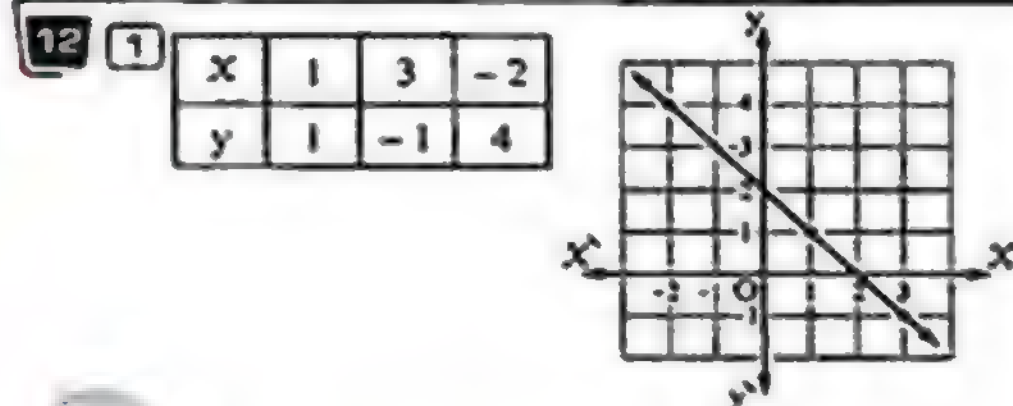
7 $\because (3, 1)$ satisfies the relation $y - 3x = a$
 $\therefore 1 - 3 \times 3 = a \quad \therefore a = -8$

8 $\because (-3, 2)$ satisfies the relation $3x + by = 1$
 $\therefore 3 \times (-3) + b \times 2 = 1 \quad \therefore 2b = 9 + 1$
 $\therefore 2b = 10 \quad \therefore b = 5$

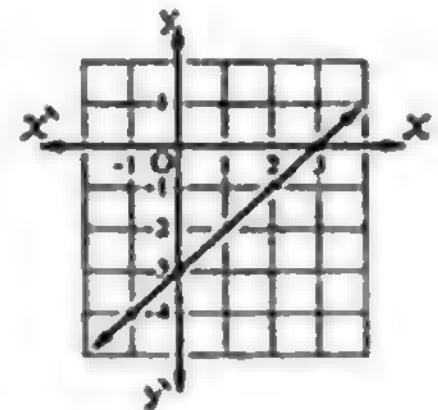
9 $\because (3, a)$ satisfies the relation $y - 2x = 4$
 $\therefore a - 2 \times 3 = 4 \quad \therefore a = 10$

10 $\because (k, 2k)$ satisfies the relation $x + y = 15$
 $\therefore k + 2k = 15 \quad \therefore 3k = 15 \quad \therefore k = 5$

11 1 $x = 3$ 2 $y = \text{zero}$



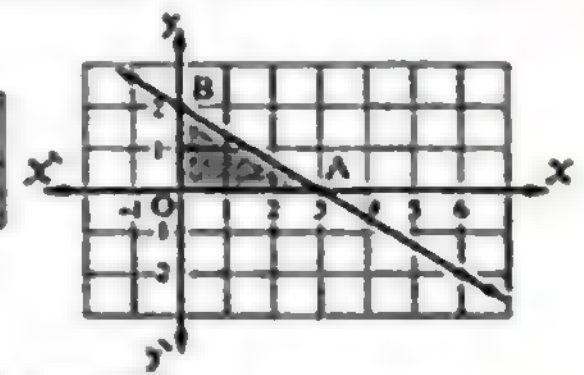
2	X	0	3	-1
	y	-3	0	-4



From (3) to (8) represent the relations graphically by yourself.

13 $y = -\frac{2}{3}x + 2$

X	0	3	6
y	2	0	-2



From the graph :

The area of $\Delta OAB = \frac{1}{2} \times 3 \times 2 = 3$ square units

14 \because the straight line intersects x -axis at $(3, b)$

$\therefore b = 0$

$\therefore (3, 0)$ satisfies the relation $2x - y = a$

$\therefore 2 \times 3 - 0 = a$

$\therefore a = 6$

15 1 d 2 b 3 c 4 b

5 a 6 c 7 c 8 c

9 c 10 c 11 c 12 d

16 Let the first number be x and the second be y

$\therefore 2x + y = 12 \quad \therefore y = 12 - 2x$

 \because the two numbers are even natural numbers. $\therefore x$ has the values 0, 2, 4, 6 then we can register the different possibilities to the two numbers in the following table :

X	0	2	4	6
y	12	8	4	0

17 Let the length of the rectangle = x cm. and the width = y cm. $\therefore x > y$

\because the perimeter of the rectangle = 14 cm.

$\therefore 2(x + y) = 14 \quad \therefore x + y = 7$

we can record the different possibilities of the length and the width of the rectangle in the opposite table :

X	6	5	4
y	1	2	3

18 Let the number of bills of L.E. 5 be X , then its value = $5X$

and let the number of bills of L.E. 20 be y then its value = $20y$

$\therefore 5X + 20y = 65$ where X and y are natural numbers.

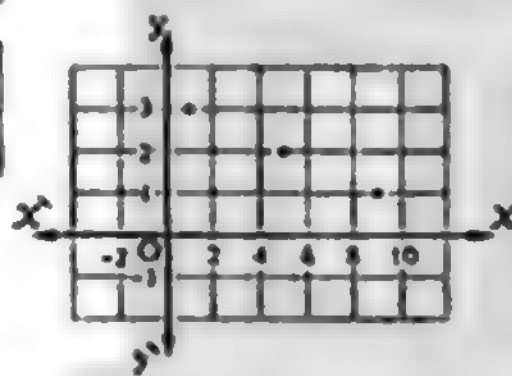
$\therefore X + 4y = 13 \quad \therefore y = \frac{13-X}{4}$

$\therefore X \leq 10$, $(13-X)$ is divisible by 4

i.e. X has the values 9, 5 and 1

then we can write the different possibilities in the following table :

X	1	5	9
y	3	2	1



19 Let the store sold in this week X computer's table and y chairs.

$\therefore 100X + 50y = 500$

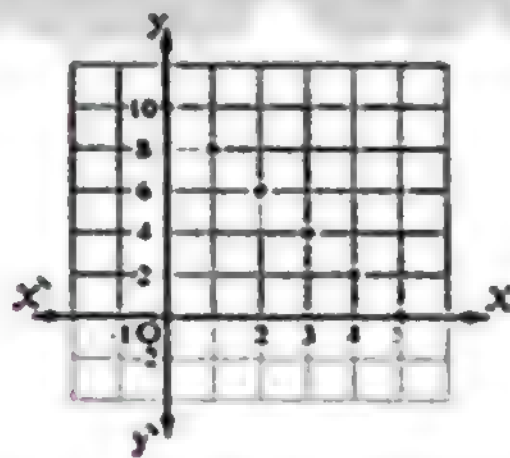
where X and y are natural numbers

$\therefore 2X + y = 10 \quad \therefore y = 10 - 2X$

$\therefore X$ is not more than 5

\therefore We can write the expectations which represent the number of computer's tables and the number of chairs in the following table :

X	0	1	2	3	4	5
y	10	8	6	4	2	0



20 Let the length of any of the two congruent sides in the triangle be X cm. and the length of the third side be y cm.

\therefore the perimeter of the triangle = 19

$\therefore 2X + y = 19 \quad \therefore y = 19 - 2X$

$\therefore X$ and y are positive integers then X is not more than 9 and from the inequality of the triangle then X has the values 5, 6, 7, 8 and 9

then we can write all the possibilities in the following table :

X	5	6	7	8	9
y	9	7	5	3	1

Answers of Exercise 12

1

figure (1) the slope is positive.

figure (2) the slope is negative.

figure (3) the slope is undefined.

figure (4) the slope equals zero.

2

1 negative 2 zero 3 undefined 4 positive

3

1 zero 2 undefined 3 X-axis 4 \overline{BC} or \overline{AC}

4

1 the slope of $\overline{AB} = \frac{4-3}{3-1} = \frac{1}{2}$

2 the slope of $\overline{AB} = \frac{0-2}{5-1} = -\frac{1}{2}$

3 the slope of $\overline{AB} = \frac{5-2}{6-3} = 1$

4 the slope of $\overline{AB} = \frac{-1+1}{4-2} = 0$

5 the slope of $\overline{AB} = \frac{3-3}{2-1} = 0$

6 the slope of $\overline{AB} = \frac{4-2}{5-5} = \frac{2}{0}$ undefined

7 the slope of $\overline{AB} = \frac{2+1}{3-3} = \frac{3}{0}$ undefined

8 the slope of $\overline{AB} = \frac{1+2}{4-3} = \frac{3}{1} = 3$

9 the slope of $\overline{AB} = \frac{1-3}{2+1} = \frac{-2}{3}$

10 the slope of $\overline{NK} = \frac{-7+2}{-1-4} = \frac{-5}{-5} = 1$

11 the slope of $\overline{EO} = \frac{0+1}{0+3} = \frac{1}{3}$

12 the slope of $\overline{AB} = \frac{-1+9}{-1+6} = \frac{8}{5}$

5

1 Taking the two points $(0, 0)$, $(1, 2)$ which lie on the straight line we find that

the slope = $\frac{2-0}{1-0} = 2$

2 Taking the two points $(0, -1)$, $(-2, 3)$ which lie on the straight line we find that

the slope = $\frac{3-(-1)}{-2-0} = \frac{4}{-2} = -2$

Algebra and Statistics

6

$\therefore m(\angle M) = 45^\circ \therefore \Delta MNL$ is an isosceles triangle.

$\therefore ML = LN$

\therefore the length of $\overline{ML} = 4$ units.

\therefore the length of $\overline{LN} = 4$ units.

$\therefore N = (3, 6)$

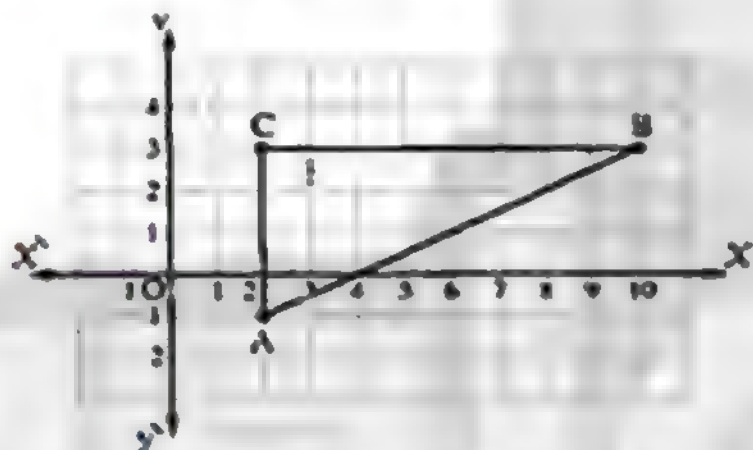
\therefore the slope of $\overline{MN} = \frac{6-2}{3-7} = \frac{4}{-4} = -1$

7

the slope of $\overline{AB} = \frac{3+1}{10-2} = \frac{1}{2}$

the slope of $\overline{BC} = \frac{3-3}{2-10} = \text{zero}$

the slope of $\overline{AC} = \frac{3+1}{2-2} = \frac{4}{0}$ (undefined)



from the graph we find that ΔABC is right-angled.

8

\therefore the slope of the straight line which passes through the two points $(1, 3)$ and $(3, k)$ equals 3

$$\therefore \frac{k-3}{3-1} = 3$$

$$\therefore \frac{k-3}{2} = 3$$

$$\therefore k-3 = 6$$

$$\therefore k = 9$$

9

\therefore the slope of the straight line which passes through the two points $(3, c)$ and $(5, -2)$ equals -3

$$\therefore \frac{-2-c}{5-3} = -3$$

$$\therefore \frac{-2-c}{2} = -3$$

$$\therefore -2-c = -6$$

$$\therefore c = 4$$

10

$$-2 = \frac{2-4}{x-(-1)}$$

$$\therefore -2 = \frac{-2}{x+1}$$

$$\therefore x+1 = 1$$

$$\therefore x = 0$$

11

$$\therefore \frac{-1-y}{3-(-2)} = -0.6$$

$$\therefore \frac{-1-y}{5} = -0.6$$

$$\therefore -1-y = -3$$

$$\therefore y = 2$$

12

\therefore the straight line is parallel to X-axis

\therefore the slope = zero

$$\therefore \frac{k-4}{2-3} = \text{zero}$$

$$\therefore k-4 = \text{zero}$$

$$\therefore k = 4$$

13

\therefore the straight line is parallel to y-axis

\therefore the slope is undefined

$$\therefore x_2 - x_1 = \text{zero}$$

$$\therefore 6 - 2x = 0$$

$$\therefore -2x = -6$$

$$\therefore x = 3$$

14

\therefore the straight line is perpendicular to y-axis

\therefore the straight line is parallel to X-axis

\therefore the slope = zero

$$\therefore y_2 - y_1 = \text{zero}$$

$$\therefore 3y - 6 = 0$$

$$\therefore 3y = 6$$

$$\therefore y = 2$$

15

\therefore the slope of the straight line passing through the two points $(-5, 11)$ and $(0, 8) = \frac{8-11}{0-(-5)} = \frac{-3}{5}$ (1)

\therefore the slope of the straight line passing through the two points $(0, 8)$ and $(5, 5) = \frac{5-8}{5-0} = \frac{-3}{5}$ (2)

from (1) and (2) we find that the three points are collinear.

(lying on the straight line whose slope = $-\frac{3}{5}$)

16

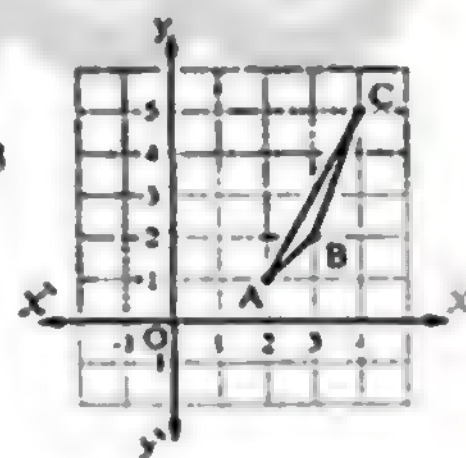
$$\text{the slope of } \overline{AB} = \frac{2-1}{3-2} = 1$$

$$\therefore \text{the slope of } \overline{BC} = \frac{5-2}{4-3} = 3$$

and the slope of \overline{AC}

$$= \frac{5-1}{4-2} = 2$$

we observe that the three points are not collinear.



17

$$\text{1} \therefore \text{the slope of } \overline{AB} = \frac{2-1}{2-1} = 1$$

$$\therefore \text{the slope of } \overline{BC} = \frac{-3-2}{-3-2} = 1$$

\therefore the slope of \overline{AB} = the slope of \overline{BC}
and the point B is a common point.

\therefore the points A, B and C are collinear.

- 2 ∴ the slope of $\overline{AB} = \frac{7 - (-3)}{-6 - 4} = \frac{10}{-10} = -1$
 ∴ the slope of $\overline{BC} = \frac{-4 - 7}{5 - (-6)} = \frac{-11}{11} = -1$
 ∴ the slope of \overline{AB} = the slope of \overline{BC} and the point B is a common point.
 ∴ the points A, B and C are collinear.

- 3 ∴ the slope of $\overline{AB} = \frac{4 - 12}{2 - (-2)} = \frac{-8}{4} = -2$
 ∴ the slope of $\overline{BC} = \frac{-4 - 4}{6 - 2} = \frac{-8}{4} = -2$
 ∴ the slope of \overline{AB} = the slope of \overline{BC} and the point B is a common point.
 ∴ the points A, B and C are collinear.

18

- 1 ∴ the slope of $\overline{AB} = \frac{0 - 1}{3 - 2} = \frac{-1}{1} = -1$
 ∴ the slope of $\overline{BC} = \frac{-1 - 0}{5 - 3} = \frac{-1}{2}$
 ∴ the slope of $\overline{AB} \neq$ the slope of \overline{BC}
 ∴ the points A, B and C are not collinear.

- 2 ∴ the slope of $\overline{AB} = \frac{1 - 2}{3 - (-1)} = \frac{-1}{4}$
 ∴ the slope of $\overline{BC} = \frac{2 - 1}{7 - 3} = \frac{1}{4}$
 ∴ the slope of $\overline{AB} \neq$ the slope of \overline{BC}
 ∴ the points A, B and C are not collinear.

- 3 ∴ the slope of $\overline{AB} = \frac{2 - (-3)}{2 - 0} = \frac{5}{2}$
 ∴ the slope of $\overline{BC} = \frac{-3 - 2}{-3 - 2} = 1$
 ∴ the slope of $\overline{AB} \neq$ the slope of \overline{BC}
 ∴ the points A, B and C are not collinear.

19

- ∴ the slope of $\overline{AB} = \frac{5 - 3}{2 + 1} = \frac{2}{3}$
 ∴ the slope of $\overline{BC} = \frac{5 - 1}{2 - 8} = \frac{4}{-6} = -\frac{2}{3}$
 ∴ the slope of $\overline{AB} \neq$ the slope of \overline{BC}
 ∴ $C \notin \overline{AB}$

20

- ∴ the slope of the straight line which passes through the two points (4, 1) and (-2, 7)
 $= \frac{7 - 1}{-2 - 4} = \frac{6}{-6} = -1$

- ∴ the slope of the straight line which passes through the two points (-2, 7) and (3, y)
 $= \frac{y - 7}{3 - (-2)} = \frac{y - 7}{5}$
 ∴ the three points are collinear.
 $\therefore \frac{y - 7}{5} = -1$ ∴ $y - 7 = -5$
 $\therefore y = -5 + 7$ ∴ $y = 2$

21

- ∴ the slope of the straight line which passes through the two points (3, -1) and (X, 1) equals $\frac{2}{3}$
 $\therefore \frac{1 - (-1)}{X - 3} = \frac{2}{3}$ ∴ $\frac{2}{X - 3} = \frac{2}{3}$
 $\therefore X - 3 = 3$ ∴ $X = 6$
 ∴ the slope of the straight line which passes through the two points (3, -1) and (9, y) equals $\frac{2}{3}$
 $\therefore \frac{y - (-1)}{9 - 3} = \frac{2}{3}$ ∴ $\frac{y + 1}{6} = \frac{2}{3}$ ∴ $3y + 3 = 12$
 $\therefore 3y = 9$ ∴ $y = 3$

Answers of Exercise 13

1

- ∴ The uniform velocity = $\frac{\text{the covered distance}}{\text{the taken time}}$
 $= \frac{180}{3} = 60 \text{ km/hr.}$
 ∴ the covered distance = the taken time \times the uniform velocity = $60 \times 5 = 300 \text{ km.}$

2

- ∴ The rate of consumption of fuel
 $= \frac{\text{the amount of consumed fuel}}{\text{time}}$
 $= \frac{247}{3} = \frac{247}{300} \text{ litre/hr.}$
 ∴ the consumed amount =
 The rate of consumption \times time
 $= \frac{247}{300} \times 10 = 8\frac{7}{30} \text{ litre}$

3

- 1 At the moment of starting the motion, the body is at a distance of 2 metres from the fixed point.
 At $t = 6$, the body is at a distance of 8 metres.
 Taking the two points (0, 2) and (6, 8) on the straight line which represents the relation between t and d

Algebra and Statistics

$$\therefore \text{the slope} = \frac{8-2}{6-0} = \frac{6}{6} = 1$$

it represents the velocity of the body within a going trip.

- 2 At the moment of starting the motion, the body is at a distance of 12 metres from the fixed point.

At $t = 6$ the body is at a distance of 2 metres. Taking the two points $(0, 12)$ and $(6, 2)$ on the straight line representing the relation between t and d

$$\therefore \text{the slope} = \frac{2-12}{6-0} = \frac{-10}{6} = -\frac{5}{3}$$

It represents the velocity of the body within the returning back.

- 3 On starting the motion, the body is at a distance of 8 metres from the fixed point.

At $t = 6$ the body is at a distance of 8 metres.

\therefore the straight line representing the relation is horizontal. \therefore The slope = zero

It means that the body is rest.

4

Taking two points on the straight line representing the relation between t and d say $(0, 50)$ and $(4, 150)$

\therefore the uniform velocity = the slope of the straight line

$$= \frac{150-50}{4-0} = \frac{100}{4} = 25 \text{ km/h.}$$

5

- 1 Taking two points on the straight line representing the relation between t and d say $(0, 50)$ and $(2, 200)$

\therefore the uniform velocity = the slope of the straight line $= \frac{200-50}{2-0} = 75 \text{ km/h.}$

- 2 from the graph :

The car is at a distance = 275 km. from the point O after passing 3 hours from the moment of beginning the motion.

6

- 1 The velocity within the first 3 hours = the slope of the straight line $\overline{OB} = \frac{125}{3} = 41\frac{2}{3} \text{ km/h}$

The velocity within the next 2 hours = the slope of the straight line $\overline{BC} = \frac{125}{2} = 62\frac{1}{2} \text{ km/h}$

- 2 The average velocity within the all trip

$$= \frac{\text{total distance}}{\text{total time}} = \frac{250}{5} = 50 \text{ km/h}$$

7

- 1 The velocity within the first 3 hours = the slope of the straight line $= \frac{60-20}{3-0} = \frac{40}{3} = 13\frac{1}{3} \text{ km/h.}$

- 2 The velocity within the next 4 hours = the slope of the straight line $= \frac{0-60}{7-3} = \frac{-60}{4} = -15 \text{ km/h.}$

The negative sign means that the bicycle returns back with velocity 15 km/h.

The total distance = $40 + 60 = 100 \text{ km.}$

8

- 1 The slope of the straight line \overline{AB}

$$= \frac{60-20}{4-0} = \frac{40}{4} = 10$$

It means the increasing of the capital within the first 4 years with rate equals 10 thousands pounds/year.

$$\text{The slope of } \overline{BC} = \frac{60-60}{6-4} = \text{zero}$$

It means constancy of the capital within the fifth and sixth years.

$$\text{The slope of } \overline{CD} = \frac{50-60}{8-6} = \frac{-10}{2} = -5$$

It means decreasing of the capital within the 7th and 8th years with rate = 5 thousands/year.

- 2 The starting capital of the company = 20 thousand pounds.

9

- 1 The slope of $\overline{AB} = \frac{125-50}{8-0} = \frac{75}{8} = 9\frac{3}{8}$

It means that the increase in height goes with respect to the increase in age.

$$\text{The slope of } \overline{BC} = \frac{175-125}{18-8} = \frac{50}{10} = 5$$

It means that the increase in height goes with respect to the increase in age but with a rate less than the rate within the first 8 years.

$$\text{The slope of } \overline{CD} = \frac{175-175}{22-18} = 0$$

It denotes the constancy in height inspite of the increase in age after 18 years.

- 2 \therefore the height of the person at age 30 years = 175 cm. and the height of the person at age 8 years = 125 cm. \therefore the difference = $175 - 125 = 50 \text{ cm.}$

10

- 1 The greatest capacity of the tank = 70 litre.

- 2 The tank will be empty after 30 hours.

- 3 The remained fuel = 35 litre.

- 4 taking the two points $(0, 70)$, $(30, 0)$ on the straight line representing the relation.

∴ The rate of consumption of the fuel =
 The slope of the straight line = $\frac{70-0}{0-30} = -2\frac{1}{3}$ litre/h.
 The negative sign means the rate of consumption.
 i.e. the amount of fuel is consumed with rate $2\frac{1}{3}$ Litre/hr

11

1 100 pages.

2 taking the two points (0, 100) and (3, 40) on the straight line representing the relation.

we find that the rate of decreasing the number of pages = the slope of the straight line
 $= \frac{40-100}{3-0} = \frac{-60}{3} = -20$ page/h

The negative sign expresses the decreasing in the number of remained pages with rate 20 page/h.

3 ∴ the remained pages decreases with rate 20 page/h.

∴ There are 100 pages.

∴ The person finishes reading the book after

$$\frac{100}{20} = 5 \text{ hours.}$$

12

1 The depth of the well before beginning digging = 5 m.

2 The depth of the well after finishing digging = 40 m.

3 The total time taken in digging = 10 h.

4 The average of digging the well in the first 5 hours
 = the slope of the straight line = $\frac{27.5-5}{5-0} = 4.5$ m/h.

5 The average of digging in the last two hours
 = the slope of the straight line
 $= \frac{40-27.5}{10-8} = 6.25$ m/h.

13

1 The velocity during the going trip

$$= \text{the slope of the straight line} = \frac{60-0}{3-0} = 20 \text{ km/h.}$$

2 The average velocity during returning back

$$= \frac{\text{total distance}}{\text{total time}} = \frac{60}{5} = 12 \text{ km/h.}$$

3 It means that the bicycle stopped within the 6th hour from the beginning.

14

Let the covered distance be d km

The amount of the remained fuel in the tank be y litre.

At the beginning the covered distance = zero

The amount of remained fuel = 40 litre

We express this by the point A (0, 40)

∴ After covering distance.

The amount of remained fuel

$$= \frac{3}{4} \times 40 = 30 \text{ litre.}$$

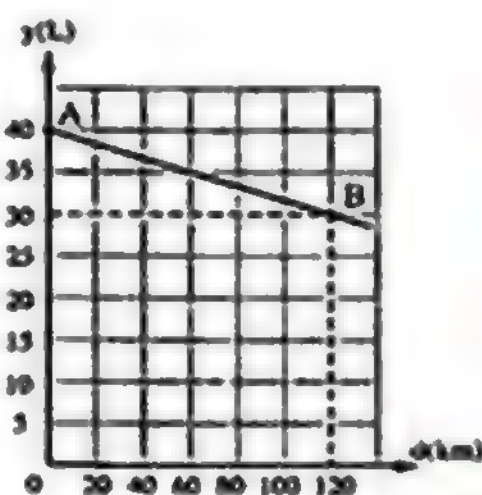
we express this by the point B (120, 30)

∴ the rate of decreasing the amount of fuel

= the slope of \overline{AB}

$$= \frac{30-40}{120-0} = \frac{-10}{120} = -\frac{1}{12}$$

∴ the amount of fuel decreases with the rate one litre for every 12 km.

∴ The distance covered by the car when the tank becomes empty = $12 \times 40 = 480$ km.

15

1 100 km.

2 the train A took 2 hours

the train B took $2\frac{1}{2}$ hours.

3 The average speed = $\frac{\text{total distance}}{\text{total time}}$
 with respect to the train A

$$\text{The average speed} = \frac{100}{2} = 50 \text{ km/h.}$$

with respect to the train B

$$\text{the average speed} = \frac{100}{2.5} = 40 \text{ km/h.}$$

4 It means that the train A was at rest from half past ten till half past eleven.

16

1 Tortoise

2 The velocity of the tortoise = $\frac{\text{the covered distance}}{\text{the taken time}}$
 $= \frac{100}{60} = 1\frac{2}{3}$ metre / minute

3 The average velocity of the rabbit = $\frac{\text{total distance}}{\text{total time}}$
 $= \frac{100}{65} = 1\frac{7}{13}$ metre / minute

4 It means that the rabbit was at rest from the tenth minute to 60th minute.

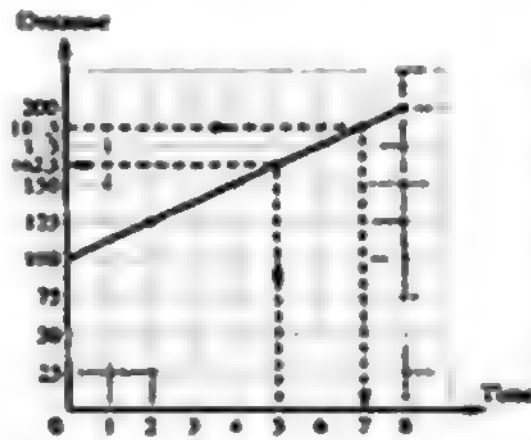
17

1 the velocity of the bicycle = the slope of the straight line = $\frac{200-125}{8-2} = 12.5$ km/h.

2 300 minutes = 5 hours,
 then the bicycle is at distance = 162.5 km.

Algebra and Statistics

3 7 hours.

4 from the graph.
the starting point is
far from the fixed
point = 100 km.

Answers of exams on unit two

Model 1

1

1 c

2 a

3 c

4 b

5 c

6 c

2

1 (4, 0)

2 4

3 2

4 undefined

5 zero

3

[a] Represent by yourself. [b] Prove by yourself.

4

[a] Represent by yourself
the area of $\triangle OAB = 3$ square units.[b] $y = \frac{-1}{2}$

5

1 $13 \frac{1}{3}$ km/hr.

2 15 km/hr.

3 100 km.

Model 2

1

1 d

2 b

3 d

4 c

5 b

6 b

2

1 zero

2 -1

3 $(-\frac{5}{2}, 0)$

4 undefined

5 collinear

3

[a] Represent by yourself. [b] $a = -6$, $b = \text{zero}$

4

[a] $a = -2$

[b] Prove by yourself.

5

First : 1 2 m.

2 8 m.

Second : 1

Answers of unit three

Answers of Exercise 14

Sets	Tallies	Freq.	Sets	Freq.
25 -	###	5	25 -	5
30 -	### ## III	13	30 -	13
35 -	### ## ## I	16	35 -	16
40 -	###	5	40 -	5
45 -	I	1	45 -	1
Total		40	Total	40

Sets	Tallies	Freq.	Sets	Freq.
30 -	IIII	4	30 -	4
40 -	###	5	40 -	5
50 -	### II	7	50 -	7
60 -	### III	8	60 -	8
70 -	### I	6	70 -	6
80 -	IIII	4	80 -	4
90 -	### I	6	90 -	6
Total		40	Total	40

The set which has the highest frequency is (X) -
The sets which have the lowest frequency
are (X) - , (X) -

Sets	Tallies	Freq.	Sets	Freq.
20 -	III	3	20 -	3
24 -	II	2	24 -	2
28 -	### I	6	28 -	6
32 -	### II	7	32 -	7
36 -	### ## II	12	36 -	12
Total		30	Total	30

2 12 students.

Sets	Tallies	Freq.	Sets	Freq.
0 -	II	2	0 -	2
4 -	### II	7	4 -	7
8 -	### ## II	12	8 -	12
12 -	### ## ##	15	12 -	15
16 -	IIII	4	16 -	4
Total		40	Total	40

The percentage of those who obtained 12 marks
at least = $\frac{15}{40} \times 100 = 47.5\%$

5

1 The least height = 112 cm. and the greatest
height = 199 cm.

The range = $199 - 112 = 87$ cm.

2

Sets	Tallies	Freq.	Sets	Freq.
110 -	II	2	110 -	2
120 -	III	3	120 -	3
130 -	III	3	130 -	3
140 -	### I	6	140 -	6
150 -	### IIII	9	150 -	9
160 -	### III	8	160 -	8
170 -	### II	7	170 -	7
180 -	### II	7	180 -	7
190 -	###	5	190 -	5
Total		50	Total	50

6

Sets	Tallies	Freq.	Sets	Freq.
165 -	### III	8	165 -	8
170 -	### ##	10	170 -	10
175 -	### ## ##	15	175 -	15
180 -	### I	6	180 -	6
185 -	### ##	10	185 -	10
190 -	IIII	4	190 -	4
195 -	I	1	195 -	1
200 -	I	1	200 -	1
Total		55	Total	55

1 39 soldiers. 2 22 soldiers.

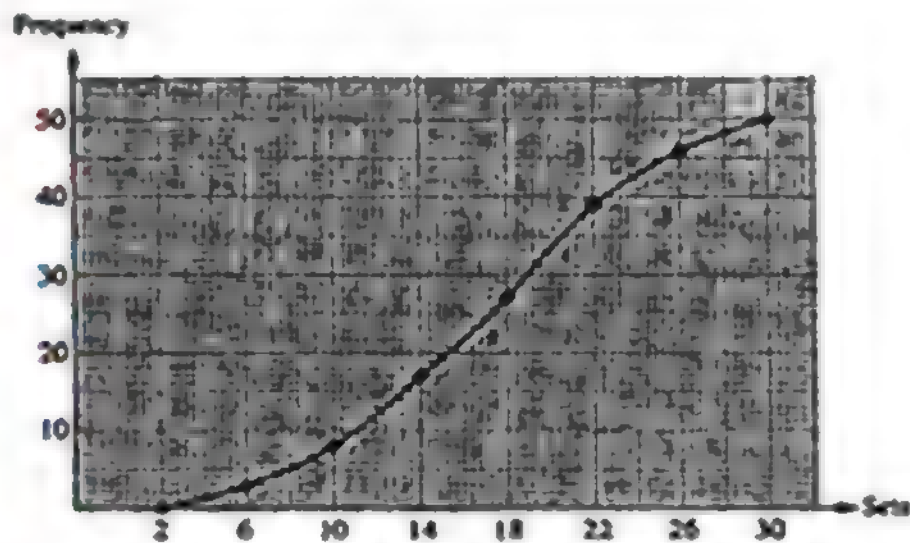
Answers of Exercise 15

First : Problems on the ascending cumulative
frequency curve.

1

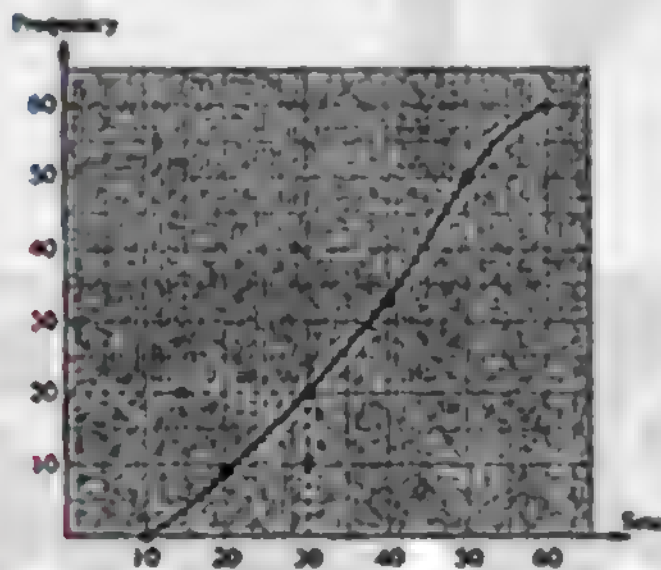
The upper boundaries of sets	ascending cumulative frequency
less than 2	0
less than 6	3
less than 10	8
less than 14	17
less than 18	27
less than 22	39
less than 26	46
less than 30	50

Algebra and Statistics



2

The upper boundaries of sets	Ascending cumulative frequency
less than 10	0
less than 20	9
less than 30	20
less than 40	33
less than 50	50
less than 60	60



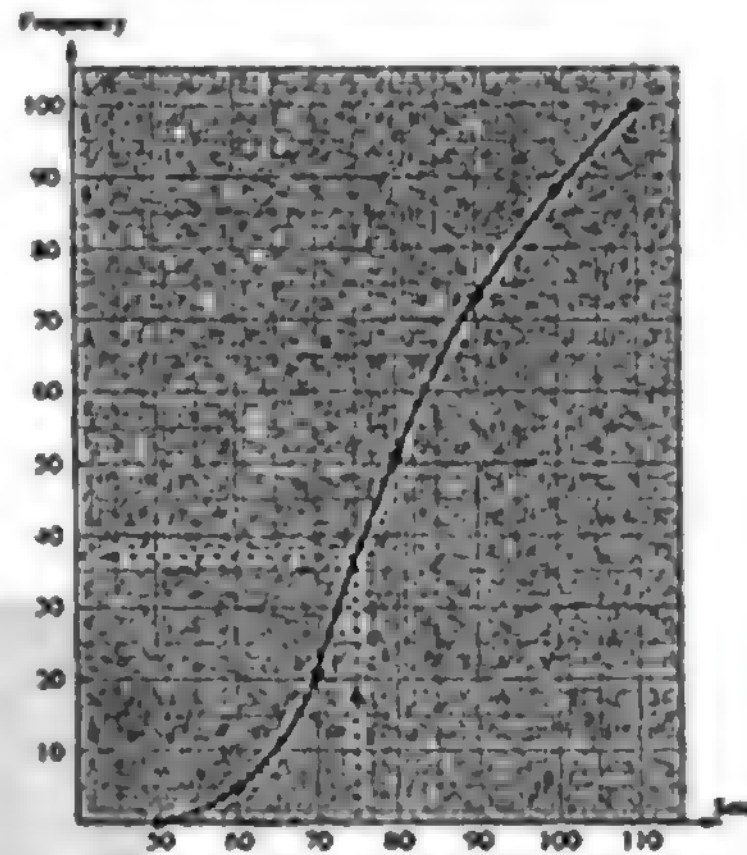
From the graph :

The number of failed pupils = 20 pupils.

3

1

The upper boundaries of sets	Ascending cumulative frequency
less than 50	0
less than 60	5
less than 70	21
less than 80	51
less than 90	73
less than 100	88
less than 110	100



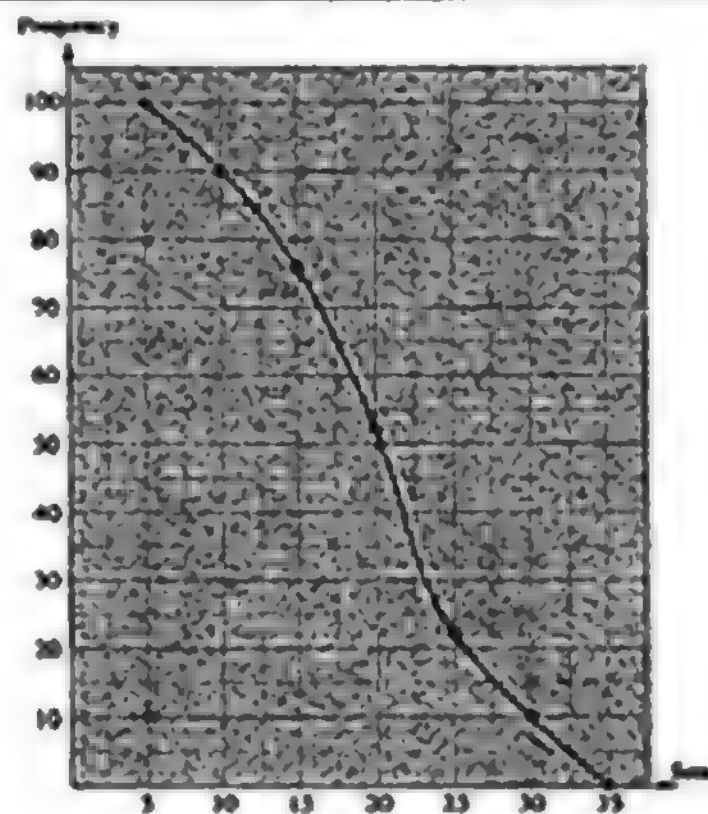
2 From the graph : The number of factories which work less than 75 hours = 37 factories.

3 The percentage of the number of factories which work less than 75 hours

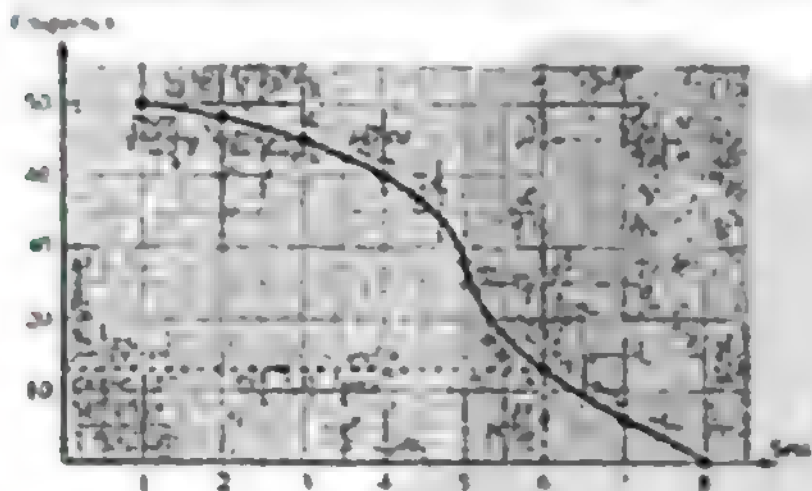
$$= \frac{37}{100} \times 100\% = 37\%$$

Second : Problems on the descending cumulative frequency curve.

The lower limits of sets	Descending cumulative frequency
5 and more	100
10 and more	90
15 and more	76
20 and more	52
25 and more	22
30 and more	10
35 and more	0



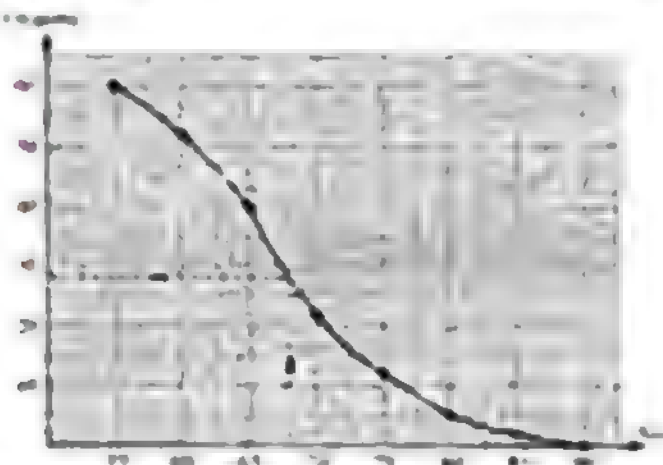
The lower boundaries of sets	Descending cumulative frequency
1 and more	50
2 and more	48
3 and more	45
4 and more	40
5 and more	28
6 and more	13
7 and more	6
8 and more	0



2. From the graph : The number of pupils who study 6 hours and more daily = 13 pupils.
3. The percentage of the number of pupils who study 6 hours and more daily = $\frac{13}{50} \times 100\% = 26\%$.

6. The missed value in the table = 10

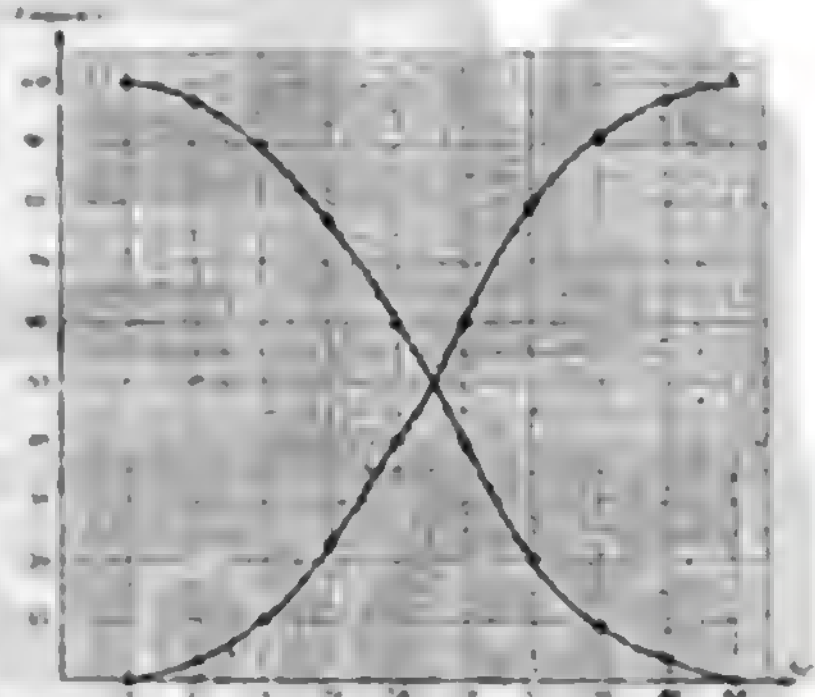
The lower limits of sets	Descending cumulative frequency
55 and more	60
60 and more	52
65 and more	40
70 and more	22
75 and more	12
80 and more	5
85 and more	2
90 and more	0



From the graph :
The number of persons whose weights are 68 kg. and more = 28 persons

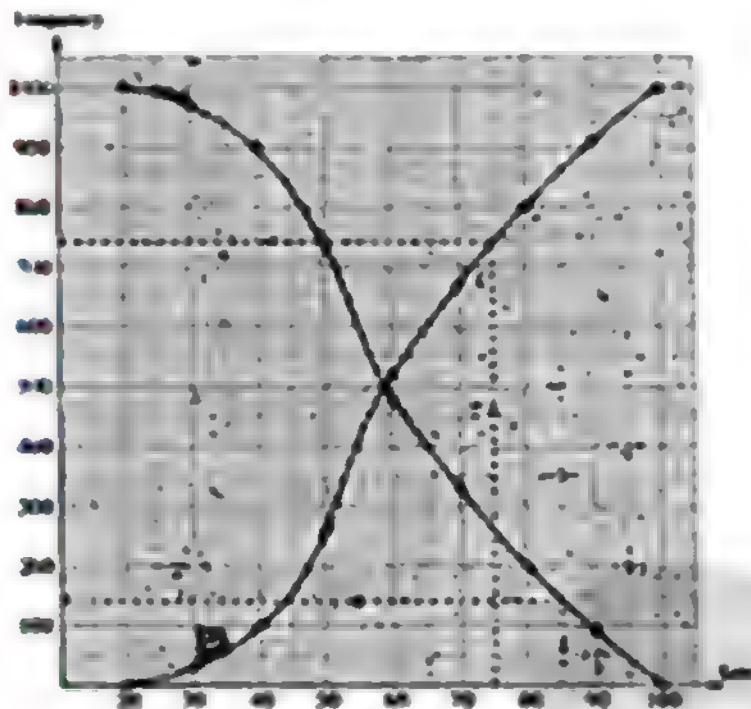
Third : Problems on the two curves together.

The upper limits of sets	Ascending cumulative frequency	The lower limits of sets	Descending cumulative frequency
less than 8	0	8 and more	100
less than 12	4	12 and more	96
less than 16	11	16 and more	89
less than 20	23	20 and more	77
less than 24	41	24 and more	59
less than 28	61	28 and more	39
less than 32	80	32 and more	20
less than 36	91	36 and more	9
less than 40	97	40 and more	3
less than 44	100	44 and more	0



The upper boundaries of sets	Ascending cumulative frequency	The lower boundaries of sets	Descending cumulative frequency
less than 20	0	20 and more	1000
less than 30	30	30 and more	970
less than 40	100	40 and more	900
less than 50	260	50 and more	740
less than 60	520	60 and more	480
less than 70	670	70 and more	330
less than 80	800	80 and more	200
less than 90	910	90 and more	90
less than 100	1000	100 and more	0

Algebra and Statistics

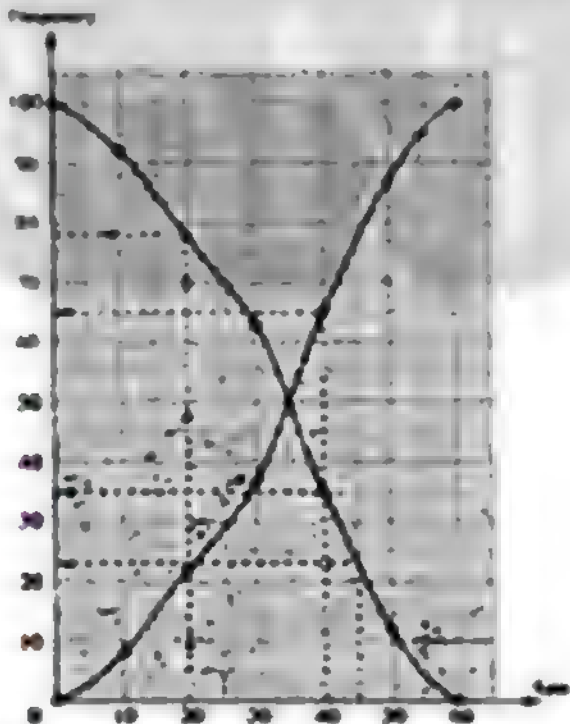


2) 740 students. 3) 140 students.

1

The upper boundaries of sets	Ascending cumulative frequency	The lower boundaries of sets	Descending cumulative frequency
less than 0	0	0 and more	100
less than 10	8	10 and more	92
less than 20	22	20 and more	78
less than 30	37	30 and more	63
less than 40	65	40 and more	35
less than 50	88	50 and more	12
less than 60	100	60 and more	0

2



3) From the graph : The number of students who got less than 40 marks = 65 students and the number of students who got 40 marks or more = 35 students.

4) The number of students who got 20 marks or more = 78 and their percentage = $\frac{78}{100} \times 100\% = 78\%$

5) The number of students who got 45 marks or more = 23 students and their percentage = $\frac{23}{100} \times 100\% = 23\%$

36

10

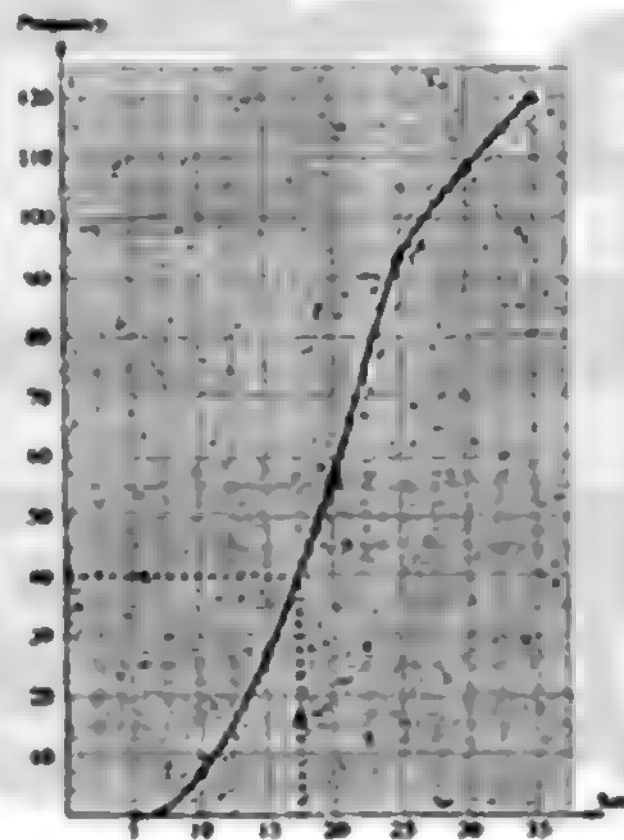
1 The frequency distribution table.

Sets	5 -	10 -	15 -	20 -	25 -	30 -	Total
Frequency	7	20	29	37	15	12	120

2 The ascending cumulative frequency table.

The upper limits of sets	Ascending cumulative frequency
less than 5	0
less than 10	7
less than 15	27
less than 20	56
less than 25	93
less than 30	108
less than 35	120

3



4) From the graph : The number of workers whose experience years are less than 17.5 years = 40 workers.

Answers of Exercise 16

1

1 The sum of values
Number of values

2 Its lower limit, its upper limit.

3 10

4 11

5 14

6 3940

2

1 c

2 d

3 c

4 b

5 a

Answers of Unit 3

3

Sets	Centre of Sets "X"	Frequency "f"	Center of sets x frequency "X x f"
5 -	10	6	60
15 -	20	8	160
25 -	30	4	120
35 -	40	2	80
Total		20	420

$$\therefore \text{The mean} = \frac{420}{20} = 21$$

4 1

Sets	"X"	"f"	"X x f"
10 -	15	1	15
20 -	25	2	50
30 -	35	4	140
40 -	45	2	90
50 -	55	1	55
Total		10	350

$$\therefore \text{The mean of marks of students} = \frac{350}{10} = 35 \text{ marks.}$$

2 The number of failed students = 3 students.

5

Sets	"X"	"f"	"X x f"
16 -	18	10	180
20 -	22	15	330
24 -	26	22	572
28 -	30	25	750
32 -	34	20	680
36 -	38	8	304
Total		100	2816

$$\therefore \text{The mean} = \frac{2816}{100} = 28.16$$

6

Sets	"X"	"f"	"X x f"
15 -	20	2	40
25 -	30	3	90
35 -	40	5	200
45 -	50	8	400
55 -	60	6	360
65 -	70	4	280
75 -	80	2	160
Total		30	1530

$$\therefore \text{The mean} = \frac{1530}{30} = 51$$

7

Sets	"X"	"f"	"X x f"
140 -	142	12	1704
144 -	146	20	2920
148 -	150	38	5700
152 -	154	22	3388
156 -	158	17	2686
160 -	162	11	1782
Total		120	18180

$$\therefore \text{The mean} = \frac{18180}{120} = 151.5 \text{ cm.}$$

8 1

Sets	"X"	"f"	"X x f"
1 -	1.5	2	3
2 -	2.5	3	7.5
3 -	3.5	5	17.5
4 -	4.5	12	54
5 -	5.5	15	82.5
6 -	6.5	7	45.5
7 -	7.5	6	45
Total		50	255

2 The mean of the number of hours of study

$$= \frac{255}{50} = 5.1 \text{ hours.}$$

2 The number of pupils who study less than 4 hours daily = 2 + 3 + 5 = 10 pupils.

9

1 25 - , 10

2

Sets	"X"	"f"	"X x f"
5 -	10	3	30
15 -	20	10	200
25 -	30	12	360
35 -	40	10	400
45 -	50	5	250
Total		40	1240

$$\therefore \text{The mean} = \frac{1240}{40} = 31 \text{ marks.}$$

3 The number of students whose marks are not less than 35 = 15 students.

Algebra and Statistics

10 The missed number is 5

Sets	"X"	"f"	"X × f"
6 -	8	2	16
10 -	12	3	36
14 -	16	5	80
18 -	20	8	160
22 -	24	6	144
26 -	28	4	112
30 -	32	2	64
Total		30	612

$$\therefore \text{The mean} = \frac{612}{30} = 20.4 \text{ kg.}$$

11

$$1) X = 30$$

$$+ k + 2 = 100 - (10 + 17 + 20 + 32 + 4) = 17$$

$$\therefore k = 15$$

2

Sets	"X"	"f"	"X × f"
10 -	15	10	150
20 -	25	17	425
30 -	35	20	700
40 -	45	32	1440
50 -	55	17	935
60 -	65	4	260
Total		100	3910

$$\therefore \text{The mean} = \frac{3910}{100} = 39.1$$

12

$$1) 3k + 4k = 50 - (7 + 10 + 8 + 4)$$

$$\therefore 7k = 21 \quad \therefore k = \frac{21}{7} = 3$$

2

Sets	"X"	"f"	"X × f"
30 -	32.5	7	227.5
35 -	37.5	9	337.5
40 -	42.5	12	510
45 -	47.5	10	475
50 -	52.5	8	420
55 -	57.5	4	230
Total		50	2200

$$\therefore \text{The mean} = \frac{2200}{50} = 44 \text{ kg.}$$

13

$$1) k - 2 = 50 - (4 + 5 + 8 + 7 + 5 + 1)$$

$$\therefore k - 2 = 20 \quad \therefore k = 22$$

2

Sets	"X"	"f"	"X × f"
2 -	4	4	16
6 -	8	5	40
10 -	12	8	96
14 -	16	20	320
18 -	20	7	140
22 -	24	5	120
26 -	28	1	28
Total		50	760

$$\therefore \text{The mean} = \frac{760}{50} = 15.2 \text{ days.}$$

14

\therefore The total of marks of the student in 5 months
 $= 5 \times 23.8 = 119$ marks.

\therefore let the required mark of the sixth month be X

$$\therefore \frac{119 + X}{6} = 24 \quad \therefore 119 + X = 144$$

$$\therefore X = 144 - 119 = 25 \text{ marks.}$$

\therefore The mark of the student in the 6th month is 25

15

\therefore The total of marks of Magdi in 4 exams
 $= 4 \times 16 = 64$ marks.

$$\therefore \text{let the required mark be X} \quad \therefore \frac{64 + X}{5} = 18$$

$$\therefore 64 + X = 90 \quad \therefore X = 90 - 64 \quad \therefore X = 26 \text{ marks.}$$

\therefore The mark of Magdi in the 5th exam should be 26 marks.

16

$$1) a = \frac{0 + 4}{2} = 2 \quad , b = \frac{90}{6} = 15 \quad , c = \frac{300}{30} = 10$$

$$\therefore \frac{4 + d}{2} = 6 \quad \therefore d = 8$$

$$, e = \frac{16 + 12}{2} = 14 \quad , f = \frac{16 + 20}{2} = 18$$

$$, X = 10 \times 18 = 180$$

$$, y = 1140 - (10 + 90 + 300 + 180) = 560$$

$$, r = \frac{560}{14} = 40$$

$$, m = 5 + 15 + 30 + 40 + 10 = 100$$

$$2) \text{The mean} = \frac{1140}{100} = 11.4 \text{ marks.}$$

Answers of Exercise 17

1

1 4

2 6

3 The third

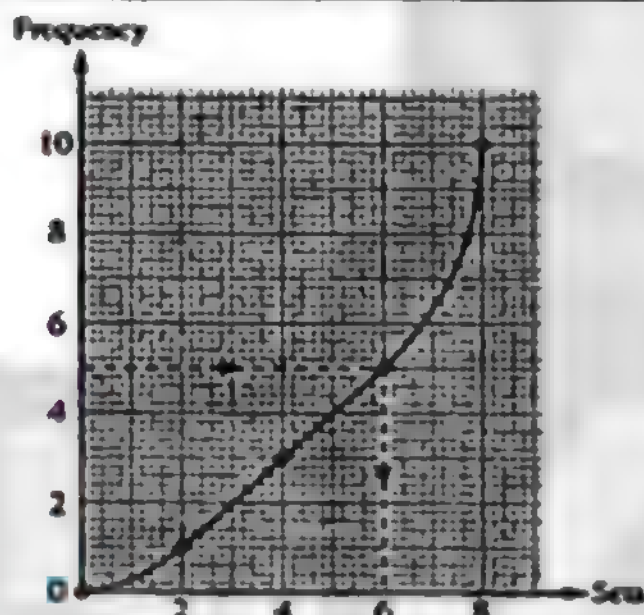
4 7

5 10

6 The median

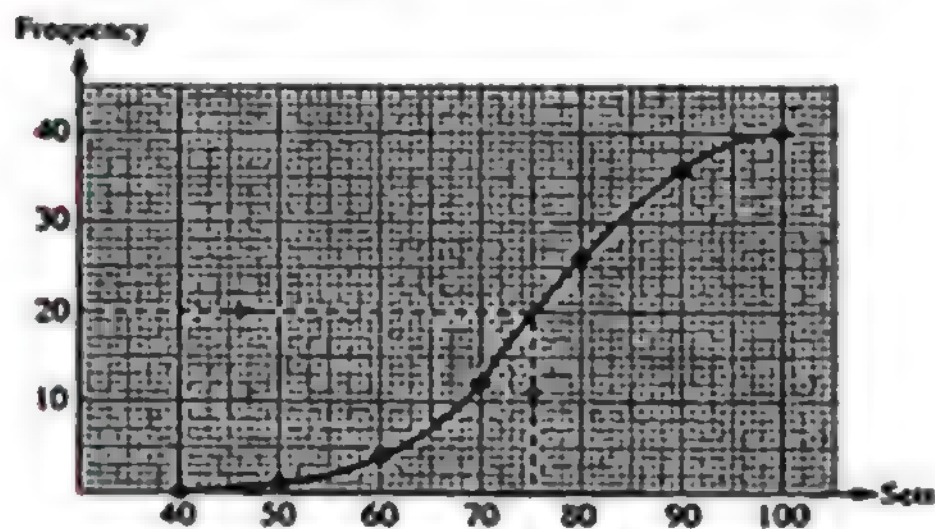
2

The upper limits of sets	Ascending cumulative frequency
less than 0	0
less than 2	1
less than 4	3
less than 6	5
less than 8	10


 \therefore The order of the median = $\frac{10}{2} = 5$
 \therefore The median = 6

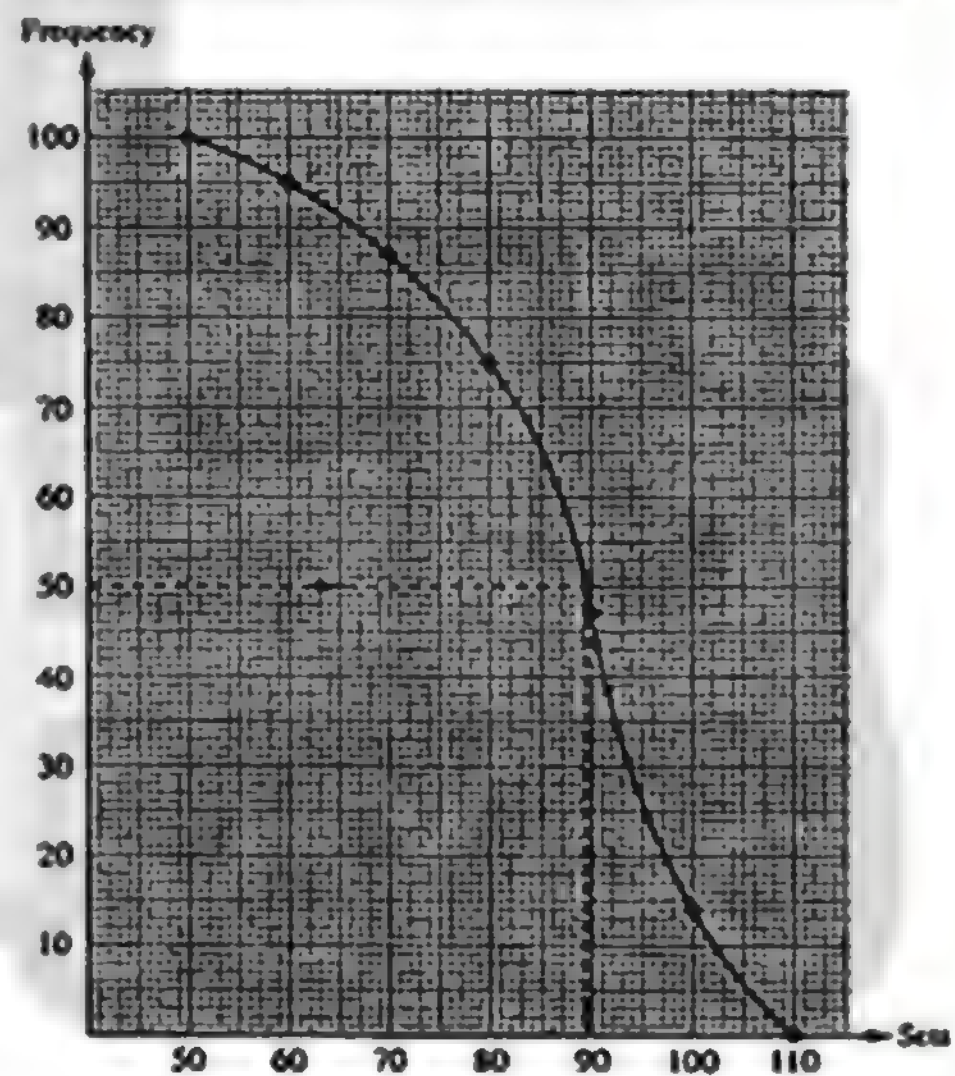
3

The upper boundaries of sets	Ascending cumulative frequency
less than 40	0
less than 50	1
less than 60	4
less than 70	12
less than 80	26
less than 90	36
less than 100	40


 \therefore The order of the median = $\frac{40}{2} = 20$
 \therefore The percentage of intelligence = 75%.

4

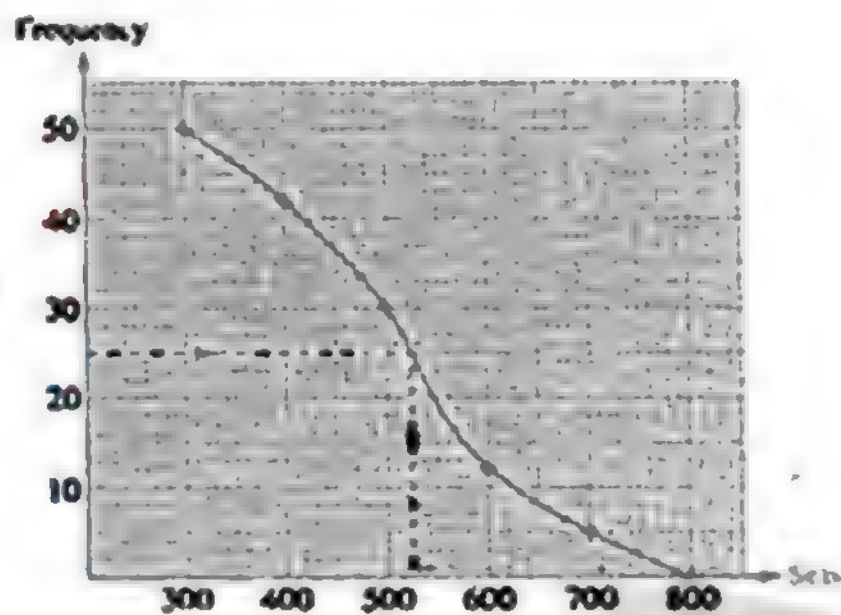
The lower boundaries of sets	Descending cumulative frequency
50 and more	100
60 and more	95
70 and more	87
80 and more	75
90 and more	47
100 and more	14
110 and more	0


 \therefore The order of the median = $\frac{100}{2} = 50$
 \therefore The median of working hours = 89.5 hours

5

The lower boundaries of sets	Descending cumulative frequency
300 and more	50
400 and more	42
500 and more	30
600 and more	12
700 and more	5
800 and more	0

Algebra and Statistics

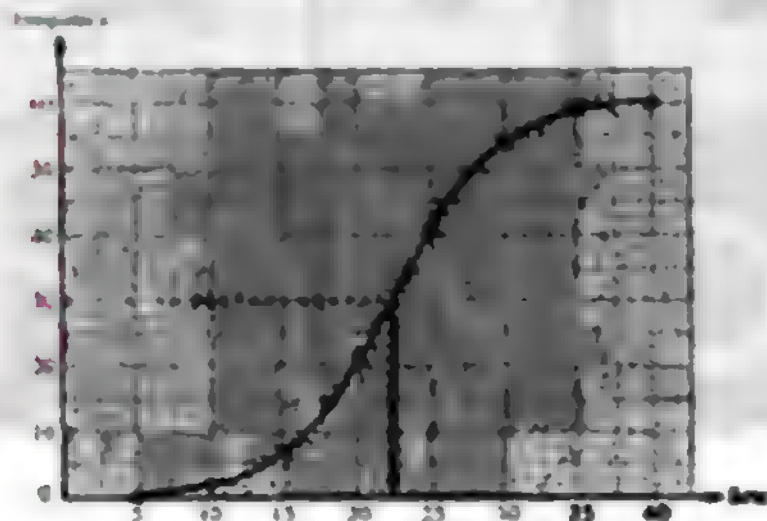


\therefore The order of the median = $\frac{50}{2} = 25$

\therefore The median wage = 520 pounds.

6

The upper limits of sets	Ascending cumulative frequency
less than 5	0
less than 10	2
less than 15	7
less than 20	21
less than 25	41
less than 30	54
less than 35	59
less than 40	60

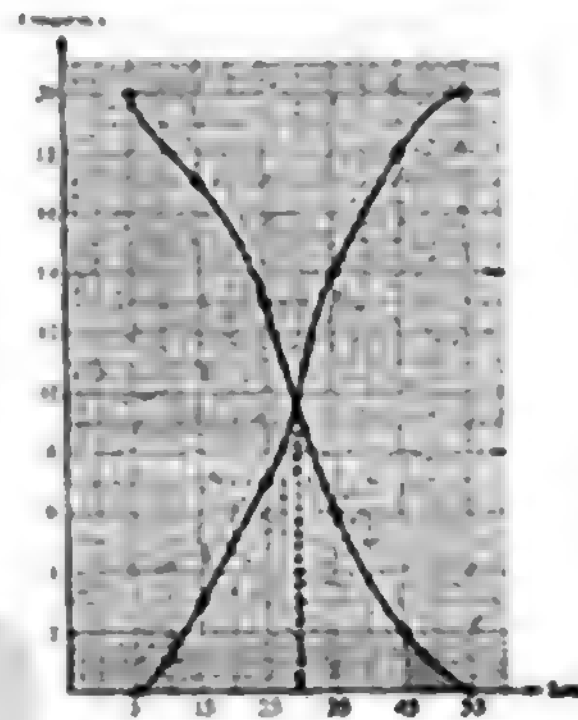


\therefore The order of the median = $\frac{60}{2} = 30$

\therefore The median mark = 22 marks.

7

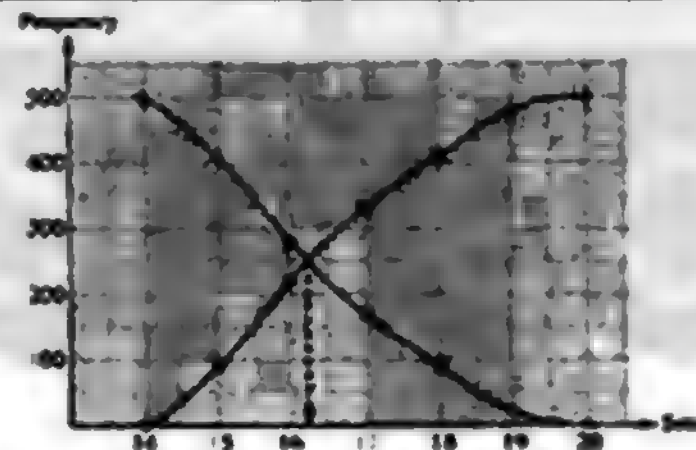
The upper limits of sets	Ascending cumulative frequency	The lower limits of sets	Descending cumulative frequency
less than 5	0	5 and more	20
less than 15	3	15 and more	17
less than 25	7	25 and more	13
less than 35	14	35 and more	6
less than 45	18	45 and more	2
less than 55	20	55 and more	0



From the graph we find that the median = 29 kg.

8

The upper boundaries of sets	Ascending cumulative frequency	The lower boundaries of sets	Descending cumulative frequency
less than 14	0	14 and more	500
less than 15	90	15 and more	410
less than 16	220	16 and more	280
less than 17	330	17 and more	170
less than 18	410	18 and more	90
less than 19	480	19 and more	20
less than 20	500	20 and more	0

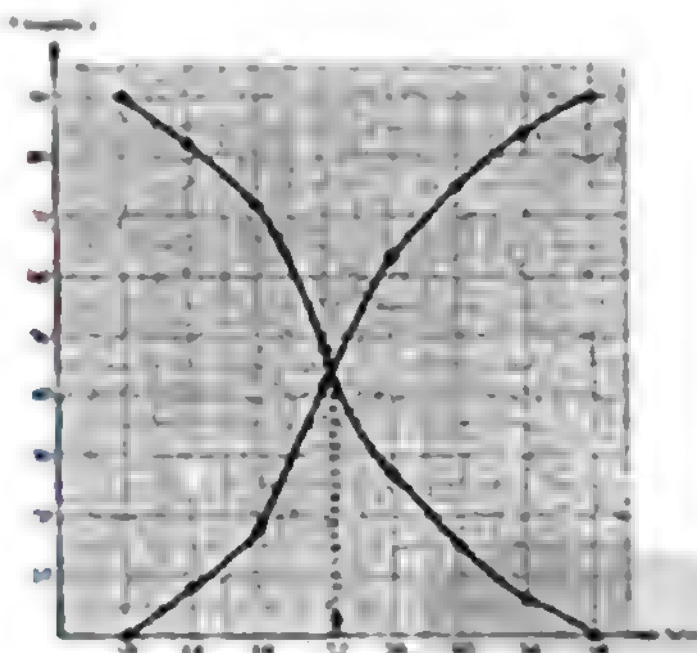


\therefore The median age = 16.3 years.

9

The upper limits of sets	Ascending cumulative frequency	The lower limits of sets	Descending cumulative frequency
less than 10	0	10 and more	90
less than 14	8	14 and more	82
less than 18	18	18 and more	72
less than 22	42	22 and more	48
less than 26	63	26 and more	27
less than 30	75	30 and more	15
less than 34	84	34 and more	6
less than 38	90	38 and more	0

Answers of Unit 3



From the graph we find that the median mark = 22.5 marks

10

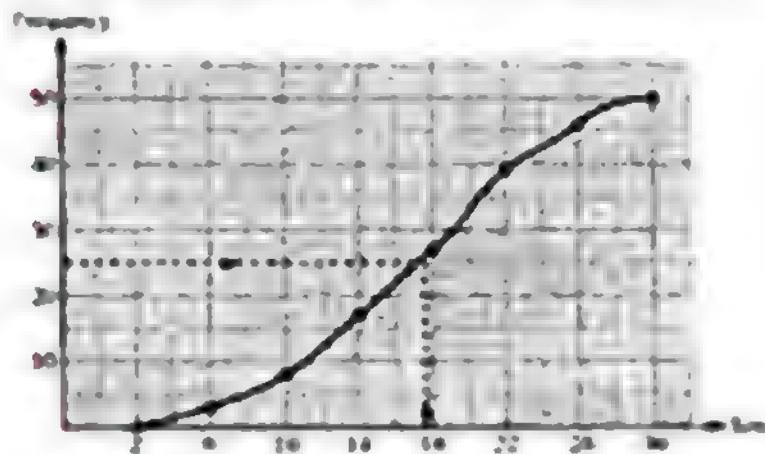
1

Sets	"X"	"f"	"X × f"
2 -	4	3	12
6 -	8	5	40
10 -	12	9	108
14 -	16	10	160
18 -	20	12	240
22 -	24	7	168
26 -	28	4	112
Total		50	840

$$\therefore \text{The mean} = \frac{840}{50} = 16.8$$

2 We form the ascending cumulative frequency table.

The upper limits of sets	Ascending cumulative frequency
less than 2	0
less than 6	3
less than 10	8
less than 14	17
less than 18	27
less than 22	39
less than 26	46
less than 30	50



$$\therefore \text{The order of the median} = \frac{50}{2} = 25$$

$$\therefore \text{The median} = 17.6$$

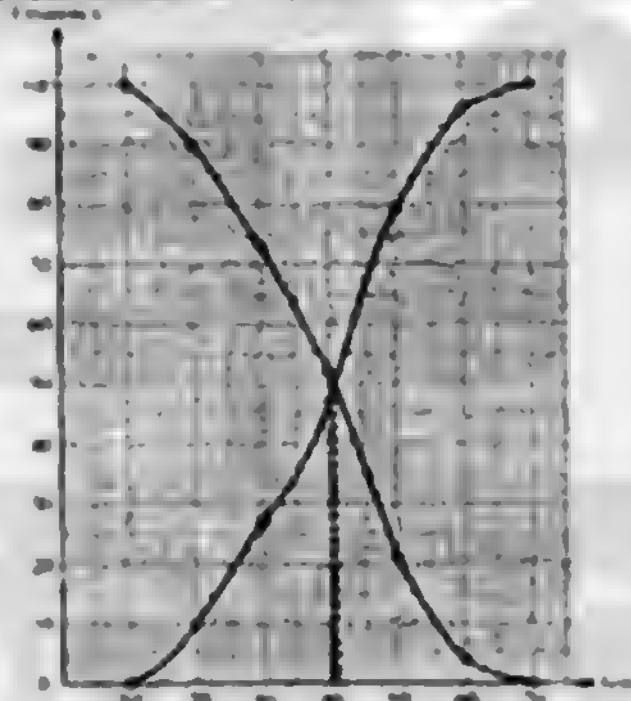
11

$$1] X = 30 \cdot k + 2 = 100 - (10 + 17 + 20 + 32 + 4)$$

$$\therefore k + 2 = 17 \quad \therefore k = 15$$

2

The upper limits of sets	Ascending cumulative frequency	The lower limits of sets	Descending cumulative frequency
less than 10	0	10 and more	100
less than 20	10	20 and more	90
less than 30	27	30 and more	73
less than 40	47	40 and more	53
less than 50	79	50 and more	21
less than 60	96	60 and more	4
less than 70	100	70 and more	0

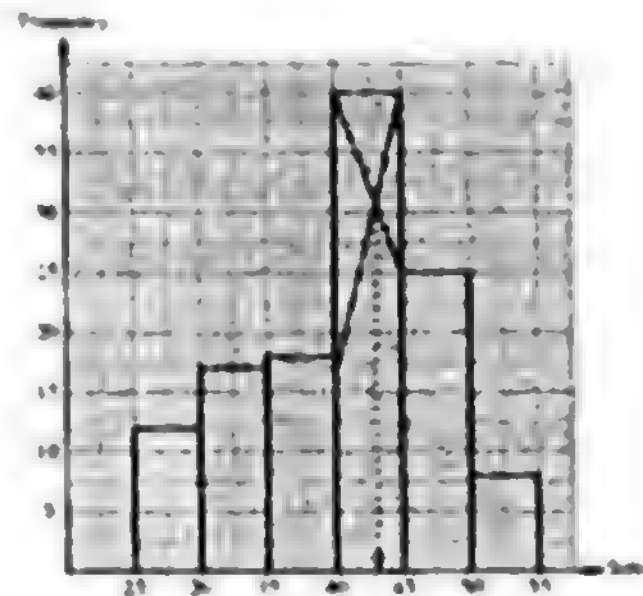


The median = 41

Answers of Exercise 18

- 1 1 the most common value in the set. 2 5
3 8 4 3 5 6 6 2

2

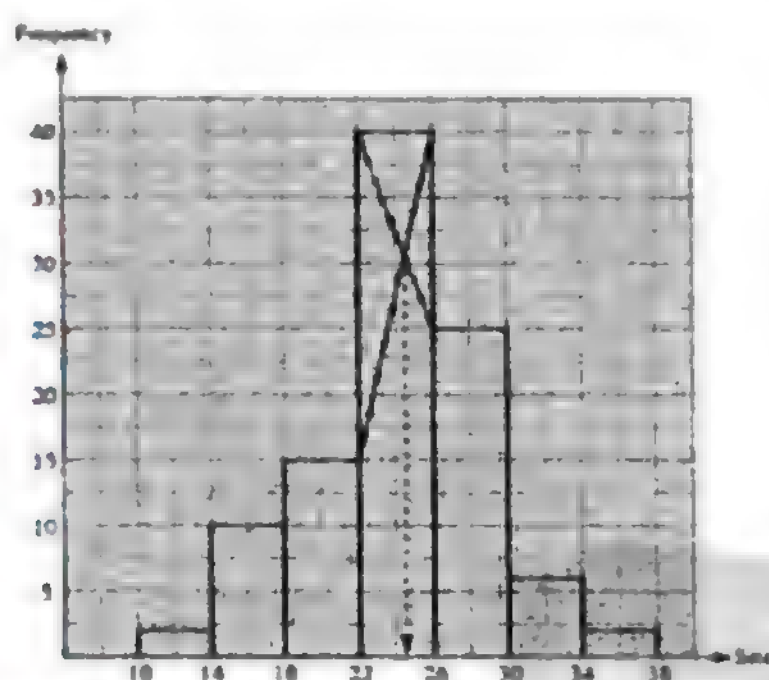


From the graph : The mode age = 43 years.

41

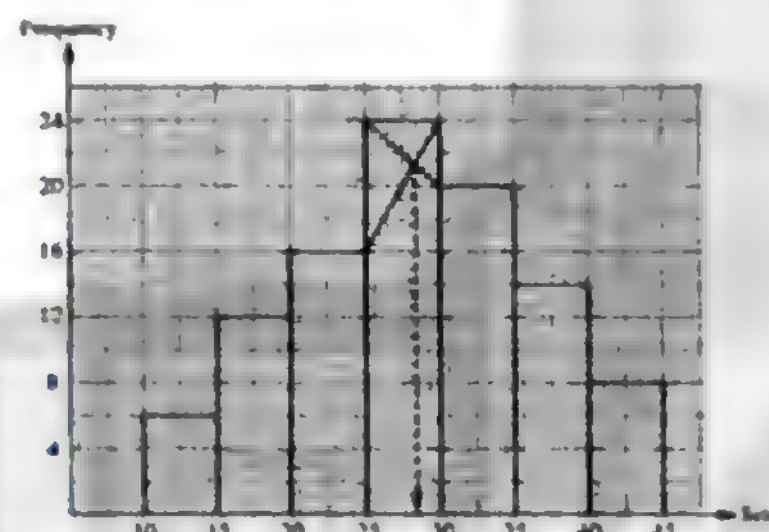
Algebra and Statistics

3



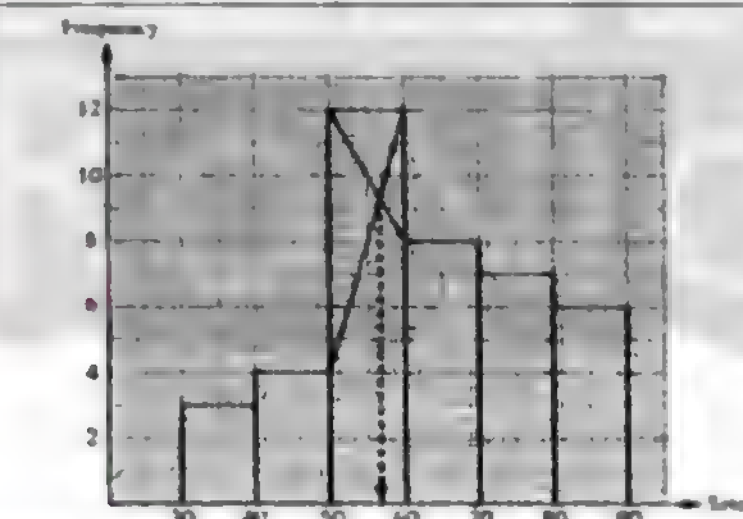
From the graph : the mode mark ≈ 24.5 marks.

4



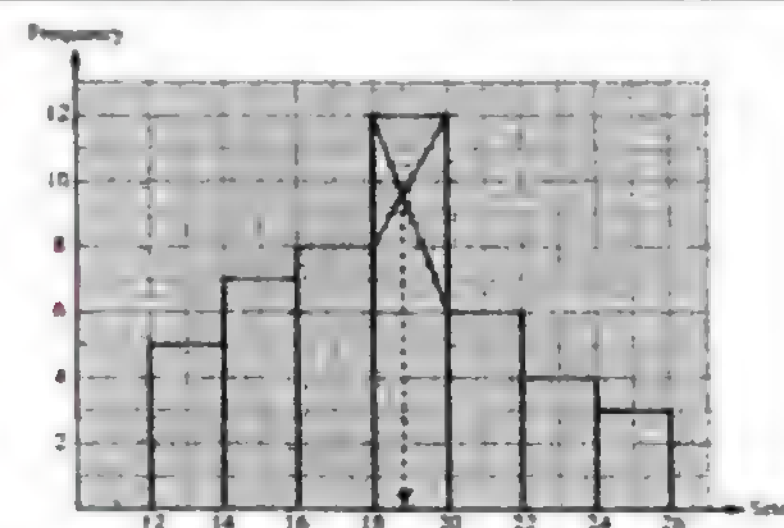
From the graph : The mode wage ≈ 28.5 pounds.

5



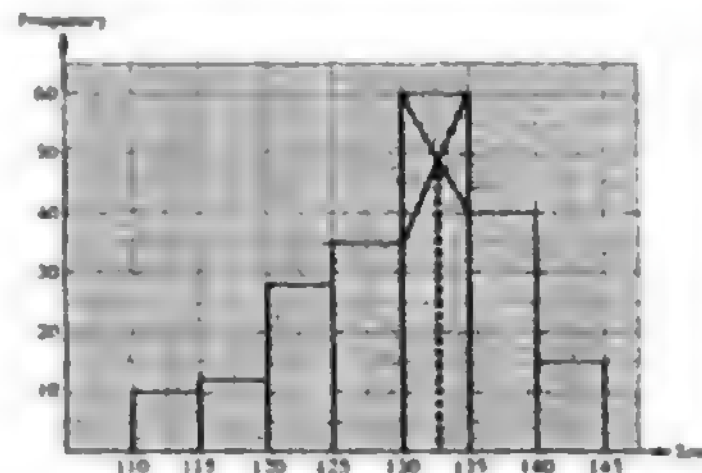
From the graph : The mode = 57

6



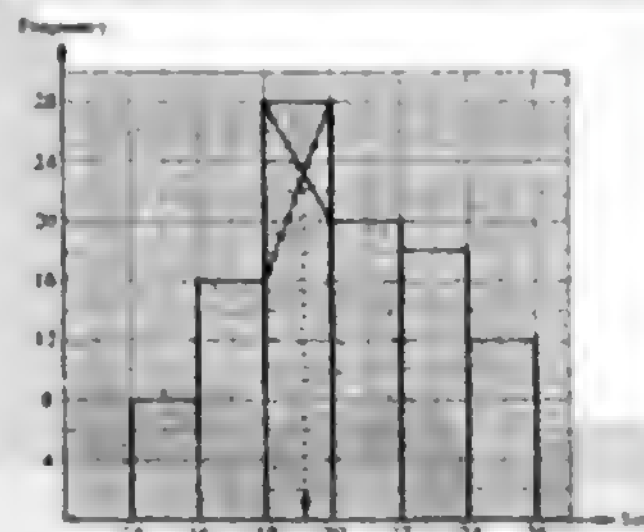
From the graph : the mode age ≈ 18.8 years.

7



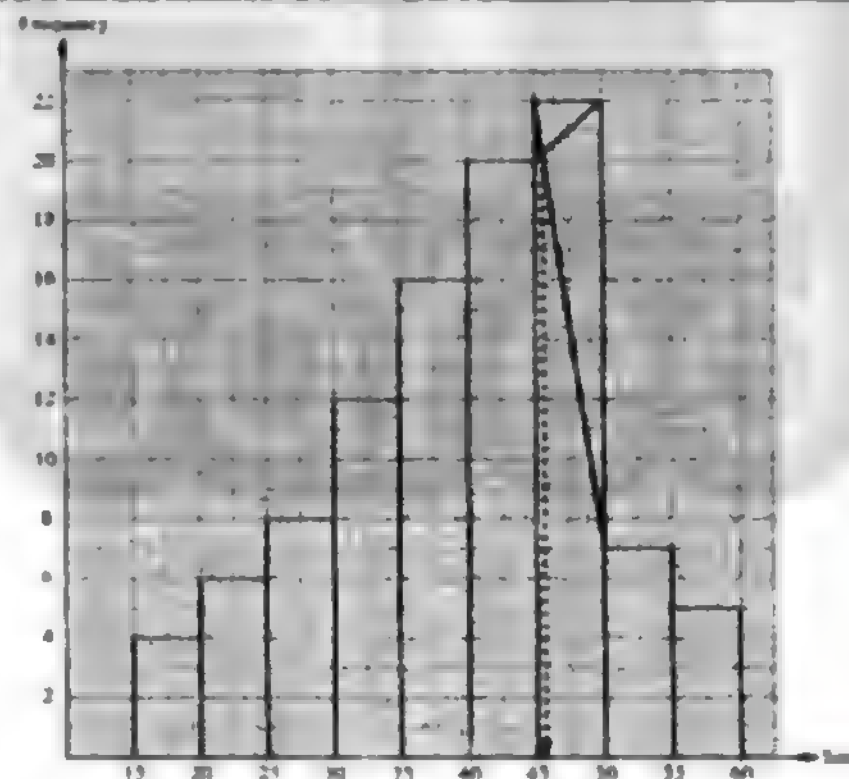
From the graph : the mode height ≈ 132.75 cm.

8



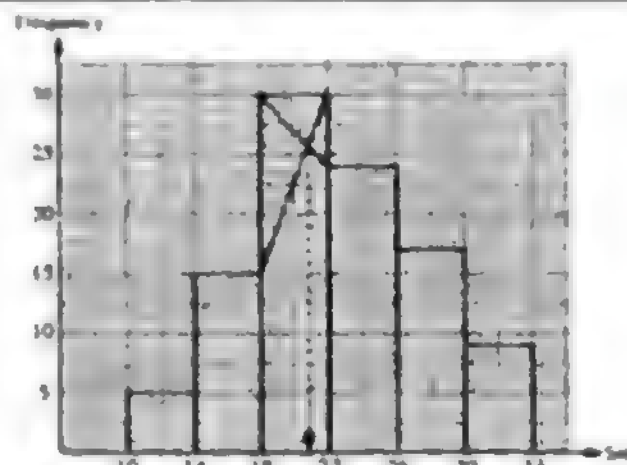
From the graph :
the mode of the amount of milk ≈ 19.2 gallons.

9



From the graph : the mode mark ≈ 45.5 marks.

10



From the graph : the mode weight ≈ 20.8 kg.

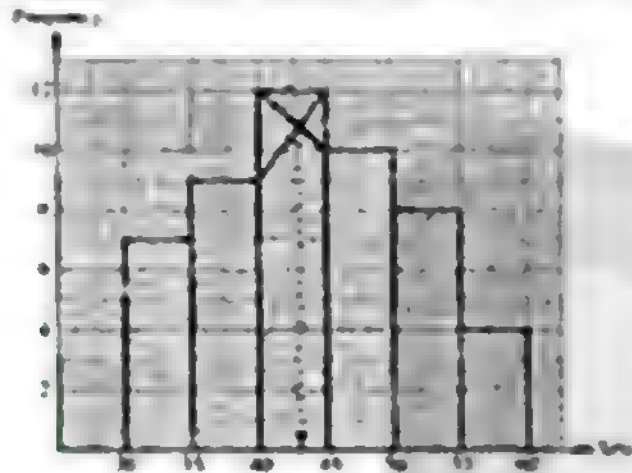
11

$$1 \therefore k + 4 + 3k + 4k + 3k + 1 + 3k - 1 + k + 1 = 50$$

$$\therefore 15k + 5 = 50 \quad \therefore 15k = 45 \quad \therefore k = 3$$

2

Weight in kg.	30 -	35 -	40 -	45 -	50 -	55 -	Total
number of students	7	9	12	10	8	4	50



From the graph : The mode weight = 43 kg.

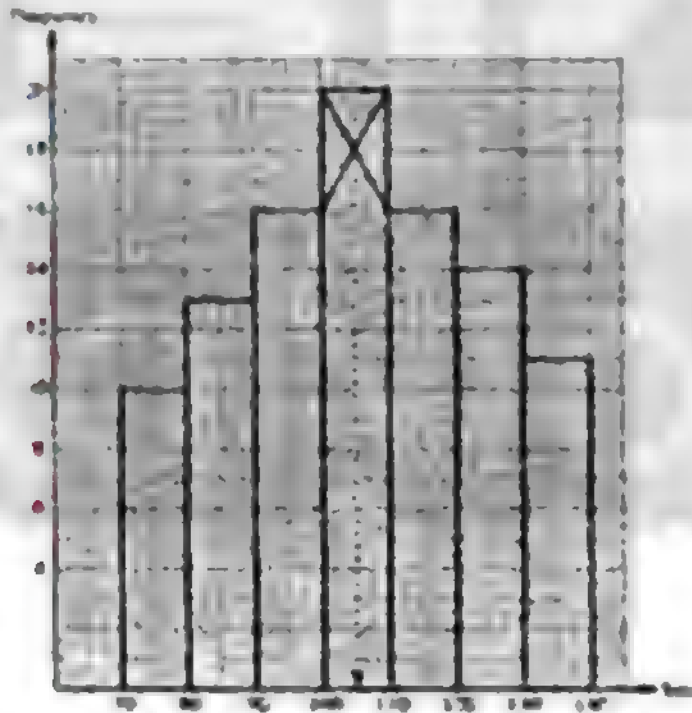
12

$$1 \quad X = 110$$

$$\therefore k - 4 = 100 - (10 + 13 + 20 + 16 + 14 + 11)$$

$$\therefore k - 4 = 16 \quad \therefore k = 20$$

2



From the graph : The mode wage = 105 pounds.

13

1

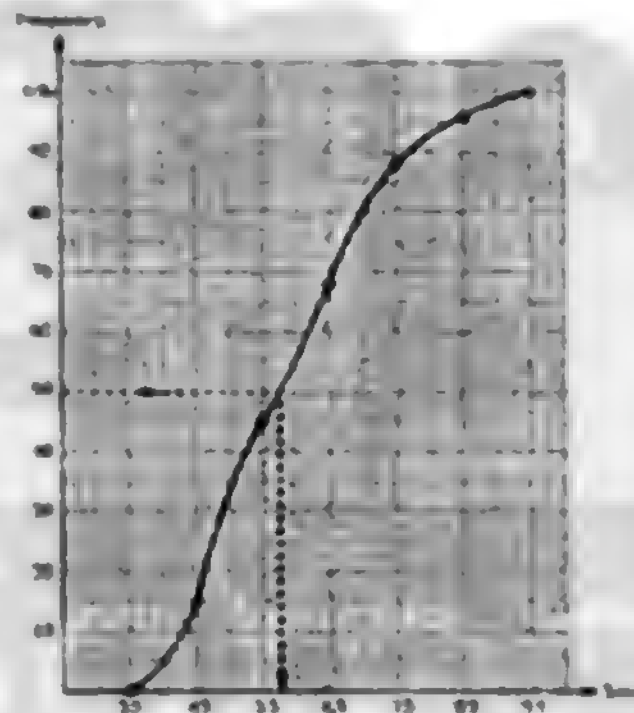
Sets	"X"	"f"	"X × f"
35 -	40	15	600
45 -	50	30	1500
55 -	60	23	1380
65 -	70	20	1400
75 -	80	8	640
85 -	90	4	360
Total		100	5880

$$\therefore \text{The mean of working hours} = \frac{5880}{100} = 58.8 \text{ hours.}$$

2 We form the ascending cumulative frequency table as follows :

The upper limits of sets	Ascending cumulative frequency
less than 35	0
less than 45	15
less than 55	45
less than 65	68
less than 75	88
less than 85	96
less than 95	100

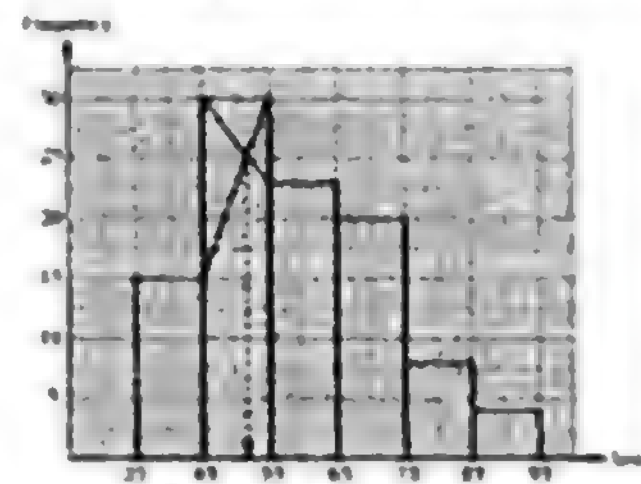
Then we draw the ascending cumulative frequency curve as follows :



$$\therefore \text{The order of the median} = \frac{100}{2} = 50$$

$$\therefore \text{The median} = 57.5 \text{ hours.}$$

3 We graph the histogram of the distribution as follows :



From the graph :

we find that the mode = 52 hours.

Algebra and Statistics

14

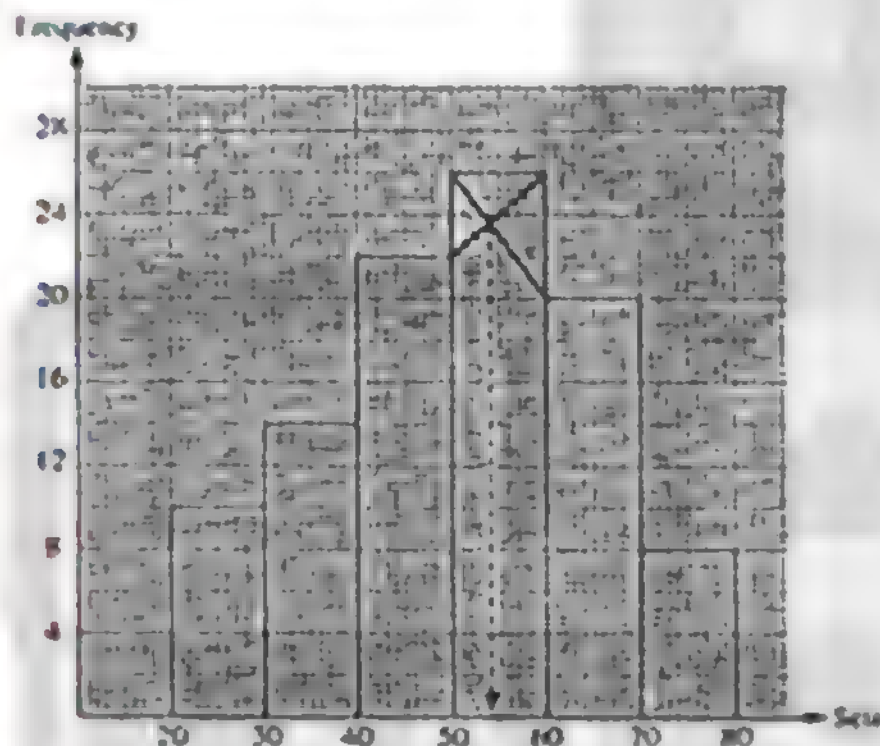
$$1) k = 100 - (10 + 22 + 26 + 20 + 8) = 14$$

2

Sets	"x"	"f"	"x × f"
20 -	25	10	250
30 -	35	14	490
40 -	45	22	990
50 -	55	26	1430
60 -	65	20	1300
70 -	75	8	600
Total		100	5060

$$\therefore \text{The mean} = \frac{5060}{100} = 50.6 \text{ pounds.}$$

3



From the graph : The mode value = 54 pounds.

15

$$1) \therefore 3k + 4k = 50 - (7 + 10 + 8 + 4)$$

$$\therefore 7k = 21 \quad \therefore k = \frac{21}{7} = 3$$

2

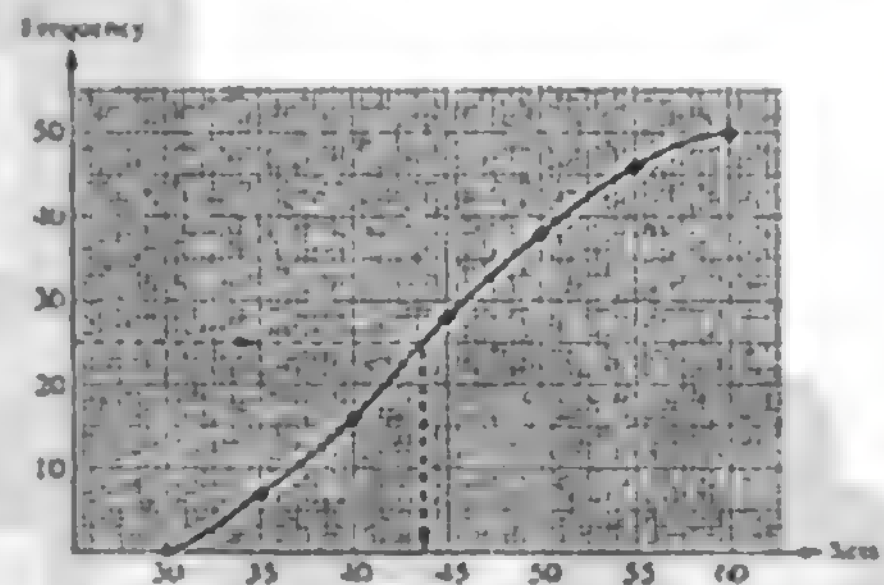
Sets	"x"	"f"	"x × f"
30 -	32.5	7	227.5
35 -	37.5	9	337.5
40 -	42.5	12	510
45 -	47.5	10	475
50 -	52.5	8	420
55 -	57.5	4	230
Total		50	2200

$$\therefore \text{The mean} = \frac{2200}{50} = 44 \text{ kg.}$$

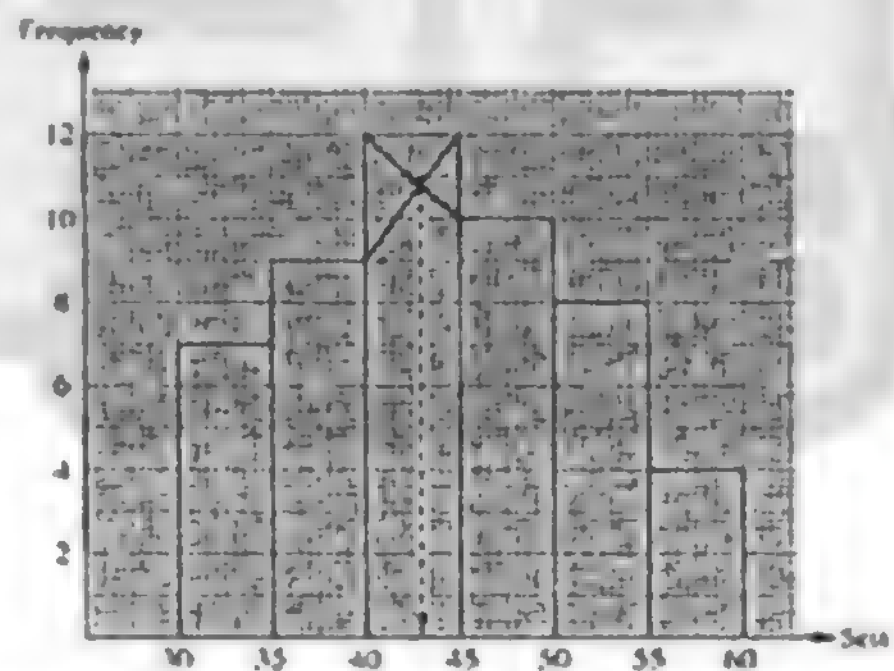
44

3

The upper limits of sets	Ascending cumulative frequency
less than 30	0
less than 35	7
less than 40	16
less than 45	28
less than 50	38
less than 55	46
less than 60	50



4



From the graph : The mode weight = 43 kg.

$$5) \text{ The order of the median} = \frac{50}{2} = 25$$

$$\therefore \text{The median} = 43.5 \text{ kg.}$$

Answers of exams on unit three

Model 1

1

1 b

2 c

3 a

4 c

5 d

2

1 9

2 3

3 the median

4 4

5 4

3

1 35 marks

2 3 students

4

1 $k = 8, m = 4$

2 The median = 5.6

5

The mode = 55 marks.

Model 2

1

1 a

2 b

3 c

4 c

5 d

2

1 the order of the median

2 the mode

3 7140

4 17

5 5

3

The arithmetic mean = 31

4

1 $k = 4, m = 3$

2 The median = 5

5

Graph by yourself, the mode age = 43 years.

Answers of accumulative basic skills

1

1 6

2 0

3 15

4 154

5 21

6 $\frac{2}{3}$

7 7500

8 4

9 12

10 9

11 27

12 $6, 8, 2$

2

1 c

2 c

3 a

4 a

5 d

6 c

7 d

8 b

9 d

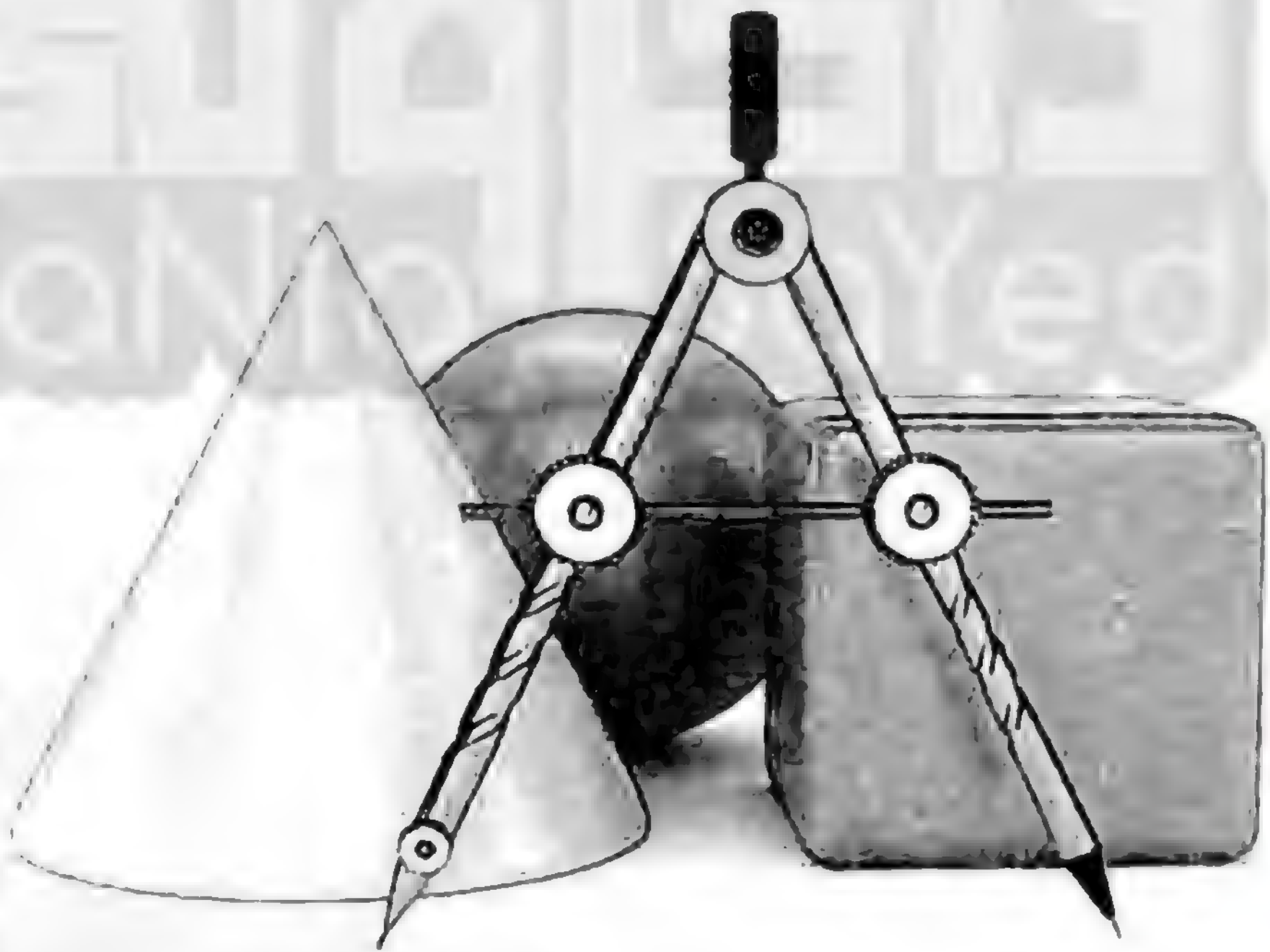
10 d

11 c

12 a

**Guide
Answers**

of Geometry Exercises



Answers of revision exercise

1

- 1 5 cm, 3 cm, 110° , 70°
 2 4 cm, 14 cm. 3 106° , 14 cm.
 4 6.3 cm. 5 16 cm, 77°
 6 5 cm, 60° 7 2 cm, 3 cm, 8 cm.
 8 16 cm, 135° , 110°

2

- 1 $X = 115^\circ$, $y = 65^\circ$, $z = 115^\circ$
 2 $X = 80^\circ$, $y = 25^\circ$, $z = 75^\circ$
 3 $X = 35^\circ$, $y = 27^\circ$, $z = 35^\circ$
 4 $X = 30^\circ$, $y = 30^\circ$, $z = 120^\circ$
 5 $X = 138^\circ$, $y = 42^\circ$, $z = 138^\circ$
 6 $X = 75^\circ$, $y = 45^\circ$, $z = 30^\circ$

3

- 1 $AB = 35$ cm. 2 $AB = \frac{1}{2} BD = 7.5$ cm.
 3 $AB = 2 AX = 14$ cm.
 4 $AB = \frac{1}{3} BC$ $\therefore AB = \frac{1}{3} \times 12 = 4$ cm.
 5 $AB = 2 BC = 2 AD$ $\therefore AB = 2 \times 45 = 90$ cm.
 6 $\therefore M$ is the midpoint of \overline{BD} , $BM = 6$ cm.
 $\therefore BD = 12$ cm.
 \therefore In $\triangle DBC$ which is right-angled at D
 $\therefore DC = \sqrt{(BC)^2 - (DB)^2} = \sqrt{225 - 144} = 9$ cm.
 $\therefore AB = DC = 9$ cm.

4

- Fig (1) : $X = 44^\circ$, $y = 88^\circ$, $z = 46^\circ$
 Fig (2) : $X = 90^\circ$, $y = 45^\circ$, $z = 90^\circ$
 Fig (3) : $X = 35^\circ$, $y = 110^\circ$, $z = 70^\circ$
 Fig (4) : $X = 60^\circ$, $y = 30^\circ$, $z = 60^\circ$

5

- 1 E is the midpoint of \overline{AC} 2 $\overline{DE} \parallel \overline{BC}$
 3 3 cm. 4 90° 5 12 cm.
 6 17 cm. 7 2.5 cm. 8 3 cm.
 9 3 cm, 45°

Answers of unit four

Answers of Exercise 1

1

- 1 a median 2 3 3 one point
 4 1 : 2 5 2 : 1 6 4

2

- 1 8 cm, 5 cm. 2 6 cm, 4 cm, $\frac{1}{3}$, $\frac{2}{3}$
 3 6 cm, 3 cm, 4 cm. 4 5 cm, 12 cm, 27 cm.

3

- $\therefore \overline{AD}$, \overline{BE} are two medians in $\triangle ABC$,
 $\overline{AD} \cap \overline{BE} = \{M\}$
 $\therefore M$ is the point of concurrence of the medians of $\triangle ABC$
 $\therefore MD = \frac{1}{3} AD = \frac{1}{3} \times 6 = 2$ cm. (1)
 $ME = \frac{1}{3} BE = \frac{1}{3} \times 9 = 3$ cm. (2)
 $\therefore D$ is the midpoint of \overline{BC} , E is the midpoint
 of \overline{AC} in $\triangle ABC$
 $\therefore DE = \frac{1}{2} AB = \frac{1}{2} \times 9 = 4.5$ cm. (3)
 From (1), (2) and (3):
 \therefore The perimeter of $\triangle MDE = 2 + 3 + 4.5 = 9.5$ cm.
 (The req.)

4

- $\therefore D$ is the midpoint of \overline{AB}
 E is the midpoint of \overline{AC}
 $\therefore BC = 2 DE$ $\therefore BC = 8$ cm.
 $\therefore M$ is the intersection point of medians of $\triangle ABC$
 $\therefore MC = 2 DM$ $\therefore MC = 6$ cm.
 $BM = \frac{2}{3} BE$ $\therefore BM = 4$ cm.
 \therefore The perimeter of $\triangle BMC = 8 + 6 + 4 = 18$ cm.
 (The req.)

5

- $\therefore M$ is the intersection point of the medians of $\triangle ABC$
 $\therefore XM = \frac{1}{2} MC = 4$ cm.
 \therefore The perimeter of $\triangle MXY = 4 + 5 + 3 = 12$ cm.
 (First req.)
 $AM = 2 MY = 6$ cm.
 $\therefore X$ is the midpoint of \overline{AB} ,
 Y is the midpoint of \overline{BC}

Geometry

$$\therefore AC = 2XY = 10 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle MAC = 6 + 8 + 10 = 24 \text{ cm.}$$

(Second req.)

6

$\therefore F$ is the midpoint of \overline{AB}

$\therefore E$ is the midpoint of \overline{AC}

$\therefore \overline{BE}$, \overline{CF} are two medians in $\triangle ABC$

$\therefore M$ is the intersection point of the medians of $\triangle ABC$

$$\therefore ME = \frac{1}{2} MB = 2 \text{ cm.} \quad (1)$$

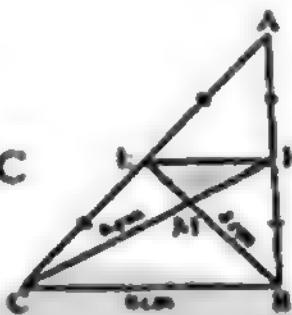
$$\therefore MF = \frac{1}{2} MC = 3 \text{ cm.} \quad (2)$$

$\therefore F$ is the midpoint of \overline{AB} , E is the midpoint of \overline{AC}

$$\therefore FE = \frac{1}{2} BC = 4 \text{ cm.} \quad (3)$$

From (1), (2) and (3):

$$\therefore \text{The perimeter of } \triangle MFE = 2 + 3 + 4 = 9 \text{ cm. (The req.)}$$



7

$\therefore F$ is the midpoint of \overline{AB}

$\therefore E$ is the midpoint of \overline{AC}

$\therefore \overline{BE}$, \overline{CF} are two medians in $\triangle ABC$

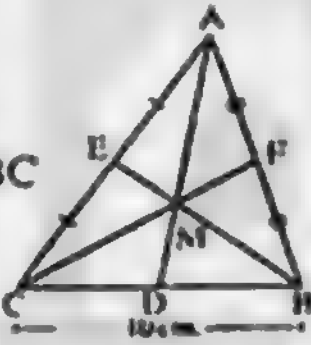
$\therefore M$ is the intersection point of the medians of $\triangle ABC$

$\therefore \overline{AD}$ is a median in $\triangle ABC$

$\therefore D$ is the midpoint of \overline{BC}

$$\therefore BD = \frac{10}{2} = 5 \text{ cm.} \quad (\text{First req.})$$

$$\therefore AM = \frac{2}{3} AD = \frac{2}{3} \times 12 = 8 \text{ cm.} \quad (\text{Second req.})$$



8

$\therefore M$ is the intersection point of medians of $\triangle ABC$

$$\therefore MF = \frac{1}{2} AM \quad (1)$$

$$\therefore MD = \frac{1}{2} MC \quad (2)$$

$\therefore D$ is the midpoint of \overline{AB} ,

F is the midpoint of \overline{BC} in $\triangle ABC$

$$\therefore DF = \frac{1}{2} AC \quad (3)$$

By adding (1), (2) and (3):

$$\therefore MF + MD + DF = \frac{1}{2} AM + \frac{1}{2} MC + \frac{1}{2} AC$$

\therefore The perimeter of $\triangle MFD$

$$= \frac{1}{2} (AM + MC + AC)$$

$$= \frac{1}{2} \text{ the perimeter of } \triangle AMC$$

$$= \frac{1}{2} \times 36 = 18 \text{ cm.}$$

(The req.)

9

$\therefore E$ is the midpoint of \overline{BC}

$\therefore O$ is the midpoint of \overline{AC}

$\therefore \overline{AE}$, \overline{BO} are two medians in $\triangle ABC$

$\therefore M$ is the intersection point of the medians of $\triangle ABC$

$\therefore \overline{CD}$ is a median in $\triangle ABC$

$$\therefore AM = \frac{2}{3} AE, \quad BM = \frac{2}{3} BO, \quad CM = \frac{2}{3} CD$$

$$\therefore AM + BM + CM = 18$$

$$\therefore \frac{2}{3} AE + \frac{2}{3} BO + \frac{2}{3} CD = 18$$

$$\therefore \frac{2}{3} (AE + BO + CD) = 18$$

$$\therefore AE + BO + CD = 18 \times \frac{3}{2} = 27 \text{ cm.} \quad (\text{The req.})$$

10

$\therefore M$ is the point of concurrence of the medians of $\triangle ABC$

$\therefore \overline{CD}$ is a median in $\triangle ABC$

$$\therefore DM = \frac{1}{2} MC = 3 \text{ cm.}$$

$\therefore \triangle AMD$ is a right-angled triangle at M

$$\therefore (AM)^2 = (AD)^2 - (DM)^2 = 25 - 9 = 16$$

$$\therefore AM = 4 \text{ cm.}$$

$$\therefore ME = \frac{1}{2} AM = 2 \text{ cm.} \quad (\text{The req.})$$

11

$\therefore ABCD$ is a parallelogram

\therefore The two diagonals bisect each other

$\therefore M$ is the midpoint of \overline{AC}

$\therefore \overline{DM}$ is a median in $\triangle ADC$

$$\therefore DE = 2 EM$$

$\therefore E$ is the intersection point of the medians of $\triangle ADC$

$\therefore E \in \overline{FC}$

$$\therefore \overline{CF}$$
 is a median in $\triangle ACD \quad \therefore AF = FD \text{ (Q.E.D.)}$

12

\therefore The two diagonals of the rectangle bisect each other

$\therefore M$ is the midpoint of \overline{AC}

$\therefore \overline{BM}$ is a median in $\triangle ABC$

$\therefore E$ is the midpoint of \overline{AB}

$\therefore \overline{CE}$ is a median in $\triangle ABC$

$$\therefore \overline{CE} \cap \overline{BM} = \{F\}$$

$\therefore F$ is the intersection point of the medians of $\triangle ABC$

(First req.)

$$\therefore BF = \frac{2}{3} BM \quad \therefore 4 = \frac{2}{3} BM \quad \therefore BM = 6 \text{ cm.}$$

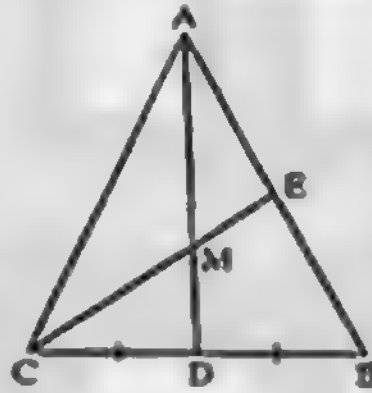
- ∴ The two diagonals of the rectangle are equal in length and bisect each other
 ∴ $AM = BM = 6 \text{ cm.}$ (Second req.)

13

- ∴ D is the midpoint of \overline{BC}
 ∴ \overline{AD} is a median in $\triangle ABC$
 ∴ $AM = \frac{2}{3} AD$
 ∴ M is the intersection point of the medians of $\triangle ABC$
 ∴ \overline{CF} is a median in $\triangle ABC$
 ∴ F is the midpoint of \overline{AB} ∴ $BF = \frac{1}{2} AB$
 ∴ $AC = AB$ ∴ $BF = \frac{1}{2} AC$ (Q.E.D.)

14

- ∴ D is the midpoint of \overline{BC}
 ∴ \overline{AD} is a median in $\triangle ABC$
 ∴ $AM = 2 MD$
 ∴ M is the intersection point of the medians of $\triangle ABC$
 ∴ $M \in \overline{CE}$
 ∴ \overline{CE} is a median in $\triangle ABC$
 ∴ $EM = \frac{1}{3} EC = \frac{1}{3} \times 12 = 4 \text{ cm.}$ (The req.)



15

- ∴ O is the midpoint of \overline{AC}
 ∴ \overline{BO} is a median in $\triangle ABC$
 ∴ $BO = 3 MO$
 ∴ M is the intersection point of the medians of $\triangle ABC$
 ∴ \overline{AE} is a median in $\triangle ABC$
 ∴ E is the midpoint of \overline{BC}
 ∴ $BE = EC$ ∴ $x + 3 = 2x - 1$
 ∴ $3 + 1 = 2x - x$ ∴ $x = 4$
 ∴ $BE = EC = 7 \text{ cm.}$
 ∴ $BC = 14 \text{ cm.}$ (The req.)

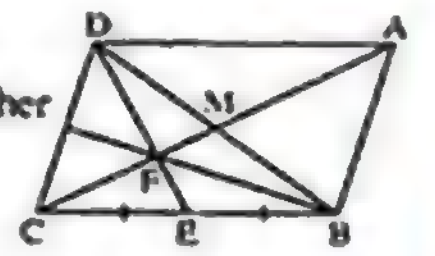
16

- ∴ M is the point of concurrence of the medians of $\triangle ABC$
 ∴ \overline{CD} is a median in $\triangle ABC$
 ∴ D is the midpoint of \overline{AB}
 In $\triangle AMB$:
 ∴ D is the midpoint of \overline{AB} , E is the midpoint of \overline{BM}
 ∴ \overline{MD} , \overline{AE} are two medians in $\triangle AMB$

- ∴ N is the point of concurrence of the medians of $\triangle AMB$
 ∴ $MN = 2 ND$ ∴ $x + 3 = 2(x - 1)$
 ∴ $x + 3 = 2x - 2$ ∴ $3 + 2 = 2x - x$
 ∴ $x = 5$
 ∴ $ND = 5 - 1 = 4 \text{ cm.}$, $MN = 5 + 3 = 8 \text{ cm.}$
 ∴ $MD = ND + MN = 12 \text{ cm.}$
 ∴ \overline{CD} is a median in $\triangle ABC$
 ∴ $MC = 2 MD = 24 \text{ cm.}$ (The req.)

17

- ∴ ABCD is a parallelogram
 ∴ The two diagonals bisect each other
 ∴ M is the midpoint of \overline{BD}
 ∴ \overline{CM} is a median in $\triangle DBC$
 ∴ E is the midpoint of \overline{BC}
 ∴ \overline{DE} is a median in $\triangle DBC$
 ∴ F is the intersection point of the medians of $\triangle DBC$
 ∴ \overline{BF} bisects \overline{CD} (Q.E.D. 1)
 ∴ $CF = \frac{2}{3} CM$, ∴ $CM = \frac{1}{2} AC$
 ∴ $CF = \frac{2}{3} \times \frac{1}{2} AC = \frac{1}{3} AC$ (Q.E.D. 2)



18

- ∴ \overline{AD} and \overline{BE} are medians in $\triangle ABC$
 ∴ M is the intersection point of the medians of $\triangle ABC$
 ∴ $M \in \overline{CF}$
 ∴ \overline{CF} is a median in $\triangle ABC$
 ∴ F is the midpoint of \overline{AB}
 In $\triangle ABM$:
 ∴ F is the midpoint of \overline{AB} , N is the midpoint of \overline{BM}
 ∴ $\overline{NF} \parallel \overline{AM}$ ∴ $\overline{NF} \parallel \overline{MD}$ (1)
 In $\triangle BMC$:
 ∴ D is the midpoint of \overline{BC} , N is the midpoint of \overline{BM}
 ∴ $\overline{ND} \parallel \overline{CM}$ ∴ $\overline{ND} \parallel \overline{MF}$ (2)
 From (1) and (2):
 ∴ The figure FNMD is a parallelogram. (Q.E.D.)

19

- ∴ D is the midpoint of \overline{BC}
 ∴ \overline{AD} is a median in $\triangle ABC$
 ∴ $AM = 2 MD$, $M \in \overline{AD}$
 ∴ M is the intersection point of the medians of $\triangle ABC$
 ∴ $M \in \overline{BE}$
 ∴ \overline{BE} is a median in $\triangle ABC$ ∴ $BM = 2 ME$

Geometry

$$\therefore BM = 4 \text{ cm.}$$

$$\therefore BE = 2 + 4 = 6 \text{ cm.}$$

$\therefore \Delta BCE$ in which :

D is the midpoint of \overline{BC} , $\overline{DF} \parallel \overline{BE}$

$\therefore F$ is the midpoint of \overline{EC}

$$\therefore DF = \frac{1}{2} BE = 3 \text{ cm.}$$

(The req.)

20

$\therefore D$ is the midpoint of \overline{BC} , $\overline{DF} \parallel \overline{AC}$

$\therefore F$ is the midpoint of \overline{AB} $\therefore DF = \frac{1}{2} AC$

In ΔABD :

$\therefore E$ is the midpoint of \overline{BD} , F is the midpoint of \overline{AB}

$\therefore \overline{AE}$ and \overline{DF} are medians in ΔABD

$\therefore M$ is the intersection point of the medians of ΔABD

$$\therefore DM = \frac{2}{3} DF \quad , \therefore DF = \frac{1}{2} AC$$

$$\therefore DM = \frac{2}{3} \times \frac{1}{2} AC = \frac{1}{3} AC \quad (\text{Q.E.D.})$$

21

$\therefore \overline{CD}$ and \overline{BE} are two medians in ΔABC

$\therefore M$ is the intersection point of the medians of ΔABC

$\therefore \overline{AF}$ is a median in ΔABC

$\therefore F$ is the midpoint of \overline{BC}

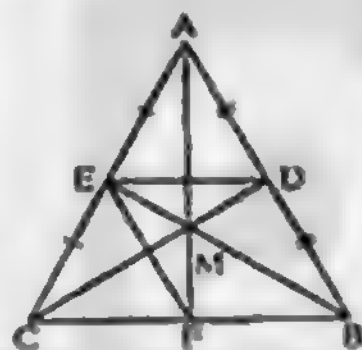
$\therefore E$ is the midpoint of \overline{AC}

$$\therefore \overline{FE} \parallel \overline{AB} , FE = \frac{1}{2} AB$$

$$\therefore \overline{FE} \parallel \overline{BD} , FE = BD$$

$\therefore DBFE$ is a parallelogram

(Q.E.D.)



22

From ΔAXD :

$\therefore B$ is the midpoint of \overline{AX} ,

$\overline{BC} \parallel \overline{AD}$ (ABCD is a parallelogram)

$$\therefore BY = \frac{1}{2} AD , \therefore BC = AD$$

(two opposite sides in the parallelogram ABCD)

$$\therefore BY = \frac{1}{2} BC \quad \therefore Y \text{ is the midpoint of } \overline{BC}$$

$\therefore \overline{DY}$ is a median in ΔDBC (1)

$\therefore M$ is the midpoint of \overline{BD} (the intersection point of the diagonals of the parallelogram)

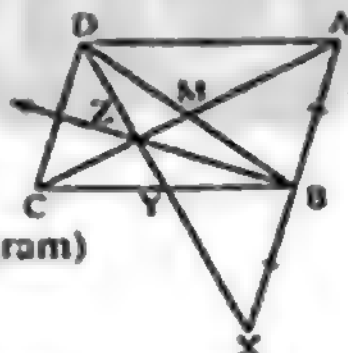
$\therefore \overline{CM}$ is a median in ΔDBC (2)

From (1) and (2) :

$$\therefore \overline{CM} \cap \overline{DY} = \{Z\}$$

$\therefore Z$ is the intersection point of the medians of ΔDBC

$\therefore \overline{BZ}$ intersects \overline{DC} at the midpoint of \overline{DC} (Q.E.D.)



Answers of Exercise 2

1

1) 3

3) right

5) twice

2) half the length of the hypotenuse

4) half the length of the hypotenuse

2

1) 3

2) 10

3) 8

4) 18, 9, $\frac{1}{3}$, 3

5) 5, 5, 15

6) 8, 9, 10, 27

3

In ΔADC :

$\therefore m(\angle D) = 90^\circ$, E is the midpoint of \overline{AC}

$$\therefore DE = \frac{1}{2} AC \quad (1)$$

In ΔABC :

$\therefore m(\angle B) = 90^\circ$, $m(\angle ACB) = 30^\circ$

$$\therefore AB = \frac{1}{2} AC \quad (2)$$

From (1) and (2) :

$$\therefore AB = DE \quad (\text{Q.E.D.})$$

In ΔLXZ :

$\therefore D$ is the midpoint of \overline{LX} , E is the midpoint of \overline{LZ}

$$\therefore DE = \frac{1}{2} XZ \quad (1)$$

From ΔXYZ :

$\therefore m(\angle Y) = 90^\circ$, M is the midpoint of \overline{XZ}

$$\therefore YM = \frac{1}{2} XZ \quad (2)$$

From (1) and (2) :

$$\therefore DE = YM \quad (\text{Q.E.D.})$$

5

In ΔACD :

$\therefore E$ is the midpoint of \overline{AD} , F is the midpoint of \overline{CD}

$$\therefore EF = \frac{1}{2} AC \quad \therefore AC = 8 \text{ cm.}$$

In ΔABC :

$\therefore m(\angle B) = 90^\circ$, $m(\angle ACB) = 30^\circ$

$$\therefore AB = \frac{1}{2} AC = 4 \text{ cm.} \quad (\text{The req.})$$

6

In ΔABC :

$\therefore m(\angle BAC) = 90^\circ$, D is the midpoint of \overline{BC}

$$\therefore BC = 2AD = 2 \times 3 = 6 \text{ cm.}$$

In $\triangle CBE$:

$$\therefore m(\angle CBE) = 90^\circ, m(\angle E) = 30^\circ$$

$$\therefore EC = 2 BC = 2 \times 6 = 12 \text{ cm.}$$

$\therefore F$ is the midpoint of \overline{CE}

$$\therefore BF = \frac{1}{2} EC = \frac{1}{2} \times 12 = 6 \text{ cm.} \quad (\text{The req.})$$

7

In $\triangle ABC$:

$$\therefore m(\angle B) = 90^\circ, m(\angle ACB) = 60^\circ$$

$$\therefore m(\angle CAB) = 30^\circ \quad \therefore BC = \frac{1}{2} AC$$

$$\therefore DE = BC \quad \therefore DE = \frac{1}{2} AC$$

$\therefore \overline{DE}$ is a median in $\triangle ACD$

$$\therefore m(\angle ADC) = 90^\circ \quad (\text{Q.E.D.})$$

8

In $\triangle ABC$:

$$\therefore m(\angle B) = 90^\circ, m(\angle ACB) = 30^\circ$$

$$\therefore AB = \frac{1}{2} AC$$

$$\therefore AB = DE = 5 \text{ cm.} \quad \therefore DE = \frac{1}{2} AC$$

$\therefore \overline{DE}$ is a median in $\triangle ACD$

$$\therefore m(\angle ADC) = 90^\circ \quad (\text{Q.E.D.})$$

9

In $\triangle ABD$:

$$\therefore m(\angle A) = 90^\circ, M \text{ is the midpoint of } \overline{BD}$$

$$\therefore AM = \frac{1}{2} BD$$

$$\therefore CM = AM \quad \therefore CM = \frac{1}{2} BD$$

$\therefore \overline{CM}$ is a median in $\triangle DBC$

$$\therefore m(\angle BCD) = 90^\circ \quad (\text{Q.E.D.})$$

10

In $\triangle DBC$:

$$\therefore E \text{ is the midpoint of } \overline{BC}, \overline{EF} \parallel \overline{BD}$$

$$\therefore EF = \frac{1}{2} BD$$

$$\therefore AM = EF \quad \therefore AM = \frac{1}{2} BD$$

$\therefore \overline{AM}$ is a median in $\triangle ABD$

$$\therefore m(\angle BAD) = 90^\circ \quad (\text{Q.E.D.})$$

11

$$\therefore \angle ADC \text{ is an exterior angle of } \triangle ABD$$

$$\therefore m(\angle ADC) = 33^\circ + 27^\circ = 60^\circ$$

\therefore In $\triangle ADC$:

$$m(\angle DAC) = 180^\circ - (60^\circ + 90^\circ) = 30^\circ$$

$$\therefore DC = \frac{1}{2} AD$$

$$\therefore AD = 8 \text{ cm.} \quad (\text{The req.})$$

12

In $\triangle ABE$:

$$\therefore m(\angle A) = 30^\circ, m(\angle B) = 90^\circ$$

$$\therefore BE = \frac{1}{2} AE \quad \therefore BE = 2 \text{ cm.} \quad (1)$$

$$m(\angle AEB) = 180^\circ - (30^\circ + 90^\circ) = 60^\circ$$

$\therefore E \in \overline{BC}$

$$\therefore m(\angle DEC) = 180^\circ - (60^\circ + 60^\circ) = 60^\circ$$

\therefore In $\triangle DEC$:

$$m(\angle D) = 180^\circ - (60^\circ + 90^\circ) = 30^\circ$$

$$\therefore CE = \frac{1}{2} DE \quad \therefore EC = 5 \text{ cm.} \quad (2)$$

Adding (1) and (2):

$$\therefore BC = 2 + 5 = 7 \text{ cm.} \quad (\text{The req.})$$

13

In $\triangle ADB$:

$$\therefore m(\angle ADB) = 90^\circ, AE = EB$$

$$\therefore DE = \frac{1}{2} AB$$

Similarly in $\triangle ACB$:

$$\therefore m(\angle ACB) = 90^\circ, AE = EB$$

$$\therefore CE = \frac{1}{2} AB \quad \therefore DE = CE$$

$$\therefore \triangle CED \text{ is an isosceles triangle.} \quad (\text{Q.E.D.})$$

14

In $\triangle LYE$:

$$\therefore m(\angle YLE) = 90^\circ, m(\angle E) = 30^\circ$$

$$\therefore LY = \frac{1}{2} YE = 5 \text{ cm.}$$

In $\triangle ZYX$:

$$\therefore m(\angle ZYX) = 90^\circ, L \text{ is the midpoint of } \overline{ZX}$$

$$\therefore YL = \frac{1}{2} ZX \quad \therefore ZX = 10 \text{ cm.} \quad (\text{The req.})$$

15

In $\triangle ABC$:

$$\therefore m(\angle ABC) = 90^\circ, m(\angle C) = 30^\circ$$

$$\therefore AC = 2 AB = 14 \text{ cm.}$$

$\therefore D$ is the midpoint of \overline{AC}

$$\therefore BD = \frac{1}{2} AC = 7 \text{ cm.}$$

Geometry

• in $\triangle DEC$:

$$\therefore m(\angle DEC) = 90^\circ, m(\angle C) = 30^\circ$$

$$\therefore DE = \frac{1}{2} DC, \therefore DC = \frac{1}{2} AC = 7 \text{ cm.}$$

$$\therefore DE = 3.5 \text{ cm.} \quad (\text{The req.})$$

16

In $\triangle ABC$:

$$\therefore m(\angle ABC) = 90^\circ, m(\angle C) = 30^\circ$$

$$\therefore AB = \frac{1}{2} AC = 4 \text{ cm.}$$

$\therefore X$ is the midpoint of \overline{AB} , Y is the midpoint of \overline{BC}

$$\therefore XY = \frac{1}{2} AC = 4 \text{ cm.}$$

In $\triangle XBY$:

$$\therefore m(\angle XBY) = 90^\circ$$

$\therefore Z$ is the midpoint of \overline{XY}

$$\therefore BZ = \frac{1}{2} XY = 2 \text{ cm.} \quad (\text{The req.})$$

17

In $\triangle MED$:

$$\therefore m(\angle MED) = 90^\circ \therefore (MD)^2 = 3^2 + 4^2 = 25$$

$$\therefore MD = \sqrt{25} = 5 \text{ cm.}$$

$\therefore M$ is the point of concurrence of the medians of $\triangle ABC$

$$\therefore AD = 3 MD = 15 \text{ cm.}$$

$$\therefore m(\angle BAC) = 90^\circ$$

$\therefore \overline{AD}$ is a median in $\triangle ABC$

$$\therefore BC = 2 AD = 30 \text{ cm.} \quad (\text{The req.})$$

18

In $\triangle ABC$:

$$\therefore m(\angle BAC) = 90^\circ$$

$$\therefore (BC)^2 = (12)^2 + (9)^2 = 225$$

$$\therefore BC = \sqrt{225} = 15 \text{ cm.}$$

$\therefore \overline{AD}$ is a median in $\triangle ABC$, $m(\angle BAC) = 90^\circ$

$$\therefore AD = \frac{1}{2} BC = 7\frac{1}{2} \text{ cm.}$$

$\therefore M$ is the point of concurrence of the medians of $\triangle ABC$

$$\therefore AM = \frac{2}{3} AD = 5 \text{ cm.} \quad (\text{The req.})$$

19

$\therefore ABCD$ is a parallelogram

$$\therefore m(\angle C) = m(\angle A) = 60^\circ$$

\therefore In $\triangle DEC$:

$$m(\angle EDC) = 180^\circ - (60^\circ + 90^\circ) = 30^\circ$$

$$\therefore CE = \frac{1}{2} DC \quad \therefore DC = 8 \text{ cm.}$$

\therefore The perimeter of the parallelogram $ABCD$

$$= (12 + 8) \times 2 = 40 \text{ cm.} \quad (\text{The req.})$$

20

$$\therefore m(\angle BAD) = 90^\circ, m(\angle BAE) = 30^\circ$$

$$\therefore m(\angle DAF) = 60^\circ$$

From $\triangle AFD$:

$$\therefore m(\angle AFD) = 90^\circ, m(\angle DAF) = 60^\circ$$

$$\therefore m(\angle ADF) = 30^\circ \quad \therefore AD = 2 AF = 8 \text{ cm.}$$

$$\therefore \text{The area of the square} = 64 \text{ cm}^2 \quad (\text{The req.})$$

21

In $\triangle BCE$:

$$\therefore m(\angle EBC) = 30^\circ$$

$$\therefore CE = \frac{1}{2} BE \quad (1)$$

$$\therefore m(\angle EBC) = 30^\circ \quad \therefore m(\angle ABE) = 60^\circ$$

\therefore In $\triangle ABE$:

$$m(\angle EAB) = 30^\circ$$

$$\therefore m(\angle AEB) = 90^\circ$$

$$\therefore BE = \frac{1}{2} AB \quad (2)$$

From (1) and (2):

$$\therefore CE = \frac{1}{2} \times \frac{1}{2} AB = \frac{1}{4} AB \quad (\text{Q.E.D.})$$

22

In $\triangle ABC$:

$$\therefore m(\angle ABC) = 90^\circ, m(\angle A) = 30^\circ$$

$$\therefore AC = 2 BC = 16 \text{ cm.}$$

$$\therefore m(\angle C) = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$$

\therefore in $\triangle BCD$:

$$\therefore m(\angle BDC) = 90^\circ, m(\angle C) = 60^\circ$$

$$\therefore m(\angle CBD) = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$$

$$\therefore CD = \frac{1}{2} BC = 4 \text{ cm.}$$

$$\therefore AD = AC - CD = 16 - 4 = 12 \text{ cm.} \quad (\text{The req.})$$

23

In $\triangle ABC$:

$$\therefore m(\angle ABC) = 90^\circ, m(\angle C) = 30^\circ$$

$$\therefore m(\angle A) = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$$

\therefore In $\triangle ABE$:

$$\therefore m(\angle AEB) = 90^\circ, m(\angle A) = 60^\circ$$

$\therefore m(\angle ABE) = 30^\circ \quad \therefore AB = 2AE = 8 \text{ cm.}$
 In $\triangle ABC$:
 $AC = 2AB = 16 \text{ cm.}$
 $\therefore \overline{BD}$ is a median in $\triangle ABC$
 $\therefore BD = 8 \text{ cm.}, AD = 8 \text{ cm.}$
 \therefore The perimeter of $\triangle ABD = 8 + 8 + 8 = 24 \text{ cm.}$ (The req.)

24

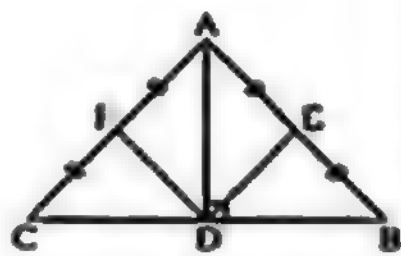
In $\triangle ABC$:
 $\therefore m(\angle B) = 30^\circ, m(\angle C) = 90^\circ$
 $\therefore AC = \frac{1}{2} AB$
 $\therefore E$ is the midpoint of \overline{BC}
 $\therefore O$ is the midpoint of \overline{AC}
 $\therefore EO = \frac{1}{2} AB \quad \therefore EO = AC$
 In $\triangle DEO$:
 $\therefore X$ is the midpoint of \overline{DE}
 $\therefore Y$ is the midpoint of \overline{DO}
 $\therefore XY = \frac{1}{2} EO \quad \therefore XY = \frac{1}{2} AC$ (Q.E.D.)

25

$\therefore ABCD$ is a parallelogram $\therefore \overline{AD} \parallel \overline{BC}$
 $\therefore \overline{XY} \parallel \overline{BC} \quad \therefore \overline{AD} \parallel \overline{XY} \parallel \overline{BC}$
 $\therefore \overline{AB}$ and \overline{EF} are transversals for them
 $\therefore EZ = ZF$
 In $\triangle EYF$:
 $\therefore m(\angle EYF) = 90^\circ, Z$ is the midpoint of \overline{EF}
 $\therefore YZ = \frac{1}{2} EF$ (Q.E.D.)

26

In $\triangle ADB$:
 $\therefore m(\angle ADB) = 90^\circ$
 $\therefore E$ is the midpoint of \overline{AB}
 $\therefore DE = \frac{1}{2} AB$
 In $\triangle ADC$:
 $\therefore m(\angle ADC) = 90^\circ$
 $\therefore F$ is the midpoint of $\overline{AC} \quad \therefore DF = \frac{1}{2} AC$
 $\therefore DE + DF = \frac{1}{2} AB + \frac{1}{2} AC$ but $AB = AC$ (Given)
 $\therefore DE + DF = \frac{1}{2} AB + \frac{1}{2} AB = AB$ (Q.E.D.)

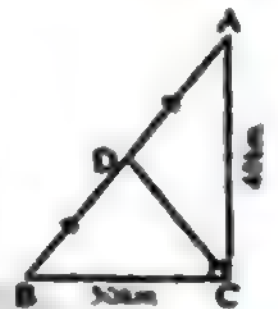


27

In $\triangle ABC$:
 $\therefore \overline{EO} \parallel \overline{AC}, E$ is the midpoint of \overline{AB}
 $\therefore O$ is the midpoint of \overline{BC}
 $\therefore BC = 4 + 12 = 16 \text{ cm.}$
 $\therefore BO = \frac{1}{2} BC = 8 \text{ cm.}$
 $\therefore DO = 8 - 4 = 4 \text{ cm.}$
 $\therefore BD = DO$
 $\therefore \overline{EO} \parallel \overline{AC}, \overline{AB}$ is a transversal
 $\therefore m(\angle BEO) = m(\angle A) = 90^\circ$ (corresponding angles)
 $\therefore ED = \frac{1}{2} BO = 4 \text{ cm.}$ (The req.)

28

Let the service station lie at the point D which is the midpoint of \overline{AB}
 \therefore The road length = the length of \overline{CD}

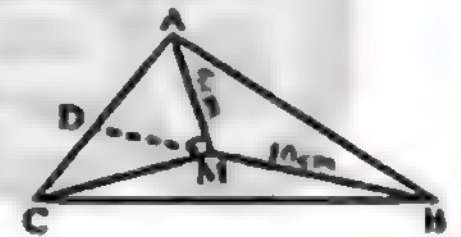


In $\triangle ACB$:
 $\therefore m(\angle ACB) = 90^\circ$
 $\therefore (AB)^2 = (AC)^2 + (BC)^2 = 1600 + 900 = 2500$
 $\therefore AB = 50 \text{ km.}$
 $\therefore D$ is the midpoint of \overline{AB}
 $\therefore CD = \frac{1}{2} AB = \frac{1}{2} \times 50 = 25 \text{ km.}$
 \therefore The length of the road 25 km. (The req.)

29

Constr : Draw \overline{BM} to intersect \overline{AC} at D

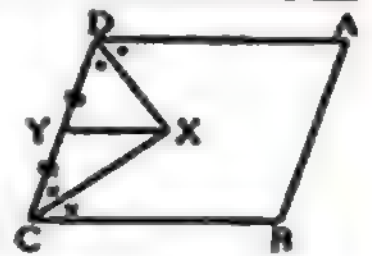
Proof : $\therefore M$ is the point of concurrence of the medians of $\triangle ABC$



$\therefore MD = \frac{1}{2} BM = 5 \text{ cm.}$
 In $\triangle AMC$: $\therefore m(\angle AMC) = 90^\circ$
 $\therefore \overline{MD}$ is a median
 $\therefore MD = \frac{1}{2} AC \quad \therefore AC = 10 \text{ cm.}$ (First req.)
 In $\triangle BMC$: $\therefore m(\angle BMC) = 90^\circ$
 $\therefore (MC)^2 = (10)^2 - (6)^2 = 64$
 $\therefore MC = \sqrt{64} = 8 \text{ cm.}$ (Second req.)

30

$\therefore \overline{DA} \parallel \overline{CB}$
 $\therefore \overline{DC}$ is a transversal
 $\therefore m(\angle ADC) + m(\angle DCB) = 180^\circ$
 $\therefore \frac{1}{2} m(\angle ADC) + \frac{1}{2} m(\angle DCB) = 90^\circ$
 $\therefore m(\angle XDC) + m(\angle DCX) = 90^\circ$



Geometry

but the sum of the measures of the interior angles of a triangle $XDC = 180^\circ$

$$\therefore m(\angle DXC) = 90^\circ, \therefore DY = YC$$

$$\therefore XY = \frac{1}{2} DC \quad \therefore XY = YC \quad (\text{Q.E.D.})$$

Answers of Exercise 3

1

$$\text{① } x = 50^\circ \quad \text{② } x = 56^\circ \quad \text{③ } y = 63^\circ$$

$$\text{④ } l = 65^\circ, z = 50^\circ \quad \text{⑤ } x = 54^\circ, y = 117^\circ$$

$$\text{⑥ } x = 69^\circ, y = 111^\circ \quad \text{⑦ } x = 120^\circ$$

$$\text{⑧ } x = 63^\circ, y = 54^\circ$$

2

$$\text{① congruent} \quad \text{② } 60^\circ \quad \text{③ } F$$

$$\text{④ } 50^\circ \quad \text{⑤ } 70^\circ \quad \text{⑥ } C, 50^\circ$$

3

$$\text{① } b \quad \text{② } c \quad \text{③ } b \quad \text{④ } a \quad \text{⑤ } b$$

4

In $\triangle ABC$:

$$\therefore AB = AC$$

$$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 40^\circ}{2} = 70^\circ \quad (\text{First req.})$$

$$\therefore m(\angle ABC) = m(\angle ACB) + \angle ABD \text{ supplements } \angle ABC$$

$$+ \angle ACE \text{ supplements } \angle ACB$$

\therefore The supplementaries of the congruent angles are congruent

$$\therefore \angle ABD = \angle ACE \quad (\text{Second req.})$$

5

From $\triangle ABC$:

$$\therefore AB = AC \quad \therefore m(\angle B) = m(\angle ACB) = 70^\circ$$

$$\therefore m(\angle BAC) = 180^\circ - (2 \times 70^\circ) = 40^\circ$$

In $\triangle ACD$:

$$\therefore AC = CD \quad \therefore m(\angle CAD) = m(\angle D)$$

$\therefore \angle ACB$ is an exterior angle of $\triangle ACD$

$$\therefore m(\angle ACB) = m(\angle CAD) + m(\angle D)$$

$$\therefore m(\angle CAD) = \frac{70^\circ}{2} = 35^\circ$$

$$\therefore m(\angle BAD) = m(\angle BAC) + m(\angle CAD)$$

$$= 40^\circ + 35^\circ = 75^\circ \quad (\text{The req.})$$

6

$\therefore \angle ACD$ is an exterior angle of $\triangle ABC$

$$\therefore m(\angle ACD) = 30^\circ + 40^\circ = 70^\circ$$

From $\triangle ACD$:

$$\therefore AC = AD$$

$$\therefore m(\angle D) = m(\angle ACD) = 70^\circ \quad (\text{First req.})$$

$$\therefore m(\angle CAD) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ \quad (\text{Second req.})$$

7

$\therefore \triangle ACD$ is an equilateral triangle

$$\therefore m(\angle CAD) = 60^\circ \quad (1)$$

From $\triangle ABC$:

$$\therefore AB = BC$$

$$\therefore m(\angle BAC) = m(\angle BCA) = \frac{180^\circ - 40^\circ}{2} = 70^\circ \quad (2)$$

From (1) and (2):

$$\therefore m(\angle BAD) = 60^\circ + 70^\circ = 130^\circ \quad (\text{The req.})$$

8

In $\triangle ABD$:

$$\therefore AB = AD$$

$$\therefore m(\angle ADB) = m(\angle ABD) = \frac{180^\circ - 120^\circ}{2} = 30^\circ \quad (\text{First req.})$$

$\therefore \overline{AD} \parallel \overline{BC}$, \overline{DC} is a transversal to them

$$\therefore m(\angle C) + m(\angle ADC) = 180^\circ$$

$$\therefore m(\angle C) = 180^\circ - (65^\circ + 30^\circ) = 85^\circ \quad (\text{Second req.})$$

9

$\therefore \overline{AD} \parallel \overline{BC}$, \overline{AC} is a transversal to them

$$\therefore m(\angle C) = m(\angle DAC) = 30^\circ \quad (\text{alternate angles})$$

In $\triangle ABC$:

$$\therefore AC = BC$$

$$\therefore m(\angle CAB) = m(\angle B) = \frac{180^\circ - 30^\circ}{2} = 75^\circ \quad (\text{The req.})$$

10

$\therefore \triangle DEC$ is an equilateral triangle

$$\therefore m(\angle ECD) = 60^\circ \quad (1)$$

From $\triangle ABC$:

$$\therefore AB = AC \quad \therefore m(\angle B) = m(\angle ACB)$$

$$\therefore m(\angle B) + m(\angle ACB) = 180^\circ - 80^\circ = 100^\circ$$

$$\therefore m(\angle B) = m(\angle ACB) = \frac{100^\circ}{2} = 50^\circ \quad (2)$$

From (1) and (2):

$$\therefore m(\angle BCD) = 50^\circ + 60^\circ = 110^\circ \quad (\text{The req.})$$

Answers of Unit 4

11

In ΔABC :

$$\therefore BA = BC, m(\angle B) = 80^\circ$$

$$\therefore m(\angle BAC) = m(\angle BCA) = \frac{180^\circ - 80^\circ}{2} = 50^\circ$$

$$\therefore m(\angle DAC) = 114^\circ - 50^\circ = 64^\circ$$

In ΔADC :

$$\therefore DA = DC, m(\angle DAC) = 64^\circ$$

$$\therefore m(\angle ADC) = 180^\circ - (64^\circ \times 2) = 52^\circ \quad (\text{The req.})$$

12

From ΔABC :

$$\therefore AB = AC$$

$$\therefore m(\angle B) = m(\angle BCA)$$

$$\therefore m(\angle B) + m(\angle BCA) = 180^\circ - 48^\circ = 132^\circ$$

$$\therefore m(\angle B) = m(\angle BCA) = \frac{132^\circ}{2} = 66^\circ \quad (\text{First req.})$$

$$\therefore \overline{CD} \text{ bisects } \angle ACB$$

$$\therefore m(\angle BCD) = \frac{66^\circ}{2} = 33^\circ \quad (\text{Second req.})$$

13

$$\therefore \Delta ABC \text{ is an equilateral triangle}$$

$$\therefore m(\angle ABC) = m(\angle ACB) = 60^\circ$$

$$\therefore \frac{1}{2} m(\angle ABC) = \frac{1}{2} m(\angle ACB) = 30^\circ$$

$$\therefore \overline{BD} \text{ bisects } \angle ABC, \overline{CD} \text{ bisects } \angle ACB$$

$$\therefore m(\angle DBC) = m(\angle DCB) = 30^\circ$$

$$\therefore \text{From } \Delta DBC:$$

$$m(\angle D) = 180^\circ - (2 \times 30^\circ) = 120^\circ \quad (\text{The req.})$$

14

$$\therefore \Delta ABC \text{ is an equilateral triangle}$$

$$\therefore m(\angle ABC) = 60^\circ \quad (1)$$

$$\text{From } \Delta DBC:$$

$$\therefore DB = DC, m(\angle D) = 100^\circ$$

$$\therefore m(\angle DBC) = m(\angle DCB) = \frac{180^\circ - 100^\circ}{2} = 40^\circ \quad (2)$$

$$\text{From (1) and (2):}$$

$$\therefore m(\angle ABD) = m(\angle ABC) - m(\angle DBC) \\ = 60^\circ - 40^\circ = 20^\circ \quad (\text{The req.})$$

15

$$\therefore \Delta ABC \text{ is an equilateral triangle}$$

$$\therefore m(\angle ACB) = m(\angle B) = m(\angle BAC) = 60^\circ$$

$$\therefore m(\angle ACD) = 120^\circ$$

In ΔACD :

$$\therefore AC = CD$$

$$\therefore m(\angle CAD) = m(\angle D)$$

$$\therefore m(\angle CAD) + m(\angle D) = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore m(\angle CAD) = \frac{60^\circ}{2} = 30^\circ$$

$$\therefore m(\angle BAD) = 60^\circ + 30^\circ = 90^\circ$$

$$\therefore \overline{BA} \perp \overline{AD} \quad (\text{Q.E.D.})$$

16

From ΔABC :

$$\therefore AB = AC$$

$$\therefore m(\angle B) = m(\angle C)$$

$$\therefore \Delta \Delta ABD, ACE \text{ in them:}$$

$$\begin{cases} AB = AC \\ m(\angle B) = m(\angle C) \\ BD = EC \end{cases}$$

$$\therefore \Delta ABD \cong \Delta ACE \text{ then we deduce that } AD = AE$$

$$\therefore \Delta ADE \text{ is an isosceles triangle} \quad (\text{Q.E.D. 1})$$

$$\therefore m(\angle ADE) = m(\angle AED)$$

$$\therefore \angle ADE = \angle AED \quad (\text{Q.E.D. 2})$$

17

$$\therefore \Delta \Delta ADE, BCE \text{ in them:}$$

$$\begin{cases} AD = CB \\ AE = EB \\ m(\angle A) = m(\angle B) \end{cases}$$

$$\therefore \Delta ADE \cong \Delta BCE, \text{ then we deduce that } DE = CE$$

$$\text{In } \Delta DEC:$$

$$\therefore DE = CE \quad \therefore m(\angle EDC) = m(\angle ECD)$$

$$\therefore m(\angle DEC) = 40^\circ$$

$$\therefore m(\angle EDC) + m(\angle ECD) = 180^\circ - 40^\circ = 140^\circ$$

$$\therefore m(\angle EDC) = \frac{140^\circ}{2} = 70^\circ \quad (\text{The req.})$$

18

$$\therefore \angle LZX \text{ is an exterior angle of } \Delta XYZ$$

$$\therefore m(\angle X) + m(\angle Y) = 130^\circ$$

$$\therefore ZX = ZY \quad \therefore m(\angle X) = m(\angle Y)$$

$$\therefore m(\angle Y) = \frac{130^\circ}{2} = 65^\circ$$

$$\therefore \overline{LM} \parallel \overline{XY}, \overline{LY} \text{ is a transversal to them}$$

$$\therefore m(\angle MLY) = m(\angle Y) = 65^\circ \quad (\text{The req.})$$

19

$$\therefore \overline{AE} \parallel \overline{BC} \text{ and } \overline{BD} \text{ is a transversal to them}$$

$$\therefore m(\angle B) = m(\angle DAE) \text{ (corresponding angles)}$$

Geometry

$\therefore \overline{AE} \parallel \overline{BC}$, \overline{AC} is a transversal to them.
 $\therefore m(\angle C) = m(\angle EAC)$ (alternate angles)
 but $m(\angle B) = m(\angle C)$ because $AB = AC$
 $\therefore m(\angle DAE) = m(\angle EAC)$
 i.e. \overline{AE} bisects $\angle DAC$ (Q.E.D.)

20

$\therefore B \in \overline{AD}$
 $\therefore m(\angle ABC) + m(\angle CBE) + m(\angle EBD) = 180^\circ$ (1)
 \therefore The sum of measures of the angles of the triangle = 180°
 $\therefore m(\angle ABC) + m(\angle A) + m(\angle C) = 180^\circ$ (2)
 From (1) and (2):
 $\therefore m(\angle CBE) + m(\angle EBD) = m(\angle A) + m(\angle C)$
 $\therefore m(\angle CBE) = m(\angle EBD)$ (Given)
 $\therefore m(\angle A) = m(\angle C)$ (because $BA = BC$)
 $\therefore m(\angle CBE) = m(\angle C)$ and they are alternate angles
 $\therefore \overline{BE} \parallel \overline{AC}$ (Q.E.D.)

21

In $\triangle DEC$:
 $\therefore DE = DC$
 $\therefore m(\angle DEC) = m(\angle C) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$
 $\therefore \overline{AD} \parallel \overline{EC}$, \overline{DE} is a transversal to them
 $\therefore m(\angle ADE) = m(\angle DEC) = 70^\circ$ (alternate angles)
 $\therefore AD = AE$
 $\therefore m(\angle AED) = m(\angle ADE) = 70^\circ$ (First req.)
 In $\triangle AED$:
 $\therefore m(\angle EAD) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$
 $\therefore m(\angle BAD) = m(\angle C) = 70^\circ$
 (from properties of the parallelogram)
 $\therefore m(\angle BAE) = 70^\circ - 40^\circ = 30^\circ$ (Second req.)

22

In $\triangle DBC$: $\therefore DB = DC$
 $\therefore m(\angle DCB) = m(\angle DBC) = \frac{180^\circ - 140^\circ}{2} = 20^\circ$
 $\therefore \overline{DE} \parallel \overline{BC}$, \overline{DC} is a transversal.
 $\therefore m(\angle EDC) = m(\angle DCB) = 20^\circ$ (alternate angles)
 In $\triangle DCE$: $\therefore DE = EC$
 $\therefore m(\angle DCE) = m(\angle EDC) = 20^\circ$
 $\therefore m(\angle ACB) = 20^\circ + 20^\circ = 40^\circ$
 From $\triangle ABC$:
 $m(\angle A) = 180^\circ - (20^\circ + 40^\circ) = 120^\circ$ (The req.)

23

From $\triangle ABC$:

$\therefore AB = AC \quad \therefore m(\angle B) = m(\angle C)$
 $\therefore 2x + 13 = 3x - 17 \quad \therefore x = 30^\circ$
 $\therefore m(\angle B) = m(\angle C) = 2 \times 30 + 13 = 73^\circ$
 $\therefore m(\angle A) = 180^\circ - (73^\circ + 73^\circ) = 34^\circ$ (The req.)

24

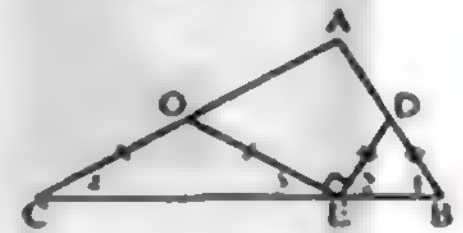
1 $x = 60^\circ, y = 121^\circ$ 2 $x = 45^\circ, y = 105^\circ$
 3 $x = 50^\circ, y = 32^\circ, z = 124^\circ$
 4 $x = 75^\circ, y = 15^\circ$ 5 $x = 25^\circ, y = 92^\circ$
 6 $y = 110^\circ, l = 40^\circ, z = 70^\circ$
 7 $x = 30^\circ, y = 40^\circ$ 8 $x = 70^\circ, y = 50^\circ$
 9 $x = 120^\circ$ 10 $x = 40^\circ$
 11 $x = 100^\circ$ 12 $x = 15^\circ$

25

1 3 cm. 2 5° 3 5°
 4 66.5° 5 7° 6 22°

26

In $\triangle DBE$:
 $\therefore DB = DE$
 $\therefore m(\angle 1) = m(\angle 2)$
 In $\triangle OEC$:
 $\therefore OE = OC \quad \therefore m(\angle 3) = m(\angle 4)$
 $\therefore E \in \overline{BC}$, $m(\angle DEO) = 90^\circ$
 $\therefore m(\angle 2) + m(\angle 3) = 90^\circ$
 $\therefore m(\angle 1) + m(\angle 4) = 90^\circ$
 In $\triangle ABC$:
 $\therefore m(\angle A) = 180^\circ - 90^\circ = 90^\circ$ (The req.)



27

In $\triangle EBD$ and $\triangle CBD$:

$\begin{cases} \overline{BD} \text{ is a common side} \\ m(\angle EBD) = m(\angle CBD) \\ m(\angle EDB) = m(\angle CDB) \end{cases}$
 $\therefore \triangle EBD \cong \triangle CBD$, then we deduce that:
 $BE = BC$, $m(\angle BED) = m(\angle C)$
 $\therefore BA = BC \quad \therefore BA = BE$
 $\therefore m(\angle A) = m(\angle BEA)$
 $\therefore m(\angle BEA) + m(\angle BED) = 180^\circ$
 $\therefore m(\angle A) + m(\angle C) = 180^\circ$ (Q.E.D.)

Answers of Unit 4

28

 $\Delta \Delta$ XYM , MZL in them :

$$\begin{cases} XY = MZ \\ YM = LZ \\ m(\angle Y) = m(\angle Z) = 90^\circ \end{cases}$$

 $\therefore \Delta$ XYM \cong Δ MZL , then we deduce that

$$XM = ML, m(\angle XMY) = m(\angle MLZ)$$

 $\therefore \angle$ MLZ complements \angle LMZ $\therefore \angle$ XMY complements \angle LMZ

$$\therefore m(\angle XML) = 90^\circ$$

 \therefore From Δ XLM :

$$\therefore MX = ML, m(\angle XML) = 90^\circ$$

$$\therefore m(\angle MXL) = m(\angle MLX) = \frac{180^\circ - 90^\circ}{2} = 45^\circ \quad (\text{The req.})$$

29

$$\therefore \overline{AB} \parallel \overline{CD}$$

 $\therefore \overline{AC}$ is a transversal to them

$$\therefore m(\angle BAC) + m(\angle ACD) = 180^\circ$$

(interior angles in the same side of the transversal)

 $\therefore \overline{AE}$ bisects \angle BAC , \overline{CE} bisects \angle ACD

$$\therefore m(\angle EAC) + m(\angle ECA) = 90^\circ$$

From Δ AEC :

$$\therefore m(\angle AEC) = 180^\circ - 90^\circ = 90^\circ$$

$$\therefore E \in \overline{BD}$$

$$\therefore m(\angle BEA) = 180^\circ - (90^\circ + 24^\circ) = 66^\circ$$

In Δ ABE :

$$\therefore BE = BA$$

$$\therefore m(\angle BEA) = m(\angle BAE) = 66^\circ$$

$$\therefore m(\angle ABE) = 180^\circ - 2 \times 66^\circ = 48^\circ \quad (\text{The req.})$$

30

 $\therefore \Delta$ ABD is an equilateral triangle

$$\therefore m(\angle ABD) = 60^\circ$$

 $\therefore \Delta$ CBD is an isosceles triangle where $CB = CD$

$$\therefore m(\angle CBD) = m(\angle CDB) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$$

$$\therefore m(\angle ABC) = 60^\circ + 65^\circ = 125^\circ \quad (\text{The req.})$$

31

From Δ BDC :

$$\therefore BD = CD$$

$$\therefore m(\angle DBC) = m(\angle BCD) \quad (1)$$

 $\therefore \angle$ ADB is an exterior angle of Δ CBD

$$\therefore m(\angle ADB) = m(\angle DBC) + m(\angle BCD)$$

$$\text{from (1)} : m(\angle ADB) = 2m(\angle BCD) \quad (2)$$

In Δ ABD :

$$\therefore AB = AD$$

$$\therefore m(\angle ABD) = m(\angle ADB)$$

from (2)

$$\therefore m(\angle ABD) = m(\angle ADB) = 2m(\angle BCD)$$

 $\therefore \angle$ BAE is an exterior angle of Δ ABD

$$\therefore m(\angle BAE) = m(\angle ABD) + m(\angle ADB)$$

$$= 2m(\angle BCD) + 2m(\angle BCD)$$

$$= 4m(\angle BCD) \quad (\text{Q.E.D.})$$

32

In Δ ABC :

$$\therefore BC = BA$$

$$\therefore m(\angle A) = m(\angle 1) = x$$

 $\therefore \angle$ 2 is an exterior angle of Δ ABC

$$\therefore m(\angle 2) = m(\angle A) + m(\angle 1) = x + x = 2x$$

In Δ DBC : $\therefore CB = CD$

$$\therefore m(\angle 3) = m(\angle 2) = 2x$$

 $\therefore \angle$ 4 is an exterior angle of Δ ACD

$$\therefore m(\angle 4) = m(\angle A) + m(\angle 3) = x + 2x = 3x \quad (1)$$

$$\therefore m(\angle DEB) = 180^\circ - 126^\circ = 54^\circ$$

In Δ CDE : $\therefore DC = DE$

$$\therefore m(\angle 4) = m(\angle DEB) = 54^\circ \quad (2)$$

From (1) and (2) : $\therefore 3x = 54^\circ$

$$\therefore x = \frac{54^\circ}{3} = 18^\circ \quad (\text{The req.})$$

Answers of Exercise 4

1

$$\text{1} \quad AB = AC$$

$$\text{2} \quad YX = YZ$$

$$\text{3} \quad XY = XZ$$

$$\text{4} \quad AB = AC = BC$$

$$\text{5} \quad ML = MN$$

$$\text{6} \quad BA = BC$$

$$\text{7} \quad ZX = ZY$$

$$\text{8} \quad CB = CA$$

$$\text{9} \quad AC = AB$$

2

$$\text{1} \quad \text{congruent , isosceles}$$

$$\text{2} \quad \text{equilateral}$$

$$\text{3} \quad \text{isosceles}$$

$$\text{4} \quad \text{isosceles}$$

$$\text{5} \quad \text{equilateral}$$

$$\text{6} \quad 6$$

Geometry

3

$\therefore B \in \overline{DC} \quad \therefore m(\angle ABC) = 180^\circ - 125^\circ = 55^\circ$
 In $\triangle ABC$: $m(\angle C) = 180^\circ - (55^\circ + 70^\circ) = 55^\circ$
 $\therefore m(\angle ABC) = m(\angle C)$
 $\therefore AB = AC$
 $\therefore \triangle ABC$ is an isosceles triangle. (Q.E.D.)

4

$\therefore Y \in \overline{ZL} \quad \therefore m(\angle XYZ) = 180^\circ - 120^\circ = 60^\circ$
 $\therefore XY = XZ$
 $\therefore \triangle XYZ$ is an equilateral triangle. (Q.E.D.)

5

$\therefore B \in \overline{AD} \quad \therefore m(\angle ABC) = 180^\circ - 120^\circ = 60^\circ$
 Similarly: $m(\angle ACB) = 60^\circ$
 $\therefore m(\angle A) = 180^\circ - (60^\circ + 60^\circ) = 60^\circ$
 $\therefore m(\angle A) = m(\angle ABC) = m(\angle ACB)$
 $\therefore \triangle ABC$ is an equilateral triangle. (Q.E.D.)

6

$\therefore \overline{AD} \parallel \overline{BC}$, \overline{DB} is a transversal to them
 $\therefore m(\angle DBC) = m(\angle ADB) = 40^\circ$ (alternate angles)
 In $\triangle DBC$: $m(\angle C) = 180^\circ - (100^\circ + 40^\circ) = 40^\circ$
 $\therefore m(\angle DBC) = m(\angle C) \quad \therefore DB = DC$
 $\therefore \triangle DBC$ is an isosceles triangle. (Q.E.D.)

7

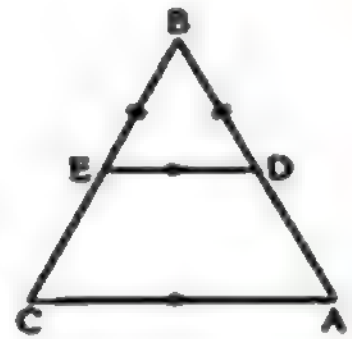
$\therefore \overline{XY} \parallel \overline{AC}$, \overline{AB} is a transversal to them
 $\therefore m(\angle A) = m(\angle ABX) = 62^\circ$ (alternate angles)
 $\therefore m(\angle ABC) = 180^\circ - (62^\circ + 56^\circ) = 62^\circ$
 $\therefore m(\angle ABC) = m(\angle A)$
 $\therefore CA = CB$ (Q.E.D.)

8

$\therefore AB = AC \quad \therefore m(\angle B) = m(\angle C)$ (1)
 $\therefore \overline{XY} \parallel \overline{BC}$, \overline{AB} is a transversal to them
 $\therefore m(\angle AXY) = m(\angle B)$ (corresponding angles) (2)
 Similarly $m(\angle AYX) = m(\angle C)$ (3)
 From (1), (2) and (3):
 $\therefore m(\angle AXY) = m(\angle AYX) \quad \therefore AX = AY$
 $\therefore \triangle AXY$ is an isosceles triangle (Q.E.D. 1)
 $\therefore AB = AC$, $AX = AY$ subtracting
 $\therefore XB = YC$ (Q.E.D. 2)

9

From $\triangle EBD$: $\therefore DB = EB$
 $\therefore m(\angle BDE) = m(\angle BED)$ (1)
 $\therefore \overline{DE} \parallel \overline{AC}$, \overline{AD} is a transversal to them
 $\therefore m(\angle A) = m(\angle BDE)$ (corresponding angles) (2)
 Similarly $m(\angle C) = m(\angle BED)$ (3)
 From (1), (2) and (3): $\therefore m(\angle A) = m(\angle C)$
 $\therefore AB = BC$ (Q.E.D.)



10

$\therefore MB = MC \quad \therefore m(\angle B) = m(\angle C)$ (1)
 $\therefore \overline{AD} \parallel \overline{BC}$ and \overline{AC} is a transversal to them
 $\therefore m(\angle A) = m(\angle C)$ (alternate angles) (2)
 similarly $m(\angle D) = m(\angle B)$ (3)
 from (1), (2) and (3) $\therefore m(\angle A) = m(\angle D)$
 $\therefore MA = MD$ (Q.E.D.)

11

$\therefore B \in \overline{AE} \quad \therefore \angle ABC$ supplements $\angle EBC$
 similarly $\angle ACB$ supplements $\angle ACD$
 $\therefore m(\angle EBC) = m(\angle ACD)$
 $\therefore m(\angle ABC) = m(\angle ACB)$
 $\therefore AB = AC = 8 \text{ cm.}$
 \therefore The perimeter of $\triangle ABC = 8 + 8 + 10 = 26 \text{ cm.}$
 (The req.)

12

$\therefore AB = AC \quad \therefore m(\angle B) = m(\angle C)$ (1)
 $\therefore \overline{AB} \parallel \overline{DE}$, \overline{BE} is a transversal to them
 $\therefore m(\angle B) = m(\angle DEF)$ (corresponding angles) (2)
 similarly $m(\angle C) = m(\angle DFE)$ (3)
 from (1), (2) and (3)
 $\therefore m(\angle DEF) = m(\angle DFE)$
 $\therefore DE = DF$ (Q.E.D. 1)
 In $\triangle ABC$, DEF
 $\therefore m(\angle B) = m(\angle DEF)$, $m(\angle C) = m(\angle DFE)$
 $\therefore m(\angle BAC) = m(\angle EDF)$ (Q.E.D. 2)

13

$\therefore \overline{ED} \parallel \overline{BC}$, \overline{DB} is a transversal to them
 $\therefore m(\angle EDB) = m(\angle DBC)$ (alternate angles)

but $m(\angle FBI) = m(\angle DBC)$

$\therefore m(\angle EDB) = m(\angle EBD)$

$\therefore EB = ED$

$\therefore \triangle EBD$ is an isosceles triangle

(Q.E.D.)

14

$\therefore \overline{AE} \parallel \overline{BC}$ and \overline{DB} is a transversal to them

$\therefore m(\angle DAE) = m(\angle B)$ (corresponding angles)

$\therefore \overline{AE} \parallel \overline{BC}$, \overline{AC} is a transversal to them

$\therefore m(\angle EAC) = m(\angle C)$ (alternate angles)

but $m(\angle DAE) = m(\angle EAC)$

$\therefore m(\angle B) = m(\angle C)$

$\therefore AB = AC$

(Q.E.D.)

15

$\therefore m(\angle ABC) = m(\angle ACB)$

$\therefore AB = AC$

$\therefore \triangle ADB \cong \triangle AEC$

in them $\begin{cases} AB = AC \\ DB = EC \\ m(\angle D) = m(\angle E) = 90^\circ \end{cases}$

$\therefore \triangle ADB \cong \triangle AEC$

$\therefore m(\angle DAB) = m(\angle CAE)$

(Q.E.D.)

16

In $\triangle YZX$:

$\therefore YZ = YX \quad \therefore m(\angle Z) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$

$\therefore m(\angle ZMX) = 50^\circ + 15^\circ = 65^\circ$

\therefore In $\triangle MZX$: $m(\angle Z) = m(\angle ZMX) \quad \therefore MX = ZX$

$\therefore \triangle MZX$ is an isosceles triangle (Q.E.D.)

17

In $\triangle ABC$: $\therefore AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC) = \frac{180^\circ - 70^\circ}{2} = 55^\circ$

$\therefore m(\angle MCA) = 25^\circ \quad \therefore m(\angle MCB) = 55^\circ - 25^\circ = 30^\circ$

$\therefore m(\angle MBC) = m(\angle MCB) \quad \therefore MB = MC$

$\therefore \triangle MBC$ is an isosceles triangle. (Q.E.D.)

18

$\therefore \angle ADC$ is an exterior angle of $\triangle ADB$

$\therefore m(\angle ADC) = 40^\circ + 30^\circ = 70^\circ$

$\therefore AD = AC \quad \therefore m(\angle C) = m(\angle ADC) = 70^\circ$

\therefore In $\triangle ABC$: $m(\angle BAC) = 180^\circ - (40^\circ + 70^\circ) = 70^\circ$

$\therefore m(\angle BAC) = m(\angle C) \quad \therefore AB = BC$ (Q.E.D.)

19

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

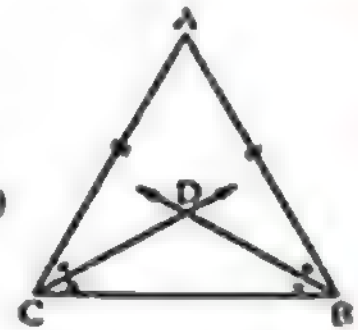
$\therefore \frac{1}{2} m(\angle ABC) = \frac{1}{2} m(\angle ACB)$

$\therefore m(\angle DBC) = \frac{1}{2} m(\angle ABC)$

$m(\angle DCB) = \frac{1}{2} m(\angle ACB)$

$\therefore m(\angle DBC) = m(\angle DCB) \quad \therefore DB = DC$

$\therefore \triangle DBC$ is an isosceles triangle (Q.E.D.)



20

$\therefore \triangle ABC$ is an equilateral triangle

$\therefore m(\angle ACB) = 60^\circ$

$\therefore \angle ACB$ is an exterior angle of $\triangle DCF$

$\therefore m(\angle D) = 60^\circ - 30^\circ = 30^\circ$

$\therefore m(\angle D) = m(\angle F)$

$\therefore CD = CF$

$\therefore \triangle DCF$ is an isosceles triangle

(Q.E.D.)

21

$\therefore DA = DC$

$\therefore m(\angle C) = m(\angle DAC) = 30^\circ$

$\therefore \angle ADB$ is an exterior angle of $\triangle ADC$

$\therefore m(\angle ADB) = 30^\circ + 30^\circ = 60^\circ$

$\therefore DA = DB$

$\therefore \triangle ABD$ is an equilateral triangle

(Q.E.D. 1)

$\therefore m(\angle BAD) = 60^\circ, m(\angle DAC) = 30^\circ$

$\therefore m(\angle BAC) = 90^\circ$

$\therefore \triangle ABC$ is a right-angled triangle

(Q.E.D. 2)

22

$\therefore \overline{ED} \parallel \overline{AC}$, \overline{EC} is a transversal to them

$\therefore m(\angle DEC) = m(\angle ACE)$ (alternate angles)

$\therefore m(\angle DEC) = m(\angle AEC)$

$\therefore m(\angle ACE) = m(\angle AEC)$

$\therefore AE = AC$

(1)

$\therefore \overline{DE} \parallel \overline{AC}$, \overline{AB} is a transversal to them

$\therefore m(\angle A) = m(\angle BED) = 60^\circ$

(corresponding angles)

(2)

from (1) and (2)

$\therefore \triangle AEC$ is an equilateral triangle.

(Q.E.D.)

23

In $\triangle ABC$: $m(\angle ACB) = 180^\circ - (60^\circ + 90^\circ) = 30^\circ$

In $\triangle ECD$: $m(\angle ECD) = 180^\circ - (30^\circ + 90^\circ) = 60^\circ$

Geometry

$$\therefore C \in \overline{BD}$$

$$\therefore m(\angle ACE) = 180^\circ - (30^\circ + 60^\circ) = 90^\circ$$

$$\text{In } \triangle ACE: m(\angle CAE) = 180^\circ - (90^\circ + 45^\circ) = 45^\circ$$

$$\therefore m(\angle CAE) = m(\angle CEA) = 45^\circ \quad \therefore CA = CE$$

$$\text{In } \triangle ECD: \therefore m(\angle D) = 90^\circ, m(\angle CED) = 30^\circ$$

$$\therefore CE = 2 CD = 6 \text{ cm.}$$

$$\text{but } AC = CE$$

$$\therefore AC = 6 \text{ cm.}$$

(The req.)

24

$$\text{In } \triangle ADE: \therefore \angle ADE = \angle AED \quad \therefore AD = AE$$

$$D \in \overline{BC}, E \in \overline{BC}$$

$$\therefore \angle ADB \text{ supplements } \angle ADE,$$

$$\angle AEC \text{ supplements } \angle AED$$

$$\text{but } m(\angle ADE) = m(\angle AED)$$

$$\therefore m(\angle ADB) = m(\angle AEC)$$

(supplementaries of the congruent angles are congruent)

$$\therefore \triangle ADB \cong \triangle AEC \text{ in them:}$$

$$\begin{cases} m(\angle ADB) = m(\angle AEC) \\ AD = AE \\ BD = CE \end{cases}$$

$$\therefore \triangle ADB \cong \triangle AEC$$

we deduce that $AB = AC$

$$\therefore \triangle ABC \text{ is an isosceles triangle.} \quad (\text{Q.E.D.})$$

25

$$\therefore BY = ZC \text{ then adding } YC \text{ to both sides}$$

$$\therefore BC = ZY$$

$$\therefore \triangle ABC \cong \triangle XZY \text{ in them:}$$

$$\begin{cases} AB = XZ \\ BC = ZY \\ m(\angle B) = m(\angle Z) \end{cases}$$

$$\therefore \triangle ABC \cong \triangle XZY$$

we deduce that $m(\angle ACB) = m(\angle XYZ)$

$$\therefore EC = EY$$

$$\therefore \triangle EYC \text{ is an isosceles triangle.} \quad (\text{Q.E.D.})$$

26

$$\text{In } \triangle BMC:$$

$$\therefore m(\angle MBC) = m(\angle MCB)$$

$$\therefore MB = MC$$

$$\therefore m(\angle ABM) = m(\angle MCD)$$

(complementaries of equal angles in measure are equal in measure)

$$\therefore \triangle ABM \cong \triangle DCM \text{ in them:}$$

$$\begin{cases} AB = DC \text{ (two sides in a square)} \\ BM = CM \text{ (proved)} \\ m(\angle ABM) = m(\angle DCM) \text{ (proved)} \end{cases}$$

$$\therefore \triangle ABM \cong \triangle DCM \text{ we deduce that } AM = DM$$

$$\therefore \triangle AMD \text{ is an isosceles triangle} \quad (\text{Q.E.D.})$$

27

$$\text{In } \triangle ABF \cong \triangle AME: m(\angle B) = m(\angle AME) = 90^\circ$$

$$m(\angle BAF) = m(\angle MAE) \text{ (AE bisects } \angle BAC)$$

$$\therefore m(\angle AFB) = m(\angle E) \quad (1)$$

$$\therefore \overline{AD} \parallel \overline{BF}, \overline{AF} \text{ is a transversal to them.}$$

$$\therefore m(\angle DAE) = m(\angle AFB) \text{ (alternate angles)} \quad (2)$$

from (1) and (2)

$$\therefore m(\angle E) = m(\angle DAE)$$

$$\therefore DA = DE \quad (\text{Q.E.D.})$$

28

$$\therefore m(\angle EAM) = m(\angle EMA) \quad \therefore EA = EM$$

$$\therefore \overline{AE} \text{ is a median in } \triangle ABD, m(\angle BAD) = 90^\circ$$

$$\therefore AE = \frac{1}{2} BD \quad \therefore EM = \frac{1}{2} BD \quad (1)$$

$$\therefore E \text{ is the midpoint of } \overline{BD}, \overline{EM} \parallel \overline{BC}$$

$$\therefore EM = \frac{1}{2} BC \quad (2)$$

from (1) and (2)

$$\therefore \frac{1}{2} BD = \frac{1}{2} BC \quad \therefore BD = BC \quad (\text{Q.E.D.})$$

29

$$\therefore m(\angle B) = m(\angle C) \quad \therefore AB = AC$$

$$\therefore 2x - 1 = x + 3 \quad \therefore 2x - x = 3 + 1 \quad \therefore x = 4$$

$$\therefore AB = AC = 2 \times 4 - 1 = 7 \text{ cm.}, BC = 9 - 4 = 5 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle ABC = 7 + 7 + 5 = 19 \text{ cm. (The req.)}$$

30

$$1 \mid \therefore 3x + x + 50^\circ + 30^\circ = 180^\circ$$

$$\therefore 4x + 80^\circ = 180^\circ$$

$$\therefore 4x = 180^\circ - 80^\circ = 100^\circ \quad \therefore x = \frac{100^\circ}{4} = 25^\circ$$

$$\therefore m(\angle A) = 3 \times 25^\circ = 75^\circ,$$

$$m(\angle B) = 25^\circ + 50^\circ = 75^\circ$$

$$\therefore m(\angle A) = m(\angle B) \quad \therefore CB = CA$$

Answers of Unit 4

2 $\therefore 2z + 3z - 10^\circ + z + 40^\circ = 180^\circ$
 $\therefore 6z + 30^\circ = 180^\circ$
 $\therefore 6z = 180^\circ - 30^\circ = 150^\circ \therefore z = \frac{150^\circ}{6} = 25^\circ$
 $\therefore m(\angle B) = 3 \times 25^\circ - 10^\circ = 65^\circ$
 $\therefore m(\angle C) = 25^\circ + 40^\circ = 65^\circ$
 $\therefore m(\angle B) = m(\angle C) \therefore AB = AC$

3 $\therefore \angle DBC$ is an exterior angle of $\triangle ABC$
 $\therefore 3x = x - 20^\circ + x + 70^\circ$
 $\therefore 3x = 2x + 50^\circ \therefore x = 50^\circ$
 $\therefore m(\angle A) = 50^\circ - 20^\circ = 30^\circ$
 $m(\angle C) = 50^\circ + 70^\circ = 120^\circ$
 $\therefore m(\angle ABC) = 180^\circ - (30^\circ + 120^\circ) = 30^\circ$
 $\therefore m(\angle A) = m(\angle ABC) \therefore CB = CA$

31

1 b 2 c 3 b

32

$\therefore \overline{AB} \parallel \overline{CE}$
 $\therefore m(\angle A) = m(\angle ACE) = 40^\circ$
 $\therefore m(\angle BCE) = 80^\circ$
 $\therefore m(\angle BCA) = 80^\circ - 40^\circ = 40^\circ$
 $\therefore m(\angle A) = m(\angle BCA) = 40^\circ$
 $\therefore BC = BA = 2.4 \text{ km.}$ (The req.)

33

$\therefore Y \in \overline{XB}, m(\angle BYC) = 120^\circ$
 $\therefore m(\angle XYZ) = 60^\circ$
 $\therefore XY = YZ$
 $\therefore \triangle XYZ$ is an equilateral triangle
 $\therefore XY = YZ = XZ$
 $\therefore BY = ZC = AX$
 Adding $\therefore BY + XY = YZ + ZC = AX + XZ$
 $\therefore BX = YC = AZ$
 $\therefore X \in \overline{AZ}$
 $\therefore m(\angle AXB) = 180^\circ - 60^\circ = 120^\circ$
 similarly $\therefore Z \in \overline{YC} \therefore m(\angle AZC) = 120^\circ$
 $\therefore \triangle AXB, \triangle BYC$ in them:
 $\begin{cases} AX = BY \text{ (given)} \\ XB = YC \text{ (proved)} \\ m(\angle AXB) = m(\angle BYC) = 120^\circ \text{ (proved)} \end{cases}$

$\therefore \triangle AXB \cong \triangle BYC$ we deduce that $AB = BC$
 similarly $\triangle AXB \cong \triangle CZA$ we deduce that
 $AB = AC \therefore AB = AC = BC$
 $\therefore \triangle ABC$ is an equilateral triangle (Q.E.D.)

Answers of Exercise 5

1

- 1 An axis of symmetry. 2 3 3 1 4 zero
 5 Bisects it and it is perpendicular to the base.
 6 Bisects the base and is perpendicular to it.
 7 Bisects each of the base and the vertex angle.
 8 The straight line perpendicular to it at its middle.
 9 at equal distances 10 AC, BC
 11 3 12 1 13 3 14 1 15 30°

2

- 1 35° 2 70° 3 55° 4 2 5 \overline{AD}

3

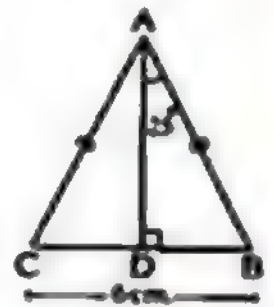
- 1 30° 2 55° 3 60° 4 4 5 8
 6 $4\sqrt{3}$ 7 3 8 1 9 $16\sqrt{3}$

4

$\therefore BA = BC, \overline{BD} \perp \overline{AC}$
 $\therefore \overline{BD}$ bisects each of $\angle ABC, \overline{AC}$
 $\therefore AC = 2AD = 40 \text{ cm.}$
 $\therefore m(\angle DBC) = \frac{1}{2} m(\angle ABC) = 45^\circ$ (1) (First req.)
 $\therefore \triangle ABC$ in which $m(\angle B) = 90^\circ, BA = BC$
 $\therefore m(\angle C) = 45^\circ$ (2)
 From (1) and (2) $\therefore DB = DC$
 $\therefore \triangle DBC$ is an isosceles triangle (Second req.)

5

$\therefore AB = AC, \overline{AD} \perp \overline{BC}$
 $\therefore \overline{AD}$ bisects each of $\angle BAC, \overline{BC}$
 $\therefore BD = \frac{1}{2} BC = 3 \text{ cm.}$ (First req.)
 \therefore In $\triangle ABD: m(\angle B)$
 $= 180^\circ - (90^\circ + 25^\circ) = 65^\circ$ (Second req.)

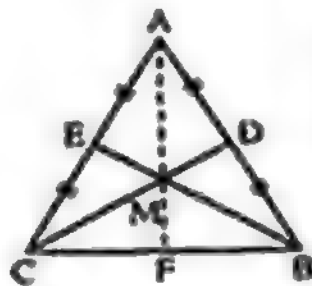


Geometry

6

In $\triangle ABC$: $AB = AC$,

M is the point of intersection of its medians

 $\therefore \overline{AF}$ is a median of $\triangle ABC$ $\therefore \overline{AM} \perp \overline{BC}$ $\therefore \overline{AM}$ bisects $\angle BAC$ 

(Q.E.D. 1)

(Q.E.D. 2)

7

In $\triangle ABC$: $\because AB = AC$, $\overline{AD} \perp \overline{BC}$ $\therefore BC = 2 \times 5 = 10$ cm.

(First req.)

In the right-angled triangle ADB at D

 $AD = \sqrt{(13)^2 - (5)^2} = 12$ cm. \therefore The area of $\triangle ABC = \frac{1}{2} \times 10 \times 12 = 60$ cm²

(Second req.)

8

 $\because AB = AC$, $\overline{AD} \perp \overline{BC} \therefore BD = \frac{1}{2} BC = 5$ cm. $\therefore m(\angle BAC) = 2 \times 30^\circ = 60^\circ$ $\therefore \triangle ABC$ is an equilateral triangle $\therefore AB = 10$ cm. \therefore In $\triangle ADB$ which is right-angled at D $\therefore AD = \sqrt{(10)^2 - (5)^2} = 5\sqrt{3}$ cm.

(First req.)

The number of axes of symmetry of $\triangle ABC = 3$

(Second req.)

The area of $\triangle ABC = \frac{1}{2} \times 10 \times 5\sqrt{3} = 25\sqrt{3}$ cm²

(Third req.)

9

In $\triangle ABC$: $\because AB = AC$, \overline{AE} bisects $\angle BAC$ $\therefore BE = \frac{1}{2} BC$

(Q.E.D.1)

 $\therefore \overline{AE} \perp \overline{BC}$ $\therefore \overline{AE}$ is the axis of symmetry of \overline{BC} , $D \in \overline{AE}$ $\therefore BD = CD$

(Q.E.D.2)

10

 $\because C \in \overline{BD}$, $m(\angle ACD) = 130^\circ$ $\therefore m(\angle ACB) = 180^\circ - 130^\circ = 50^\circ$ From $\triangle ABC$: $m(\angle B) = 180^\circ - (80^\circ + 50^\circ) = 50^\circ$ $\therefore m(\angle B) = m(\angle ACB)$ $\therefore \triangle ABC$ is an isosceles triangle $\therefore \overline{AE}$ bisects $\angle BAC$ $\therefore \overline{AE} \perp \overline{BC}$, E is the midpoint of \overline{BC}

(Q.E.D.)

11

 $\therefore m(\angle ABX) = m(\angle ACY)$ $\therefore m(\angle ABC) = m(\angle ACB)$

(The supplementaries of congruent angles are congruent)

 $\therefore AB = AC$ $\therefore \overline{AD}$ is a median of $\triangle ABC$ which is isosceles $\therefore \overline{AD} \perp \overline{BC}$

(Q.E.D.)

12

 $\because \overline{AD} \parallel \overline{BC}$, \overline{DB} is a transversal to them $\therefore m(\angle ADB) = m(\angle DBC)$ (alternate angles)but $m(\angle ABD) = m(\angle DBC)$ $\therefore m(\angle ADB) = m(\angle ABD)$ \therefore In $\triangle ABD$: $AB = AD$

(Q.E.D.1)

 $\therefore \overline{AE}$ bisects $\angle BAD \therefore \overline{AE} \perp \overline{BD}$

(Q.E.D.2)

 $\therefore BE = ED$

(Q.E.D.3)

13

In $\triangle ACD$: $\therefore E$ is the midpoint of \overline{AD} $\therefore \overline{CE} \perp \overline{AD}$ $\therefore DC = AC$ $\therefore \triangle ACD$ is an isosceles triangle. $\therefore \angle ADC$ is an exterior angle of $\triangle ADB$ $\therefore m(\angle ADC) = 20^\circ + 30^\circ = 50^\circ$ From $\triangle CDE$: $m(\angle DCE) = 180^\circ - (90^\circ + 50^\circ) = 40^\circ$ $\therefore \overline{CE}$ bisects $\angle ACD$ $\therefore m(\angle ACE) = m(\angle DCE) = 40^\circ$

(The req.)

14

In $\triangle ADC$: $\therefore E$ is the midpoint of \overline{DC} $\therefore \overline{AE} \perp \overline{DC}$ $\therefore AD = AC$ $\therefore \triangle ADC$ is an isosceles triangle $\therefore m(\angle ADC) = m(\angle C) = 70^\circ$ $\therefore \angle ADC$ is an exterior angle of $\triangle ABD$ $\therefore m(\angle ADC) = m(\angle B) + m(\angle BAD)$ $\therefore BD = AC$, $AD = AC$ $\therefore m(\angle B) = m(\angle BAD) = \frac{70^\circ}{2} = 35^\circ$

(The req.)

15

In $\triangle XYL$: $\because XL = XY$, M is the midpoint of \overline{LY} $\therefore \overline{XM}$ is the axis of \overline{LY}

similarly in $\triangle ZYL$, \overline{ZM} is the axis of \overline{LY}
 $\therefore X, M$ and Z are on the same straight line. (Q.E.D.)

16

$\therefore AB = AC \therefore A \in$ the axis of \overline{BC} (1)

$\therefore m(\angle ABC) = m(\angle ACB)$,

$\therefore m(\angle ABD) = m(\angle ACD)$

by subtracting :

$\therefore m(\angle DBC) = m(\angle DCB)$

$\therefore DB = DC \therefore D \in$ the axis of \overline{BC} (2)

From (1) and (2) :

$\therefore \overline{AD}$ is the axis of \overline{BC} (Q.E.D.)

17

$\therefore \overline{AD}$ bisects the base of $\triangle ABC$ which is an isosceles triangle

$\therefore \overline{AD} \perp \overline{BC} \therefore m(\angle ADB) = 90^\circ$

$\therefore \overline{XY} \parallel \overline{BC}$, \overline{AD} is a transversal to them

$\therefore m(\angle YAD) = m(\angle ADB) = 90^\circ$ (alternate angles)

$\therefore \overline{AD} \perp \overline{XY}$ (Q.E.D.)

18

$\therefore AB = AC, EB = EC \therefore \overline{AE}$ is the axis of \overline{BC}

$\therefore BD = DC$ (First req.)

$\therefore DC = 3$ cm.

In $\triangle ADC$ which is right-angled at D

$\therefore AD = \sqrt{(10)^2 - (3)^2} = \sqrt{100 - 9} = \sqrt{91}$ cm. (Second req.)

19 Constr. :

Draw $\overline{MF} \perp \overline{BC}$ to meet \overline{BC} at F and \overline{AD} at E

Proof: $\therefore \overline{AD} \parallel \overline{BC}$, \overline{AC} is a transversal to them

$\therefore m(\angle A) = m(\angle C)$ similarly $m(\angle B) = m(\angle D)$

$\therefore MB = MC \therefore m(\angle B) = m(\angle C)$

$\therefore m(\angle A) = m(\angle D) \therefore AM = DM$

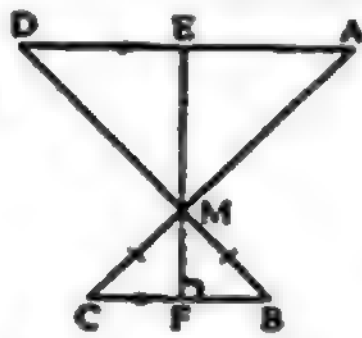
$\therefore \triangle AMD$ is an isosceles triangle (Q.E.D.1)

In $\triangle MBC$: $\therefore MB = MC, \overline{MF} \perp \overline{BC}$

$\therefore \overline{MF}$ is the axis of symmetry of $\triangle MBC$

$\therefore \overline{AD} \parallel \overline{BC}$, \overline{FE} is a transversal to them

$\therefore m(\angle AEM) = m(\angle BFM) = 90^\circ$



$\therefore \overline{ME} \perp \overline{AD}$

$\therefore MA = MD \therefore \overline{ME}$ is the axis of $\triangle AMD$

$\therefore \overline{EF}$ is the axis of symmetry of each of $\triangle AMD$

$\triangle BMC$ (Q.E.D.2)

20

$\therefore AB = AC$

$\therefore m(\angle 1) = m(\angle 4)$ (1)

$\therefore m(\angle DBC) = 180^\circ - m(\angle 1)$ (2)

$\therefore m(\angle BCE) = 180^\circ - m(\angle 4)$ (3)

From (1), (2) and (3)

$\therefore m(\angle DBC) = m(\angle BCE)$

$\therefore \frac{1}{2} m(\angle DBC) = \frac{1}{2} m(\angle BCE)$

$\therefore m(\angle 2) = m(\angle 5)$

$\therefore FB = FC \therefore \triangle BFC$ is an isosceles triangle (Q.E.D.1)

$\therefore AB = AC, FB = FC$

$\therefore \overline{AF}$ is the axis of \overline{BC} (Q.E.D.2)



21

Constr. : Draw $\overline{BD}, \overline{BE}$

Proof: $\triangle ABE, CBD$ in them:

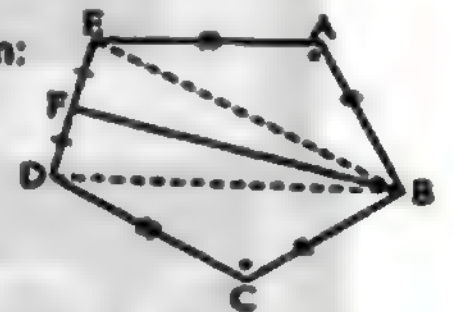
$$\begin{cases} m(\angle A) = m(\angle C) \\ AB = CB, \\ AE = CD \end{cases}$$

$\therefore \triangle ABE \cong \triangle CBD$

\therefore then we deduce that: $BE = BD$

$\therefore \overline{BF}$ is a median of $\triangle BED$ which is isosceles

$\therefore \overline{BF} \perp \overline{DE}$ (Q.E.D.)



22

In $\triangle ABD$:

$\therefore E$ is the midpoint of \overline{AB}

$\therefore \overline{DE} \perp \overline{AB} \therefore DA = DB$

$\therefore m(\angle A) = m(\angle ABD)$ (1)

\therefore in $\triangle DBC$:

$\therefore O$ is the midpoint of \overline{BC}

$\therefore \overline{DO} \perp \overline{BC} \therefore DB = DC$

$\therefore m(\angle DBC) = m(\angle C)$ (2)

$\therefore m(\angle ABD) + m(\angle DBC) = 130^\circ$ (3)

From (1), (2) and (3):

$\therefore m(\angle A) + m(\angle C) = 130^\circ$

From the quadrilateral $ABCD$

$\therefore m(\angle ADC) = 360^\circ - (130^\circ + 130^\circ) = 100^\circ$ (The req.)

Geometry

23

- 1 c 2 b 3 c 4 b 5 a 6 b

24

In $\triangle ABD$:

$$\therefore m(\angle BDA) = 90^\circ,$$

$$AD = \frac{1}{2} AB$$

$$\therefore m(\angle B) = 30^\circ, m(\angle BAD) = 60^\circ$$

In $\triangle ABC$:

$$\therefore \overline{AD} \perp \overline{BC}$$

$$\therefore \overline{AD} \text{ bisects } \angle BAC$$

$$\therefore m(\angle BAC) = 60^\circ \times 2 = 120^\circ$$

$$\therefore BA = CA$$

$$\therefore m(\angle C) = m(\angle B) = 30^\circ$$

(First req.)

$$\therefore \triangle ABD \text{ is right-angled at D}$$

$$\therefore (BD)^2 = (AB)^2 - (AD)^2$$

$$\therefore (BD)^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\therefore BD = \sqrt{\frac{3}{4}} = \frac{1}{2} \sqrt{3} \text{ km.}$$

$$\therefore \overline{AD} \perp \overline{BC}$$

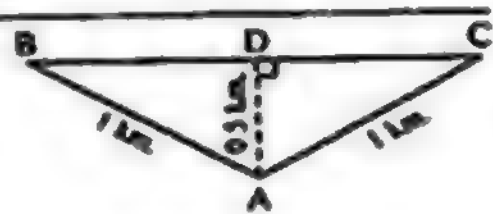
$$\therefore D \text{ is the midpoint of } \overline{BC}$$

$$\therefore BC = 2 \times \frac{1}{2} \sqrt{3} = \sqrt{3} \text{ km.}$$

$$\therefore \text{The distance} = 1 + \sqrt{3} + 1 = (2 + \sqrt{3}) \text{ km.}$$

$$= 4 \text{ km.}$$

(Second req.)



25

Constr.:

Draw \overline{AD} , \overline{AC}

Proof:

$$\therefore ABCDE \text{ is a regular pentagon}$$

$$\therefore \text{The measure of each interior angle} = 108^\circ$$

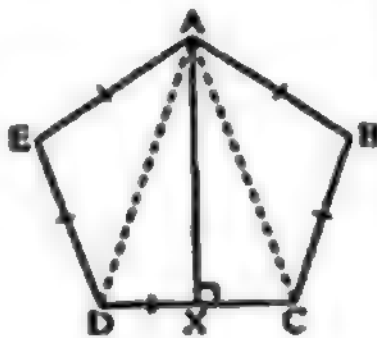
$$\therefore AB = BC = CD = DE = EA$$

$$\text{In } \triangle ABC: \therefore AB = BC, m(\angle ABC) = 108^\circ$$

$$\therefore m(\angle BAC) = \frac{180^\circ - 108^\circ}{2} = 36^\circ$$

$$\therefore \triangle ABC, \triangle AED \text{ in them:}$$

$$\begin{cases} AB = AE \\ BC = ED \\ m(\angle ABC) = m(\angle AED) = 108^\circ \end{cases}$$



$$\therefore \triangle ABC \cong \triangle AED$$

$$\text{Then we deduce that: } m(\angle EAD) = m(\angle BAC) = 36^\circ$$

$$\therefore AC = AD \therefore \triangle ADC \text{ is an isosceles triangle}$$

$$\therefore \overline{AX} \perp \overline{CD}$$

$$\therefore m(\angle CAX) = m(\angle DAX)$$

$$= \frac{108^\circ - (36^\circ + 36^\circ)}{2} = 18^\circ$$

(The req.)

Answers of exams on unit four

Model 1

1

1 b

2 d

3 d

4 c

5 b

6 b

2

1 4

2 half the length of the hypotenuse

3 bisects the vertex angle, perpendicular to the base

4 the angle at this vertex is right

5 $65^\circ, 50^\circ$

3

[a] The perimeter of $\triangle DEF = 22 \frac{1}{2}$ cm.

[b] Prove by yourself.

4

[a] $EF = 6$ cm.[b] $m(\angle ABD) = 125^\circ$

5

[a] The perimeter of $\triangle MBC = 26$ cm.

[b] Prove by yourself.

Model 2

1

1 c

4 a

2 c

3 b

3 b

6 b

2

1 perpendicular to the base , bisects it.

2 equal

3 8

4 80° , 130°

3

[a] Prove by yourself.

[b] 1 BD = 10 cm.

2 Prove by yourself

4

[a] BD = 6 cm. , MD = 2 cm.

[b] $m(\angle MLY) = 70^\circ$

5

[a] The perimeter of the triangle = 19 cm.

[b] Prove by yourself , CD = 3 cm. , AD = $\sqrt{91}$ cm.

Geometry

Answers of unit five

Answers of Exercise 6

1

- 1 > 2 > 3 < 4 <
5 < 6 > 7 > , <

2

- 1 $m(\angle 1) < m(\angle 3)$ 2 $m(\angle 4) < m(\angle 2)$
3 $m(\angle 3) < m(\angle 5)$ 4 $m(\angle 6) < m(\angle 2)$
5 $m(\angle 1) < m(\angle 3) < m(\angle 5)$
6 $m(\angle 3) < m(\angle 5) < m(\angle 7)$
7 $m(\angle 1) < m(\angle 3) < m(\angle 5) < m(\angle 7)$

3

- $\therefore \overline{AD} \perp \overline{BC}$ from its midpoint
 $\therefore \overline{AD}$ is the axis of symmetry of \overline{BC}
 $\therefore AB = AC$
 $\therefore E \in \overline{AB}$ $\therefore AB > AE$
 $\therefore AC > AE$ (Q.E.D.)

4

- $\therefore \overline{AB} \parallel \overline{CD}$, \overline{BC} is a transversal
 $\therefore m(\angle BCD) = m(\angle ABC)$ (alternate angles)
 $\therefore m(\angle BCD) + m(\angle ACB) > m(\angle ABC)$
 $\therefore m(\angle ACD) > m(\angle ABC)$ (1) (Q.E.D.1)
 $\therefore E \in \overline{CD}$ $\therefore \angle ADE$ is an exterior angle of $\triangle ACD$
 $\therefore m(\angle ADE) > m(\angle ACD)$ (2)
From (1) and (2):
 $\therefore m(\angle ADE) > m(\angle ABC)$ (Q.E.D.2)

5

- $\therefore E \in \overline{CB}$ \therefore The exterior angle $\angle ABE$ of $\triangle ABC$
 $\therefore m(\angle ABE) > m(\angle A)$ (1)
 $\triangle ABM$, $\triangle CDM$ in them:
 $\begin{cases} AM = MC \\ MB = MD \\ m(\angle AMB) = m(\angle DMC) \end{cases}$ (V.O.A)
 $\therefore \triangle ABM \cong \triangle CDM$, then we deduce that
 $m(\angle A) = m(\angle ACD)$ and from (1):
 $\therefore m(\angle ABE) > m(\angle ACD)$ (Q.E.D.)

66

6

- \therefore The figure is a parallelogram
 $\therefore AD = BC$, $AB = CD$
 $\therefore DX < BY$ $\therefore AX > CY$
 $\therefore AX + AB > CY + CD$ (Q.E.D.)

7

- $\therefore D \in \overline{AB}$ $\therefore \angle ADC$ is an exterior angle of $\triangle DBC$
 $\therefore m(\angle ADC) > m(\angle B)$
But $m(\angle ADC) = m(\angle ACD)$
because $\triangle ADC$ in which $AD = AC$
 $\therefore m(\angle ACD) > m(\angle B)$
 $\therefore m(\angle ACD) + m(\angle DCB) > m(\angle B)$
 $\therefore m(\angle ACB) > m(\angle B)$ (Q.E.D.)

8

- In $\triangle AXY$: $\therefore m(\angle AXY) = m(\angle AYX)$
 $\therefore AX = AY$ (1)
 $\therefore AC > AB$ $\therefore AY + YC > AX + XB$ (2)
From (1) and (2):
 $\therefore YC > XB$ (Q.E.D.)

9

- $\therefore \angle ADC$ is an exterior angle of $\triangle DBC$
 $\therefore m(\angle ADC) > m(\angle B)$
But $m(\angle B) = m(\angle ACB)$
(because $AB = AC$ in $\triangle ABC$)
 $\therefore m(\angle ADC) > m(\angle ACB)$ (Q.E.D.)

10

- $\therefore m(\angle ACB) > m(\angle ABC)$
 \therefore The supplement
of $\angle ABC >$ the supplement of $\angle ACB$
 $\therefore m(\angle ABD) > m(\angle ACE)$
 $\therefore \frac{1}{2}m(\angle ABD) > \frac{1}{2}m(\angle ACE)$
i.e. $m(\angle ABX) > m(\angle ACY)$ (Q.E.D.)

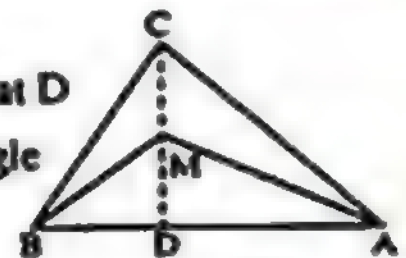
11

Const: Draw \overline{CM} to intersect \overline{BA} at D Proof: $\therefore \angle AMD$ is an exterior angle of $\triangle AMC$

$$\therefore m(\angle AMD) > m(\angle ACM) \quad (1)$$

 $\therefore \angle BMD$ is an exterior angle of $\triangle CMB$

$$\therefore m(\angle BMD) > m(\angle BCM) \quad (2)$$



Adding (1) and (2)

$$\begin{aligned} \therefore m(\angle AMD) + m(\angle BMD) &> m(\angle ACM) \\ &+ m(\angle BCM) \\ \therefore m(\angle AMB) &> m(\angle C) \end{aligned} \quad (\text{Q.E.D.})$$

12

$$\begin{aligned} \therefore m(\angle B) &> m(\angle C) \\ \therefore m(\angle B) + \frac{1}{2}m(\angle BAC) &> m(\angle C) + \frac{1}{2}m(\angle BAC) \\ \therefore m(\angle B) + m(\angle BAD) &> m(\angle C) + m(\angle CAD) \\ \text{but } m(\angle B) + m(\angle BAD) &= m(\angle CDA) \\ (\text{an exterior angle of } \triangle ABD), \\ m(\angle C) + m(\angle CAD) &= m(\angle BDA) \\ (\text{an exterior angle of } \triangle ACD) \\ \therefore m(\angle ADC) &> m(\angle ADB) \\ \therefore \text{Their sum} &= 180^\circ \quad \therefore m(\angle ADC) > \frac{180^\circ}{2} \\ \text{i.e. } m(\angle ADC) &> 90^\circ \\ \text{i.e. } \angle ADC &\text{ is an obtuse angle.} \end{aligned} \quad (\text{Q.E.D.})$$

13

$$\begin{aligned} \therefore AC &= AD \quad \therefore m(\angle D) = m(\angle ACD) \\ \therefore m(\angle ACB) &> m(\angle ABC) \\ \therefore m(\angle ACB) + m(\angle ACD) &> m(\angle ABC) + m(\angle D) \\ \therefore m(\angle BCD) &> m(\angle B) + m(\angle D) \\ \text{but the sum of measures of the interior angles} \\ \text{of } \triangle BCD &= 180^\circ \\ \therefore m(\angle BCD) &> \frac{180^\circ}{2} \quad \text{i.e. } m(\angle BCD) > 90^\circ \\ \text{i.e. } \angle BCD &\text{ is an obtuse angle.} \end{aligned} \quad (\text{Q.E.D.})$$

Answers of Exercise 7

1

- 1 The angle of the greater measure
2 $\angle A$ 3 $m(\angle D)$
4 $m(\angle A) < m(\angle B) < m(\angle C)$

2

- 1 $> > >$ 2 $< < <$ 3 $> > >$

3

- 1 $\therefore \overline{BC}$ is the longest side
 $\therefore \angle A$ is the greatest angle in measure
 $\therefore \overline{AC}$ is the shortest side
 $\therefore \angle B$ is the smallest angle in measure
 \therefore The ascending order of measures of the angles is :
 $m(\angle B) < m(\angle C)$ and $m(\angle A)$

- 2 $\therefore \overline{BC}$ is the longest side
 $\therefore \angle A$ is the greatest angle in measure
 $\therefore \overline{AB}$ is the shortest side
 $\therefore \angle C$ is the smallest angle in measure
 \therefore The ascending order of the measures of the angles is : $m(\angle C) < m(\angle B)$ and $m(\angle A)$

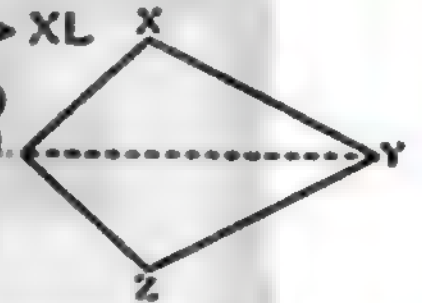
4

$$\begin{aligned} \text{In } \triangle ABC : \therefore AC &> AB \\ \therefore m(\angle ABC) &> m(\angle ACB) \quad (1) \\ \text{In } \triangle BDC : \therefore DB &= DC \\ \therefore m(\angle DBC) &= m(\angle DCB) \quad (2) \\ \text{Adding (1) and (2) :} \\ \therefore m(\angle ABC) + m(\angle DBC) &> m(\angle ACB) + m(\angle DCB) \\ \therefore m(\angle ABD) &> m(\angle ACD) \end{aligned} \quad (\text{Q.E.D.})$$

5

Construction : Draw \overline{YL}

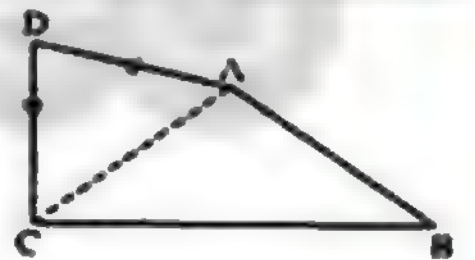
$$\begin{aligned} \text{Proof : In } \triangle XYL \quad \therefore XY &> XL \\ \therefore m(\angle XLY) &> m(\angle XYL) \quad (1) \\ \text{In } \triangle ZYL : \therefore YZ &> ZL \\ \therefore m(\angle ZLY) &> m(\angle ZYL) \quad (2) \\ \text{Adding (1) and (2) :} \\ \therefore m(\angle XLY) + m(\angle ZLY) &> m(\angle XYL) + m(\angle ZYL) \\ \therefore m(\angle XLZ) &> m(\angle XYZ) \end{aligned} \quad (\text{Q.E.D.})$$



6

Construction : Draw \overline{AC}

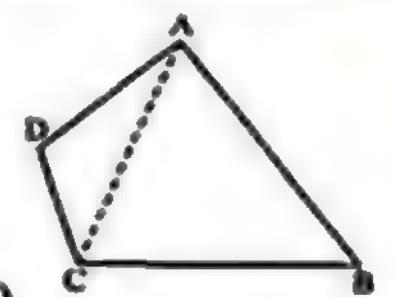
$$\begin{aligned} \text{Proof : In } \triangle ABC \\ \therefore BC &> AB \\ \therefore m(\angle BAC) &> m(\angle ACB) \quad (1) \\ \text{In } \triangle DAC : \therefore DA &= DC \\ \therefore m(\angle DAC) &= m(\angle DCA) \quad (2) \\ \text{Adding (1) and (2) :} \\ \therefore m(\angle BAC) + m(\angle DAC) &> m(\angle ACB) + m(\angle DCA) \\ \therefore m(\angle BAD) &> m(\angle BCD) \end{aligned} \quad (\text{Q.E.D.})$$



7

Construction : Draw \overline{AC}

$$\begin{aligned} \text{Proof : In } \triangle ABC \\ \therefore AB &> BC \\ \therefore m(\angle ACB) &> m(\angle BAC) \quad (1) \end{aligned}$$



Geometry

In $\triangle ADC$: $\therefore AD > DC$

$$\therefore m(\angle ACD) > m(\angle CAD) \quad (2)$$

Adding (1) and (2) :

$$\therefore m(\angle BCD) > m(\angle BAD) \quad (Q.E.D.)$$

8

In $\triangle MBC$: $\therefore MC > MB$

$$\therefore m(\angle MBC) > m(\angle MCB)$$

$$\therefore m(\angle MBC) = \frac{1}{2} m(\angle ABC)$$

$$m(\angle MCB) = \frac{1}{2} m(\angle ACB)$$

$$\therefore \frac{1}{2} m(\angle ABC) > \frac{1}{2} m(\angle ACB)$$

$$\therefore m(\angle ABC) > m(\angle ACB) \quad (Q.E.D.)$$

9

In $\triangle DBC$: $\therefore DB > DC$

$$\therefore m(\angle DCB) > m(\angle DBC)$$

In $\triangle ABC$: $\therefore AB = AC$

$$\therefore m(\angle ACB) = m(\angle ABC)$$

$$\therefore m(\angle ACB) - m(\angle DCB) < m(\angle ABC) - m(\angle DBC)$$

$$\therefore m(\angle ACD) < m(\angle ABD)$$

$$i.e. m(\angle ABD) > m(\angle ACD) \quad (Q.E.D.)$$

10

In $\triangle ABC$: $\therefore AB > AC \therefore m(\angle C) > m(\angle B) \quad (1)$

$\therefore \overline{XY} \parallel \overline{BC}$ and \overline{AC} is a transversal

$$\therefore m(\angle AYX) = m(\angle C) \text{ (Corresponding angles)} \quad (2)$$

Similarly : $\therefore \overline{XY} \parallel \overline{BC}$, \overline{AB} is a transversal

$$\therefore m(\angle AXY) = m(\angle B) \quad (3)$$

From (1) , (2) and (3) :

$$\therefore m(\angle AYX) > m(\angle AXY) \quad (Q.E.D.)$$

11

$$\therefore AB > AC \quad \therefore m(\angle C) > m(\angle B)$$

But $m(\angle C) = m(\angle AED)$ (corresponding angles)

$$m(\angle B) = m(\angle ADE) \text{ (corresponding angles)}$$

$$\therefore m(\angle AED) > m(\angle ADE)$$

In $\triangle ADE$:

$$\therefore m(\angle A) = 90^\circ$$

$$\therefore m(\angle AED) + m(\angle ADE) = 90^\circ$$

$$\therefore m(\angle AED) > m(\angle ADE)$$

$$\therefore m(\angle AED) > \frac{90^\circ}{2}$$

$$\therefore m(\angle AED) > 45^\circ \quad (Q.E.D.)$$

12

$\therefore AB > AC$, $BD = CE$ Subtracting $\therefore AD > AE$

\therefore In $\triangle ADE$: $\therefore AD > AE$

$$\therefore m(\angle AED) > m(\angle ADE) \quad (Q.E.D.)$$

13

In $\triangle ABC$: $\therefore AC > AB$

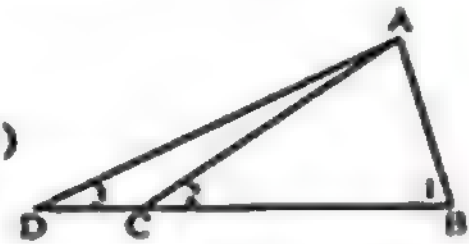
$$\therefore m(\angle 1) > m(\angle 2) \quad (1)$$

But $\angle 2$ is an exterior angle of $\triangle ACD$

$$\therefore m(\angle 2) > m(\angle 3) \quad (2)$$

From (1) and (2) : $\therefore m(\angle 1) > m(\angle 3)$

$$\therefore m(\angle ABD) > m(\angle D) \quad (Q.E.D.)$$



14

$\therefore \triangle ABC$ is an equilateral triangle

$$\therefore m(\angle ABC) = m(\angle ACB) = 60^\circ$$

$$\therefore m(\angle EBC) < m(\angle ECB) \quad \text{Subtracting}$$

$$\therefore m(\angle ABC) - m(\angle EBC) > m(\angle ACB) - m(\angle ECB)$$

$$\therefore m(\angle ABE) > m(\angle ACE) \quad (1) \quad (Q.E.D.1)$$

$$\therefore m(\angle A) = m(\angle B)$$

$$\therefore m(\angle A) = m(\angle ABE) + m(\angle EBC)$$

$$\therefore m(\angle A) > m(\angle ABE) \text{ and from (1) :}$$

$$\therefore m(\angle A) > m(\angle ABE) > m(\angle ACE) \quad (Q.E.D.2)$$

15

In $\triangle XBC$: $\therefore XC > XB$

$$\therefore m(\angle XBC) > m(\angle XCB)$$

$\therefore ABCD$ is a rectangle

$$i.e. m(\angle ABC) = m(\angle DCB) = 90^\circ$$

$$\therefore 90^\circ - m(\angle XBC) < 90^\circ - m(\angle XCB)$$

$$\therefore m(\angle ABX) < m(\angle XCD) \quad (Q.E.D.)$$

16

In $\triangle ADE$: $\therefore AD = 5 \text{ cm. , } AE = 3 \text{ cm. } \therefore AD > AE$

$$\therefore m(\angle AED) > m(\angle ADE)$$

From the equilateral triangle ABC we find that

$$m(\angle A) = 60^\circ$$

$$\therefore m(\angle AED) + m(\angle ADE) = 180^\circ - 60^\circ = 120^\circ$$

$$\therefore m(\angle AED) > 60^\circ \quad (Q.E.D.)$$

17

In $\triangle ABC$: $\therefore AB > AC \therefore m(\angle ACB) > m(\angle ABC)$

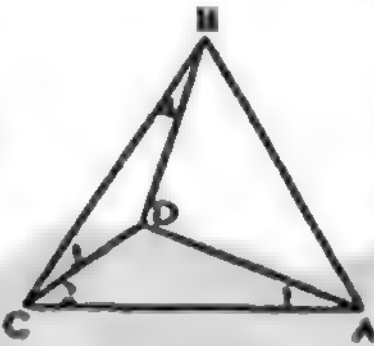
$$\therefore 180^\circ - m(\angle ACB) < 180^\circ - m(\angle ABC)$$

Answers of Unit 5

$\therefore D \in \overline{AB}, E \in \overline{AC} \therefore m(\angle BCE) < m(\angle DBC)$
 $\therefore \overline{BF}$ bisects $\angle DBC$, \overline{CF} bisects $\angle BCE$
 $\therefore m(\angle BCF) < m(\angle FBC)$
 $\therefore m(\angle FBC) > m(\angle BCF)$ (Q.E.D.)

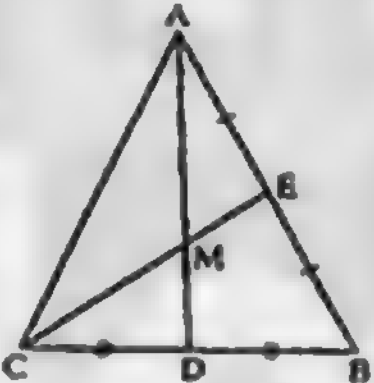
18

In $\triangle DBC$: $\therefore DB > DC$
 $\therefore m(\angle 3) > m(\angle 4)$ (1)
 In $\triangle DAC$: $\therefore DA > DC$
 $\therefore m(\angle 2) > m(\angle 1)$ (2)
 From (1) and (2) and adding:
 $\therefore m(\angle 3) + m(\angle 2) > m(\angle 4) + m(\angle 1)$
 $\therefore m(\angle ACB) > m(\angle DBC) + m(\angle DAC)$ (Q.E.D.)



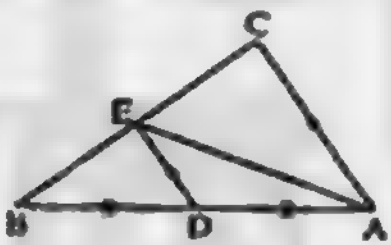
19

$\therefore \overline{AD}, \overline{CE}$ are two medians of $\triangle ABC$ intersecting at M
 $\therefore AM = 2MD, MC = 2ME$
 $\therefore MD > ME \therefore AM > MC$
 thus in $\triangle AMC$
 $\therefore m(\angle CAM) < m(\angle MCA)$ (Q.E.D.)



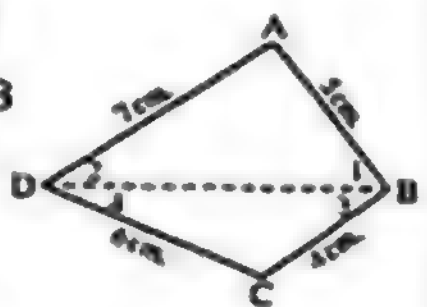
20

In $\triangle ABC$: $\therefore D$ is the midpoint of $\overline{AB}, \overline{DE} \parallel \overline{AC}$
 $\therefore DE = \frac{1}{2} AC$
 $\therefore AD = \frac{1}{2} AB, \therefore AB > AC$
 $\therefore \frac{1}{2} AB > \frac{1}{2} AC$
 $\therefore AD > DE \therefore m(\angle AED) > m(\angle DAE)$
 But $m(\angle AED) = m(\angle CAE)$ (alternate angles)
 $\therefore m(\angle CAE) > m(\angle DAE)$ (Q.E.D.)



21

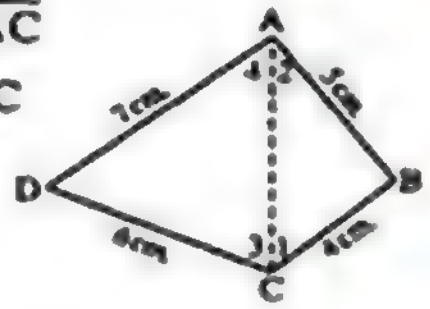
First construction: Draw \overline{BD}
 Proof: In $\triangle ABD$: $\therefore AD > AB$
 $\therefore m(\angle 1) > m(\angle 2)$ (1)
 In $\triangle CBD$: $\therefore CD > CB$
 $\therefore m(\angle 3) > m(\angle 4)$ (2)
 Adding (1) and (2):
 $\therefore m(\angle 1) + m(\angle 3) > m(\angle 2) + m(\angle 4)$
 $\therefore m(\angle ABC) > m(\angle ADC)$ (Q.E.D.1)

Second construction: Draw \overline{AC} Proof: In $\triangle ABC$: $\therefore BA > BC$ $\therefore m(\angle 1) > m(\angle 2)$ (3)In $\triangle ADC$: $\therefore AD > DC$ $\therefore m(\angle 3) > m(\angle 4)$ (4)

Adding (3) and (4):

 $\therefore m(\angle 1) + m(\angle 3) > m(\angle 2) + m(\angle 4)$ $\therefore m(\angle BCD) > m(\angle BAD)$ (Q.E.D.2)

\therefore The sum of measure of the interior angles of the quadrilateral = 360°
 and from the two preceding requirements

 $\therefore m(\angle B) + m(\angle C) > \frac{360^\circ}{2}$ $\therefore m(\angle B) + m(\angle C) > 180^\circ$ (Q.E.D.3)

22

 $\therefore \overline{AE}$ is a median in $\triangle ABD, m(\angle A) = 90^\circ$ $\therefore AE = \frac{1}{2} BD$ $\therefore E$ is the midpoint of $\overline{BD}, \overline{EX} \parallel \overline{AC}$ $\therefore EX = \frac{1}{2} DC$ $\therefore AE > EX$ $\therefore \frac{1}{2} BD > \frac{1}{2} DC$ $\therefore BD > DC$ $\therefore m(\angle C) > m(\angle DBC)$ (Q.E.D.)

23

In $\triangle ABM$: $\therefore AM > BM \therefore m(\angle ABM) > m(\angle A)$ (1) $\therefore AM = CM \therefore$ In $\triangle CBM$: $MC > MB$ $\therefore m(\angle MBC) > m(\angle C)$ (2)

Adding (1) and (2):

 $\therefore m(\angle ABM) + m(\angle MBC) > m(\angle A) + m(\angle C)$ $\therefore m(\angle ABC) > m(\angle A) + m(\angle C)$ $\therefore \angle ABC$ is an obtuse angle. (Q.E.D.)

24

In $\triangle ABD$: $\therefore m(\angle B) = 90^\circ - m(\angle BAD)$ (1)From $\triangle ACD$: $m(\angle C) = 90^\circ - m(\angle CAD)$ (2)From $\triangle ABC$: $AC > AB \therefore m(\angle B) > m(\angle C)$ (3)

From (1), (2) and (3):

 $\therefore 90^\circ - m(\angle BAD) > 90^\circ - m(\angle CAD)$ $\therefore m(\angle BAD) < m(\angle CAD)$ (Q.E.D.)

Geometry

25

In $\triangle ABC$: $\because AC > AB$

$$\therefore m(\angle B) > m(\angle C)$$

$$\therefore m(\angle BAD) = m(\angle DAC) \\ (\overline{AD} \text{ bisects } \angle A)$$

$$\therefore m(\angle B) + m(\angle BAD) > m(\angle C) + m(\angle DAC)$$

 $\therefore \angle ADC$ is an exterior angle of $\triangle ABD$

$$\therefore m(\angle ADC) = m(\angle B) + m(\angle BAD)$$

 $\therefore \angle ADB$ is an exterior angle of $\triangle ADC$

$$\therefore m(\angle ADB) = m(\angle C) + m(\angle DAC)$$

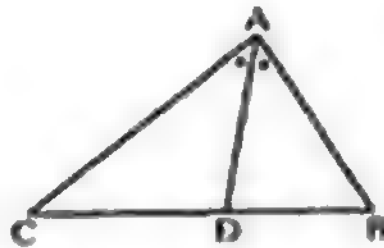
$$\therefore m(\angle ADC) > m(\angle ADB)$$

$$\therefore m(\angle ADC) + m(\angle ADB) = 180^\circ$$

$$\therefore m(\angle ADC) > \frac{180^\circ}{2} \quad \text{i.e. } m(\angle ADC) > 90^\circ$$

i.e. $\angle ADC$ is an obtuse angle.

(Q.E.D.)



26

Let $\overline{CA} \cap \overline{DB} = \{M\}$ \therefore The two diagonals in the parallelogram bisect each other

$$\therefore AC > BD \quad \therefore MA > MD, \\ MC > MD.$$

From $\triangle AMD$: $\therefore AM > MD$

$$\therefore m(\angle 2) > m(\angle 1) \quad (1)$$

From $\triangle DMC$: $\therefore MC > MD$

$$\therefore m(\angle 3) > m(\angle 4) \quad (2)$$

Adding (1) and (2):

$$\therefore m(\angle 2) + m(\angle 3) > m(\angle 1) + m(\angle 4)$$

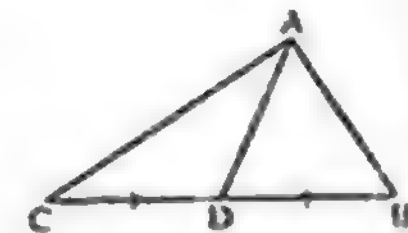
$$\therefore m(\angle D) > m(\angle 1) + m(\angle 4)$$

 \therefore In $\triangle ADC$:

$$\therefore m(\angle D) > m(\angle CAD) + m(\angle ACD)$$

 $\therefore \angle D$ is an obtuse angle.

(Q.E.D.)



27

 \therefore The perimeter of $\triangle ACD$

$$= CD + DA + AC$$

The perimeter of $\triangle ABD$

$$= BD + DA + AB$$

 \therefore The perimeter of $\triangle ACD$ > The perimeter of $\triangle ABD$

$$\therefore CD + DA + AC > BD + DA + AB$$

But $CD = BD$

$$\therefore AC > AB \quad \therefore m(\angle B) > m(\angle C)$$

(Q.E.D.)

28

Construction: Draw $\overline{DE} \parallel \overline{AC}$
to intersect \overline{AB} at EProof: In $\triangle ABC$: $\because \overline{DE} \parallel \overline{AC}$,D is the midpoint of \overline{BC} \therefore E is the midpoint of \overline{BA}

$$\therefore AE = \frac{1}{2} AB, DE = \frac{1}{2} AC$$

$$\therefore AB > AC$$

$$\therefore AE > DE$$

$$\therefore m(\angle 2) > m(\angle 3) \quad (1)$$

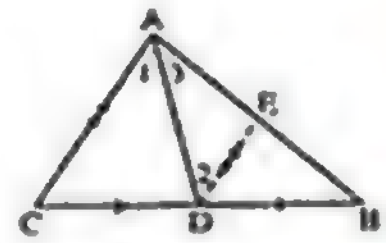
 $\because \overline{DE} \parallel \overline{AC}$, \overline{AD} is a transversal to them

$$\therefore m(\angle 1) = m(\angle 2) \text{ (Alternate angles)}$$

$$\text{From (1): } \therefore m(\angle 1) > m(\angle 3)$$

$$\therefore m(\angle BAD) < m(\angle CAD)$$

(Q.E.D.)



Answers of Exercise B

1

1. A side greater in length than that opposite to the other angle, greater in measure than the measure of the angle opposite to the other side.

2. The shortest side.

3. The hypotenuse.

4. The length of the line segment drawn from the given point perpendicular to the given straight line.

5. \overline{AB} 6. \overline{AC} 7. \overline{BC}

2

1. c

2. a

3. d

3

1. $> > <$ 2. $> > >$ 3. $> > > >$ 4. $> < < >$

YZ < XY < XZ

AC > AB > BC

4

 $\because \overline{AE} \parallel \overline{BC}$, \overline{AC} is a transversal

$$\therefore m(\angle C) = m(\angle EAC) = 30^\circ \text{ (alternate angles) (1)}$$

 $\because \overline{AE} \parallel \overline{BC}$, \overline{AB} is a transversal

$$\therefore m(\angle B) = m(\angle DAE) = 70^\circ$$

(corresponding angles) (2)

$$\text{From (1) and (2): } \therefore m(\angle B) > m(\angle C)$$

$$\therefore AC > AB$$

(Q.E.D.)

12

$$\begin{aligned} \because C \in \overline{AE} \quad \therefore m(\angle ACB) &= 180^\circ - 120^\circ = 60^\circ \\ \because B \in \overline{CD} \quad \therefore m(\angle ABC) &= 180^\circ - 110^\circ = 70^\circ \\ \therefore m(\angle A) &= 180^\circ - (60^\circ + 70^\circ) = 50^\circ \\ \therefore m(\angle ACB) &> m(\angle A) \quad \therefore AB > BC \quad (\text{Q.E.D.}) \end{aligned}$$

13

$$\begin{aligned} \text{In } \triangle ABC: \because AB &= AC \\ \therefore m(\angle ACB) &= m(\angle B) = 65^\circ \\ \therefore m(\angle DCB) &= 65^\circ + 20^\circ = 85^\circ \\ \text{In } \triangle DBC: \therefore m(\angle D) &= 180^\circ - (65^\circ + 85^\circ) = 30^\circ \\ \therefore \text{In } \triangle DAC: m(\angle D) &> m(\angle ACD) \\ \therefore AC &> AD \text{ but } AB = AC \\ \therefore AB &> AD \quad (\text{Q.E.D.}) \end{aligned}$$

14

$$\begin{aligned} \text{In } \triangle DBC: \because DB &= DC \\ \therefore m(\angle B) &= m(\angle DCB) = \frac{180^\circ - 100^\circ}{2} = 40^\circ \\ \because \overline{CD} \text{ bisects } \angle ACB \quad \therefore m(\angle ACD) &= 40^\circ \\ \because D \in \overline{AB} \\ \therefore m(\angle ADC) &= 180^\circ - 100^\circ = 80^\circ \\ \therefore \text{In } \triangle ADC: m(\angle A) &= 180^\circ - (40^\circ + 80^\circ) = 60^\circ \\ \therefore m(\angle ADC) &> m(\angle A) \\ \therefore AC &> DC \text{ but } DC = DB \\ \therefore AC &> DB \quad (\text{Q.E.D.}) \end{aligned}$$

15

$$\begin{aligned} \because \overline{AD} \parallel \overline{BC}, \overline{AC} \text{ is a transversal} \\ \therefore m(\angle ACB) &= m(\angle DAC) = 30^\circ \text{ (alternate angles)} \\ \text{In } \triangle ABC: \because m(\angle BAC) &> m(\angle ACB) \\ \therefore BC &> AB \quad (\text{Q.E.D.}) \end{aligned}$$

16

$$\begin{aligned} \text{In } \triangle ACM: \because m(\angle C) &= 90^\circ \quad \therefore AM > CM \quad (1) \\ \text{In } \triangle BDM: \because m(\angle D) &= 90^\circ \quad \therefore BM > DM \quad (2) \\ \text{Adding (1) and (2): } \therefore AM + MB &> CM + MD \\ \therefore AB &> CD \quad (\text{Q.E.D.}) \end{aligned}$$

17

$$\begin{aligned} \text{In } \triangle ABC: \because AB &= AC \\ \therefore m(\angle ABC) &= m(\angle ACB) \\ \because m(\angle ABM) &< m(\angle ACM) \end{aligned}$$

$$\therefore m(\angle ABC) - m(\angle ABM) > m(\angle ACB) - m(\angle ACM)$$

$$\therefore m(\angle MBC) > m(\angle MCB)$$

$$\text{From } \triangle MBC: \therefore MC > MB \quad (\text{Q.E.D.})$$

18

$$\begin{aligned} \text{In } \triangle ABC: \because \angle B \text{ is an obtuse angle} \\ \therefore m(\angle B) &> m(\angle C) \quad (1) \\ \because \overline{DE} \parallel \overline{BC}, \overline{DB} \text{ is a transversal} \\ \therefore m(\angle ADE) &= m(\angle B) \text{ (corresponding angles)} \quad (2) \\ \because \overline{DE} \parallel \overline{BC}, \overline{EC} \text{ is a transversal} \\ \therefore m(\angle AED) &= m(\angle C) \text{ (corresponding angles)} \quad (3) \\ \text{From (1), (2) and (3):} \\ \therefore m(\angle ADE) &> m(\angle AED) \\ \therefore AE &> AD \quad (\text{Q.E.D.}) \end{aligned}$$

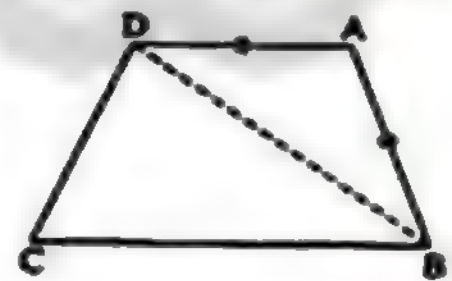
19

$$\begin{aligned} \text{In } \triangle ABC: \because AB &> AC \quad \therefore m(\angle C) > m(\angle B) \quad (1) \\ \because \overline{DE} \parallel \overline{BC} \text{ and } \overline{DC} \text{ is a transversal} \\ \therefore m(\angle D) &= m(\angle C) \text{ (alternate angles)} \quad (2) \\ \because \overline{DE} \parallel \overline{BC}, \overline{BE} \text{ is a transversal} \\ \therefore m(\angle E) &= m(\angle B) \text{ (alternate angles)} \quad (3) \\ \text{From (1), (2) and (3):} \\ \therefore m(\angle D) &> m(\angle E) \text{ and from } \triangle ADE \\ \therefore AE &> AD \quad (\text{Q.E.D.}) \end{aligned}$$

20

Const.: Draw \overline{BD} Proof: In $\triangle ADB$

$$\begin{aligned} \because AD &= AB \\ \therefore m(\angle ADB) &= m(\angle ABD) \\ \because m(\angle ADC) &> m(\angle ABC) \\ \therefore m(\angle ADC) - m(\angle ADB) &> m(\angle ABC) - m(\angle ABD) \\ \therefore m(\angle BDC) &> m(\angle DBC) \\ \therefore \text{In } \triangle BDC: BC &> CD \quad (\text{Q.E.D.}) \end{aligned}$$



21

$$\begin{aligned} \text{In } \triangle ABC: \because AB &> AC \\ \therefore m(\angle ABC) &< m(\angle ACB) \\ \because B \in \overline{AD}, C \in \overline{AE} \end{aligned}$$

Geometry

$\therefore 180^\circ - m(\angle ABC) > 180^\circ - m(\angle ACB)$
 $\therefore m(\angle CBD) > m(\angle BCE)$
 $\therefore \overline{BF}$ bisects $\angle DBC$, \overline{CF} bisects $\angle BCE$
 $\therefore m(\angle FBC) > m(\angle BCF)$ (Q.E.D.1)
 $\therefore CF > BF$ (Q.E.D.2)

17

In $\triangle ABD$: $\therefore BD = AD$
 $\therefore m(\angle BAD) = m(\angle B)$
 $\therefore m(\angle BAD) + m(\angle DAC) > m(\angle B)$
 $\therefore m(\angle BAC) > m(\angle B) \therefore BC > AC$ (Q.E.D.)

18

In $\triangle DBC$: $\therefore m(\angle B) > m(\angle DCB)$
 $\therefore DC > DB$ but $DB = AD \therefore DC > AD$
 \therefore In $\triangle ADC$: $m(\angle A) > m(\angle ACD)$ (Q.E.D.1)
 $\therefore m(\angle BDC) = 180^\circ - (70^\circ + 50^\circ) = 60^\circ$
 $\therefore \angle BCD$ is an exterior angle of $\triangle ADC$
 $\therefore m(\angle BDC) = m(\angle A) + m(\angle ACD) = 60^\circ$
 $\therefore m(\angle A) > m(\angle ACD) \therefore m(\angle ACD) < 30^\circ$
 $\therefore m(\angle ACD) + m(\angle DCB) < 30^\circ + 50^\circ$
 $\therefore m(\angle ACB) < 80^\circ$
 $\therefore \angle ACB$ is an acute angle (Q.E.D.2)

19

In $\triangle AFB$: $\therefore FA = FB$
 $\therefore m(\angle FBA) = m(\angle FAB) = 50^\circ$ (1)
 $\therefore \angle AFD$ is an exterior angle of $\triangle AFB$
 $\therefore m(\angle AFD) = 50^\circ + 50^\circ = 100^\circ$ (2)
 \therefore In $\triangle AFD$
 $\therefore FA = FD \therefore m(\angle FDA) = \frac{180^\circ - 100^\circ}{2} = 40^\circ$
 From (1) and (2): \therefore In $\triangle ABD$
 $m(\angle ABD) > m(\angle ADB)$
 $\therefore AD > AB$ (Q.E.D.1)
 In $\triangle ABD$: $\therefore \overline{AF}$ is a median $\therefore AF = \frac{1}{2}BD$
 $\therefore m(\angle DAB) = 90^\circ$
 $\therefore \overline{BC}$ is a hypotenuse of $\triangle BAC$
 $\therefore BC > AC$ (Q.E.D.2)

20

$\therefore \angle ADB$ is an exterior angle of $\triangle ADC$
 $\therefore m(\angle ADB) > m(\angle C)$
 $\therefore m(\angle C) = m(\angle B)$ ($AB = AC$ in $\triangle ABC$)
 $\therefore m(\angle ADB) > m(\angle B)$
 And from $\triangle ABD$: $AB > AD$ (Q.E.D.)

21

$\triangle ABD \cong \triangle AED$ in them
 $m(\angle B) = m(\angle AED)$, $m(\angle BAD) = m(\angle DAE)$
 $\therefore m(\angle ADB) = m(\angle ADE)$
 $\therefore \triangle ABD \cong \triangle AED$
 In them $\begin{cases} m(\angle BAD) = m(\angle DAE) \\ m(\angle ADB) = m(\angle ADE) \\ \overline{AD} \text{ is a common side} \end{cases}$
 $\therefore \triangle ABD \cong \triangle AED$ then we deduce that
 $BD = DE$ (Q.E.D.1)
 \therefore In $\triangle DEC$
 $\therefore m(\angle DEC) = 90^\circ \therefore DC > DE$
 $\therefore DE = DB \therefore DC > DB$ (Q.E.D.2)

22

$\therefore m(\angle AXC) = 180^\circ - 110^\circ = 70^\circ$
 $\therefore \triangle ACD$ in which $m(\angle ADC) > m(\angle C)$
 $\therefore AC > AD$ (1)
 $\therefore \triangle ADB$ is an obtuse-angled at D
 $\therefore AB > AD$ (2)
 By adding (1) and (2): $\therefore AB + AC > 2AD$ (Q.E.D.)

23

In $\triangle ABC$: $\therefore m(\angle B) = 90^\circ$
 \therefore The hypotenuse \overline{AC} is the longest side
 $\therefore AB < AC$, $BC < AC$
 By adding: $\therefore AB + BC < 2AC$ (Q.E.D.)

24

$\therefore \overline{AD} \parallel \overline{CE}$, \overline{AC} is a transversal
 $\therefore m(\angle DAC) = m(\angle ACE)$
 $\therefore m(\angle BCE) > m(\angle DAC)$
 $\therefore m(\angle BCE) > m(\angle BAD)$ (\overline{AD} bisects $\angle BAC$) (1)
 $\therefore \overline{AD} \parallel \overline{CE}$ and \overline{BE} is a transversal
 $\therefore m(\angle BAD) = m(\angle E)$ (corresponding angles) (2)
 From (1) and (2):
 $\therefore m(\angle BCE) > m(\angle E)$ and from $\triangle BCE$
 $\therefore BE > BC$ (Q.E.D.)

Answers of Unit 5

25

$\therefore \Delta XYM$ is right-angled at Y
 $\therefore \angle XMY$ is an acute angle
 $\therefore \angle XMZ$ is an obtuse angle
 $\therefore \Delta XMZ$ is an obtuse-angled at M
 $\therefore XZ > XM$

(Q.E.D.)

26

In ΔABC : $\therefore AB > BC$ $\therefore m(\angle C) > m(\angle A)$ (1) $\therefore m(\angle ABC) = 90^\circ$ $\therefore \angle A$ complements $\angle C$ (2)in ΔABD : $\therefore m(\angle ADB) = 90^\circ$ $\therefore \angle A$ complements $\angle ABD$ (3)

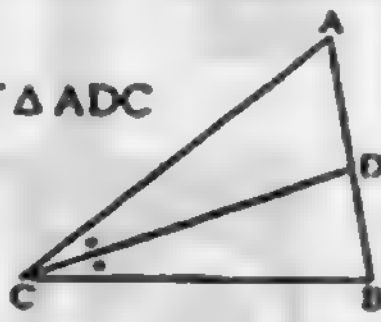
From (2) and (3):

 $\therefore m(\angle C) = m(\angle ABD)$

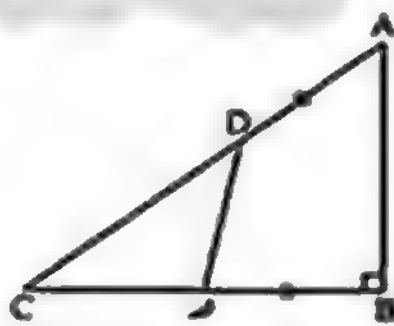
from (1):

 $\therefore m(\angle ABD) > m(\angle A)$ \therefore In ΔABD : $AD > BD$ (Q.E.D.)

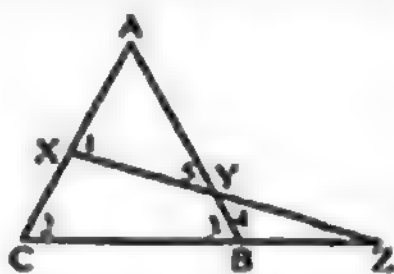
27

 $\therefore \angle BDC$ is an exterior angle of ΔADC $\therefore m(\angle BDC) > m(\angle ACD)$ $\therefore m(\angle BCD) = m(\angle ACD)$ $\therefore m(\angle BDC) > m(\angle BCD)$ In ΔDBC : $\therefore BC > BD$ (Q.E.D.)

28

In ΔABC : $\therefore m(\angle B) = 90^\circ$ $\therefore AC > BC$ $\therefore AD = BE$ $\therefore AC - AD > BC - BE$ $\therefore DC > EC$ \therefore In ΔDEC : $\therefore DC > EC$ $\therefore m(\angle CED) > m(\angle CDE)$ (Q.E.D.)

29

 $\therefore \angle 1$ is an exterior angle of ΔXZC $\therefore m(\angle 1) > m(\angle 2)$ But $m(\angle 3) = m(\angle 2)$, ($AB = AC$ in ΔABC) $\therefore m(\angle 1) > m(\angle 3)$ But $\angle 3$ is an exterior angle of ΔYZB $\therefore m(\angle 3) > m(\angle 4) \therefore m(\angle 1) > m(\angle 4)$ $\therefore m(\angle 4) = m(\angle 5)$ (V.O.A.) $\therefore m(\angle 1) > m(\angle 5)$ and from ΔAYX : $\therefore AY > AX$ (Q.E.D.)

30

 $\therefore m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$ $\therefore 5x + 2^\circ + 6x - 10^\circ + x + 20^\circ = 180^\circ$ $\therefore 12x + 12^\circ = 180^\circ \therefore 12x = 180^\circ - 12^\circ = 168^\circ$ $\therefore x = \frac{168^\circ}{12} = 14^\circ \therefore m(\angle A) = 5 \times 14^\circ + 2^\circ = 72^\circ$ $\therefore m(\angle B) = 6 \times 14^\circ - 10^\circ = 74^\circ$ $m(\angle C) = 14^\circ + 20^\circ = 34^\circ$ $\therefore AB < BC < AC$ (The req.)

31

In ΔABC : $\therefore AB < AC$ $\therefore m(\angle ACB) < m(\angle ABC)$ $\therefore \frac{1}{2} m(\angle ACB) < \frac{1}{2} m(\angle ABC)$ $\therefore m(\angle MCB) < m(\angle MBC)$ (1)And from ΔMBC : $MB < MC$ (2) $\therefore \overline{XY} \parallel \overline{CB}$, \overline{XB} is a transversal $\therefore m(\angle X) = m(\angle MBC)$ (alternate angles) (3) $\therefore \overline{XY} \parallel \overline{BC}$, \overline{CY} is a transversal $\therefore m(\angle Y) = m(\angle MCB)$ (alternate angles) \therefore In ΔXMY from (1), (2) and (3): $\therefore m(\angle Y) < m(\angle X) \therefore XM < MY$ (4)By adding (2) and (4): $\therefore MB + MX < MC + MY$ $\therefore BX < CY$ (Q.E.D.)

Answers of Exercise 9

1

[1] $\therefore 3 + 4 < 9$ \therefore lengths are not suitable[2] $\therefore 5 + 7 > 8$ \therefore lengths are suitable[3] $\therefore 4 + 6 = 10$ \therefore lengths are not suitable[4] $\therefore 6 + 8 > 13$ \therefore lengths are suitable[5] $\therefore 3 + 4 > 5$ \therefore lengths are suitable[6] $\therefore 9 + 9 < 19$ \therefore lengths are not suitable

Geometry

2

Let the length of the third side be l

$$① \because 9 - 6 < l < 9 + 6 \therefore 3 < l < 15$$

$$\therefore l \in]3, 15[$$

$$② \because 3 - 3 < l < 3 + 3 \therefore 0 < l < 6$$

$$\therefore l \in]0, 6[$$

$$③ \because 3.2 - 2.9 < l < 3.2 + 2.9$$

$$\therefore 0.3 < l < 6.1 \therefore l \in]0.3, 6.1[$$

$$④ \because 7.3 - 5.7 < l < 7.3 + 5.7$$

$$\therefore 1.6 < l < 13 \therefore l \in]1.6, 13[$$

3

$$① \text{ b} \quad ② \text{ b} \quad ③ \text{ c} \quad ④ \text{ d} \quad ⑤ \text{ a}$$

$$⑥ \text{ b} \quad ⑦ \text{ d} \quad ⑧ \text{ b} \quad ⑨ \text{ a}$$

4

In $\triangle XLY$: $XL + LY > XY$ (The triangle inequality)

$$\text{But } XL = LZ \therefore LZ + LY > XY$$

$$\therefore YZ > XY \quad (\text{Q.E.D.})$$

5

In $\triangle ABC$:

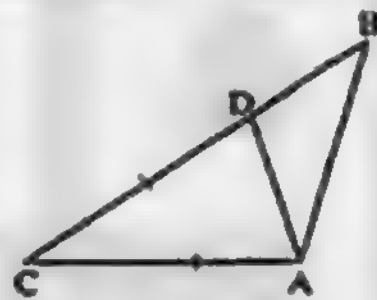
$$\therefore CA + AB > BC$$

(triangle inequality)

$$\therefore CA + AB > BD + DC$$

$$\text{But } CA = DC$$

$$\therefore AB > BD \quad (\text{Q.E.D.})$$



6

From $\triangle ABD$:

$$AD + DB > AB$$

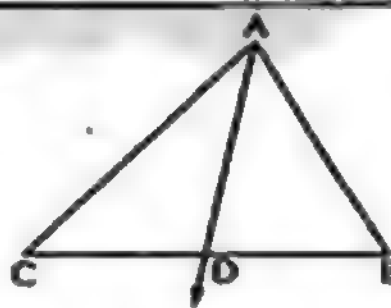
(triangle inequality) (1)

From $\triangle ADC$: $AD + DC > AC$

(Triangle inequality) (2)

Adding (1) and (2):

$$\therefore BD + DC + 2AD > AB + AC \quad (\text{Q.E.D.})$$



7

From $\triangle ABM$: $MA + MB > AB$

(Triangle inequality) (1)

From $\triangle BMC$: $MB + MC > BC$

(Triangle inequality) (2)

From $\triangle AMC$: $MA + MC > AC$ (Triangle inequality) (3)

Adding (1), (2) and (3):

$$\therefore 2MA + 2MB + 2MC > AB + BC + AC$$

$$\therefore MA + MB + MC > \frac{1}{2} \text{ the perimeter of } \triangle ABC$$

(Q.E.D.)

8

From $\triangle AEZ$: $AE + AZ > EZ$ (Triangle inequality) (1)From $\triangle EBF$:

$$EB + BF > EF \quad (\text{Triangle inequality}) \quad (2)$$

From $\triangle ZFC$: $ZC + CF > ZF$ (Triangle inequality) (3)

Adding (1), (2) and (3):

$$\therefore AB + AC + BC > EZ + EF + ZF$$

$$\therefore \text{The perimeter of } \triangle ABC > \text{The perimeter of } \triangle EFZ$$

(Q.E.D.)

9

In $\triangle DAC$: $DA + DC > AC$ (1)In $\triangle DBC$: $DB + DC > BC$ (2)In $\triangle DBA$: $DB + DA > AB$ (3)

Adding (1), (2) and (3):

$$\therefore 2(DA + DB + DC) > AC + BC + AB$$

$$\therefore AC + BC + AB < 2(DA + DB + DC)$$

$$\therefore \text{The perimeter of } \triangle ABC < 2(DA + DB + DC)$$

(Q.E.D.)

10

Assuming that ABC is a triangle

$$\therefore AB < AC + BC \text{ adding AB to both sides}$$

$$\therefore 2AB < AC + BC + AB$$

$$\therefore AB < \frac{1}{2} \text{ the perimeter of } \triangle ABC$$

$$\therefore \text{The length of any side in the triangle is less than the half of the perimeter of the triangle} \quad (\text{Q.E.D.})$$

11

Construction: Draw \overline{AC} Proof: From $\triangle ABC$

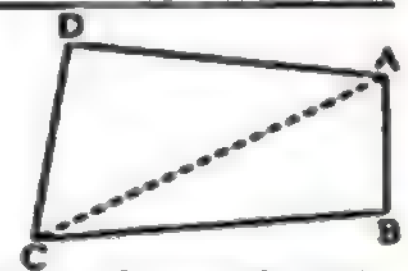
$$AB + BC > AC$$

(Triangle inequality) (1)

From $\triangle ADC$:

$$AC + CD > AD$$

(Triangle inequality) (2)

From (1) and (2): $\therefore AB + BC + CD > AD$ (Q.E.D.)

12

Let ABCD be a quadrilateral

$$\text{In } \triangle ABC : AB + BC > AC \quad (1)$$

$$\text{In } \triangle BCD : BC + CD > BD \quad (2)$$

$$\text{In } \triangle ACD : AD + CD > AC \quad (3)$$

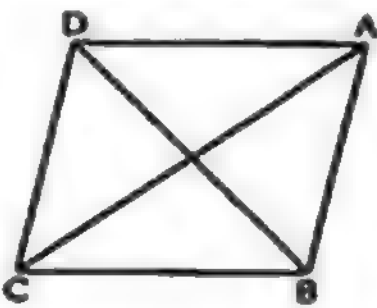
$$\text{In } \triangle ABD : AB + AD > BD \quad (4)$$

Adding (1), (2), (3), and (4):

$$\therefore 2AB + 2BC + 2CD + 2AD > 2AC + 2BD$$

$$\therefore AB + BC + CD + AD > AC + BD$$

\therefore The sum of lengths of the two diagonals in any convex quadrilateral is less than the perimeter of the quadrilateral (Q.E.D.)



13

Let ABCD be a quadrilateral,
 $\overline{AC} \cap \overline{BD} = \{M\}$

$$\text{From } \triangle ABM : AB < MA + MB \quad (1)$$

$$\text{From } \triangle BMC : BC < MB + MC \quad (2)$$

$$\text{From } \triangle CMD : CD < MC + MD \quad (3)$$

$$\text{From } \triangle AMD : AD < MA + MD \quad (4)$$

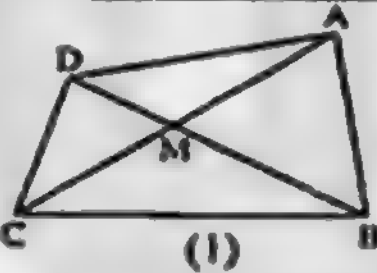
Adding (1), (2), (3) and (4)

$$\therefore AB + BC + CD + AD < 2MA + 2MC + 2MB + 2MD$$

$$\therefore AB + BC + CD + DA < 2(MA + MC) + 2(MB + MD)$$

$$\therefore AB + BC + CD + DA < 2(AC + BD)$$

\therefore The perimeter of the quadrilateral ABCD < twice the sum of lengths of the two diagonals. (Q.E.D.)



14

Construction :

Draw \overline{BM} to cut \overline{AC} at D

Proof :

In $\triangle BDC$:

$$BC + DC > BD \quad (\text{Triangle inequality})$$

$$\therefore BC + DC > BM + MD \quad (1)$$

$$\therefore \text{In } \triangle AMD : AD + MD > AM \quad (\text{Triangle inequality})$$

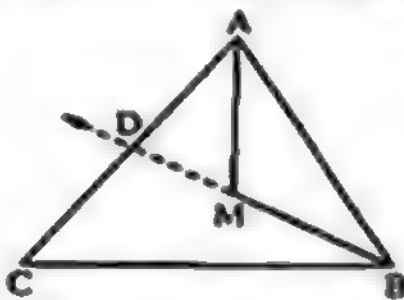
$$\therefore AD > AM - MD \quad (2)$$

Adding (1), (2):

$$\therefore BC + AD + DC > BM + MD + AM - MD$$

$$\therefore BC + AC > BM + AM$$

$$\therefore AM + MB < BC + AC \quad (\text{Q.E.D.})$$



Another solution :

Construction :

Draw \overline{XY} Passing through

the point M where

$$X \in \overline{AC}, Y \in \overline{BC}$$

Proof : In $\triangle CXY$

$$CY + CX > XM + MY \quad \text{adding } BY \text{ and } AX$$

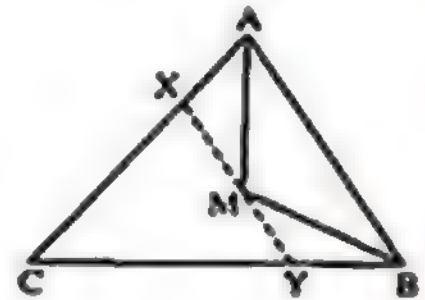
$$\therefore CY + BY + CX + AX > XM + AX + MY + BY$$

$$\therefore BC + AC > XM + AX + MY + BY$$

$$\therefore XM + AX > AM, MY + BY > MB$$

$$\therefore BC + AC > AM + MB$$

$$\therefore AM + MB < AC + BC \quad (\text{Q.E.D.})$$



15

Construction :

Extend \overline{AF} as its length
 to D then draw \overline{CD}

Proof : $\triangle AFB \cong \triangle DFC$ in them :

$$\begin{cases} AF = DF \text{ const.} \\ BF = FC \text{ (given)} \\ m(\angle AFB) = m(\angle DFC) \text{ (V.O.A.)} \end{cases}$$

 \therefore The two triangles are congruentthen we deduce that $AB = DC$ but in $\triangle ACD$ we find that

$$AC + CD > AD \quad (\text{triangle inequality})$$

$$\therefore AC + AB > AD$$

$$\therefore AD = 2AF$$

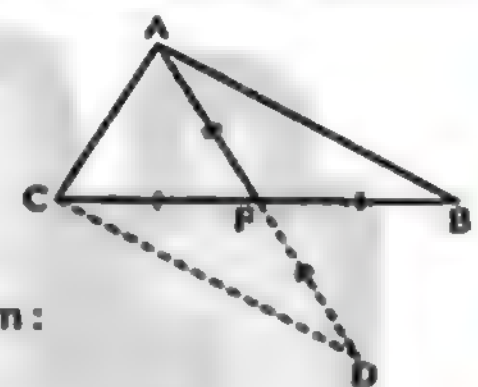
$$\therefore AC + AB > 2AF \quad (1) \text{ (Q.E.D.1)}$$

$$\text{From } \triangle ABC : \therefore AB + AC > BC$$

$$\text{i.e. } AB + AC > 2BF \quad (2)$$

$$\text{Adding (1) and (2) : } 2AB + 2AC > 2AF + 2BF$$

$$\text{Dividing by 2 : } \therefore AB + AC > AF + BF \quad (\text{Q.E.D.2})$$



Geometry

Answers of exams on unit five

Model 1

1

1 b

2 a

3 a

4 a

5 c

6 b

2

1 greater in measure than the angle opposite to the other side.

2 the hypotenuse

3 $\angle C$ 4 $<$

5 2, 8

3

[a] The order is : AB, AC, BC

[b] Prove by yourself.

4

[a] Prove by yourself.

[b] Prove by yourself.

5

[a] Prove by yourself.

[b] Prove by yourself.

Model 2

1

1 c

2 c

3 a

4 d

5 d

6 a

2

1 a side greater in length than that opposite to the other angle.

2 BC

3 BC, AC

4 BC

5 $<$

3

[a] Prove by yourself.

[b] Prove by yourself.

4

[a] The order is : $m(\angle A)$, $m(\angle B)$ and $m(\angle C)$

[b] Prove by yourself.

5

[a] Prove by yourself.

[b] Prove by yourself.

Answers of accumulative basic skills

1

1 $2\sqrt{10}$

2 2 : 3

3 5

4 150°

5 18

6 54

7 $\frac{1}{2}P - y$ 8 $5\sqrt{3}$ 9 108°

10 60

11 9

12 19

2

1 (b)

2 (c)

3 (a)

4 (c)

5 (a)

6 (c)

7 (d)

8 (a)

9 (c)

10 (b)

11 (d)

12 (c)

Guide
Answers

of The Notebook

- Quizzes.
- Final Examinations.



Algebra and Statistics

Answers of the quizzes
on Algebra and Statistics

Quiz ①

1

1 d

2 b

3 c

2

[a] 1 {3}

2 {1}

[b] 20 cm.

Quiz ②

1

1 c

2 c

3 a

2

[a] Prove by yourself.

[b] Represent by yourself.

Quiz ③

1

1 \emptyset 2 \emptyset

3 -125

2

[a] 1 $X = 2.4$ or $X = -2.4$ 2 $X = 4.7$ [b] $\sqrt{2}, \sqrt{3}, \sqrt{5}$ (There are other numbers)

Quiz ④

1

1 c

2 b

3 b

2

[a] 1 $X \cup Y =]-\infty, 4[$ 2 $X \cap Y = [-2, 1[$ 3 $X - Y =]-\infty, -2[$ 4 $\bar{X} = [1, \infty[$ [b] $\{-\sqrt{32}, \sqrt{32}\}$ 

Quiz ⑤

1

1 b

2 d

3 b

2

[a] 1 $\sqrt{3} - 3$ 2 $-2 + \sqrt{3}$

[b]

1 $X \cap Y =]1, 5[$ 2 $X \cup Y = [-1, 7[$ 3 $X - Y = [-1, 1]$

Quiz ⑥

1

1 c

2 c

3 d

2

[a] 0

[b] $\{-\sqrt{3}, \sqrt{3}\}$

Quiz ⑦

1

1 $\sqrt{3} - \sqrt{2}$

2 2

3 $]1, 5[$

2

[a] 12

[b]

 $\mathbb{R} - [1, 2]$

Quiz ⑧

1

1 b

2 d

3 c

2

[a] $\frac{1}{2}$ [b] $\sqrt{2} - \sqrt[3]{9}$

Quiz ⑨

1

1 a

2 c

3 c

Answers of Quizzes

2

[a] $18\pi \text{ cm}^2$

[b] 20

Quiz 10

1

[1] c

[2] d

[3] c

2

[a] $[-2, 1[$ [b] $\sqrt{5}$

Quiz 11

1

[1] c

[2] b

[3] a

2

[a] $(0, -2), (3, 0), (6, 2)$

, there are other solutions , represent by yourself.

[b] $a = -3, b = 0$

Quiz 12

1

[1] -125

[2] $\frac{1}{3}$

[3] undefined

2

[a] Represent by yourself

, the area of $\Delta AOB = 4$ square units

[b] Prove by yourself.

Quiz 13

1

[1] c

[2] b

[3] c

2

[a] 14 cm.

[b] [1] 40 litre

[2] $\frac{1}{3}$ litre/min.

[3] After 2 hr.

Quiz 14

1

[1] c

[2] b

[3] a

2

Answer by yourself.

Quiz 15

1

[1] b

[2] c

[3] c

2

[1] 20 workers

[2] Draw by yourself.

Quiz 16

1

[1] 3

[2] $]-\infty, -3[$

[3] 1230

2

[a] [1] $k = 22$

[2] The arithmetic mean = 50.6

[b] $\sqrt{7}$

Quiz 17

1

[1] c

[2] d

[3] c

2

15 approximately

Quiz 18

1

[1] 2

[2] $2\sqrt[3]{4}$

[3] 0

2

[a] [1] $x = 30, k = 5$

[2] 24.5 approximately.

[b] Prove by yourself , 1

Algebra and Statistics

Answers of school book model examinations on algebra and statistics

Model 1

1

1 $\{-1\}$

2 20

3 $[-2, 2]$

4 24

5 $\sqrt{3} - \sqrt{2}$

2

1 d

2 c

3 c

4 c

5 a

6 b

3

$$\begin{aligned} \text{[a]} \sqrt{2 \times 9} + \sqrt[3]{2 \times 27} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{2 \times 8} \\ = 3\sqrt{2} + 3\sqrt[3]{2} - 3\sqrt{2} - \frac{1}{2} \times 2\sqrt[3]{2} \\ = 2\sqrt[3]{2} \end{aligned}$$

$$\begin{aligned} \text{[b]} \because X &= \frac{3}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{3(\sqrt{5}+\sqrt{2})}{5-2} \\ &= \sqrt{5}+\sqrt{2} \end{aligned}$$

 $\therefore X$ and y are two conjugate numbers.

4

[a] \because The area of the square $= \frac{1}{2} d^2$

$\therefore \frac{1}{2} d^2 = 1089$

$\therefore d^2 = 2178$

$\therefore d = \sqrt{2178} = 33\sqrt{2} \text{ cm.}$

[b] $\because 6 \times \frac{3X+1}{6} < 6(X+1) < 6 \times \frac{X+4}{2}$

$\therefore 3X+1 < 6X+6 < 3X+12$

$\therefore 3X-3X+1 < 6X-3X+6 < 3X-3X+12$

$\therefore 1 < 3X+6 < 12 \quad \therefore 1-6 < 3X < 12-6$

$\therefore -5 < 3X < 6 \quad \therefore -\frac{5}{3} < X < 2$

$\therefore \text{The S.S.} =]-\frac{5}{3}, 2[$



5

$$\begin{aligned} \text{[a]} \text{ The volume of the cylinder} &= \pi r^2 h \\ &= (4\sqrt{2})^2 \times 9 \times \pi \\ &= 288 \pi \text{ cm}^3 \end{aligned}$$

\because the volume of the cylinder
= the volume of the sphere

$\therefore \text{The volume of the sphere} = 288 \pi \text{ cm}^3$

$\therefore \frac{4}{3} \pi r^3 = 288 \pi$

$\therefore r^3 = 288 \times \frac{3}{4} = 216$

$\therefore r = \sqrt[3]{216} = 6 \text{ cm.}$

[b]

Sets	X	f	X × f
5 -	10	7	70
15 -	20	10	200
25 -	30	12	360
35 -	40	13	520
45 -	50	8	400
Total		50	1550

$\therefore \text{The mean} = \frac{1550}{50} = 31$

Model 2

1

1 $\sqrt{3} + \sqrt{5}$

2 6

3 $3 + \sqrt{10}$

4 3

5 $]3, 4]$

2

1 b

2 a

3 b

4 c

5 c

6 d

3

$$\begin{aligned} \text{[a]} \frac{\sqrt{3}(\sqrt{5}+\sqrt{3}) + \sqrt{5}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} \\ = \frac{\sqrt{15}+3+5-\sqrt{15}}{5-3} = \frac{8}{2} = 4 \end{aligned}$$

[b] The left hand side

$$\begin{aligned} &= \sqrt[3]{2 \times 64} + \sqrt[3]{2 \times 8} - 2\sqrt[3]{2 \times 27} \\ &= 4\sqrt[3]{2} + 2\sqrt[3]{2} - 2 \times 3\sqrt[3]{2} = 6\sqrt[3]{2} - 6\sqrt[3]{2} = 0 \\ &= \text{the right hand side} \end{aligned}$$

Answers of Final Examinations

4

$$[a] \because -2-7 < 3x+7-7 \leq 10-7$$

$$\therefore -9 < 3x \leq 3$$

$$\therefore -3 < x \leq 1$$

$$\therefore \text{The S.S.} =]-3, 1]$$



$$[b] \because x = \sqrt{2+\sqrt{3}} \quad \therefore x^2 = 2 + \sqrt{3}$$

$$\therefore x^4 = (2 + \sqrt{3})^2 = 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$$

$$\therefore x^4 - 2x^2 + 1 = 7 + 4\sqrt{3} - 4 - 2\sqrt{3} + 1 = 4 + 2\sqrt{3}$$

5

[a] 20

[b]

Sets	x	f	x × f
5 -	10	4	40
15 -	20	5	100
25 -	30	6	180
35 -	40	3	120
45 -	50	2	100
Total		20	540

$$\therefore \text{The mean} = \frac{540}{20} = 27$$

Answers of model for the merge students

1

- 1 $\sqrt{3}-\sqrt{2}$ 2 $3\sqrt{6}$ 3 3
4 5 5 \emptyset

2

- 1 a 2 b 3 a 4 a 5 a

3

- 1 $\{5, -5\}$ 2 $[0, 2]$ 3 7
4 irrational

5



4

- 1 ✓ 2 ✓ 3 ✓ 4 x 5 ✓

5

$$[a] \text{ The centre} = \frac{8+4}{2} = 6$$

[b]

Sets	The centre of the set = \bar{x}	Frequency = f	$x \times f$
5 -	10	7	$10 \times 7 = 70$
15 -	20	10	$20 \times 10 = 200$
25 -	30	12	$30 \times 12 = 360$
35 -	40	13	$40 \times 13 = 520$
45 -	50	8	$50 \times 8 = 400$
Total		50	1550

$$\therefore \text{The arithmetic mean} = \frac{\text{The sum of } (x \times f)}{\text{The sum of } (f)} = \frac{1550}{50} = 31$$

Algebra and Statistics

Answers of schools examinations
on algebra and statistics

1 Cairo

- 1 (d) (b) (d)
4 (a) (a) (d)

- 2 1 [1, 3] 2 4 3 6
4]2, 3[5 0

3

$$\begin{aligned} \text{[a]} a^2 - ab + b^2 &= (a - b)^2 + ab \\ &= (\sqrt{3} + \sqrt{2} - \sqrt{3} + \sqrt{2})^2 + (\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) \\ &= (2\sqrt{2})^2 + 1 = 8 + 1 = 9 \end{aligned}$$

$$\text{[b]} \text{ (1) } \because 5x - 3 < 2x + 9 \quad \therefore 3x < 12$$

$$\therefore x < 4$$

$$\therefore \text{The S.S.} =]-\infty, 4[$$



$$\text{[2] } \because 1 \leq 3 - 2x < 5 \quad \therefore -2 \leq -2x < 2$$

$$\therefore -1 < x \leq 1$$

$$\therefore \text{The S.S.} =]-1, 1]$$



4



$$\text{[1] } M \cap J = [2, 3[\quad \text{[2] } M - J = [3, \infty[$$

$$\begin{aligned} \text{[b]} \frac{\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}} &= \frac{\sqrt{3}(\sqrt{5}+\sqrt{3}) + \sqrt{5}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} \\ &= \frac{\sqrt{15} + 3 + 5 - \sqrt{15}}{5-3} = \frac{8}{2} = 4 \end{aligned}$$

5

$$\begin{aligned} \text{[a]} 2\sqrt{9 \times 2} + \sqrt{25 \times 2} + \frac{1}{3}\sqrt{81 \times 2} \\ = 6\sqrt{2} + 5\sqrt{2} + 3\sqrt{2} = 14\sqrt{2} \end{aligned}$$

[b]

Sets	x	f	x × f
5 -	10	4	40
15 -	20	5	100
25 -	30	6	180
35 -	40	3	120
45 -	50	2	100
Total		20	540

$$\therefore \text{The mean} = \frac{540}{20} = 27$$

2 Cairo

- 1 (a) (d) (b)
4 (a) (b) (c)

- 2 1 {2, 7} 2 2 3 R
4 $\frac{2}{3}$ 5 $\{\sqrt{12}, -\sqrt{12}\}$

3

$$\begin{aligned} \text{[a]} 2\sqrt{4 \times 2} + \sqrt{25 \times 2} - \sqrt{16 \times 2} \\ = 4\sqrt{2} + 5\sqrt{2} - 4\sqrt{2} = 5\sqrt{2} \end{aligned}$$

$$\text{[b]} \because 3x - 4 \leq 5 \quad \therefore 3x \leq 9$$

$$\therefore x \leq 3$$

$$\therefore \text{The S.S.} =]-\infty, 3]$$

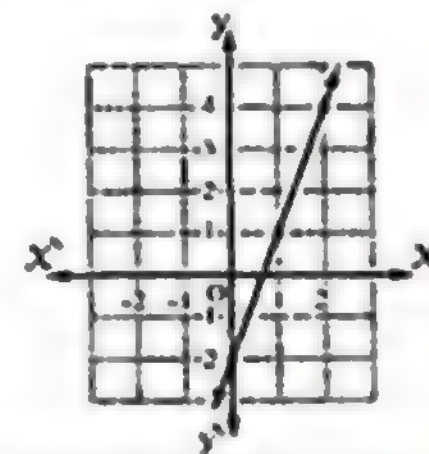


4

$$\begin{aligned} \text{[a]} \because x &= \frac{2}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} \\ &= \frac{2(\sqrt{7}+\sqrt{5})}{7-5} = \sqrt{7}+\sqrt{5} \\ \therefore (x+y)^2 &= (\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5})^2 \\ &= (2\sqrt{7})^2 = 28 \end{aligned}$$

$$\text{[b]} y = 3x - 2$$

x	0	1	2
y	-2	1	4



5

$$\text{[a]} \because \text{The volume} = \frac{4}{3} \pi r^3 \quad \therefore \frac{500}{3} \pi = \frac{4}{3} \pi r^3$$

$$\therefore r^3 = \frac{500}{3} \times \frac{3}{4} = 125$$

$$\therefore r = 5 \text{ cm.}$$

Answers of Final Examinations

[b]

Sets	x	f	$x \times f$
5 -	10	7	70
15 -	20	10	200
25 -	30	12	360
35 -	40	13	520
45 -	50	8	400
Total		50	1550

$$\therefore \text{The mean} = \frac{1550}{50} = 31$$

3 Cairo

- 1 (d) 2 (b) 3 (a)
4 (a) 5 (b) 6 (b)

- 2 1 -27 2 2 3 9
4 [3, 5] 5 7

3

[a] \therefore The volume = $\frac{4}{3} \pi r^3$ $\therefore 562.5 \pi = \frac{4}{3} \pi r^3$

$$\therefore r^3 = \frac{562.5 \times 3}{4} = 421.875 \quad \therefore r = 7.5 \text{ cm.}$$

$$\therefore \text{The area} = 4 \times \pi \times (7.5)^2 = 225 \pi \text{ cm}^2$$

[b] $\therefore X = \frac{4}{\sqrt{7} + \sqrt{3}} \times \frac{\sqrt{7} - \sqrt{3}}{\sqrt{7} - \sqrt{3}} = \frac{4(\sqrt{7} - \sqrt{3})}{7 - 3}$
 $= \sqrt{7} - \sqrt{3}$

$$\therefore X^2 - 2Xy + y^2 = (X - y)^2$$
$$= (\sqrt{7} - \sqrt{3} - \sqrt{7} - \sqrt{3})^2$$
$$= (-2\sqrt{3})^2 = 12$$

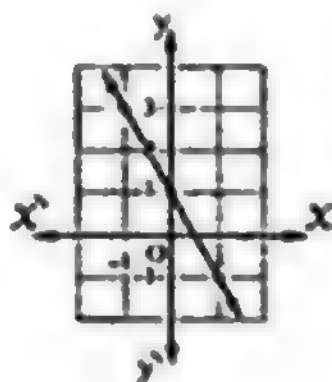
4

[a] $\therefore -1 < 3X + 5 \leq 14$ $\therefore -6 < 3X \leq 9$
 $\therefore -2 < X \leq 3$ $\therefore \text{The S.S.} =]-2, 3]$



[b] $2X + y = 1$

x	-1	0	1
y	3	1	-1



[c]



1 $A \cap B = [-1, 3[$ 2 $A - B =]-\infty, -1[$

5

[a] The slope of $\overline{AB} = \frac{5-3}{2-1} = \frac{2}{1}$

$$\therefore \text{the slope of } \overline{BC} = \frac{1-5}{8-2} = \frac{-4}{6} = \frac{-2}{3}$$

$$\therefore \text{The slope of } \overline{AB} \neq \text{the slope of } \overline{BC} \quad \therefore C \notin \overline{AB}$$

[b]

Sets	x	f	$x \times f$
5 -	10	7	70
15 -	20	10	200
25 -	30	12	360
35 -	40	13	520
45 -	50	8	400
Total		50	1550

$$\therefore \text{The mean} = \frac{1550}{50} = 31$$

4 Giza

- 1 1 8 2 [-3, 4[3 3
4 2.5 5 3

- 2 1 (b) 2 (a) 3 (c)
4 (d) 5 (b) 6 (c)

3



[a]

1 $X =]-\infty, -2[\cup]4, \infty[$

2 $X \cap Y =]1, 4[$

3 $X - Y = [-2, 1]$

[b] $\therefore 2X + 1 < 7$

$$\therefore 2X < 6$$

$$\therefore X < 3$$

$$\therefore \text{The S.S.} =]-\infty, 3[$$

4

[a] $2\sqrt{2} \times 9 + \sqrt{2} \times 25 - \sqrt{2} \times 81$
 $= 6\sqrt{2} + 5\sqrt{2} - 9\sqrt{2} = 2\sqrt{2}$

[b] $\therefore y = \frac{4}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{4(3-\sqrt{5})}{9-5} = 3-\sqrt{5}$

$$\therefore X = 3+\sqrt{5} \quad \therefore X, y \text{ are conjugate numbers.}$$

$$\therefore X^2 - 2Xy + y^2 = (X - y)^2$$

$$= (3+\sqrt{5} - 3+\sqrt{5})^2 = (2\sqrt{5})^2 = 20$$

Algebra and Statistics

5

[a] \therefore The volume of the cuboid $= 77 \times 24 \times 21$
 $= 38808 \text{ cm}^3$

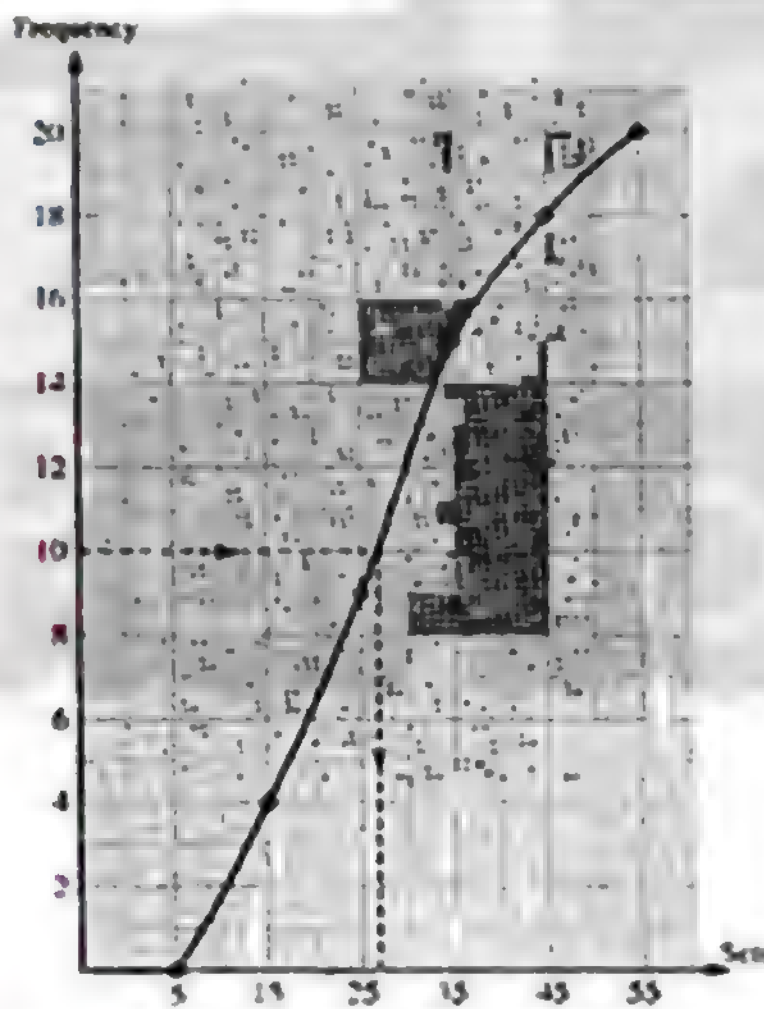
\therefore The volume of the sphere $= 38808 \text{ cm}^3$

$\therefore \frac{4}{3} \pi r^3 = 38808$

$\therefore r^3 = \frac{38808 \times 7 \times 3}{22 \times 4} = 9261 \quad \therefore r = 21 \text{ cm.}$

[b]

The upper boundaries of sets	Ascending cumulative frequency
less than 5	0
less than 15	4
less than 25	9
less than 35	15
less than 45	18
less than 55	20



\therefore The order of the median $= \frac{20}{2} = 10$

\therefore The median $= 27$

5

Giza

1

(c)

(d)

(d)

(d)

(b)

(a)

2

4

 $2x$ $1:2$

0

 $]2, 3[$

3

[a] $\therefore y = \frac{1}{x} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$
 $= \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$

$\therefore (x+y)^2 = (\sqrt{7}-\sqrt{6}+\sqrt{7}+\sqrt{6})^2$
 $= (2\sqrt{7})^2 = 28$

[b] $\therefore -15 \leq 2x-3 \leq 5 \quad \therefore -12 \leq 2x \leq 8$

$\therefore -6 \leq x \leq 4 \quad \therefore$ The S.S. $= [-6, 4]$

[c] $\sqrt[3]{27 \times 2} + 4\sqrt[3]{8 \times \frac{1}{4}} + 5\sqrt[3]{8 \times 2}$
 $= 3\sqrt[3]{2} + 4\sqrt[3]{2} + 10\sqrt[3]{2} = 17\sqrt[3]{2}$

4

[a]



[1] $X \cap Y = [1, 5]$ [2] $X \cup Y =]-\infty, 9[$

[3] $X - Y =]-\infty, 1]$

[b] The slope $= \frac{5-4}{4-2} = \frac{1}{2}$

5

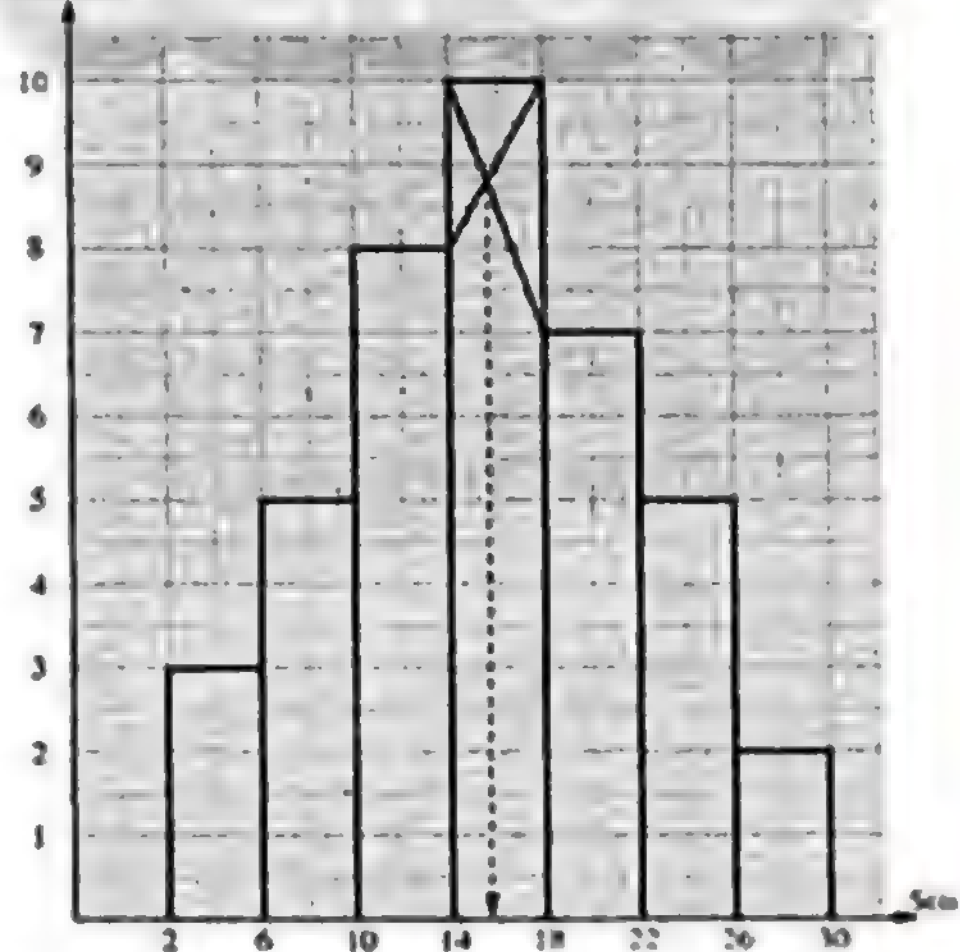
[a] $\therefore 125x^3 - 7 = 20 \quad \therefore 125x^3 = 27$

$\therefore x^3 = \frac{27}{125}$

$\therefore x = \frac{3}{5}$

\therefore The S.S. $= \left\{ \frac{3}{5} \right\}$

[b] Frequency



\therefore The mode $= 16$

6 Alexandria

- 1 (a) (b) (c)
(d) (e) (f)

- 2 (a) zero (b) 5 (c) $[-2, 2]$
(d) 3 (e) 24 cm.

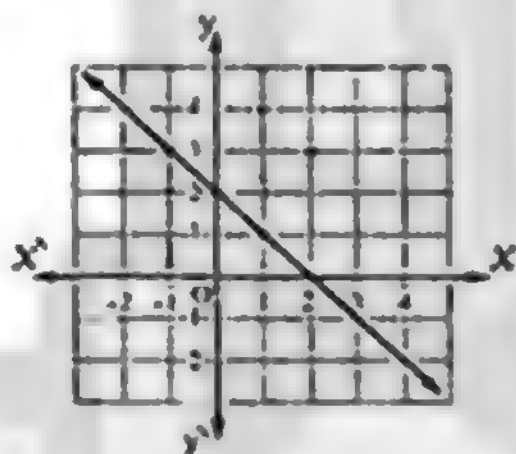
3

$$[a] \sqrt{9 \times 2} + \sqrt[3]{27 \times 2} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{8 \times 2}$$

$$= 3\sqrt{2} + 3\sqrt[3]{2} - 3\sqrt{2} - \sqrt[3]{2} = 2\sqrt[3]{2}$$

$$[b] y = 2 - x$$

x	-1	0	1
y	3	2	1



4

$$[a] \because -2 < 3x + 7 \leq 10 \quad \therefore -9 < 3x \leq 3$$

$$\therefore -3 < x \leq 1 \quad \therefore \text{The S.S.} =]-3, 1]$$



$$[b] \frac{\sqrt{3}(\sqrt{5} + \sqrt{3}) + \sqrt{5}(\sqrt{5} - \sqrt{3})}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})}$$

$$= \frac{\sqrt{15} + 3 + 5 - \sqrt{15}}{5 - 3} = \frac{8}{2} = 4$$

5

$$[a] \because (\sqrt{3})^x = (2\sqrt{2})^2 - (\sqrt{5})^2 = 8 - 5 = 3$$

$$\therefore (\sqrt{3})^x = (\sqrt{3})^2 \quad \therefore x = 2$$

Sets	x	f	x × f
5 -	10	7	70
15 -	20	10	200
25 -	30	12	360
35 -	40	13	520
45 -	50	8	400
Total		50	1550

$$\therefore \text{The mean} = \frac{1550}{50} = 31$$

7 Alexandria

- 1 (c) (b) (a)
(d) (e) (f)

- 2 $-\sqrt[3]{5}$ \mathbb{R}^* $\sqrt{a} - \sqrt{b} + 2\sqrt{a}$
(d) 2 (e) 0

3

$$[a] (1) \sqrt{16 \times 2} - \sqrt{25 \times 2} + 2\sqrt{4 \times \frac{1}{2}}$$

$$= 4\sqrt{2} - 5\sqrt{2} + 2\sqrt{2} = \sqrt{2}$$

$$(2) \sqrt[3]{8 \times 2} - \frac{1}{3}\sqrt[3]{27 \times 2} = 2\sqrt[3]{2} - \sqrt[3]{2} = \sqrt[3]{2}$$

$$[b] \because y = \frac{2}{x} = \frac{2}{\sqrt{7} + \sqrt{5}} \times \frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} - \sqrt{5}}$$

$$= \frac{2(\sqrt{7} - \sqrt{5})}{7 - 5} = \sqrt{7} - \sqrt{5}$$

$$\therefore \frac{x+y}{xy} = \frac{\sqrt{7} + \sqrt{5} + \sqrt{7} - \sqrt{5}}{(\sqrt{7} + \sqrt{5})(\sqrt{7} - \sqrt{5})} = \frac{2\sqrt{7}}{7 - 5} = \sqrt{7}$$

4

$$[a] \because -1 \leq 3 - 2x < 5 \quad \therefore -4 \leq -2x < 2$$

$$\therefore 2 \geq x > -1 \quad \therefore \text{The S.S.} =]-1, 2]$$



$$[b] \because \text{The volume} = \pi r^2 h$$

$$\therefore \pi r^2 \times r = 72\pi$$

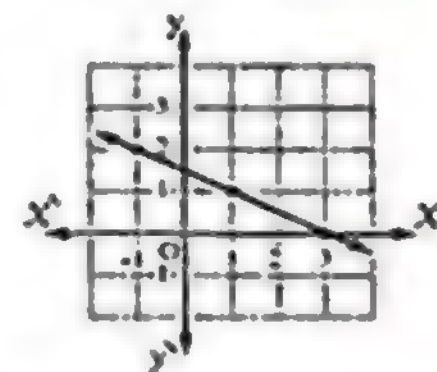
$$\therefore r^3 = 72$$

$$\therefore r = 2\sqrt[3]{9}$$

$$\therefore h = 2\sqrt[3]{9} \text{ cm.}$$

$$[c] x + 2y = 3$$

x	-1	1	3
y	2	1	0



5

$$[a] \because \text{The slope of } \overline{AB} = \frac{5-3}{2+1} = \frac{2}{3}$$

$$\therefore \text{the slope of } \overline{BC} = \frac{1-5}{8-2} = \frac{-2}{3}$$

$$\therefore \text{The slope of } \overline{AB} \neq \text{the slope of } \overline{BC}$$

$$\therefore C \notin \overline{AB}$$

Algebra and Statistics

(b)

Sets	X	f	X × f
8 -	10	4	40
12 -	14	10	140
16 -	18	16	288
20 -	22	12	264
24 -	26	8	208
Total		50	940


$$\therefore \text{The mean} = \frac{940}{50} = 18.8$$

8 El-Kalyoubia

- 1 (1) (b) (2) (c) (3) (c)
 (4) (a) (5) (a) (6) (a)

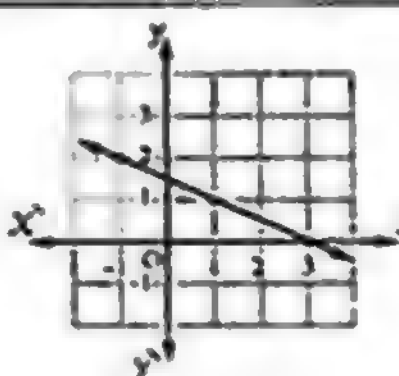
- 2 (1) $\frac{1}{4}$ (2) 0 (3) $\frac{3}{4}$
 (4) -5 (5) $\sqrt{3} - \sqrt{2}$

- 3
 (a) $\because X^3 - 1000 = 0 \quad \therefore X^3 = 1000$
 $\therefore X = \sqrt[3]{1000} = 10$
 (b) $\because \text{The area} = \pi r^2 \quad \therefore \pi r^2 = 3\pi$
 $\therefore r^2 = 3 \quad \therefore r = \sqrt{3} \text{ cm.}$
 $\therefore \text{The circumference} = 2\pi r = 2\sqrt{3}\pi \text{ cm.}$

- 4
 (a) $[2, 3[$ 
 (b) $(\sqrt{2} + 5)(3 + \sqrt{2}) = 3\sqrt{2} + 17 + 5\sqrt{2} = 17 + 8\sqrt{2}$

- 5
 (a) $X + 2y = 3$

X	-1	1	3
y	2	1	0



(b)

Sets	X	f	X × f
5 -	10	4	40
15 -	20	5	100
25 -	30	6	180
35 -	40	3	120
45 -	50	2	100
Total		20	540

$$\therefore \text{The mean} = \frac{540}{20} = 27$$

9 El-Gharbia

- 1 (1) (d) (2) (c) (3) (c)
 (4) (d) (5) (b) (6) (b)

- 2 (1) 20 (2) $\sqrt{3} - \sqrt{2}$ (3) $]3, 4]$
 (4) 4 (5) $-\frac{4}{3}$

3
 (a) (1) $\because y = \frac{2}{\sqrt{7} + \sqrt{5}} \times \frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} - \sqrt{5}} = \frac{2(\sqrt{7} - \sqrt{5})}{7 - 5}$
 $= \sqrt{7} - \sqrt{5}$

$$\therefore X = \sqrt{7} + \sqrt{5}$$

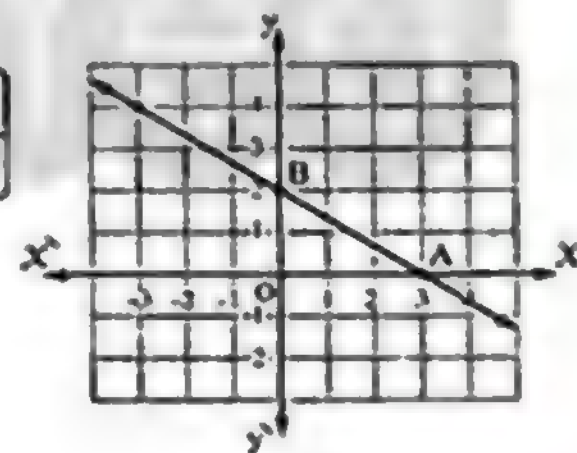
$\therefore X, y$ are conjugate numbers.

(2) $xy = (\sqrt{7} + \sqrt{5})(\sqrt{7} - \sqrt{5}) = 7 - 5 = 2$
 $\therefore (X + y)^2 = (\sqrt{7} + \sqrt{5} + \sqrt{7} - \sqrt{5})^2$
 $= (2\sqrt{7})^2 = 28$

(b) $\sqrt{4 \times 3} + \sqrt[3]{27 \times 2} - \sqrt{3} - \sqrt[3]{8 \times 2}$
 $= 2\sqrt{3} + 3\sqrt[3]{2} - \sqrt{3} - 2\sqrt[3]{2}$
 $= \sqrt{3} + \sqrt[3]{2}$

4
 (a) $2X + 3y = 6$

X	-3	0	3
y	4	2	0



From the graph :

The area of $\triangle OAB = \frac{1}{2} \times 3 \times 2 = 3$ square units.

(b) $\because 8X^3 + 7 = 8 \quad \therefore 8X^3 = 1$
 $\therefore X^3 = \frac{1}{8} \quad \therefore X = \frac{1}{2}$
 $\therefore \text{The S.S.} = \left\{ \frac{1}{2} \right\}$

5
 (a) $\because 2X - 1 \geq 5 \quad \therefore 2X \geq 6$
 $\therefore X \geq 3 \quad \therefore \text{The S.S.} = [3, \infty[$

Answers of Final Examinations

[b]

Sets	X	f	X × f
5 -	10	4	40
15 -	20	5	100
25 -	30	6	180
35 -	40	3	120
45 -	50	2	100
Total		20	540

$$\therefore \text{The mean} = \frac{540}{20} = 27$$

10 El-Dakahlia

- 1 (1) (d) (2) (d) (3) (b)
 (4) (a) (5) (c) (6) (c)

- 2 (1) $]-3, 7[$ (2) \emptyset (3) 3
 (4) 0 (5) $\frac{3}{2}$ cm.

3

$$\begin{aligned} \text{[a]} \sqrt{9 \times 2} + \sqrt[3]{27 \times 2} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{8 \times 2} \\ = 3\sqrt{2} + 3\sqrt[3]{2} - 3\sqrt{2} - \sqrt[3]{2} = 2\sqrt[3]{2} \end{aligned}$$

[b]



- 1 $X \cap Y =]1, 4]$
 2 $X - Y = [-3, 1]$

4

$$\begin{aligned} \text{[a]} \because 7 \leq -3X + 1 < 13 \quad \therefore -8 \leq -3X < 12 \\ \therefore -4 < X \leq \frac{8}{3} \quad \therefore \text{The S.S.} = \left]-4, \frac{8}{3}\right] \end{aligned}$$



$$\text{[b]} (1) \because y = \frac{1}{\sqrt{6} + \sqrt{5}} \times \frac{\sqrt{6} - \sqrt{5}}{\sqrt{6} - \sqrt{5}} = \sqrt{6} - \sqrt{5}$$

$$, x = \sqrt{6} + \sqrt{5}$$

$\therefore X, y$ are conjugate numbers.

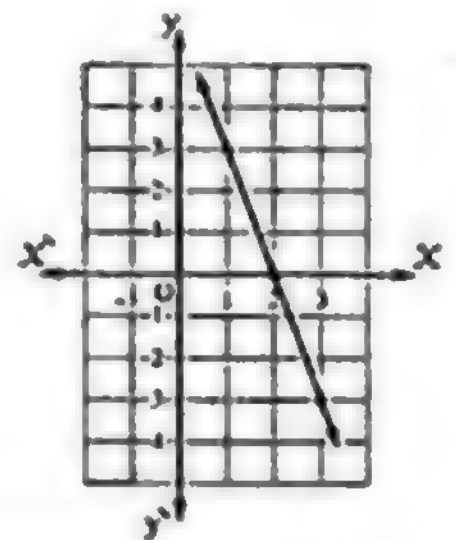
$$\begin{aligned} (2) (X - y)^2 &= (\sqrt{6} + \sqrt{5} - \sqrt{6} + \sqrt{5})^2 \\ &= (2\sqrt{5})^2 = 20 \end{aligned}$$

5

$$\text{[a]} y + 3x = 6$$

x	1	2	3
y	3	0	-3

$$\text{The slope} = \frac{-3 - 0}{3 - 2} = -3$$



[b]

Sets	X	f	X × f
10 -	15	5	75
20 -	25	15	375
30 -	35	20	700
40 -	45	25	1125
50	55	10	550
Total		75	2825

$$\therefore \text{The mean} = \frac{2825}{75} = \frac{113}{3}$$

11 Ismailia

- 1 (1) (b) (2) (c) (3) (a)
 (4) (b) (5) (d) (6) (b)

- 2 (1) 12 (2) $\sqrt{3} + \sqrt{2}$ (3) 11
 (4) 0 (5) 3

3

$$\begin{aligned} \text{[a]} \because 8 \leq 3X + 2 \leq 17 \quad \therefore 6 \leq 3X \leq 15 \\ \therefore 2 \leq X \leq 5 \quad \therefore \text{The S.S.} = [2, 5] \end{aligned}$$

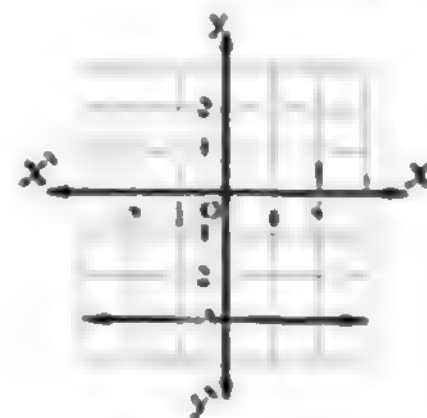


$$\begin{aligned} \text{[b]} \sqrt{36 \times 2} + 3\sqrt{9 \times 2} - \sqrt{4 \times \frac{1}{2}} \\ = 6\sqrt{2} + 9\sqrt{2} - \sqrt{2} = 14\sqrt{2} \end{aligned}$$

4

$$\begin{aligned} \text{[a]} \because \text{The volume} &= \pi r^2 h \\ \therefore 1540 &= \frac{22}{7} \times r^2 \times 10 \quad \therefore r^2 = \frac{1540 \times 7}{22 \times 10} = 49 \\ \therefore r &= 7 \text{ cm.} \quad \therefore d = 14 \text{ cm.} \end{aligned}$$

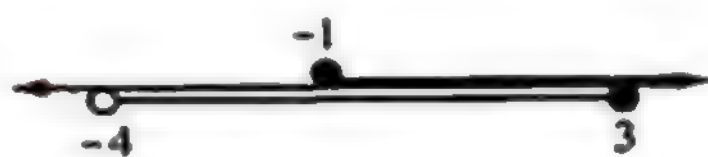
[b]



Algebra and Statistics

5

[a]



① $X \cap Y = [-1, 3]$

② $X \cup Y =]-4, \infty[$

③ $X =]-\infty, -1[$

[b]

Sets	x	f	$x \times f$
10 -	15	8	120
20 -	25	12	300
30 -	35	14	490
40 -	45	9	405
50 -	55	7	385
Total		50	1700

$$\therefore \text{The mean} = \frac{1700}{50} = 34$$

12 Damietta

1

① (b)

② (d)

③ (b)

④ (c)

⑤ (b)

⑥ (a)

2

① -2

② 5

③ the median

④ 4

⑤ 6

3

$$[a] \therefore y = \frac{3}{x} = \frac{3}{\sqrt{5} + \sqrt{2}} \times \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}} = \frac{3(\sqrt{5} - \sqrt{2})}{5 - 2}$$

$$= \sqrt{5} - \sqrt{2}$$

$$\therefore \frac{x+y}{xy} = \frac{\sqrt{5} + \sqrt{2} + \sqrt{5} - \sqrt{2}}{(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})} = \frac{2\sqrt{5}}{3}$$

[b] $\therefore -3 \leq 4x - 7 \leq 5$

$$\therefore 4 \leq 4x \leq 12$$

$$\therefore 1 \leq x \leq 3$$

$$\therefore \text{The S.S.} = [1, 3]$$

[c] $\therefore \text{The volume} = \pi r^2 h$

$$\therefore 72\pi = \pi \times r^2 \times 8$$

$$\therefore r^2 = \frac{72}{8} = 9$$

$$\therefore r = 3 \text{ cm.}$$

4

$$[a] \sqrt{25 \times 2} + \sqrt[3]{27 \times 2} - 5\sqrt{4 \times \frac{1}{2}} - \sqrt[3]{8 \times 2}$$

$$= 5\sqrt{2} + 3\sqrt[3]{2} - 5\sqrt{2} - 2\sqrt[3]{2} = \sqrt[3]{2}$$

[b]



① $X \cup Y = [-1, \infty[$

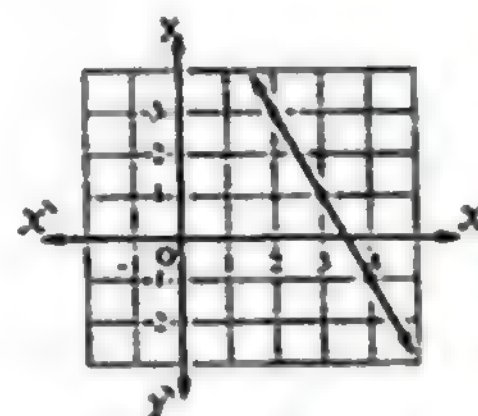
② $X \cap Y = [2, 5[$

③ $X - Y = [-1, 2[$

5

[a] $2x + y = 7$

x	2	3	4
y	3	1	-1



[b]

Sets	x	f	$x \times f$
5 -	10	4	40
15 -	20	5	100
25 -	30	6	180
35 -	40	3	120
45 -	50	2	100
Total		20	540

$$\therefore \text{The mean} = \frac{540}{20} = 27$$

13 Kafr El-Sheikh

1

① (c)

② (b)

③ (a)

④ (d)

⑤ (c)

⑥ (b)

2

① 1

② 2

③ 8.5

④ 6

⑤ fourth

3

[a] The volume $= \pi r^2 h = \frac{22}{7} \times 5^2 \times 7 = 550 \text{ cm}^3$

[b]



① $X \cap Y =]1, 5]$

② $X \cup Y =]-\infty, 7]$

③ $Y - X =]5, 7]$

[c] $\therefore 8x^3 + 7 = 8$

$$\therefore 8x^3 = 1$$

$$\therefore x^3 = \frac{1}{8}$$

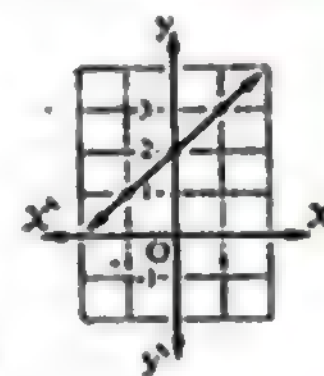
$$\therefore x = \frac{1}{2}$$

$$\therefore \text{The S.S.} = \left\{ \frac{1}{2} \right\}$$

4

[a] $y = x + 2$

x	-1	0	1
y	1	2	3



$$\therefore (-4, a) \text{ satisfies the relation}$$

$$\therefore a = -4 + 2 = -2$$

Answers of Final Examinations

$$[b] \sqrt{9 \times 2} + \sqrt{25 \times 2} - 2\sqrt{4 \times 2}$$

$$= 3\sqrt{2} + 5\sqrt{2} - 4\sqrt{2} = 4\sqrt{2}$$

$$[c] \because -8 < 3X + 1 \leq 4 \quad \therefore -9 < 3X \leq 3$$

$$\therefore -3 < X \leq 1 \quad \therefore \text{The S.S.} =]-3, 1]$$

5

$$[a] \because y = \frac{1}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

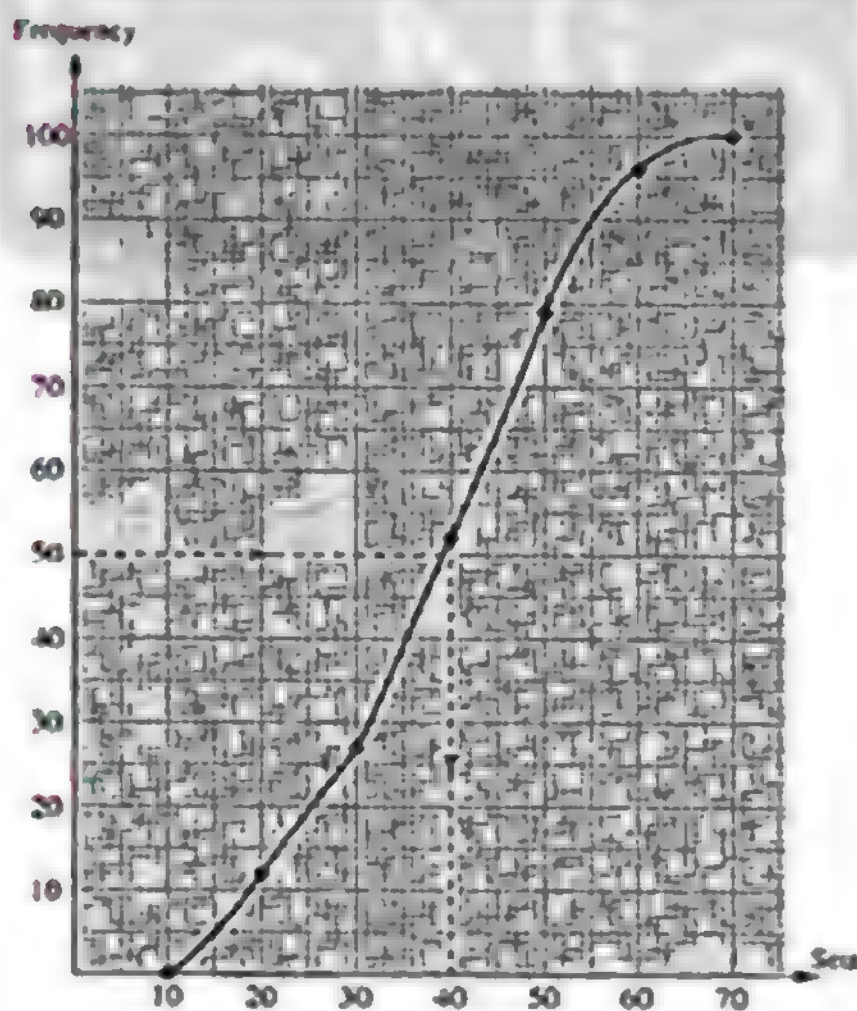
$$= \sqrt{3} - \sqrt{2}$$

$$\therefore \frac{x+y}{xy} = \frac{\sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2}}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})} = 2\sqrt{3}$$

$$[b] \text{ 1 } k = 13$$

2

The upper limits of sets	Ascending cumulative frequency
less than 10	0
less than 20	12
less than 30	27
less than 40	52
less than 50	79
less than 60	96
less than 70	100



$$\therefore \text{The order of the median} = \frac{100}{2} = 50$$

$$\therefore \text{The median} = 40$$

14 Souhag

- 1 1 (c) 2 (b) 3 (d)
4 (c) 5 (c) 6 (c)

- 2 1]1, 5[2 {0, 1, -1} 3 6X
4 5 5 8

3

$$[a] y = 2X + 1$$

$$[b] \because -2 < 3X + 7 \leq 10 \quad \therefore -9 < 3X \leq 3$$

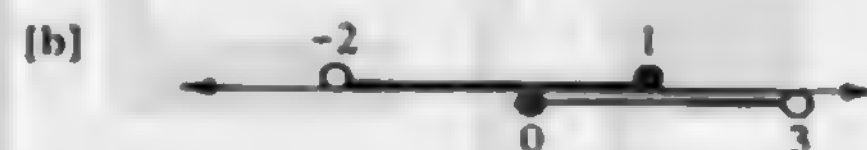
$$\therefore -3 < X \leq 1 \quad \therefore \text{The S.S.} =]-3, 1]$$



4

$$[a] \because y = \frac{1}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \sqrt{3} - \sqrt{2}$$

$$\therefore \frac{x+y}{xy} = \frac{\sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2}}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})} = 2\sqrt{3}$$



- 1 $X \cap Y = [0, 1]$ 2 $X \cup Y =]-2, 3[$
3 $X - Y =]-2, 0[$

5

$$[a] \text{ 1 } \sqrt{25 \times 2} + \sqrt{9 \times 2} - \sqrt{16 \times 2}$$

$$= 5\sqrt{2} + 3\sqrt{2} - 4\sqrt{2} = 4\sqrt{2}$$

$$\text{2 } \sqrt[3]{27 \times 2} + 4\sqrt[3]{8 \times \frac{1}{4}} + 5\sqrt[3]{8 \times 2}$$

$$= 3\sqrt[3]{2} + 4\sqrt[3]{2} + 10\sqrt[3]{2} = 17\sqrt[3]{2}$$

[b]

Sets	X	f	X × f
5 -	10	4	40
15 -	20	5	100
25 -	30	6	180
35 -	40	3	120
45 -	50	2	100
Total		20	540

$$\therefore \text{The mean} = \frac{540}{20} = 27$$

Algebra and Statistics

15 Luxor

- 1 (c) 2 (a) 3 (c)
4 (a) 5 (c) 6 (c)

- 2 (1) $]2, 7[$ (2) 5 (3) 4
(4) undefined (5) 6

3

$$[a] \sqrt{9 \times 3} - \sqrt{4 \times 3} + \sqrt{100 \times 3}$$

$$= 3\sqrt{3} - 2\sqrt{3} + 10\sqrt{3} = 11\sqrt{3}$$

$$[b] a^2 + 2ab + b^2 = (a + b)^2$$

$$= (\sqrt{5} + \sqrt{3} + \sqrt{5} - \sqrt{3})^2 = (2\sqrt{5})^2 = 20$$

4

$$[a] \because 2x + 1 \leq 7 \quad \therefore 2x \leq 6 \quad \therefore x \leq 3$$

$$\therefore \text{The S.S.} =]-\infty, 3]$$



$$[b] \text{ The volume} = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (2.1)^3$$

$$= 38.808 \text{ cm}^3.$$

5

$$[a] \text{ The slope of } \overline{AB} = \frac{3+1}{10-2} = \frac{1}{2}$$

$$\text{The slope of } \overline{BC} = \frac{3-3}{2-10} = 0$$

[b]

Sets	x	f	x × f
5 -	10	4	40
15 -	20	5	100
25 -	30	6	180
35 -	40	3	120
45 -	50	2	100
Total		20	540

$$\therefore \text{The mean} = \frac{540}{20} = 27$$

Answers of Quizzes

Answers of the quizzes
on Geometry

Quiz ①

1

- 1 one point 2 2 : 1 3 9

2

[a] The perimeter of $\Delta MBC = 26$ cm.

[b] Prove by yourself.

Quiz ②

1

- 1 half the length of the hypotenuse
2 8
3 half the length of the hypotenuse

2

[a] $BD = 4.5$ cm, $BM = 3$ cm, $AB = 4.5$ cm.

[b] Prove by yourself.

Quiz ③

1

- 1 120° 2 35° 3 right

2

[a] Prove by yourself.

[b] Prove by yourself.

Quiz ④

1

- 1 an equilateral triangle 2 AC
3 $\frac{1}{2}$

2

[a] Prove by yourself.

[b] Prove by yourself.

Quiz ⑤

1

- 1 bisects the base and is perpendicular to it
2 $\frac{1}{3}$ 3 equidistant

2

[a] $AD = 6$ cm.[b] $m(\angle MLY) = 70^\circ$

Quiz ⑥

1

- 1 the measure of any interior angle of the triangle except its adjacent angle.
2 $\frac{2}{3}$ 3 >

2

[a] Prove by yourself.

[b] Prove by yourself.

Quiz ⑦

1

- 1 to the angle of the greater measure
2 axis of symmetry
3 B, A, C

2

[a] Prove by yourself.

[b] Prove by yourself.

Quiz ⑧

1

- 1 The hypotenuse 2 \overline{AB}
3 45°

2

[a] Prove by yourself.

[b] The perimeter of the figure ADME = 13 cm.

Quiz ⑨

1

- 1 c 2 b 3 d

2

[a] Prove by yourself.

[b] Prove by yourself.

Geometry

Answers of school book model examinations on geometry

Model 1

1

- 1 The hypotenuse [2] 5 cm, 9 cm.
 3 a side greater in length than that opposite to the other angle.
 4 The angle at this vertex is right [5] equilateral

2

- 1 c [2] a [3] b
 4 b [5] a [6] d

3

[a] >

[b] $\therefore \triangle DBC$ is equilateral triangle

$$\therefore m(\angle DBC) = 60^\circ$$

In $\triangle ABC$: $AB = AC$

$$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 50^\circ}{2} = 65^\circ$$

$$\therefore m(\angle ABD) = 60^\circ + 65^\circ = 125^\circ \quad (\text{The req.})$$

[c] $\therefore \overline{AD} \parallel \overline{BC}$, \overline{AC} is a transversal

$$\therefore m(\angle ACB) = m(\angle DAC) = 50^\circ \quad (\text{alternate angles})$$

$$\text{In } \triangle ABC : m(\angle B) = 180^\circ - (70^\circ + 50^\circ) = 60^\circ$$

$$\therefore m(\angle BAC) > m(\angle B)$$

$$\therefore BC > AC \quad (\text{Q.E.D.})$$

4

[a] Theoretical

[b] In $\triangle ABC$: $\therefore AB = AC$

$$\therefore m(\angle ABC) = m(\angle ACB)$$

$$\therefore \frac{1}{2} m(\angle ABC) = \frac{1}{2} m(\angle ACB)$$

$$\therefore m(\angle DBC) = m(\angle DCB)$$

$$\therefore \triangle DBC \text{ is an isosceles triangle} \quad (\text{Q.E.D.})$$

5

[a] $\therefore \overline{AC}$ is the longest side.

$$\therefore \angle B \text{ is the greatest angle in measure}$$

$$\therefore \overline{AB} \text{ is the shortest side.}$$

$$\therefore \angle C \text{ is the smallest angle in measure}$$

$$\therefore \text{The descending order of measures of the angles is } m(\angle B), m(\angle A) \text{ and } m(\angle C) \quad (\text{Q.E.D.})$$

[b] In $\triangle ABC$: $\therefore AB > BC$

$$\therefore m(\angle ACB) > m(\angle BAC) \quad (1)$$

 $\therefore \overline{XY} \parallel \overline{BC}$, \overline{AC} is a transversal

$$\therefore m(\angle XYA) = m(\angle ACB)$$

$$(\text{corresponding angles}) \quad (2)$$

$$\text{From (1) and (2) : } \therefore m(\angle XYA) > m(\angle BAC)$$

$$\therefore AX > XY \quad (\text{Q.E.D.})$$

Model 2

1

- 1 d [2] a [3] b
 4 b [5] d [6] d

2

- 1 an isosceles triangle. [2] less than
 3 XY [4] $\frac{1}{2}$ [5] perpendicular.

3

[a] $\therefore \overline{AB}$ is the longest side.

$$\therefore \angle C \text{ is the greatest angle in measure}$$

$$\therefore \overline{CB} \text{ is the shortest side}$$

$$\therefore \angle A \text{ is the smallest angle in measure}$$

$$\therefore \text{The ascending order of measure of the angles is } m(\angle A), m(\angle B) \text{ and } m(\angle C) \quad (\text{Q.E.D.})$$

[b] In $\triangle ABC$: $\therefore m(\angle B) = 90^\circ$

$$\therefore D \text{ is the midpoint of } \overline{AC}$$

$$\therefore E \text{ is the midpoint of } \overline{BC}$$

$$\therefore \overline{BD}, \overline{AE} \text{ are two medians in } \triangle ABC$$

$$\therefore M \text{ is the intersection point of the medians of } \triangle ABC$$

$$\therefore BD = \frac{1}{2} AC = \frac{1}{2} \times 9 = 4.5 \text{ cm.}$$

$$\therefore BM = \frac{2}{3} BD = \frac{2}{3} \times 4.5 = 3 \text{ cm.}$$

$$\therefore m(\angle C) = 30^\circ$$

$$\therefore AB = \frac{1}{2} AC = \frac{1}{2} \times 9 = 4.5 \text{ cm.} \quad (\text{The req.})$$

4

[a] In $\triangle ABC$: $\therefore D$ is the midpoint of \overline{AC}

$$\therefore \overline{BD} \text{ is a median}$$

$$\therefore m(\angle ABC) = 90^\circ \quad \therefore BD = \frac{1}{2} AC \quad (1)$$

$$\text{In } \triangle BDE : \therefore m(\angle BDE) = 90^\circ, m(\angle E) = 30^\circ$$

$$\therefore BD = \frac{1}{2} BE \quad (2)$$

$$\text{From (1) and (2) : } \therefore AC = BE \quad (\text{Q.E.D.})$$

Answers of Final Examinations

[b] $\therefore \overline{AD} \parallel \overline{BC}$, \overline{AC} is a transversal

$$\therefore m(\angle ACB) = m(\angle CAD) = 30^\circ \text{ (Alternate angles)}$$

$$\text{In } \triangle ABC : m(\angle B) = 180^\circ - (70^\circ + 30^\circ) = 80^\circ$$

$$\therefore m(\angle B) > m(\angle BAC)$$

$$\therefore AC > BC$$

(Q. E. D.)

5

[a] a side greater in length than that opposite to the other angle.

[b] $\therefore \overline{AB}$ bisects $\angle YAZ$

$$\therefore m(\angle YAB) = m(\angle BAZ) \quad (1)$$

$\therefore \overline{AB} \parallel \overline{XY}$, \overline{AY} is a transversal

$$\therefore m(\angle BAY) = m(\angle AYX) \text{ (Alternate angles) } (2)$$

$\therefore \overline{AB} \parallel \overline{XY}$, \overline{ZX} is a transversal

$$\therefore m(\angle X) = m(\angle BAZ) \text{ (Corresponding angles) } (3)$$

From (1), (2) and (3): $m(\angle AYX) = m(\angle X)$

$$\therefore m(\angle AYX) + m(\angle AYZ) > m(\angle X)$$

$$\therefore m(\angle ZYX) > m(\angle X) \therefore XZ > YZ \text{ (Q.E.D.)}$$

Answers of model for the merge students

1

$$1 : 2$$

2 half the length of the hypotenuse

3 congruent

4 >

5 bisects it, perpendicular to the base.

2

1 b

2 a

3 d

4 a

5 a

3

$$\therefore m(\angle B) = 90^\circ, m(\angle C) = 30^\circ$$

$$\therefore AB = \frac{1}{2} \times AC$$

$$\therefore AC = 10 \text{ cm.}$$

4

[a] AC, AB, BC

$$[b] 1 40^\circ$$

$$2 \overline{AB}$$

5

1 ✓

2 X

3 X

4 ✓

5 ✓

Geometry

Answers of schools examinations
on geometry

1 Cairo

1

1 (c)

2 (c)

3 (c)

4 (c)

5 (b)

6 (b)

2

1 3

2 congruent

3 the hypotenuse

4 bisects the base and is perpendicular to it

5 6

3

[a] In ΔABC :

$$\therefore m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$$

$$\therefore 6x + (4x - 9) + 3(x - 2) = 180^\circ$$

$$\therefore 13x = 195^\circ \quad \therefore x = 15^\circ$$

$$\therefore m(\angle A) = 90^\circ, m(\angle B) = 51^\circ, m(\angle C) = 39^\circ$$

$$\therefore m(\angle C) < m(\angle B) < m(\angle A)$$

$$\therefore AB < AC < BC \quad (\text{The req.})$$

[b] In ΔABC : $\therefore m(\angle B) = 90^\circ, m(\angle C) = 30^\circ$

$$\therefore AB = \frac{1}{2} AC = 5 \text{ cm.}$$

$$\therefore \overline{BD} \text{ is a median} \quad \therefore BD = \frac{1}{2} AC = 5 \text{ cm.}$$

$$\therefore AD = 5 \text{ cm.}$$

$$\therefore \text{The perimeter of } \Delta ABD = 5 + 5 + 5 = 15 \text{ cm.} \quad (\text{The req.})$$

4

[a] In ΔMBC : $\therefore MB = MC$

$$\therefore m(\angle B) = m(\angle C) \quad (1)$$

 $\therefore \overline{AD} \parallel \overline{BC}, \overline{AC} \text{ is a transversal.}$

$$\therefore m(\angle A) = m(\angle C) \quad (2)$$

 $\therefore \overline{AD} \parallel \overline{BC}, \overline{BD} \text{ is a transversal.}$

$$\therefore m(\angle D) = m(\angle B) \quad (3)$$

From (1), (2) and (3):

$$\therefore m(\angle A) = m(\angle D)$$

$$\therefore MA = MD$$

 $\therefore \Delta MAD \text{ is isosceles.} \quad (\text{Q.E.D.})$
[b] In ΔABC :

$$\therefore m(\angle BAC) + m(\angle B) + m(\angle ACB) = 180^\circ$$

$$\therefore m(\angle ACB) = 180^\circ - (55^\circ + 70^\circ) = 55^\circ$$

$$\therefore m(\angle B) = m(\angle ACB) \quad \therefore AB = AC \quad (1)$$

$$\text{In } \Delta ACD: \therefore m(\angle ACD) = 90^\circ$$

$$\therefore \overline{AD} \text{ is the hypotenuse} \quad \therefore AD > AC \quad (2)$$

From (1) and (2):

$$\therefore AD > AB \quad (\text{Q.E.D.})$$

5

[a] $\therefore \Delta ABC$ is equilateral.

$$\therefore m(\angle ACB) = 60^\circ$$

$$\text{In } \Delta ACD: \therefore DC = DA, m(\angle D) = 40^\circ$$

$$\therefore m(\angle DCA) = m(\angle DAC) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$$

$$\therefore m(\angle DCB) = 70^\circ + 60^\circ = 130^\circ \quad (\text{The req.})$$

[b] In ΔABD : $\therefore AD > AB$

$$\therefore m(\angle ABD) > m(\angle ADB) \quad (1)$$

$$\text{In } \Delta BCD: \therefore CD > BC$$

$$\therefore m(\angle DBC) > m(\angle BDC) \quad (2)$$

Adding (1), (2):

$$\therefore m(\angle ABC) > m(\angle ADC) \quad (\text{Q.E.D.})$$

2 Cairo

1

1 it, the base

2 120°

3 congruent

4 2, 10

5 the hypotenuse

2

1 (a)

2 (a)

3 (c)

4 (c)

5 (c)

6 (b)

3

[a] In ΔABC : $\therefore AB = AC$

$$\therefore m(\angle B) = m(\angle C) = 50^\circ$$

In $\Delta\Delta ABY, ACX$:

$$\begin{cases} AB = AC \\ m(\angle B) = m(\angle C) \\ BY = CX \end{cases}$$

$$\therefore \Delta ABY \cong \Delta ACX \quad \therefore AY = AX$$

$$\therefore \Delta AXY \text{ is isosceles} \quad (\text{First req.})$$

$$\therefore m(\angle BAY) = m(\angle CAX) = 30^\circ$$

$$\therefore m(\angle AYB) = 180^\circ - (50^\circ + 30^\circ) = 100^\circ \quad (\text{Second req.})$$

Answers of Final Examinations

[b] $\because DA = BA, DC = BC$
 $\therefore \overline{AC}$ is the axis of \overline{BD}
 $\therefore \overline{BD} \cap \overline{AC} = \{M\}$
 $\therefore M$ is the midpoint of \overline{BD} (Q.E.D.)

[4]
 [a] In $\triangle ABC$: $\because AB > AC$
 $\therefore m(\angle ACB) > m(\angle ABC)$
 $\therefore \frac{1}{2} m(\angle ACB) > \frac{1}{2} m(\angle ABC)$
 $\therefore \overline{CD}$ bisects $\angle ACB$, \overline{BD} bisects $\angle ABC$
 $\therefore m(\angle DCB) > m(\angle DBC)$
 $\therefore BD > CD$ (Q.E.D.)

[b] $\because \triangle ACD$ is an equilateral triangle.
 $\therefore m(\angle ADC) = 60^\circ$
 In $\triangle BCD$: $\because DB = DC$
 $\therefore m(\angle B) = m(\angle BCD) = 65^\circ$
 $\therefore m(\angle BDC) = 180^\circ - 2 \times 65^\circ = 50^\circ$
 $\therefore m(\angle ADB) = 60^\circ + 50^\circ = 110^\circ$ (The req.)

[5]
 [a] $\because m(\angle ABC) = 90^\circ$, \overline{BE} is a median in $\triangle ABC$
 $\therefore BE = \frac{1}{2} AC = \frac{1}{2} \times 12 = 6$ cm.
 $\because \overline{AD}$ is a median in $\triangle ABC$
 $\therefore M$ is the point of intersection of medians
 $\therefore ME = \frac{1}{3} BE = \frac{1}{3} \times 6 = 2$ cm. (The req.)
 [b] In $\triangle EBD$: $\because ED + BD > BE$
 $\because CD = ED \therefore CD + BD > BE$
 $\therefore BC > BE$ (Q.E.D.1)
 In $\triangle ABC$: $\because AB + BC > AC$ (1)
 $\because CD = CE$ (2)
 Subtracting (2) from (1):
 $\therefore AB + BD > AE$ (Q.E.D.2)

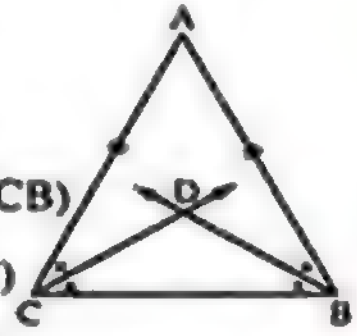
3 Cairo

[1] (c) [2] (d) [3] (a)
 [4] (a) [5] (c) [6] (b)

[2]
 [1] a side greater in length than that opposite to the other angle.

[2] bisects the base and is perpendicular to it.
 [3] congruent [4] is greater than [5] 5

[3]
 [a] $\because AB = AC$
 $\therefore m(\angle ABC) = m(\angle ACB)$
 $\therefore \frac{1}{2} m(\angle ABC) = \frac{1}{2} m(\angle ACB)$
 $\therefore m(\angle DBC) = \frac{1}{2} m(\angle ABC)$
 $m(\angle DCB) = \frac{1}{2} m(\angle ACB)$
 $\therefore m(\angle DBC) = m(\angle DCB)$
 $\therefore DB = DC$
 $\therefore \triangle DBC$ is an isosceles triangle (Q.E.D.)



[b] In $\triangle ABC$: $\because m(\angle B) = 90^\circ, m(\angle ACB) = 30^\circ$
 $\therefore AB = \frac{1}{2} AC$
 $\because AB = DE = 5$ cm. $\therefore DE = \frac{1}{2} AC$
 $\therefore \overline{DE}$ is a median in $\triangle ADC$
 $\therefore m(\angle ADC) = 90^\circ$ (Q.E.D.)

[4]
 [a] In $\triangle ABC$: $\because AB > AC$
 $\therefore m(\angle ABC) < m(\angle ACB)$
 $\therefore \overline{BE} \in \overline{AD}, \overline{CE} \in \overline{AE}$
 $\therefore 180^\circ - m(\angle ABC) > 180^\circ - m(\angle ACB)$
 $\therefore m(\angle CBD) > m(\angle BCE)$
 $\therefore \overline{BF}$ bisects $\angle DBC$, \overline{CF} bisects $\angle BCE$
 $\therefore m(\angle FBC) > m(\angle BCF)$ (Q.E.D.1)
 $\therefore CF > BF$ (Q.E.D.2)
 [b] $\because \overline{AD}, \overline{BE}$ are two medians in $\triangle ABC$
 $\therefore M$ is the point of intersection of the medians
 $\therefore MB = 2 ME = 2 \times 2 = 4$ cm.
 $\therefore MA = 2 MD = 2 \times 3 = 6$ cm.
 $\because D$ is the midpoint of \overline{BC}
 $\therefore E$ is the midpoint of \overline{AC}
 $\therefore AB = 2 DE = 2 \times 4 = 8$ cm.
 \therefore The perimeter of $\triangle MAB = 4 + 6 + 8 = 18$ cm.
 (The req.)

[5]
 [a] From $\triangle ABM$:
 $MA + MB > AB$ (Triangle inequality) (1)

Geometry

From $\triangle BMC$:

$$MB + MC > BC \text{ (Triangle inequality)} \quad (2)$$

From $\triangle AMC$:

$$MA + MC > AC \text{ (Triangle inequality)} \quad (3)$$

Adding (1), (2) and (3):

$$\begin{aligned} \therefore 2MA + 2MB + 2MC &> AB + BC + AC \\ \therefore MA + MB + MC &> \frac{1}{2} \text{ the perimeter of } \triangle ABC \end{aligned} \quad (\text{Q.E.D.})$$

[b] $\because \overline{AD} \parallel \overline{BC}$, \overline{BD} is a transversal

$$\therefore m(\angle ADB) = m(\angle DBC) \text{ (alternate angles)}$$

$$\therefore m(\angle ABD) = m(\angle DBC)$$

$$\therefore m(\angle ABD) = m(\angle ADB)$$

$$\therefore \text{In } \triangle ABD: AB = AD \quad (\text{Q.E.D.1})$$

$$\therefore \overline{AE} \text{ bisects } \angle BAD$$

$$\therefore \overline{AE} \perp \overline{BD} \quad (\text{Q.E.D.2})$$

$$\therefore E \text{ is the midpoint of } \overline{BD}$$

$$\therefore BE = ED \quad (\text{Q.E.D.3})$$

4 Giza

1

$$1 \text{ (c)} \quad 2 \text{ (d)} \quad 3 \text{ (a)}$$

$$4 \text{ (a)} \quad 5 \text{ (d)} \quad 6 \text{ (d)}$$

2

$$\begin{aligned} 1 &> & 2 &\overline{BC} \\ 3 &\text{is perpendicular to it} & 4 &3, 11 \\ 5 &18 \text{ cm.} \end{aligned}$$

3

[a] In $\triangle ABD$: $\because BA = BD$

$$\therefore m(\angle BDA) = m(\angle BAD) = 70^\circ$$

 $\therefore \triangle ACD$ is an equilateral triangle

$$\therefore m(\angle ADC) = 60^\circ$$

$$\therefore m(\angle BDC) = 70^\circ + 60^\circ = 130^\circ \quad (\text{The req.})$$

[b] $\because \overline{BE}$, \overline{CF} are two medians in $\triangle ABC$ $\therefore M$ is the point of intersection of medians

$$\therefore MF = \frac{1}{2} CM = \frac{1}{2} \times 6 = 3 \text{ cm.}$$

$$\therefore ME = \frac{1}{2} BM = \frac{1}{2} \times 5 = 2.5 \text{ cm.}$$

 $\therefore F$ is the midpoint of \overline{AB} $\therefore E$ is the midpoint of \overline{AC}

$$\therefore FE = \frac{1}{2} BC = \frac{1}{2} \times 12 = 6 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle MEF = 3 + 2.5 + 6 = 11.5 \text{ cm.} \quad (\text{The req.})$$

4

[a] In $\triangle ABC$: $\because m(\angle ABC) = 90^\circ$

$$\therefore \overline{BE} \text{ is a median} \quad \therefore BE = \frac{1}{2} AC$$

In $\triangle ACD$: $\because X$ is the midpoint of \overline{AD}

$$\therefore Y \text{ is the midpoint of } \overline{CD} \quad \therefore XY = \frac{1}{2} AC$$

$$\therefore XY = BE \quad (\text{Q.E.D.})$$

[b] $\because m(\angle DAE) = 90^\circ - 30^\circ = 60^\circ$ In $\triangle AFD$:

$$\therefore m(\angle ADF) = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$$

$$\therefore m(\angle AFD) = 90^\circ$$

$$\therefore AD = 2 AF = 2 \times 4 = 8 \text{ cm.}$$

$$\therefore \text{The area of the square } ABCD = 8 \times 8 = 64 \text{ cm}^2 \quad (\text{The req.})$$

5

[a] $\because m(\angle A) = m(\angle B) \quad \therefore AC = BC$

$$\therefore 3x - 2 = x + 6 \quad \therefore 2x = 8 \quad \therefore x = 4 \text{ cm.}$$

$$\therefore AC = BC = 10 \text{ cm.} \quad \therefore AB = 7 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle ABC = 10 + 10 + 7 = 27 \text{ cm.} \quad (\text{The req.})$$

[b] In $\triangle ABC$: $\because AB > AC$

$$\therefore m(\angle ABC) < m(\angle ACB)$$

$$\therefore \overline{BE} \perp \overline{AD}, \overline{CE} \perp \overline{AE}$$

$$\therefore 180^\circ - m(\angle ABC) > 180^\circ - m(\angle ACB)$$

$$\therefore m(\angle CBD) > m(\angle BCE)$$

$$\therefore \overline{BF} \text{ bisects } \angle DBC, \overline{CF} \text{ bisects } \angle BCE$$

$$\therefore m(\angle FBC) > m(\angle BCF) \quad (\text{Q.E.D.1})$$

$$\therefore CF > BF \quad (\text{Q.E.D.2})$$

5 Giza

1

$$1 \text{ (b)} \quad 2 \text{ (b)} \quad 3 \text{ (c)}$$

$$4 \text{ (d)} \quad 5 \text{ (c)} \quad 6 \text{ (a)}$$

2

$$1 \text{ bisects the base, is perpendicular to it}$$

$$2 \text{ 6 cm.} \quad 3 \text{ } 120^\circ$$

$$4 \text{ 3, 11} \quad 5 \text{ } 90^\circ$$

3

[a] In $\triangle ABD$: $\because AD = BD$

$$\therefore m(\angle BAD) = m(\angle ABD) = 40^\circ$$

Answers of Final Examinations

$\therefore m(\angle ADB) = 180^\circ - 2 \times 40^\circ = 100^\circ$
 $\therefore m(\angle ABD) < m(\angle ADB)$
 $\therefore AD < AB$ (Q.E.D.1)
 In $\triangle ABE$: $\therefore AD = \frac{1}{2} BE$
 $\therefore \overline{AD}$ is a median
 $\therefore m(\angle BAE) = 90^\circ$
 $\therefore \overline{BC}$ is a hypotenuse of $\triangle BAC$
 $\therefore BC > AC$ (Q.E.D.2)

[b] In $\triangle ABC$: $\therefore AB = AC$
 $\therefore m(\angle B) = m(\angle C)$
 $\therefore \frac{1}{2} m(\angle B) = \frac{1}{2} m(\angle C)$
 $\therefore m(\angle CBD) = m(\angle BCD)$
 $\therefore BD = CD$
 $\therefore \triangle DBC$ is isosceles. (Q.E.D.)

[a] In $\triangle ABC$:
 $\therefore m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$
 $\therefore 6x + (4x - 9) + 3(x - 2) = 180^\circ$
 $\therefore 6x + 4x - 9 + 3x - 6 = 180^\circ$
 $\therefore 13x = 195^\circ \quad \therefore x = 15^\circ$
 $\therefore m(\angle A) = 90^\circ, m(\angle B) = 51^\circ$
 $\therefore m(\angle C) = 39^\circ$
 $\therefore m(\angle C) < m(\angle B) < m(\angle A)$
 $\therefore AB < AC < BC$ (The req.)

[b] $\therefore \overline{AC}$ is a median in $\triangle ABD$
 $\therefore AC = \frac{1}{2} BD \quad \therefore m(\angle BAD) = 90^\circ$
 $\therefore \overline{AB} \perp \overline{AD}$ (Q.E.D.)

[a] In $\triangle ABC$: $\therefore X$ is the midpoint of \overline{AB}
 $\therefore Y$ is the midpoint of \overline{BC}
 $\therefore XY = \frac{1}{2} AC = \frac{1}{2} \times 24 = 12 \text{ cm.}$
 In $\triangle XBY$: $\therefore m(\angle XBY) = 90^\circ$
 $\therefore \overline{BY}$ is a median in $\triangle XBY$
 $\therefore BY = \frac{1}{2} XY = \frac{1}{2} \times 12 = 6 \text{ cm.}$ (The req.)
 [b] $\therefore \overline{BN}, \overline{CF}$ are two medians in $\triangle ABC$
 $\therefore M$ is the point of intersection of medians
 $\therefore MF = \frac{1}{3} CF = \frac{1}{3} \times 10.5 = 3.5 \text{ cm.}$

$\therefore MN = \frac{1}{2} BM = \frac{1}{2} \times 6 = 3 \text{ cm.}$
 $\therefore AF = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4 \text{ cm.}$
 $\therefore AN = \frac{1}{2} AC = \frac{1}{2} \times 12 = 6 \text{ cm.}$
 \therefore The perimeter of $\triangle FMN = 3.5 + 3 + 4 + 6$
 $= 16.5 \text{ cm.}$ (The req.)

6 Alexandria

1 25° 2 AC 3 120°
 4 2 5 < 6 concurrent

2 1 (a) 2 (a) 3 (d)
 4 (b) 5 (c)

[a] $\therefore m(\angle C) = 180^\circ - (90^\circ + 75^\circ) = 15^\circ$
 $\therefore m(\angle B) > m(\angle A) > m(\angle C)$
 $\therefore AC > BC > AB$ (The req.)
 [b] $\therefore \overline{YM}, \overline{ZL}$ are two medians in $\triangle XYZ$
 $\therefore N$ is the point of intersection of medians
 $\therefore NL = \frac{1}{3} LZ = \frac{1}{3} \times 15 = 5 \text{ cm.}$
 $\therefore YN = \frac{2}{3} YM = \frac{2}{3} \times 18 = 12 \text{ cm.}$
 $\therefore YL = \frac{1}{2} XY = \frac{1}{2} \times 20 = 10 \text{ cm.}$
 \therefore The perimeter of $\triangle NLY = 5 + 12 + 10$
 $= 27 \text{ cm.}$ (The req.)

[a] In $\triangle ABC$:
 $\therefore m(\angle ABC) = 90^\circ, m(\angle ACB) = 30^\circ$
 $\therefore AB = \frac{1}{2} AC$
 $\therefore AB = DE \quad \therefore DE = \frac{1}{2} AC$
 $\therefore \overline{DE}$ is a median in $\triangle ADC$
 $\therefore m(\angle ADC) = 90^\circ$ (Q.E.D.)

[b] Construction: Draw \overline{AC}

Proof: In $\triangle ABC$

$\therefore AB > BC$

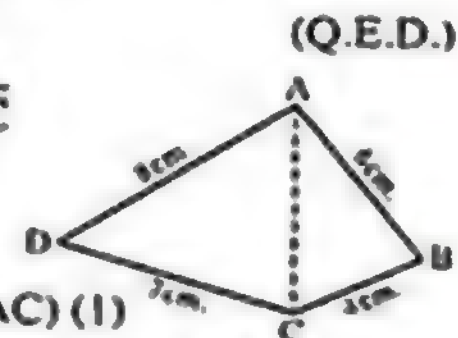
$\therefore m(\angle ACB) > m(\angle BAC)$ (1)

In $\triangle ACD$: $\therefore AD > CD$

$\therefore m(\angle ACD) > m(\angle CAD)$ (2)

Adding (1) + (2):

$\therefore m(\angle BCD) > m(\angle BAD)$ (Q.E.D.)



Geometry

5

[a] $\because \overline{DE} \parallel \overline{BC}$, \overline{BD} is a transversal

$$\therefore m(\angle CBD) = m(\angle BDE) \quad (\text{alternate angles})$$

$$\therefore m(\angle EBD) = m(\angle CBD)$$

$$\therefore m(\angle EBD) = m(\angle BDE)$$

$$\therefore BE = DE$$

 $\therefore \triangle EBD$ is an isosceles triangle. (Q.E.D.)[b] $\because \triangle ABC$ is an equilateral triangle

$$\therefore m(\angle BAC) = 60^\circ$$

$$\text{In } \triangle ACD: \because AD = CD, m(\angle D) = 96^\circ$$

$$\therefore m(\angle DAC) = m(\angle DCA) = \frac{180^\circ - 96^\circ}{2} = 42^\circ$$

$$\therefore m(\angle DAB) = 60^\circ + 42^\circ = 102^\circ \quad (\text{The req.})$$

7 Alexandria

1

1 (b)

2 (b)

3 (b)

4 (b)

5 (d)

6 (c)

2

1 right

2 greater than

3 8

4 3, 11

5 40°

3

[a] In $\triangle ABC$: $\because X$ is the midpoint of \overline{AB} $\therefore Y$ is the midpoint of \overline{BC}

$$\therefore XY = \frac{1}{2} AC = \frac{1}{2} \times 20 = 10 \text{ cm.}$$

$$\text{In } \triangle XBY: \because m(\angle XBY) = 90^\circ$$

 $\therefore \overline{BD}$ is a median

$$\therefore BD = \frac{1}{2} XY = \frac{1}{2} \times 10 = 5 \text{ cm.} \quad (\text{The req.})$$

[b] $\because \overline{XY} \parallel \overline{AC}$, \overline{AB} is a transversal.

$$\therefore m(\angle A) = m(\angle ABX) = 62^\circ \quad (\text{alternate angles})$$

$$\therefore m(\angle ABC) = 180^\circ - (56^\circ + 62^\circ) = 62^\circ$$

$$\therefore m(\angle ABC) = m(\angle BAC)$$

$$\therefore AC = BC$$

(Q.E.D.)

4

[a] $\because \overline{BE}$, \overline{CD} are two medians in $\triangle ABC$ $\therefore M$ is the point of intersection of medians

$$\therefore ME = \frac{1}{2} BM = \frac{1}{2} \times 4 = 2 \text{ cm.}$$

$$\therefore MD = \frac{1}{3} DC = \frac{1}{3} \times 9 = 3 \text{ cm.}$$

 $\because D$ is the midpoint of \overline{AB} $\therefore E$ is the midpoint of \overline{AC}

$$\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 8 = 4 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle DME = 2 + 3 + 4 = 9 \text{ cm.}$$

(The req.)

[b] $\because m(\angle EAD) = 90^\circ - 30^\circ = 60^\circ$ In $\triangle ADF$:

$$\therefore m(\angle ADF) = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$$

$$\therefore m(\angle AFD) = 90^\circ$$

$$\therefore AD = 2 AF = 2 \times 4 = 8 \text{ cm.}$$

$$\therefore \text{The area of the square } ABCD = 8 \times 8 = 64 \text{ cm}^2$$

(The req.)

5

[a] $\because \overline{AD} \parallel \overline{BC}$, \overline{AB} is a transversal

$$\therefore m(\angle BAD) + m(\angle B) = 180^\circ \quad (\text{interior angles})$$

$$\therefore m(\angle B) = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore m(\angle BAC) > m(\angle B)$$

$$\therefore BC > AC$$

(Q.E.D.)

[b] $\because \triangle ACD$ is an equilateral triangle.

$$\therefore m(\angle ADC) = 60^\circ$$

 \therefore in $\triangle ABD: \because AB = BD$

$$\therefore m(\angle BDA) = m(\angle BAD) = 70^\circ$$

$$\therefore m(\angle BDC) = 60^\circ + 70^\circ = 130^\circ \quad (\text{The req.})$$

8 El-Kalyoubia

1

1 (b)

2 (d)

3 (a)

4 (b)

5 (d)

6 (a)

2

1 concurrent

2 hypotenuse

3 an isosceles

4 >

5 3, 15

3

$$[a] \because m(\angle B) > m(\angle C) > m(\angle A)$$

$$\therefore AC > AB > BC$$

(The req.)

[b] In $\triangle ABC: \because AB = BC$

$$\therefore m(\angle A) = m(\angle C)$$

 $\therefore \overline{XY} \parallel \overline{AC}$, \overline{AB} is a transversal

$$\therefore m(\angle BXY) = m(\angle A) \quad (\text{corresponding angles})$$

Answers of Final Examinations

$\therefore \overline{XY} \parallel \overline{AC}$, \overline{BC} is a transversal
 $\therefore m(\angle BYX) = m(\angle C)$ (corresponding angles)
 $\therefore m(\angle A) = m(\angle C)$
 $\therefore m(\angle BXY) = m(\angle BYX)$
 In $\triangle BXY$: $\therefore BX = BY$ (Q.E.D.)

4

[a] In $\triangle XYL$: $\therefore YX = LX$, $LM = MY$
 $\therefore \overline{XM}$ is the axis of \overline{YL}
 In $\triangle YZL$: $\therefore YZ = LZ$, $LM = MY$
 $\therefore \overline{ZM}$ is the axis of \overline{YL}
 $\therefore X, M, Z$ are on the same straight line (Q.E.D.)

[b] In $\triangle ADB$: $\therefore DB = DA$
 $\therefore m(\angle B) = m(\angle BAD)$ (1)
 In $\triangle ADC$: $\therefore DC = DA$
 $\therefore m(\angle C) = m(\angle CAD)$ (2)
 \therefore in $\triangle ABC$: $\therefore AB > AC$
 $\therefore m(\angle C) > m(\angle B)$ (3)
 From (1), (2), (3):
 $\therefore m(\angle BAD) < m(\angle CAD)$ (Q.E.D.)

5

[a] In $\triangle ABC$:
 $\therefore m(\angle ABC) = 90^\circ$, \overline{BD} is a median
 $\therefore BD = \frac{1}{2} AC = \frac{1}{2} \times 9 = 4.5$ cm.
 $\therefore \overline{AE}$ and \overline{BD} are two medians in $\triangle ABC$
 $\therefore M$ is the point of intersection of medians
 $\therefore BM = \frac{2}{3} BD = \frac{2}{3} \times 4.5 = 3$ cm.
 $\therefore MD = \frac{1}{3} BD = \frac{1}{3} \times 4.5 = 1.5$ cm.
 $\therefore m(\angle C) = 30^\circ$
 $\therefore AB = \frac{1}{2} AC = \frac{1}{2} \times 9 = 4.5$ cm. (The req.)

[b] $\therefore m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$
 $\therefore 2x + x + 40 + 3x - 10 = 180^\circ$
 $\therefore 6x = 150^\circ \quad \therefore x = 25^\circ$
 $\therefore m(\angle A) = 50^\circ$, $m(\angle B) = 65^\circ$
 $\therefore m(\angle C) = 65^\circ \quad \therefore m(\angle B) = m(\angle C)$
 $\therefore AB = AC$ (Q.E.D.)

9 El-Sharkia

1

- 1 (a) 2 (b) 3 (c)
 4 (d) 5 (a) 6 (b)

2

- 1 5 2 90° 3 the hypotenuse
 4 0
 5 bisects it, is perpendicular to the base.

3

[a] In $\triangle XYZ$:
 $\therefore m(\angle YXZ) = 180^\circ - (70^\circ + 30^\circ) = 80^\circ$
 $\therefore \overline{XL}$ bisects $\angle YXZ$
 $\therefore m(\angle LXZ) = m(\angle LXY) = 80^\circ \div 2 = 40^\circ$
 In $\triangle XLZ$:
 $\therefore m(\angle XLZ) = 180^\circ - (70^\circ + 40^\circ) = 70^\circ$ (First req.)
 $\therefore m(\angle XLZ) = m(\angle Z) \quad \therefore XL = XZ$
 $\therefore \triangle XLZ$ is isosceles. (Second req.)

[b] In $\triangle ADC$: $\therefore AD = DC$
 $\therefore m(\angle CAD) = m(\angle ACD)$ (1)
 In $\triangle ABC$: $\therefore BC > AB$
 $\therefore m(\angle BAC) > m(\angle ACB)$ (2)
 Adding (1), (2):
 $\therefore m(\angle BAD) > m(\angle BCD)$ (Q.E.D.)

4

[a] $\therefore \overline{AY}$, \overline{BX} are two medians in $\triangle ABC$
 $\therefore M$ is the point of intersection of medians
 $\therefore MY = \frac{1}{2} AM = \frac{1}{2} \times 5 = 2.5$ cm.
 $\therefore MX = \frac{1}{3} BX = \frac{1}{3} \times 6 = 2$ cm.
 $\therefore X$ is the midpoint of \overline{AC}
 $\therefore Y$ is the midpoint of \overline{BC}
 $\therefore XY = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4$ cm.
 \therefore The perimeter of $\triangle XMY = 2.5 + 2 + 4 = 8.5$ cm. (The req.)

[b] $\therefore \angle ACD$ is an exterior angle of $\triangle ABC$
 $\therefore m(\angle A) + m(\angle B) = 140^\circ$

Geometry

$\therefore AC = BC$
 $\therefore m(\angle A) = m(\angle B)$
 $\therefore m(\angle B) = \frac{140^\circ}{2} = 70^\circ$
 $\therefore \overline{AB} \parallel \overline{DE}$, \overline{BD} is a transversal
 $\therefore m(\angle BDE) = m(\angle B) = 70^\circ$ (alternate angles)
 (The req.)

5

[a] $\therefore \overline{AD} \parallel \overline{BC}$, \overline{BD} is a transversal.
 $\therefore m(\angle ADB) = m(\angle DBC)$ (alternate angles)
 $\therefore m(\angle ABD) = m(\angle DBC)$
 $\therefore m(\angle ABD) = m(\angle ADB)$
 \therefore In $\triangle ABD$: $AD = AB$ (Q.E.D.1)
 $\therefore \overline{AE}$ bisects $\angle BAD$
 $\therefore \overline{AE} \perp \overline{BD}$ (Q.E.D.2)

[b] \therefore The point $A \in \overline{BE}$
 $\therefore m(\angle BAC) = 180^\circ - (75^\circ + 35^\circ) = 70^\circ$
 $\therefore \overline{AD} \parallel \overline{BC}$, \overline{AC} is a transversal
 $\therefore m(\angle C) = m(\angle CAD) = 35^\circ$ (alternate angles)
 $\therefore m(\angle BAC) > m(\angle C)$
 $\therefore BC > AB$ (Q.E.D.)

10 El-Monofia

1

- 1 (a) 2 (a) 3 (b)
 4 (c) 5 (c) 6 (c)

2

- 1 congruent 2 260° 3 3
 4 < 5 is perpendicular to it

3

[a] $\therefore \overline{BE}$, \overline{CD} are two medians in $\triangle ABC$
 $\therefore F$ is the point of intersection of medians
 $\therefore FE = \frac{1}{2} FB = \frac{1}{2} \times 6 = 3$ cm.
 $\therefore FD = \frac{1}{2} FC = \frac{1}{2} \times 4 = 2$ cm.
 $\therefore D$ is the midpoint of \overline{AB}
 $\therefore E$ is the midpoint of \overline{AC}
 $\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 8 = 4$ cm.
 \therefore The perimeter of $\triangle DFE = 3 + 2 + 4 = 9$ cm.
 (The req.)

[b] In $\triangle ABD$: $\therefore AD = BD$
 $\therefore m(\angle BAD) = m(\angle ABD) = 35^\circ$
 $\therefore \angle ADC$ is an exterior angle of $\triangle ABD$
 $\therefore m(\angle ADC) = 35^\circ + 35^\circ = 70^\circ$
 In $\triangle ADC$: $\therefore AC = AD$
 $\therefore m(\angle C) = m(\angle ADC) = 70^\circ$
 $\therefore m(\angle CAD) = 180^\circ - 2 \times 70^\circ = 40^\circ$
 $\therefore m(\angle BAC) = 35^\circ + 40^\circ = 75^\circ$ (The req.)

4

[a] In $\triangle ABC$: $\therefore AC = AB$
 $\therefore m(\angle B) = m(\angle C)$
 $\therefore \angle ADB$ is an exterior angle of $\triangle ADC$
 $\therefore m(\angle ADB) > m(\angle C)$
 $\therefore m(\angle ADB) > m(\angle B)$
 In $\triangle ADB$: $\therefore AB > AD$ (Q.E.D.)

[b] In $\triangle ABC$:
 $\therefore m(\angle C) = 180^\circ - (40^\circ + 80^\circ) = 60^\circ$
 $\therefore m(\angle B) > m(\angle C) > m(\angle A)$
 $\therefore AC > AB > BC$ (The req.)

5

In $\triangle ABC$:
 $\therefore E$ is the midpoint of \overline{AC}
 $\therefore F$ is the midpoint of \overline{AB}
 $\therefore FE = \frac{1}{2} BC$ (1)
 In $\triangle BDC$: $\therefore m(\angle BDC) = 90^\circ$
 $\therefore \overline{DG}$ is a median
 $\therefore GD = \frac{1}{2} BC = \frac{1}{2} \times 10 = 5$ cm. (2)
 $\therefore m(\angle CBD) = 30^\circ$
 $\therefore DC = \frac{1}{2} BC = \frac{1}{2} \times 10 = 5$ cm. (3)
 From (1), (2), (3): $\therefore FE = DC = GD$ (First req.)
 $\therefore CG = \frac{1}{2} BC = \frac{1}{2} \times 10 = 5$ cm.
 \therefore The perimeter of $\triangle GCD = 5 + 5 + 5 = 15$ cm.
 (Second req.)

11 El-Dakahlia

1

- 1 (a) 2 (b) 3 (d)
 4 (d) 5 (c) 6 (a)

Answers of Final Examinations

2

- 1 |
 2 bisects the base and is perpendicular to it
 3 one point 4 hypotenuse 5 100°

3

- [a] In $\triangle ABC$: $\therefore m(\angle ABC) = 90^\circ$
 $\therefore \overline{BD}$ is a median
 $\therefore BD = \frac{1}{2} AC$ (1)
 In $\triangle BDE$: $\therefore m(\angle BDE) = 90^\circ$
 $\therefore m(\angle E) = 30^\circ$
 $\therefore BD = \frac{1}{2} BE$ (2)
 From (1) & (2): $\therefore AC = BE$ (Q.E.D.)
 [b] In $\triangle ABC$: $\therefore AB = AC$
 $\therefore m(\angle ABC) = m(\angle ACB)$ (1)
 In $\triangle BCD$: $\therefore DC > DB$
 $\therefore m(\angle CBD) > m(\angle BCD)$ (2)
 Adding (1) & (2):
 $\therefore m(\angle ABD) > m(\angle ACD)$ (Q.E.D.)

4

- [a] $\therefore \overline{AD} \parallel \overline{BC}$, \overline{AB} is a transversal.
 $\therefore m(\angle B) = m(\angle BAD) = 60^\circ$ (alternate angles)
 In $\triangle ABC$:
 $\therefore m(\angle C) = 180^\circ - (50^\circ + 60^\circ) = 70^\circ$
 $\therefore m(\angle C) > m(\angle B)$
 $\therefore AB > AC$ (Q.E.D.)
 [b] $\therefore \angle ACL$ is an exterior angle of $\triangle ABC$
 $\therefore m(\angle A) + m(\angle B) = 130^\circ$
 $\therefore AC = BC$ $\therefore m(\angle A) = m(\angle B)$
 $\therefore m(\angle B) = \frac{130^\circ}{2} = 65^\circ$
 $\therefore \overline{AB} \parallel \overline{LM}$, \overline{BL} is a transversal
 $\therefore m(\angle MLC) = m(\angle B) = 65^\circ$ (alternate angles)
 (The req.)

5

- [a] In $\triangle ABC$:
 $\therefore X$ is the midpoint of \overline{AB}
 $\therefore Y$ is the midpoint of \overline{BC}
 $\therefore XY = \frac{1}{2} AC = \frac{1}{2} \times 22 = 11 \text{ cm.}$

In $\triangle XBY$: $\therefore m(\angle XBY) = 90^\circ$ $\therefore \overline{BD}$ is a median

$$\therefore BD = \frac{1}{2} XY = \frac{1}{2} \times 11 = 5.5 \text{ cm. (The req.)}$$

[b] $\therefore AB = AC$ $\therefore BE = CE$ $\therefore \overline{AE}$ is the axis of \overline{BC} $\therefore BD = CD$

(Q.E.D.)

12

Suez

1

- 1 congruent 2 equilateral 3 \overline{BC}
 4 $2:1$ 5 2

2

- 1 (d) 2 (d) 3 (b)
 4 (a) 5 (b) 6 (a)

3

- [a] In $\triangle ABC$:
 $\therefore m(\angle C) = 180^\circ - (40^\circ + 75^\circ) = 65^\circ$
 $\therefore m(\angle B) > m(\angle C) > m(\angle A)$
 $\therefore AC > AB > BC$ (The req.)
 [b] In $\triangle ABC$:
 $\therefore AB = AC$, \overline{AE} bisects $\angle BAC$
 $\therefore \overline{AE} \perp \overline{BC}$, E is the midpoint of \overline{BC}
 $\therefore \overline{AE}$ is axis of \overline{BC}
 $\therefore D \in \overline{AE}$ $\therefore BD = CD$ (Q.E.D.)

4

- [a] In $\triangle ABD$: $\therefore AD = AB$
 $\therefore m(\angle ADB) = m(\angle ABD) = 25^\circ$
 $\therefore \overline{AD} \parallel \overline{BC}$, \overline{BD} is a transversal
 $\therefore m(\angle DBC) = m(\angle ADB) = 25^\circ$
 (alternate angles)
 $\therefore X = 25^\circ$
 \therefore in $\triangle BCD$:
 $y = 180^\circ - (25^\circ + 63^\circ) = 92^\circ$ (The req.)
 [b] In $\triangle ABD$: $\therefore AB = BD = DA$
 $\therefore \triangle ABD$ is an equilateral triangle
 $\therefore m(\angle BAD) = m(\angle B)$
 $\therefore m(\angle CAD) + m(\angle BAD) > m(\angle B)$
 $\therefore m(\angle BAC) > m(\angle B)$
 $\therefore BC > AC$ (Q.E.D.)

Geometry

5

[a] In $\triangle ABC$: $\because AB > BC$

$$\therefore m(\angle ACB) > m(\angle BAC) \quad (1)$$

In $\triangle ACD$: $\because AD > CD$

$$\therefore m(\angle ACD) > m(\angle CAD) \quad (2)$$

Adding (1) , (2) :

$$\therefore m(\angle BCD) > m(\angle BAD) \quad (\text{Q.E.D.})$$

[b] In $\triangle ABC$:

$$\because m(\angle ABC) = 90^\circ, m(\angle C) = 30^\circ$$

$$\therefore AB = \frac{1}{2} AC = \frac{1}{2} \times 12 = 6 \text{ cm.}$$

 $\because \overline{BD}$ is a median

$$\therefore BD = \frac{1}{2} AC = \frac{1}{2} \times 12 = 6 \text{ cm.}$$

 $\because D$ is the midpoint of \overline{AC}

$$\therefore AD = \frac{1}{2} AC = \frac{1}{2} \times 12 = 6 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle ABD = 6 + 6 + 6 = 18 \text{ cm.}$$

(The req.)

13 El-Beheira

1

$$\text{1} \quad \frac{1}{2}$$

$$\text{2} \quad 18 \text{ cm.}$$

$$\text{3} \quad 1$$

4 a side greater in length than that opposite to the other angle.

$$\text{5} \quad 5 \text{ cm.}$$

2

$$\text{1} \quad (\text{b})$$

$$\text{2} \quad (\text{c})$$

$$\text{3} \quad (\text{d})$$

$$\text{4} \quad (\text{c})$$

$$\text{5} \quad (\text{a})$$

$$\text{6} \quad (\text{b})$$

3

[a] $\because \overline{BE}, \overline{CD}$ are two medians in $\triangle ABC$ $\therefore M$ is the point of intersection of medians

$$\therefore ME = \frac{1}{2} MB = \frac{1}{2} \times 6 = 3 \text{ cm.}$$

$$\therefore MD = \frac{1}{2} MC = \frac{1}{2} \times 8 = 4 \text{ cm.}$$

 $\because D$ is the midpoint of \overline{AB} $\therefore E$ is the midpoint of \overline{AC}

$$\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 12 = 6 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle MDE = 3 + 4 + 6 = 13 \text{ cm.}$$

(The req.)

[b] $\because AC = \frac{1}{2} BD, \overline{AC}$ is a median in $\triangle ABD$

$$\therefore m(\angle BAD) = 90^\circ \quad (\text{Q.E.D.})$$

4

[a] In $\triangle ABD$: $\because AB = AD$

$$\therefore m(\angle ABD) = m(\angle ADB) \quad (1)$$

In $\triangle BCD$: $\because CD > BC$

$$\therefore m(\angle DBC) > m(\angle BDC) \quad (2)$$

Adding (1) , (2) :

$$\therefore m(\angle ABC) > m(\angle ADC) \quad (\text{Q.E.D.})$$

[b] $\because \overline{AD} \parallel \overline{BC}, \overline{AB}$ is a transversal.

$$\therefore m(\angle ACB) = m(\angle DAC) = 35^\circ \text{ (alternate angles)}$$

In $\triangle ABC$:

$$\therefore m(\angle B) = 180^\circ - (70^\circ + 35^\circ) = 75^\circ$$

$$\therefore m(\angle B) > m(\angle BAC)$$

$$\therefore AC > BC \quad (\text{Q.E.D.})$$

5

In $\triangle ABC$: $\because AB = AC$

$$\therefore m(\angle B) = m(\angle C) = 50^\circ$$

 \therefore In $\triangle ABX, \triangle ACY$:

$$\begin{cases} AB = AC \\ m(\angle B) = m(\angle C) \\ BX = CY \end{cases}$$

$$\therefore \triangle ABX \cong \triangle ACY$$

$$\therefore AX = AY$$

 $\therefore \triangle AYX$ is isosceles

(First req.)

$$\therefore m(\angle XAB) = m(\angle YAC) = 30^\circ$$

 $\therefore \angle AXY$ is an exterior angle of $\triangle ABX$

$$\therefore m(\angle AXY) = 50^\circ + 30^\circ = 80^\circ \quad (\text{Second req.})$$

14 El-Menia

1

$$\text{1} \quad (\text{a})$$

$$\text{2} \quad (\text{a})$$

$$\text{3} \quad (\text{b})$$

$$\text{4} \quad (\text{c})$$

$$\text{5} \quad (\text{b})$$

$$\text{6} \quad (\text{d})$$

2

$$\text{1} \quad 360^\circ$$

$$\text{2} \quad \text{bisects}$$

$$\text{3} \quad \text{equilateral}$$

$$\text{4} \quad 1 : 2$$

$$\text{5} \quad >$$

3

[a] In $\triangle AMC$: $\because m(\angle C) = 90^\circ$

$$\therefore AM > CM \quad (1)$$

In $\triangle BMD$: $\because m(\angle D) = 90^\circ$

$$\therefore BM > DM \quad (2)$$

Adding (1) , (2) : $\therefore AB > CD$

(Q.E.D.)

[b] In $\triangle ABD$: $\because AB = AD$

$$\therefore m(\angle ABD) = m(\angle ADB) \quad (1)$$

Answers of Final Examinations

In $\triangle BCD$: $\because BC = CD$
 $\therefore m(\angle CBD) = m(\angle CDB)$ (2)
 Adding (1) & (2) : $\therefore m(\angle ABC) = m(\angle ADC)$
 (Q.E.D.)

4

[a] In $\triangle ABC$: $\because AB > BC$
 $\therefore m(\angle C) > m(\angle A)$ (1)
 $\because \overline{XY} \parallel \overline{BC}$, \overline{AC} is a transversal
 $\therefore m(\angle AYX) = m(\angle C)$
 (corresponding angles) (2)
 From (1) & (2) :
 $\therefore m(\angle AYX) > m(\angle A)$
 $\therefore AX > XY$ (Q.E.D.)

[b] $\because \overline{AD} \parallel \overline{BC}$, \overline{AC} is a transversal
 $\therefore m(\angle C) = m(\angle CAD) = 30^\circ$ (alternate angles)
 $\therefore AC = BC$
 $\therefore m(\angle BAC) = m(\angle B) = \frac{180^\circ - 30^\circ}{2} = 75^\circ$
 (The req.)

5

[a] In $\triangle ABC$: $\because m(\angle ABC) = 90^\circ$
 $\therefore \overline{BD}$ is a median $\therefore BD = \frac{1}{2} AC$ (1)
 In $\triangle BDE$: $\because m(\angle BDE) = 90^\circ$
 $\therefore m(\angle E) = 30^\circ$
 $\therefore BD = \frac{1}{2} BE$ (2)
 From (1) & (2) : $\therefore AC = BE$ (Q.E.D.)

[b] In $\triangle ABC$:
 $\therefore AB = AC$ $\therefore m(\angle B) = m(\angle C)$
 $\therefore \frac{1}{2} m(\angle B) = \frac{1}{2} m(\angle C)$
 $\therefore m(\angle DBC) = m(\angle DCB)$ $\therefore BD = CD$
 $\therefore \triangle DBC$ is isosceles. (Q.E.D.)

15 Qena

1

- 1 3 2 50° 3 $<$
 4 2 : 1 5 its axis of symmetry.

2

- 1 (a) 2 (a) 3 (c)
 4 (b) 5 (b)

3

[a] In $\triangle ABC$: $\because AB = AC$
 $\therefore m(\angle B) = m(\angle C)$
 $\because \overline{XY} \parallel \overline{BC}$, \overline{AB} is a transversal
 $\therefore m(\angle AXY) = m(\angle B)$ (corresponding angles)
 $\because \overline{XY} \parallel \overline{BC}$, \overline{AC} is a transversal
 $\therefore m(\angle AYX) = m(\angle C)$ (corresponding angles)
 $\therefore m(\angle B) = m(\angle C)$
 $\therefore m(\angle AXY) = m(\angle AYX)$
 In $\triangle AXY$: $\therefore AX = AY$
 $\therefore \triangle AXY$ is isosceles (Q.E.D.)
 [b] $\because m(\angle C) = 180^\circ - (40^\circ + 75^\circ) = 65^\circ$
 $\therefore m(\angle A) < m(\angle C) < m(\angle B)$
 $\therefore BC < AB < AC$ (The req.)

4

[a] In $\triangle ABC$: $\because E$ is the midpoint of \overline{AC}
 $\therefore F$ is the midpoint of \overline{AB}
 $\therefore EF = \frac{1}{2} BC = \frac{1}{2} \times 20 = 10 \text{ cm.}$
 In $\triangle ADB$: $\because m(\angle ADB) = 90^\circ$
 $\therefore \overline{DF}$ is a median
 $\therefore DF = \frac{1}{2} AB = \frac{1}{2} \times 14 = 7 \text{ cm.}$
 In $\triangle ADC$: $\because m(\angle ADC) = 90^\circ$
 $\therefore \overline{DE}$ is a median
 $\therefore DE = \frac{1}{2} AC = \frac{1}{2} \times 18 = 9 \text{ cm.}$
 \therefore The perimeter of $\triangle DEF = 10 + 7 + 9 = 26 \text{ cm.}$
 (The req.)

[b] In $\triangle ABD$: $\because AD > AB$
 $\therefore m(\angle ABD) > m(\angle ADB)$ (1)
 In $\triangle BCD$: $\because CD > BC$
 $\therefore m(\angle CBD) > m(\angle CDB)$ (2)
 Adding (1) & (2) :
 $\therefore m(\angle ABC) > m(\angle ADC)$ (Q.E.D.)

5

[a] $\because \overline{AD} \parallel \overline{BC}$, \overline{AC} is a transversal
 $\therefore m(\angle ACB) = m(\angle CAD) = 30^\circ$
 (alternate angles)
 In $\triangle ABC$:
 $\therefore m(\angle BAC) > m(\angle ACB)$
 $\therefore BC > AB$ (Q.E.D.)
 [b] 2, 12